High precision data confronting
Field theory – Monte Carlo solution:
From LEP2 to Linear Collider.

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Main Topics:

- **Status.**
- **τ-decay.**
- **τ-production.**
- **Techniques.**
- **Outlook.**

Slides are on http://home.cern.ch/wasm

INTRODUCTION

Theory versus experiment

I hope that we can agree, that at least in simplified way:
The purpose of any theory is to represent in as compact as possible and unambiguous way as large as possible domain of experimental data (past and future).

The purpose of any measurement is to confirm existing theory in new conditions, explore the limits of its validity or, eventually find where it is necessary to develop the new theory.

Each side stimulate another:

New data stimulate creation of new theories.
New theories stimulate new experiments.

At technical level:

The better, more precise are experiments, the better more precise theoretical predictions must be to draw conclusions on the differences or agreements between data and experiment.
Of course there is a lot of work needed to study internal structure of theory and equally much for experiments.

INTRODUCTION

Generalities

- To have theoretical uncertainty negligible we need theoretical calculations for 'known physics' to have uncertainty of about 1/3 of expected experimental one.
- This is of course motivated by the consideration of manadging resources.
- Theoretical calculations are extremelly simple and straightforward: they represent fraction of an effort needed to build acclelerators and detectors.
- But nonetheless we discussed projects of the scale of 10+ man-years, where one has to worry aboutmanpower coordination, availability, schedules, intermediate goals, reliability etc.

How it was/is at LEP?

INTRODUCTION

Table 1: Taken from two fermion final states section of (YR CERN 2000-009): Comparison of the typical theoretical uncertainties with the typical experimental precision tags

class of observables	theoretical uncertainty	experimental precision tag
$e^+e^- \to q\bar{q}(\gamma)$	0.26 %	0.1 % -0.2 %
$e^+e^- \to \mu^+\mu^-(\gamma)$	0.4 %	0.4 % -0.5 %
$e^+e^- \to \tau^+\tau^-(\gamma)$	0.4 %	0.4 % -0.6 %
$e^+e^- \to e^+e^-(\gamma)$ (endcap)	0.5 %	0.13 %
$e^+e^- \to e^+e^-(\gamma)$ (barrel)	2.0 %	0.21 %
$e^+e^- \to e^+e^-\gamma$	3 %	1.5 %
$e^+e^- \to l^+l^-\gamma$	1 %	1.5 %
$e^+e^- \to \nu\bar{\nu}\gamma$	4 %	0.5 %

Theoretical calculations will be completed later than last data taken !!

Future experiments will be less demanding?

Material for discussions

I will continue with the calculations for τ leptons, to illustrate possible techniques and solutions.

Skipped

In particular I will omit such calculations as small angle Bhabha, WW pair production and decay etc.

Papers on CEEX

Papers on CEEX by S. Jadach, B.F.L. Ward and Z. Wąs:

Phys. Lett. B449 (1999) 97, (CERN-TH/98-253), CERN-TH/98-235, subm. to EPJ,

Comput. Phys. Commun. 130 (2000) 260

UTHEP-99-09-01 (CERN-TH/2000-087), in preparation.

Educative introduction to exponentiation:

Z. Was, Radiative Corrections, Preprint CERN-TH 7154/94.

Papers on TAUOLA

1. R. Decker, S.Jadach, M.Jeżabek, J.H.Kuhn, Z. Was, Comput. Phys. Commun. 76 (1993) 361, ibid. 70 (1992) 69, ibid. 64 (1990) 275
2. P. Golonka, Z. Was, TPJU 10/2000, technical stuff only.
3. Alain Weinstein www home page: http://www.cithep.caltech.edu/~ajw/korb_doc.html#files
4. B. Bloch, private communications.
5. R. Decker, M. Finkemeier, P. Heiliger and H.H. Jonsson, Hep-ph/9410260, now standard 4π formfactors.
6. P. Abreu et al., Phys. Lett. B426 (1998) 411 (alternative 3π formfactors)
7. Sherry Towers aternative formfactors in K$\pi\pi$ modes, hep-ex/9908013

Motivation

I hope that we can agree, that at least in simplified way:

Physics of the τ lepton developpes fast

Precision of the measurements is already high and improving,

Different τ decay modes form background to each other

Measurements often require sophisticated cuts.

Monte Carlo solution is a must:

Due to the long τ lepton lifetime,

generation can be separated into production and decay as completely independent projects.

All spin effects of the Einstein-Rosen-Podolsky type can be handled this way.

τ's can serve as good starting example for presentation.

τ decay generation consist of:

Bookkeeping and library for phase space generation.

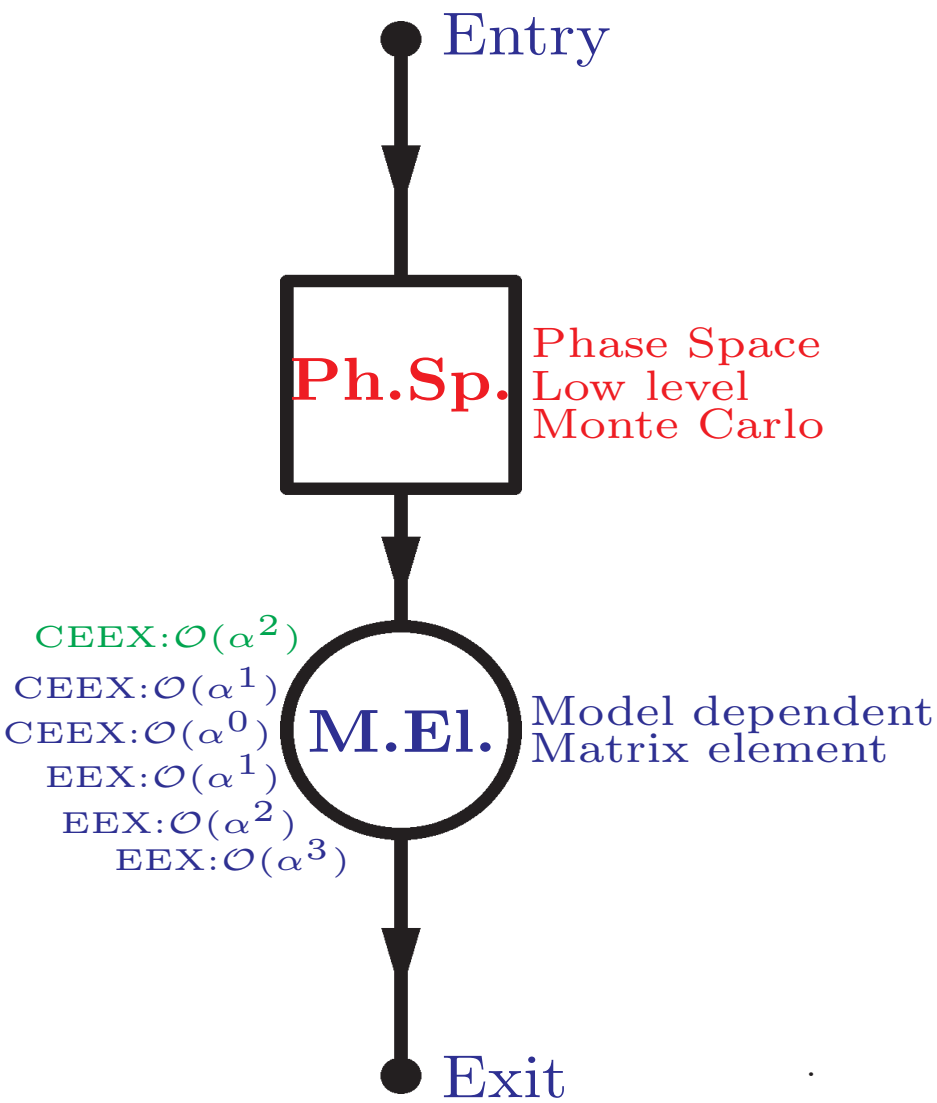
Easy to replace matrix elements or hadronic currents

τ production:

Can be rigorously controlled with the help of Electroweak Field Theory.

TERMINOLOGY

General structure of precision MC



- **Universal Phase-space Monte Carlo simulator**
- **Library of several types of e.g. QED matrix elements**
- **Library of SM/QCD formf-factors**

τ-DECAY

1. R. Decker, S.Jadach, M.Jeżabek, J.H.Kuhn, Z. Was, Comput. Phys. Commun. 76 (1993) 361, ibid. 70 (1992) 69, ibid. 64 (1990) 275
2. P. Golonka, Z. Was, TPJU 10/2000, technical stuff only.

Also:

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5. Sherry Towers aternative formfactors in K$\pi\pi$ modes, hep-ex/9908013

And as usual, references therein

I am nearly sure, I have forgotten something.

The distribution directory can be found on: http://home.cern.ch/wasm

Tauola itself

Not much progress on our side:

Elements of standard description:

- There are (at least) 22 decay modes included,
- Full phase-space – matrix-element separation,
- Modifications to matrix elements (hadronic currents) are easy and localized in well defined functions.
- Interface for complete and partial spin treatment is prepared.

Recent progress:

- Technical fix ups (essential for linux)
- Pre-coding for versions using cpp pre-compiler

Choices available

- Type of physic initialization:CPC, CLEO, ALEPH
- Type of application: stand-alone, like in KKMC, universal interface based on searching through HEPEVT common block
- utilities to replace: • random number generators and ,
- (centrally) input parameters
- Dimensionality of HEPEVT common block; PHOTOS versions are also included in this environment

τ-PRODUCTION

QUICK, UNIVERSAL, APPROXIMATE

SPECIFIC, PRECISE

Universal interface

HEPEVT common block

The work was started by one member of LHC collaboration

It is often convenient to decay τ leptons easy and quickly. We do exactly like that. Once event is generated and HEPEVT common block is filled it is enough to CALL TAUOLA(0,IDUM) and all undecayed τ leptons will be decayed and stored into HEPEVT.

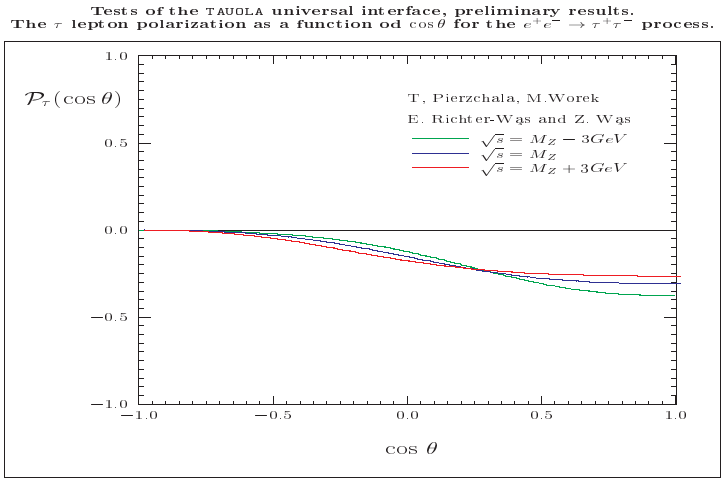
Some minimum care of the spin is taken at the longitudinal approximation:

1. Cases of τ produced from: W (virtual), Z/γ, Higgs, Charged Higgs are distinguished in spin correlations.
2. For Z/γ case τ polarization as a function of effective $\cos\theta$, s and initial state flavour is used.
3. Lots of tests need to be done, Now we have done only most elementary ones, full spin option for Z/γ is thus not yet included in the distribution file instead, polarization as from pure γ production is used.

When do we need (and have) unique event structure in C++ ?

Preliminary tests

τ-polarization in $e^+e^- \to \tau^+\tau^-$

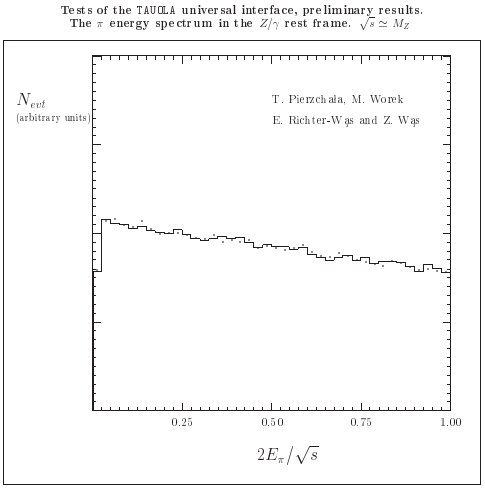


The function used in the interface is taken form KORALZ.

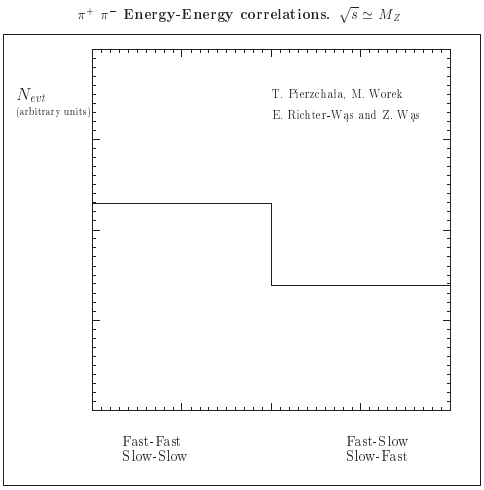
It looks after implementation (equally) OK.

Preliminary tests

Test at τ decay products level



1



2

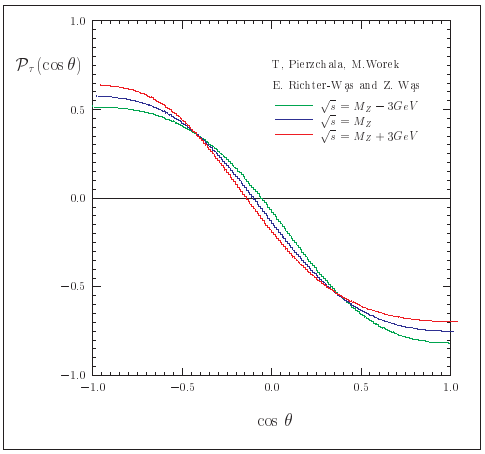
To make spin effects large, we take sample of both τ' decaying into $\pi\nu$. Standard tests are getting reproduced qualitativelly.

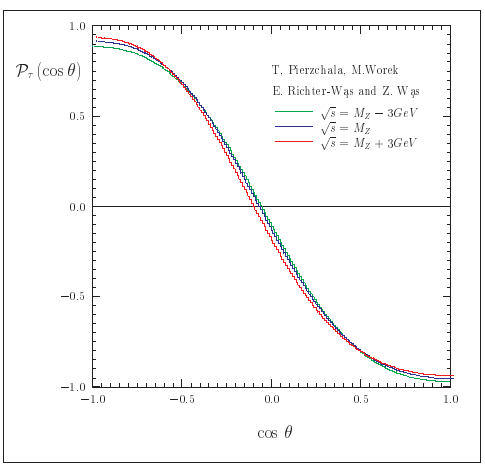
We will continue with the more sophisticated tests in future. In particular we need to test the algorithm for matching bremsstrahlung.

Preliminary tests

τ-polarization in $q\bar{q} \to \tau^+\tau^-$

Now let us use the same functions for quarks:





Coherent Exclusive Exponentiation
of 2f Processes
in e^+e^- Annihilation

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Main Topics:

- **Introduction.**
- **Physics motivation.**
- **Basic steps to CEEX.**
- **Summary, outlook.**

Papers on CEEX by S. Jadach, B.F.L. Ward and Z. Wąs:

Phys. Lett. B449 (1999) 97, (CERN-TH/98-253), CERN-TH/98-235, subm. to EPJ,

DESY-99-106 (CERN-TH/99-235), 88p., Comput. Phys. Commun. in press

UTHEP-99-09-01 (CERN-TH/2000-087), in preparation.

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Precision requirements.

The $\sim$ 0.1 % precision level means:

Present, and near future experiments of HEP do not require the precision to cross the threshold of about 0.01 % $\sim \alpha^2_{QED}$.
That is why:

Complete electroweak calculations beyond one loop are not necessary, if all possible enhancement factors of higher orders, such as: $\ln \frac{s}{m_e^2}$, $\ln \frac{M_Z}{\Gamma_Z}$, $\frac{M_t}{M_Z}$, α_s, ... are taken into account.

This required work of many:

QCD calculations, low energy hadr. contributions to vac. pol.

Electroweak calculations

Pair effects

These essential topics will/were covered in other talks.

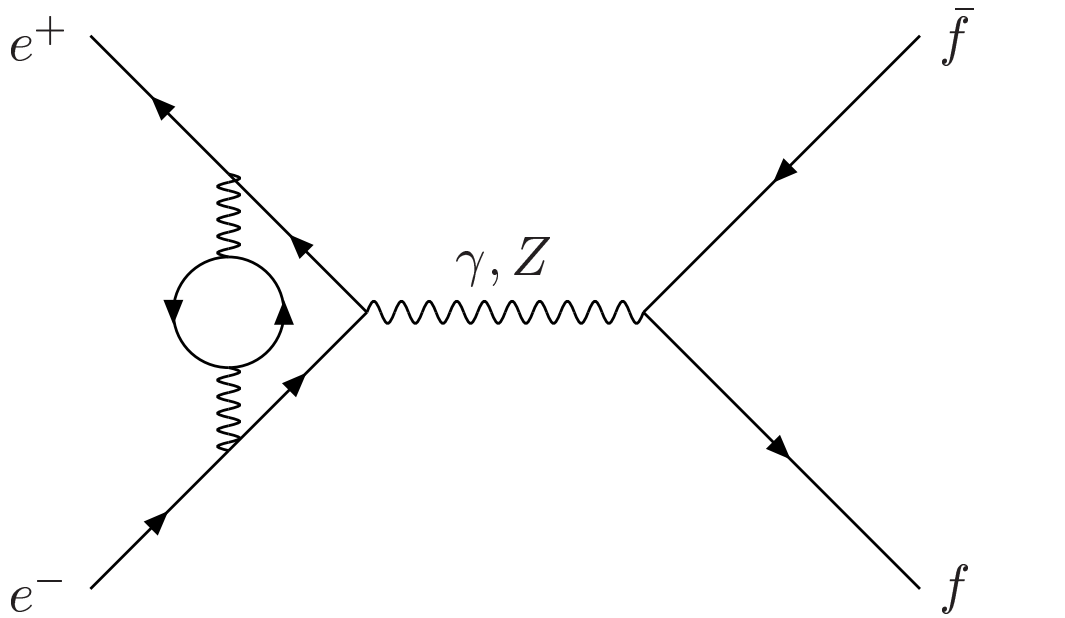
QED effects can be separated.

That is good news, because that is what makes relations with the measurement conditions most complicated.

Pair effects can not be separated from the cut off-treatment even at 0.5 % precision level, but do not need to be included in the same segment of the calculation.

Virtual pairs

A typical example of virtual pair correction



Virtual Pair effect can be added to F_1 electric form-factor using old and well known formula:

$$F_1^{pair}(s) = \sum_f \left\{ -\frac{1}{36} L_f^3 + \frac{19}{72} L_f^2 + (\frac{1}{18}\pi^2 - \frac{265}{216}) L_f + C_F \right\}, \quad (1)$$

$$C_F = \begin{cases} \frac{383}{108} - \frac{11}{6}\frac{\pi^2}{6}, & m_f = m_F, \\ -\frac{1}{3}\zeta(3) + \frac{3355}{1296} - \frac{19}{18}\frac{\pi^2}{6}, & m_f \gg m_F. \end{cases} \quad (2)$$

$$L_f = \log \frac{s}{m_f^2}, \quad (3)$$

See for instance G. Burgers, *Phys. Lett.*, **B164**, (1985), 167

m_f denotes mass of the fermion in the loop and m_F mass of the incoming fermion (typically an electron).

CEEX features

Why COHERENT?

Because: Friendly to Quantum coherence;

Coherence among Feynman diagrams: Narrow resonances,

$\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR$\oplus$FSR real emissions, etc.

Complete $\left| \sum_{diagr.}^{n} \mathcal{M}_i \right|^2$ rather then often incomplete $\sum_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j^*$

Why EXCLUSIVE?

Because: Phase-Space $\times \sum_{spin} \left|\mathcal{M}\right|^2$

Contrary to Parton-Showers, Leading-Logs, Structure-Functions

which are all inherently inclusive.

Why EXPONENTIATION?

Infrared (IR) contributions summed to $\mathcal{O}(\alpha^\infty)$

Going **beyond** Yennie-Frautschi-Suura (EEX):

Real IR singularities isolated/reorganized at the **spin amplitude**

level, while IR cancellations occur for probabilities (integrated

x-sections). Paradox???

Differences with older EEX/YFS? See next slide

EEX= Exclusive EXponentiation = variant of YFS expon.

Phase space–near factorization

$$
\begin{aligned}
& dLips_{n+1}(P) = \\
& \left[2M dk_\gamma \frac{2k_\gamma}{M} d\cos\theta d\phi \frac{1}{8(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n) \\
= \quad & \left[k_\gamma dk_\gamma d\cos\theta d\phi \frac{1}{2(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n), \qquad (4)
\end{aligned}
$$

If we have l photons accompanying n other particles, then the factor in brackets become iterated. Statistical factor $\frac{1}{l!}$ has to be introduced:

$$
dLips_{n+l}(P) = \frac{1}{l!} \prod_{i=1}^{l} \left[k_{\gamma i} dk_{\gamma i} d\cos\theta_i d\phi_i \frac{1}{2(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n). \qquad (5)
$$

If we do not ask questions about phase-space limit this look like perfect term in expansion of exponent[a].

As the leading parts of the Matrix elements squared, have the same factorization property, the exponents of the YFS expansion form.

[a] In reality it can be realized relatively easy in M.C. because of the scale symmetry present in the expression.

We get:

$$\sum_{l=0} dLips_{n+l}(P) = \exp\left[k_\gamma\, dk_\gamma\, d\cos\theta d\phi \frac{1}{2(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n) \times \mathcal{J}(P, p, l, \ldots) \quad (6)$$

Where $\mathcal{J}$ is:

- complicated,
- depends on everything
- but is calculable
- and $0 \leq \mathcal{J} \leq \mathcal{J}_{max}$!!!!

Here we omit in discussion lots of essential points:

- angular singularities,
- initial vs. final state radiation,
- etc.

We got functional exponent

We got functional exponent:

We got expression of the form of **functional** exponent (up to positive and bounded from above correction factor) possible to install in MC with for n particles plus arbitrary number of photons.

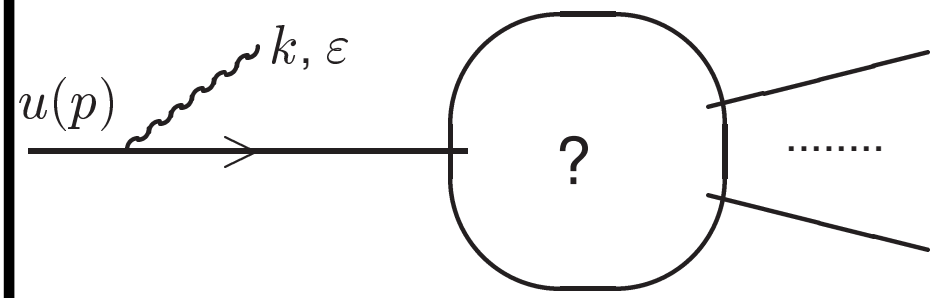
This answers:

Why EXCLUSIVE ?

Why EXPONENTIATION ? (partially)

Let us go to the matrix element ...

M.E. factorization



Feynman diagram of photon emission from initial state electron. Dots represent all other fields entering amplitude.

$$\mathcal{M} = \ldots \frac{\not{p} - \not{k} + m}{-2pk} e \not{\varepsilon} u(p,s). \tag{7}$$

$$\begin{aligned}
\mathcal{M}^{\mathcal{IR}} &= \frac{-e}{2pk} \ldots (\not{p} + m) \not{\varepsilon} u(p,s) \\
&= \frac{-e}{2pk} \ldots \Big(2p\varepsilon + \not{\varepsilon}(-\not{p} + m)\Big) u(p,s) \\
&= \frac{-e}{2pk} \ldots (2p\varepsilon) u(p,s) \\
&= e\frac{-\varepsilon p}{pk} \ldots u(p,s) \\
&= -e\frac{\varepsilon p}{pk} \mathcal{M}_B,
\end{aligned} \tag{8}$$

The same is true for all incoming and outgoing fermion lines.

Where $\mathcal{M}_B$ is (up to some transfers in propagators) Born-level spin amplitude.

M.E. factorization

We can write finally:

$$\mathcal{M} = -e\frac{\varepsilon p}{pk}\mathcal{M}_B + \mathcal{M}^1_{\mathcal{O}(k^0)}, \tag{9}$$

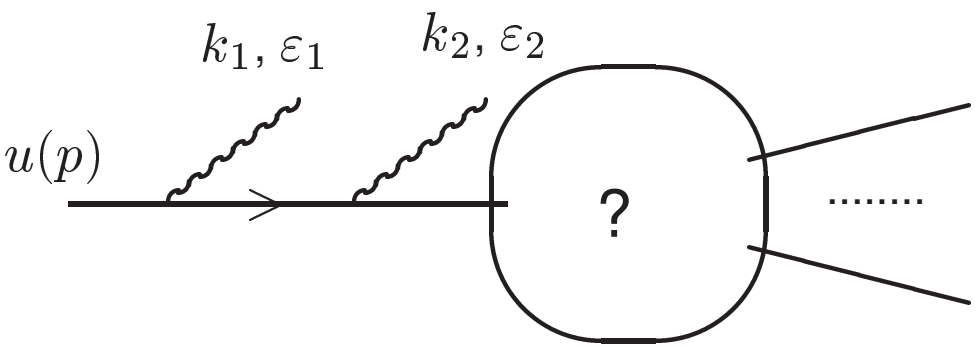
or, equally correct

$$\mathcal{M} = -e\frac{\varepsilon p}{pk}\bar{\mathcal{M}}_B + \bar{\mathcal{M}}^1_{\mathcal{O}(k^0)}, \tag{10}$$

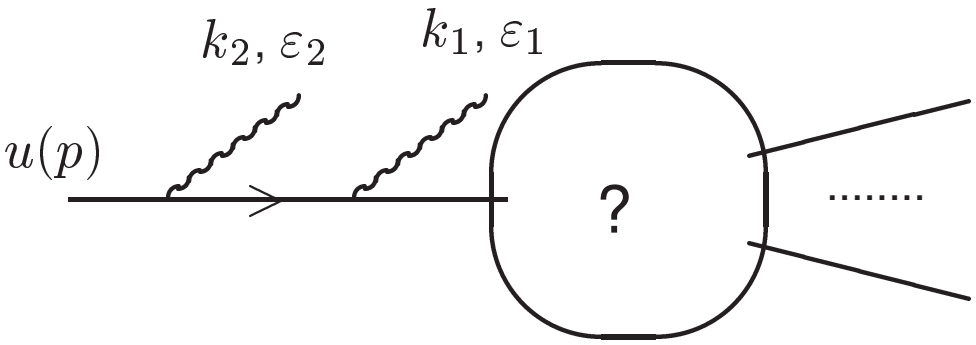
where $\bar{\mathcal{M}}_B$ means that the transfers used in denominators of propagators do feel energy loss due to k momentum

- Important for CEEX was to present consistently the first term as analytic expression and second term using spin amplitude techniques.
- We have chosen conventions simmilar to Kleiss-Stirling but with full control of complex phases.
- As by-product, this enabled calculation of density matrices for all fermions without any approximations.

M.E. factorization



a)



b)

Figure 1: *Feynman diagrams for double photon emission in the initial state from electron. Dots represent all other fields entering amplitude (initial or final).*

M.E. factorization

$$\mathcal{M}_1 = \ldots iS_F(p-k_1-k_2)(-ie\gamma_\mu)iS_F(p-k_1)(-ie\gamma_\nu)u(p,s)\;\varepsilon_2^\mu\varepsilon_1^\nu$$
$$\mathcal{M}_2 = \ldots iS_F(p-k_1-k_2)(-ie\gamma_\mu)iS_F(p-k_2)(-ie\gamma_\nu)u(p,s)\;\varepsilon_1^\mu\varepsilon_2^\nu \quad (11)$$

Summing $\mathcal{M}_1, \mathcal{M}_2$ contributions we obtain:

$$\begin{aligned}
\mathcal{M}=\mathcal{M}_1+\mathcal{M}_2 &= \left(e\frac{-\varepsilon_1 p}{k_1 p}e\frac{-\varepsilon_2 p}{k_1 p+k_2 p}+e\frac{-\varepsilon_2 p}{k_2 p}e\frac{-\varepsilon_2 p}{k_1 p+k_2 p}\right)\mathcal{M}_B \\
&= \left(e\frac{-\varepsilon_1 p}{k_1 p}\right)\left(e\frac{-\varepsilon_2 p}{k_2 p}\right)\mathcal{M}_B \\
&+ \mathcal{M}^R_{k_1}+\mathcal{M}^R_{k_2} \\
&+ \mathcal{M}^R_{k_1\times k_2}. \\
&= \left(e\frac{-\varepsilon_1 p}{k_1 p}\right)\left(e\frac{-\varepsilon_2 p}{k_2 p}\right)\mathcal{M}_B \\
&+ \left(e\frac{-\varepsilon_1 p}{k_1 p}\right)\mathcal{M}^1_{\mathcal{O}(k_2^0)}+\left(e\frac{-\varepsilon_2 p}{k_2 p}\right)\mathcal{M}^1_{\mathcal{O}(k_1^0)} \\
&+ \mathcal{M}^2_{\mathcal{O}(k_1,k_2)}. \qquad (12)
\end{aligned}$$

Note that terms in red are the infrared-finite parts of the amplitudes for 0, 1, 2 photon(s) emission: we got separation of double photon amplitude into: ● infrared part (which we need now),

●first-order-finite-times-IR, ● genuine second order.

We will come to it later !

Simmilar is true for all incoming and outgoing fermion lines, as well as for higher orders.

The Exponent

Now we can obtain an exponent for the first time. If we sum the cross sections for the configurations with 0, 1, 2, ..., photons we obtain:

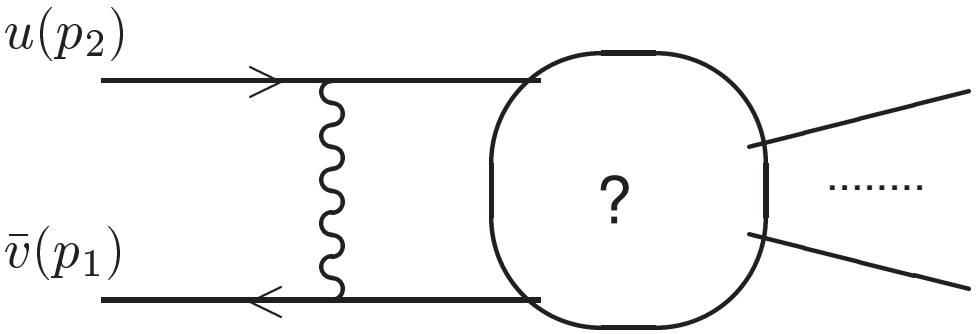
$$
\begin{aligned}
& d\sigma(p_1, p_2 \to q_1, \ldots, q_n, \text{ and photons}) \\
= \quad & \frac{1}{|v_1 - v_2|} \frac{1}{2p_1^0} \frac{1}{2p_2^0} |\mathcal{M}_B|^2 \, dLips_n(p \to q_1 \ldots q_n) \\
\times \quad & \sum_{l=0}^{\infty} \frac{1}{l!} \prod_{i=1}^{l} \left[k_i dk_i d\cos\theta_i d\phi_i \frac{1}{2(2\pi)^3} \sum_{\varepsilon_i} e^2 \left(\frac{\varepsilon_i p_2}{k_i p_2} - \frac{\varepsilon_i p_1}{k_i p_1} \right)^2 \right] \\
& \times \mathcal{J} \times (1 + higher\ oredrs) \\
= \quad & \frac{1}{|v_1 - v_2|} \frac{1}{2p_1^0} \frac{1}{2p_2^0} |\mathcal{M}_B|^2 \, dLips_n(p \to q_1 \ldots q_n) \\
\times \quad & \exp\left[kdkd\cos\theta d\phi \frac{1}{2(2\pi)^3} \sum_{\varepsilon} e^2 \left(\frac{\varepsilon p_2}{k p_2} - \frac{\varepsilon p_1}{k p_1} \right)^2 \right] \qquad (13) \\
= \quad & \times \mathcal{J} \times (1 + higher\ oredrs).
\end{aligned}
$$

Important is that the term $\mathcal{J} \times (1 + higher\ oredrs)$ can be made always positive, and bounded from above.

Infrared Catastrophy

It is nearly complete result of our presentation of exclusive exponentiation. However, it is infinite. Let us introduce the fictitious photon mass λ and replace the kdk factors of integration by: $k^2 dk \ / \ \sqrt{k^2 + \lambda^2}$, to *regularize* the infinity.

M.E. factorization-virtual



$\mathcal{M}_V$

Figure 2: *Feynman diagram for the vertex-like correction in initial state in e^+, e^- collision. Dots represent all final state fields.*

First, we will study the factorization of the virtual corrections. First, we will take a vertex-type amplitude $\mathcal{M}_V$.

$$\mathcal{M}_V^1 = \int \frac{d^4k}{(2\pi)^4}\left(-i\frac{g_{\mu\nu}}{k^2+i\epsilon}\right)\bar{v}(p_1)(-ie\gamma^\mu)i\frac{-(\not{p}_1+\not{k})+m}{(p_1+k)^2-m^2+i\epsilon}$$
$$\ldots \quad i\frac{(\not{p}_2+\not{k})+m}{(p_2+k)^2-m^2+i\epsilon}(-ie\gamma^\nu)u(p_2). \qquad (14)$$

Since we are interested, as in the real bremsstrahlung case, only in the contribution of the soft photons $k^\mu \simeq 0$ region, we will neglect the $\not{k}$ terms in the numerators of the propagators. Also, we assume that the remaining part of the diagram, noted by us as …, depends on the virtual photon momentum weakly.

M.E. factorization-virtual

Later, we can/will recover neglected parts, order by order of perturbative expansion.
We can write:

$$\mathcal{M}^1_V = \mathcal{M}_B \times B + \mathcal{M}^1_{\mathcal{O}(k^0)}$$

$$B = 2\alpha\Re\int \frac{d^4k}{(k^2+i\epsilon)}\frac{i}{(2\pi)^3}\left(\frac{2p_1-k}{2p_1k+k^2+i\epsilon}-\frac{2p_2-k}{2p_2k+k^2+i\epsilon}\right)^2 \quad (15)$$

Note that term in red is infrared-finite. We got separation into: • infrared part (which we need now), and • genuine first order.

We will come to it later !

M.E. factorization-virtual two-loops

$$\mathcal{M}^2_V = \mathcal{M}_B \times \frac{1}{2!}B^2 + B \times \mathcal{M}^1_{\mathcal{O}(k^0)} + \mathcal{M}^2_{\mathcal{O}(k^0_1,k^0_2)} \quad (16)$$

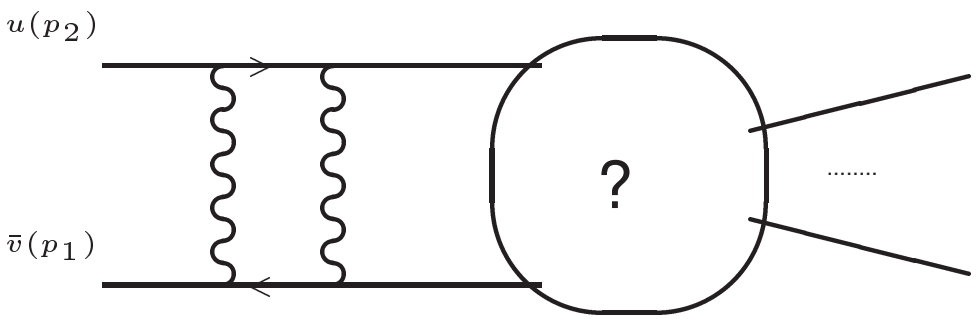
Again, terms in red are infrared-finite. We got separation into:

- infrared part (which we need now),
- genuine-first-order-times-IR and
- genuine second order.

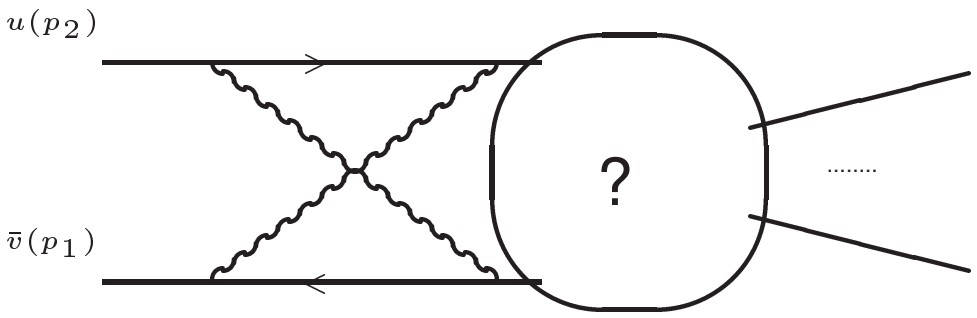
Simmilar is true for more fermionic lines and higher orders!

M.E. factorization-virtual two-loops

This was due to diagrams:



a)

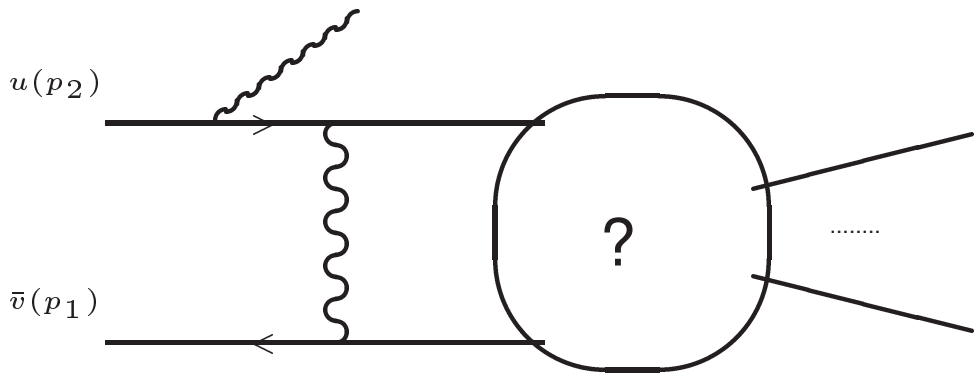


b)

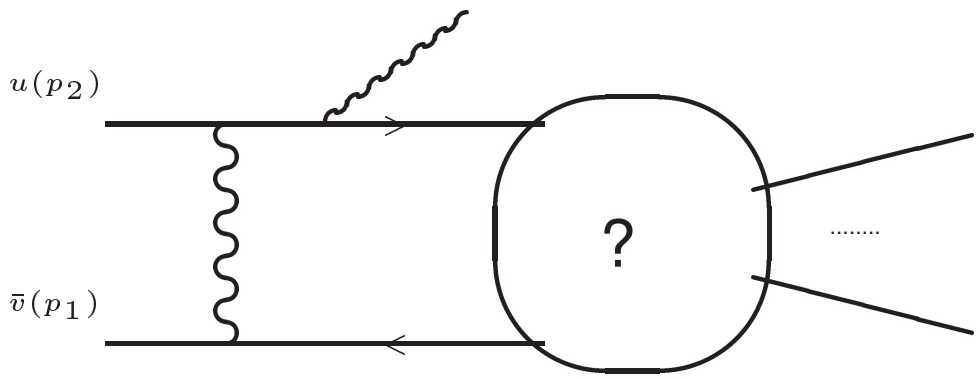
Figure 3: *Feynman diagrams for the vertex-like correction in initial state in e^+, e^- collision, second order. Dots represent all final state fields.*

M.E. factorization-virtual one-loop one-real

The diagrams:



a)



b)

Figure 4: *Feynman diagrams for the mixed real-vertex-correction emission from electron (for emission from positron diagrams are simmilar).*

M.E. factorization virtual-one-loop one-real

We get:

$$\mathcal{M}^2_{VR} = \mathcal{M}_B \times B \times \left(e\frac{-\varepsilon p_2}{kp_2}\right) + \mathcal{M}^1_{R-\mathcal{O}(k_1^0)} \times B + \mathcal{M}^1_{V-\mathcal{O}(k_1^0)}\left(e\frac{-\varepsilon p_2}{kp_2}\right) + \mathcal{M}^2_{V-\mathcal{O}(k^0),R-\mathcal{O}(k^0)} \quad (17)$$

Note that terms in red are the infrared-finite parts of the amplitudes for 0, 1-real, 1-virtual and 1-real, 1-virtual. We got separation ofamplitude into:

- infrared part (which we need now),
- first-order-real-finite times IR-virtual,
- first-order-virtual-finite times IR-real,
- genuine second order.

We will come to it later !

Simmilar is true for all incoming and outgoing fermion lines, as well as for higher orders: more virtual and real lines.

We keep photon mass $\lambda > 0$

But we ignore for a while non-leading terrms

We will ignore ISR-FSR interference also now.

We will ignore now all FSR also.

Summing all, we again obtain an exponent:

M.E. for no photon emission:

$$\mathcal{M}_{0-phot} = \mathcal{M}_B \times \sum_{n=0}^{\infty} \frac{1}{n!} B(\lambda)^n = \mathcal{M}_B \times \exp(B(\lambda)), \tag{18}$$

M.E. for 1-photon emission:

$$\mathcal{M}_{1-phot} = \Big[\mathcal{M}_B \times \exp(B(\lambda))\Big]\left(e\frac{-\varepsilon p_2}{kp_2}\right), \tag{19}$$

M.E. for n-photon emission:

$$\mathcal{M}_{n-phot} = \Big[\mathcal{M}_B \times \exp(B(\lambda))\Big]\left(e\frac{-\varepsilon p_2}{kp_2}\right)^n, \tag{20}$$

Finally exponentiation, ISR-only

$$\begin{aligned} & d\sigma(p_1, p_2 \to q_1, \ldots, q_n, \text{ and photons}) = \\ & \frac{1}{|v_1 - v_2|}\frac{1}{2p_1^0}\frac{1}{2p_2^0} \times |\mathcal{M}_B|^2 dLips_n(p \to q_1 \ldots q_n) \\ \times \quad & \exp(2B(\lambda)) \\ \times \quad & \exp\left[\frac{k^2\,dk}{k^2+\lambda^2} d\cos\theta d\phi \frac{\alpha}{(2\pi)^2}\sum_{\varepsilon}\left(\frac{\varepsilon p_2}{kp_2} - \frac{\varepsilon p_1}{kp_1}\right)^2\right] \\ \times \quad & \mathcal{J} \times (1 + higher\ orders). \end{aligned} \tag{21}$$

Summary on this sketchy calculation:

After all that we got:

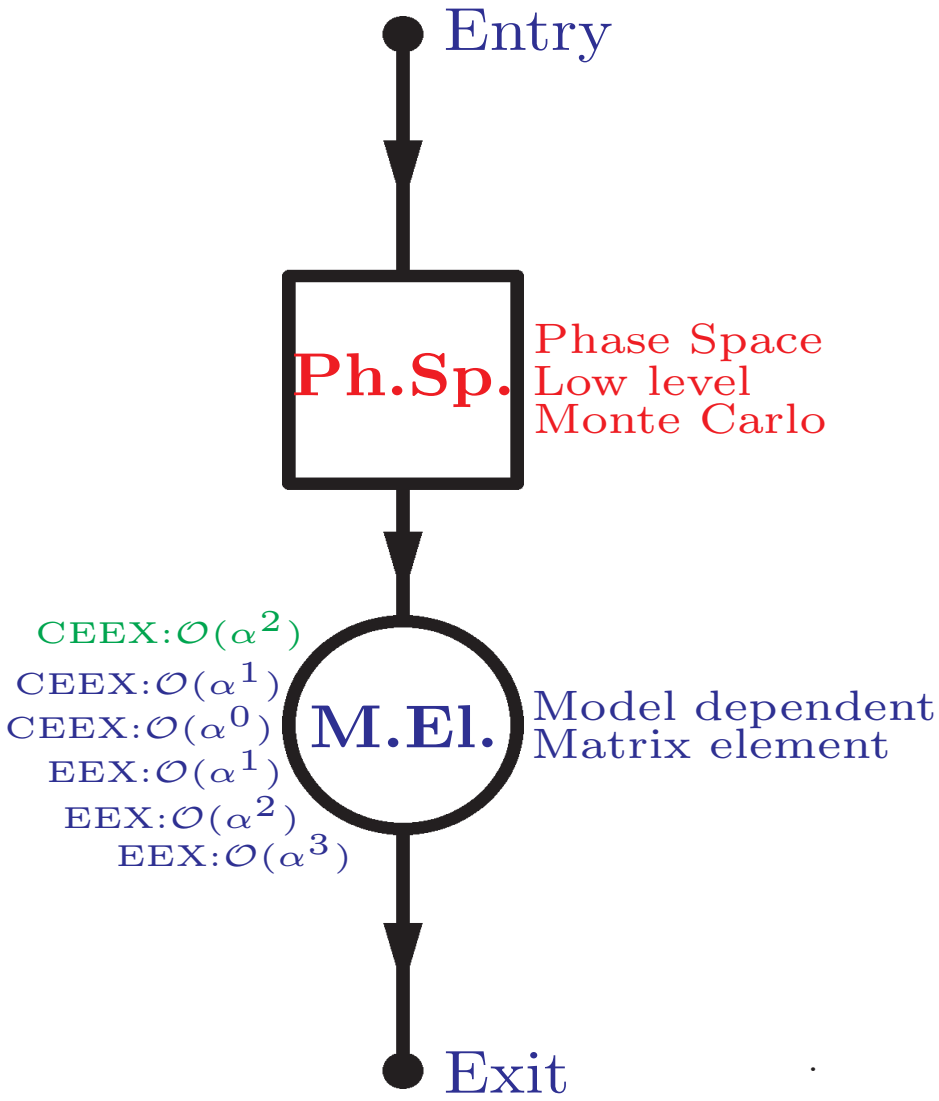
- All finite.
- Well defined and MC friendly.
- With precise relation to Feynman diagrams and phase space.
- With clear scenario how to include higher order corrs.

Basic points of generation consist of:

- Generation of total energy and number of photons, accordingly to crude distribution: phase space+soft factors..
- Further generation of all kinematical variables for photons.
- Calculation of phase space weight $\mathcal{J}$.
- Calculation of weight due to the matrix element of the given order.

TERMINOLOGY

General structure of $\mathcal{KK}$ MC (flowchart)



- **Universal Phase-space Monte Carlo simulator**
- **Library of several types of QED matrix elements**
- **Library of SM/QCD formf-factors**
- **Tau decays or hadronization come after**

CEEX schematically

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \sum_{\lambda,\sigma_1,\ldots,\sigma_n} \left| e^{B(m_\gamma)} \mathcal{M}^{\lambda}_{n,\sigma_1,\ldots,\sigma_n}(k_1,\ldots,k_n) \right|^2$$

$\mathcal{O}(\alpha^1)$ Example:

$\mathcal{M}^{\lambda}_{0} = \hat{\beta}^{\lambda}_{0}$, λ=fermion helicities,

$\mathcal{M}^{\lambda}_{1,\sigma_1}(k_1) = \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)$

$\mathcal{M}^{\lambda}_{2,\sigma_1,\sigma_2}(k_1,k_2) = \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1)$

IR-finite building blocks:

$$\hat{\beta}^{\lambda}_{0} = \left(e^{-B}\mathcal{M}^{\mathrm{Born+Virt.}}_{\lambda}\right)\Big|_{\mathcal{O}(\alpha^1)},$$

$$\hat{\beta}^{\lambda}_{1,\sigma}(k) = \mathcal{M}^{\lambda}_{1,\sigma}(k) - \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma}(k)$$

Everything in terms of $\mathcal{M}$-spin-amplitudes !!!!

Full Scale CEEX

Full Scale CEEX $\mathcal{O}(\alpha^r)$, r=1,2, master formula

Polarized total x-section:

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a+p_b; p_c, p_d, k_1, \ldots, k_n)\; e^{2\alpha \Re B_4} \sum_{\sigma_i,\lambda,\bar{\lambda}} \sum_{i,j,l,m=0}^{3}$$

$$\hat{\varepsilon}_a^i \hat{\varepsilon}_b^j \sigma^i_{\lambda_a \bar{\lambda}_a} \sigma^j_{\lambda_b \bar{\lambda}_b} \mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 \\ \lambda\, \sigma_1\, \sigma_2 \end{pmatrix} \cdots \begin{pmatrix} k_n \\ \sigma_n \end{pmatrix} \left[\mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 \\ \bar{\lambda}\, \sigma_1\, \sigma_2 \end{pmatrix} \cdots \begin{pmatrix} k_n \\ \sigma_n \end{pmatrix} \right]^{\star} \sigma^l_{\bar{\lambda}_c \lambda_c} \sigma^m_{\bar{\lambda}_d \lambda_d} \hat{h}_c^l \hat{h}_c^m$$

CEEX amplitudes:

$$\mathfrak{M}_n^{(1)} \begin{pmatrix} p\, k_1 \\ \lambda\, \sigma_1 \end{pmatrix} \cdots \begin{pmatrix} k_n \\ \sigma_n \end{pmatrix} = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}} \left\{ \beta_0^{(1)} \begin{pmatrix} p \\ \lambda \end{pmatrix}; X_\wp \right) + \sum_{j=1}^{n} \frac{\beta_{1\{\wp_j\}}^{(1)} \left(\begin{smallmatrix} p\, k_j \\ \lambda\, \sigma_j \end{smallmatrix}; X_\wp \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} \right\}$$

$$\mathfrak{M}_n^{(2)} \begin{pmatrix} p\, k_1 \\ \lambda\, \sigma_1 \end{pmatrix} \cdots \begin{pmatrix} k_n \\ \sigma_n \end{pmatrix} = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}}$$

$$\times \left\{ \beta_0^{(2)} \begin{pmatrix} p \\ \lambda \end{pmatrix}; X_\wp \right) + \sum_{j=1}^{n} \frac{\beta_{2\{\wp_j\}}^{(1)} \left(\begin{smallmatrix} p\, k_j \\ \lambda\, \sigma_j \end{smallmatrix}; X_\wp \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} + \sum_{1 \le j < l \le n} \frac{\beta_{2\{\wp_j, \wp_l\}}^{(2)} \left(\begin{smallmatrix} p\, k_j\, k_l \\ \lambda\, \sigma_j\, \sigma_l \end{smallmatrix}; X_\wp \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}} \mathfrak{s}_{[l]}^{\{\wp_l\}}} \right\}$$

For details see next slides, and for true details our papers.

CEEX features

Why COHERENT?

Because: Friendly to Quantum coherence;

Coherence among Feynman diagrams: Narrow resonances,

$\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR$\oplus$FSR real emissions, etc.

Complete $\left| \sum_{diagr.}^{n} \mathcal{M}_i \right|^2$ rather then often incomplete $\sum_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j^*$

Why EXCLUSIVE?

Because: Phase-Space $\times \sum_{spin} \left|\mathcal{M}\right|^2$

Contrary to Parton-Showers, Leading-Logs, Structure-Functions

which are all inherently inclusive.

Why EXPONENTIATION?

Infrared (IR) contributions summed to $\mathcal{O}(\alpha^{\infty})$

Going **beyond** Yennie-Frautschi-Suura (EEX):

Real IR singularities isolated/reorganized at the **spin amplitude**

level, while IR cancellations occur for probabilities (integrated

x-sections). Paradox???

Differences with older EEX/YFS? See next slide

EEX= Exclusive EXponentiation = variant of YFS expon.

$e^-(p_1,\lambda_1)+e^+(p_2,\lambda_2)\to f(q_1,\lambda_1')+\bar{f}(q_2,\lambda_2')+\gamma(k_1,\sigma_1)+\ldots+\gamma(k_n,\sigma_n)$

EEX/YFS very schematically

$$\sigma=\sum_{n=0}^{\infty}\int_{m_\gamma} d\Phi_{n+2}\, e^{Y(m_\gamma)} D_n(q_1,q_2,k_1,\ldots,k_n)$$

$\mathcal{O}(\alpha^1)$ **Example:**

$$D_0=\bar{\beta}_0$$
$$D_1(k_1)=\bar{\beta}_0\tilde{S}(k_1)+\bar{\beta}_1(k_1)$$
$$D_2(k_1,k_2)=\bar{\beta}_0\tilde{S}(k_1)\tilde{S}(k_2)+\bar{\beta}_1(k_1)\tilde{S}(k_2)+\bar{\beta}_1(k_2)\tilde{S}(k_1)$$

Real soft factors:

$$\tilde{S}(k)=\sum_\sigma|\mathfrak{s}_\sigma(k)|^2=|\mathfrak{s}_+|^2(k)+|\mathfrak{s}_-(k)|^2=-\tfrac{\alpha}{\pi}\left(\tfrac{q_1}{kq_1}-\tfrac{q_2}{kq_2}\right)^2$$

IR-finite building blocks:

$\bar{\beta}_0=\sum_\lambda|\mathcal{M}_\lambda|^2$, λ = fermion helicities, σ = photon helicity

$$\bar{\beta}_1(k)=\sum_{\lambda\sigma}|\mathcal{M}^{1-\mathrm{PHOT}}_{\lambda\sigma}|^2-\sum_\sigma|\mathfrak{s}_\sigma(k)|^2\sum_\lambda|\mathcal{M}^{\mathrm{Born}}_\lambda|^2$$

Everything in terms of $\sum_{spin}|\ldots|^2$ **!!!!**

TERMINOLOGY

EEX = Exclusive EXponentiation

Based on Yennie-Frautschi-Suura (1961)

YFS2, YFS3, KORALZ, $e^+e^- \to f\bar{f} + n\gamma$

YFS3WW, $e^+e^- \to W^+W^- + n\gamma$

YFS3ZZ, $e^+e^- \to ZZ + n\gamma$

BHLUMI, BHWIDE, $e^+e^- \to e^+e^- + n\gamma$

CEEX = Coherent Exclusive EXponentiation

New one, by Jadach-Ward-Wąs (1998), an extension of Yennie-Frautschi-Suura

$\mathcal{KK}$MC, $e^+e^- \to f\bar{f} + n\gamma,\ \ f = \mu, \tau, d, u, s, c, b$

IEX = Inclusive EXponentiation

Semi-analytical exponentiation, performed at the level of earlier integrated cross-sections:

$\mathcal{KK}$sem, CALASY, Zfitter, Gentle...

CEEX practical considerations

Practical problems

Deficiencies of YFS **Exclusive Exponentiation** (EEX) technique, as implemented in KORALZ/YFS3, BHLUMI, BHWIDE, KORALW:

- Lack of the ISR/FSR or Up/Down interferences
  (except BHWIDE, but difficult to upgrade...)
- Simplified treatment of spin, neglected transverse polarization
  (except KORALB, but no exponentiation...)
- Approximate matrix element for 2 and 3 hard large p_T photons.
  (Multiphotons are important for searches.)

The CEEX solution

New Coherent Exclusive Exponentiation CEEX solves many (all?):

- ISR$\otimes$FSR interferences are included in a natural way.
  Spin amplitudes for ISR and FSR summed squared numerically.
  (The BHWIDE approach gets cumbersome beyond first order)
- Exact treatment of fermion spin polarizations (transv.$\&$longitudinal).
  Numerical Wigner rotations of spin density matrices available.
  (Necessary for interfacing with decay M.C. simulating fermion decays).
- Exact matrix element for 2 and 3 and more photons using
  Kleiss-Stirling (KS) spinor technique.

Phenomenology of SM is important

I just can not resist to mention

the three dark sides to fight,

which take most of our time:

Physical uncertainty

- Order of perturbation expansion, - mass terms
- narrow resonance approximations
- Fits to data for vacuum polarization, - etc etc..

Technical uncertainty

- Correctness of the integration/generation algorithms
- Technical approximations in M.E. and/or phase space
- Computer bugs.

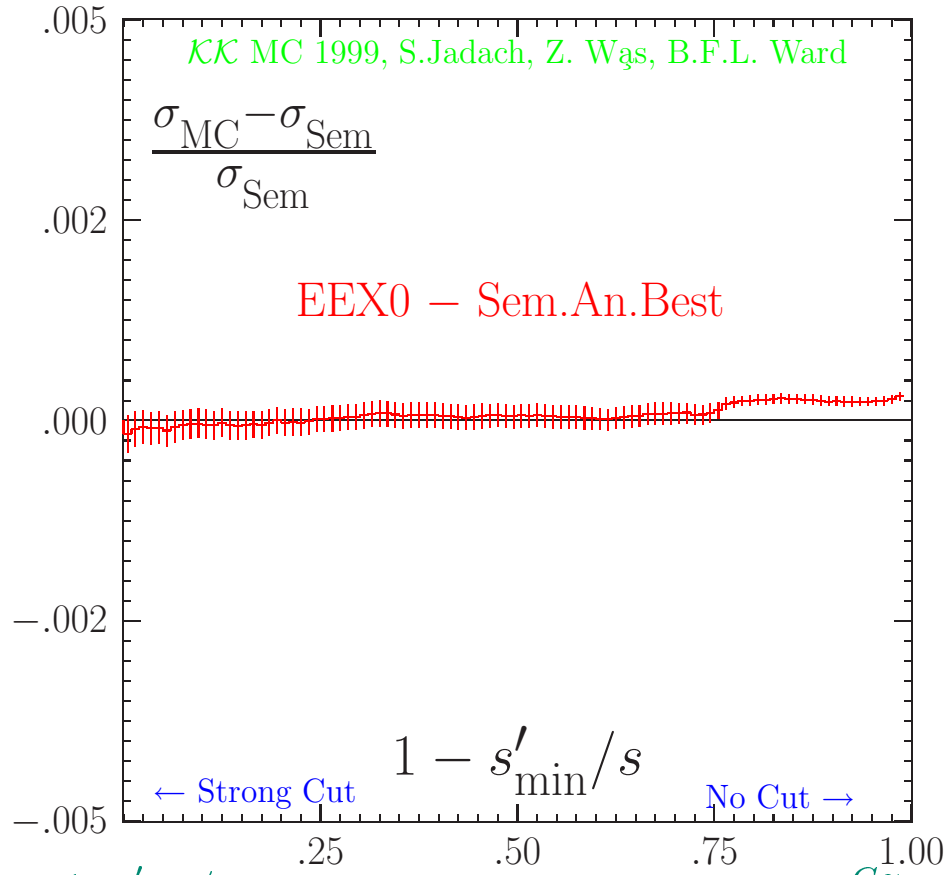
Common sense uncertainty

- Mismatch in input parameters
- Accidental misuse of codes
- All other unexpected little things, obvious but invisible.

Design and implementation of algorithms for tests is difficult part of the work, but it is necessary if one wants to exploit fully data obtained from huge effort of experimental communities.

Baseline test of the Low level MC

$|$ Matrix Element$|^2$ = Born $\times$ Product of soft factors



$$\sigma_{\rm Sem.} = \int_0^{1-s'_{\rm min}/s} dv\ \sigma^f_{\rm Born}(s(1-u)(1-v))\ \frac{e^{-C\gamma_e}}{\Gamma(1+\gamma_e)}$$
$$e^{\frac{1}{4}\gamma_e+\frac{\alpha}{\pi}\left(\frac{1}{2}+\frac{\pi^2}{3}\right)}\ \gamma_e v^{\gamma_e-1}\left[1-\frac{\gamma_e\ln(1-v)}{4}-\frac{\alpha}{\pi}\frac{\ln^2(1-v)}{2}+0\ \gamma_e^2\right]$$
$$\frac{e^{-C\gamma_f}}{\Gamma(1+\gamma_f)}e^{\frac{1}{4}\gamma_f-\frac{1}{2}\gamma_f\ln(1-u)+\frac{\alpha}{\pi}\left(\frac{1}{2}+\frac{\pi^2}{3}\right)}\gamma_f u^{\gamma_f-1}\left[1-\frac{\gamma_f\ln(1-u)}{4}\right]$$

Techn.Error=$|$MC−Sem.Anl.$|$ $\leq 2\cdot 10^{-4}$

Numerical Results: Physical Precision

PP = neglected higher orders and subleadings

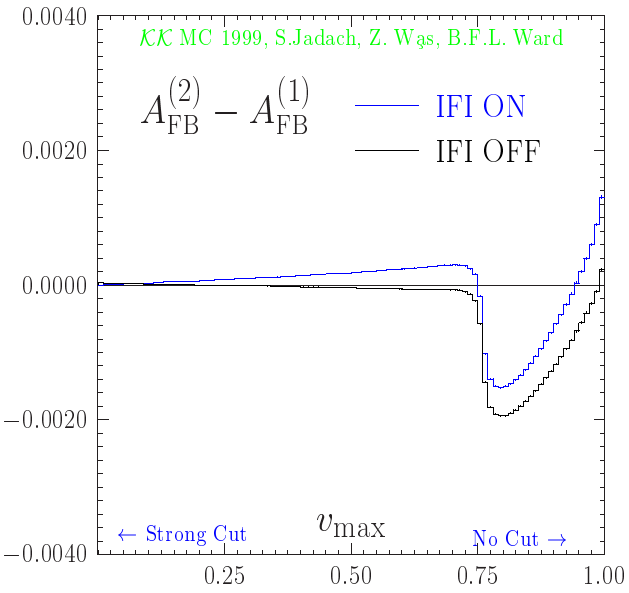
Our basic estimate: PP = $\frac{1}{2}\left\{\mathcal{O}(\alpha^2)_{\rm CEEX} - \mathcal{O}(\alpha^1)_{\rm CEEX}\right\}$

$\mathcal{KK}$ MC results for $e^+e^- \to \mu^+\mu^-$ **at 189GeV**

Photon Energy Cut: $v = 1 - s'/s < v_{\rm max}; \quad s' = m^2_{f\bar{f}}$

Angular Cut: $|\cos\theta| < 1$

$v_{\rm max}$	$\mathcal{KK}$sem Refer.	$\mathcal{O}(\alpha^3)_{\rm EEX3}$	$\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF	$\mathcal{O}(\alpha^2)_{\rm CEEX}$	KORALZ	KORALZ Interf.
	$\sigma(v_{\rm max})$ [pb], $\mathcal{KK}$ M.C. and KORALZ 1-st order					
.01	1.6712 ± .0000	1.6687 ± .0020	1.6690 ± .0020	1.7679 ± .0024	.9639 ± .0009	.1610 ± .0009
.10	2.5198 ± .0000	2.5164 ± .0023	2.5170 ± .0023	2.5967 ± .0027	2.1919 ± .0010	.0880 ± .0010
.30	3.0616 ± .0000	3.0565 ± .0024	3.0581 ± .0024	3.1190 ± .0029	2.7690 ± .0010	.0545 ± .0010
.50	3.3747 ± .0000	3.3682 ± .0025	3.3713 ± .0025	3.4203 ± .0029	3.0565 ± .0010	.0385 ± .0010
.70	3.7225 ± .0000	3.7131 ± .0025	3.7200 ± .0025	3.7596 ± .0030	3.3649 ± .0010	.0246 ± .0010
.90	7.1434 ± .0000	7.0904 ± .0024	7.1496 ± .0024	7.1789 ± .0030	6.3558 ± .0010	.0210 ± .0010
.99	7.6145 ± .0000	7.5511 ± .0024	7.6254 ± .0024	7.6542 ± .0029	6.7004 ± .0010	.0213 ± .0010
	$A_{\rm FB}(v_{\rm max})$, $\mathcal{KK}$ M.C. and KORALZ 1-st order					
.01	.5654 ± .0000	.5650 ± .0014	.5650 ± .0014	.6111 ± .0016	.5765 ± .0013	.1201 ± .0013
.10	.5664 ± .0000	.5660 ± .0011	.5660 ± .0011	.5922 ± .0012	.5784 ± .0006	.0324 ± .0006
.30	.5692 ± .0000	.5687 ± .0009	.5686 ± .0009	.5856 ± .0011	.5818 ± .0005	.0164 ± .0005
.50	.5744 ± .0000	.5738 ± .0009	.5737 ± .0009	.5863 ± .0010	.5868 ± .0005	.0112 ± .0005
.70	.5864 ± .0000	.5852 ± .0008	.5852 ± .0008	.5947 ± .0009	.5972 ± .0004	.0078 ± .0004
.90	.3105 ± .0000	.3115 ± .0004	.3096 ± .0004	.3170 ± .0005	.3260 ± .0002	.0037 ± .0002
.99	.2851 ± .0000	.2867 ± .0004	.2843 ± .0004	.2912 ± .0004	.3039 ± .0002	.0024 ± .0002



The above result imply for Physical Precision:

$\frac{\delta\sigma}{\sigma} \leq 0.2\%, \qquad \delta A_{FB} \leq 0.1\%,$

even including radiative-return on-shell Z.

This is confirmed by more tests, see later.

CEEX Numerical Results

Summary on numerical results

- **Technical Precision** $\frac{\delta\sigma}{\sigma} \leq 2 \cdot 10^{-4}$
- **Physical Precision:**
  $\frac{\delta\sigma}{\sigma} \leq 0.2\%$ **and** $\delta A_{FB} \leq 0.1\%$
- **ISR$\otimes$FSR interf. under control at LEP2 energies**

CEEX - Summary and Outlook

CEEX offers clear upgrade path for the precision calculations of the h.o. radiative corrections in the SM/QED, with the Monte Carlo implementation. It goes beyond (extends) the classical Yennie-Frautschi-Suura scheme. $\mathcal{KK}$ MC based on CEEX offers clear upgrade path for the MC event generators to Linear Collider era. CEEX is firmly based on spin amplitudes. The main profits are:

- **Interferences ISR$\otimes$FSR included and under firm control**
- **All kind of coherence effects, including narrow resonances.**
- **Complete treatment of spin (also transverse) for initial and final states.**
- **Exact M.E. for 2 high hard photons (3 photons pending).**
- **Future extension to all-angle second order Bhabha is possible.**
- **$\nu\bar{\nu}\gamma$ channel is at work and advanced.**
- **Some terms proportinal to final state mass still missing, trivial things need to be fixed for applications at CMS energies below 40 GeV.**

MC Implementation of CEEX

Comparison of $\mathcal{KK}$ MC with its predecessors

Feature	KORALB	KORALZ	$\mathcal{KK}$ 4.13	$\mathcal{KK}$ 2000+?
QED type	$\mathcal{O}(\alpha)$	EEX	CEEX, EEX	CEEX, EEX
CEEX(ISR+FSR)	none	none	$\{\alpha, \alpha L; \alpha^2 L^2, \alpha^2 L^1\}$	$\{...\alpha^2 L^1; \alpha^3 L^3\}$
EEX(ISR*FSR)	none	$\{\alpha, \alpha L, \alpha^2 L^2\}$	$\{\alpha, \alpha L, \alpha^2 L^2, \alpha^3 L^3\}$	$\{...\alpha^2 L^2, \alpha^3 L^3\}$
ISR-FSR int.	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	$\{\alpha, \alpha L\}_{\rm CEEX}$	$\{\alpha, \alpha L\}_{\rm CEEX}$
Exact bremss.	1 γ	1, 2coll. γ	1, 2, 3coll. γ	up to 3 γ
Electroweak	No Z-res.	DIZET 6.x	DIZET 6.x	New version?
Beam polar.	long+trans.	longit.	long+trans.	long+trans.
τ polar.	long+trans.	longit.	long+trans.	long+trans.
Hadronization	—	JETSET	JETSET	PYTHIA
τ decay	TAUOLA	TAUOLA	TAUOLA	TAUOLA
Inclusive mode	—	No	Yes	Yes
Beamstrahlung	—	No	Yes	Yes
Beam spread	—	No	Yes	Yes
$\nu\nu$ channel	—	Yes	No	Yes
ee channel	—	No	No	Yes
tt channel	—	No	No	yes?
WW channel	—	No	No	yes?

CEEX - Summary and Outlook

First official version 4.13 of $\mathcal{KK}$ MC available. It works for fermion pair production at LEP, LC's, τ and b factories, μ-colliders.

Check

http://home.cern.ch/jadach

http://home.cern.ch/wasm

http://hpjmiady.ifj.edu.pl/

http://atlasinfo.cern.ch/Atlas/GROUPS/PHYSICS/HIGGS/Atlfast.html

for slides, programs, manuals

Our PROGRAMS ...

BHLUMI 0.061 % precision tag

BHWIDE t-channel

KORALW multi-singularity

YFSWW LPA

YFSZZ

ATLFAST fast simmulation of the detector response

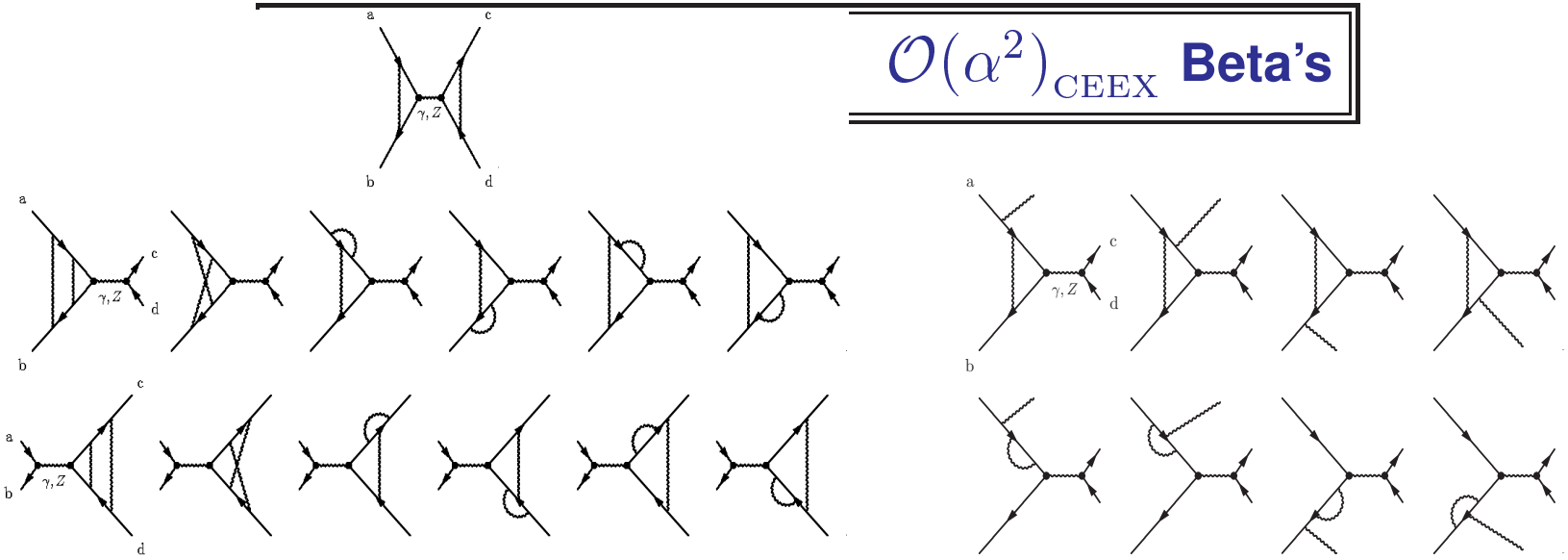
CEEX - Summary and Outlook

Warning:

Every new requirement for precision or parameter extension need the question of uncertainty to be revisited.

CEEX - additional transparencies

$\mathcal{O}(\alpha^2)_{\rm CEEX}$ Beta's



$$\beta_0^{(l)}\left(\begin{smallmatrix}p\\ \lambda\end{smallmatrix};P\right)=\mathfrak{M}_0^{(l)}\left(\begin{smallmatrix}p\\ \lambda\end{smallmatrix};P\right),\ l=0,1,2,$$

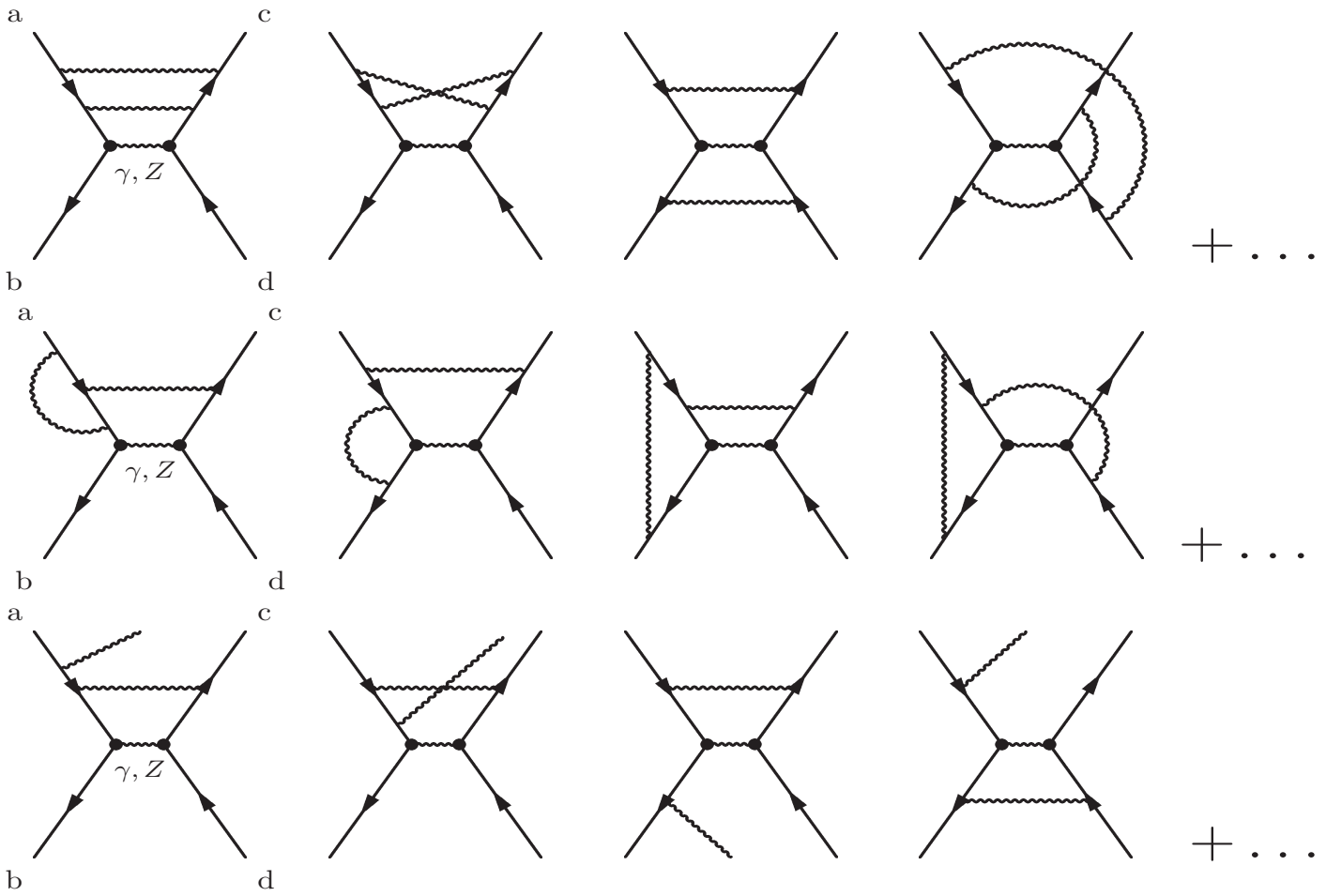
$$\beta_{1\{I\}}^{(1+l)}\left(\begin{smallmatrix}p\ k_1\\ \lambda\ \sigma_1\end{smallmatrix};P-k_1\right)=\mathfrak{M}_{1\{I\}}^{(1+l)}\left(\begin{smallmatrix}p\ k_1\\ \lambda\ \sigma_1\end{smallmatrix};P-k_1\right)-\beta_0^{(l)}\left(\begin{smallmatrix}p\\ \lambda\end{smallmatrix};P-k_1\right)\mathfrak{s}_{\sigma_1}^{\{I\}}(k_1),\ l=0,1,$$

$$\beta_{1\{F\}}^{(1+l)}\left(\begin{smallmatrix}p\ k_1\\ \lambda\ \sigma_1\end{smallmatrix};P\right)=\mathfrak{M}_{1\{F\}}^{(1+l)}\left(\begin{smallmatrix}p\ k_1\\ \lambda\ \sigma_1\end{smallmatrix};P\right)-\beta_0^{(l)}\left(\begin{smallmatrix}p\\ \lambda\end{smallmatrix};P\right)\mathfrak{s}_{\sigma_1}^{\{F\}}(k_1),\ l=0,1,$$

$$\begin{aligned}\beta_{2\{\omega_1,\omega_2\}}^{(2)}&\left(\begin{smallmatrix}p\ k_1\ k_2\\ \lambda\ \sigma_1\ \sigma_2\end{smallmatrix};X_\omega\right)=\mathfrak{M}_{2\{\omega_1,\omega_2\}}^{(2)}\left(\begin{smallmatrix}p\ k_1\ k_2\\ \lambda\ \sigma_1\ \sigma_2\end{smallmatrix};X_\omega\right)\\&-\beta_{1\{\omega_1\}}^{(1)}\left(\begin{smallmatrix}p\ k_1\\ \lambda\ \sigma_1\end{smallmatrix};X_\omega\right)\mathfrak{s}_{\sigma_2}^{\{\omega_2\}}(k_2)-\beta_{1\{\omega_2\}}^{(1)}\left(\begin{smallmatrix}p\ k_2\\ \lambda\ \sigma_2\end{smallmatrix};X_\omega\right)\mathfrak{s}_{\sigma_1}^{\{\omega_1\}}(k_1)\\&-\beta_0^{(0)}\left(\begin{smallmatrix}p\\ \lambda\end{smallmatrix};X_\omega\right)\mathfrak{s}_{\sigma_1}^{\{\omega_1\}}(k_1)\mathfrak{s}_{\sigma_2}^{\{\omega_2\}}(k_2),\end{aligned}$$

CEEX - additional transparencies

$\mathcal{O}(\alpha^2)_{\rm CEEX}$ Shopping list



Never ever combine with real emission! (Keep photon mass and publish.)

Never ever square and spin sum! In the M.C. real emission is added anyway.