Instrumentation and Diagnostics Using Schottky Signals

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Abstract

Schottky signal measurements are a widely used tool for the determination of longitudinal and transverse dynamical properties of hadron beams in circular accelerators and storage rings [1]. When applied to coasting beams, it is possible to deduce properties as the momentum distribution. the $Q_{x,y}$ values and the average betatron amplitudes. Scientific applications have been developed in the past few years, as well, namely nuclear Schottky mass spectrometry and lifetime measurements.

Schottky signals from a coasting beam are random signals which appear at every revolution harmonic and the respective betatron sidebands. Their interpretation is more or less straightforward unless the signal is perturbed by collective effects in the case of high phase space density.

Schottky signals from bunched beams reveal the synchrotron oscillation frequency, from which the effective rf voltage seen by the beam can be deduced.

The detection devices can be broad-band or narrow-band. The frequency range is usually in the range between a few hundred kHz up to about 150 MHz. In connection with stochastic cooling, Schottky signals are used at frequencies up to 8 GHz. Narrow-band devices are needed if signal-to-noise problems arise, e.g. in the case of antiproton beams. Heavy ion beams require less effort, it is relatively easy to detect single circulating highly charged ions.

1 SCHOTTKY SPECTRA

1.1 Coasting beam current

Imagine a detector at some given location in the storage ring. The beam current at this given place is the sum of the currents from each charged particle passing by. Let us assume that the beam is composed of N particles of a single species with charge qe. These particles are characterized in the longitudinal phase plane by their revolution period T_n (which we assume to be constant). Secondly, they are ordered randomly along the ring circumference. One way to parameterize this random placement is to stop the time t_n the particle passes by the detector during the revolution $k = 0$. The beam current is the sum over all particles n:

$$I(t) = qe \sum_{n=1}^{N} \sum_{k=-\infty}^{\infty} \delta(t - t_n - kT_n) \qquad (1)$$

The frequency spectrum, i.e. the Fourier transform of this current is an infinite train of delta functions, as well,

$$\tilde{I}(\Omega) = qe \sum_{n=1}^{N} \omega_n \sum_{k=-\infty}^{\infty} \delta(\Omega - k\omega_n) \exp(-i\Omega t_n) \qquad (2)$$

i.e. the current spectrum consists of peaks at each harmonic of the revolution frequency $\omega_n = 2\pi/T_n$ with a random phase. The mathematics of this Fourier correspondence can be found in most textbooks on signal theory. Because of the random phase, the expectation value of $\tilde{I}(\Omega)$ vanishes. However, the cancellation of the phases is never complete due to the finite number of particles in the beam. Therefore the power spectrum of the random process $I(t)$ can be detected, as we shall see. Before doing so, let us look at the distribution of the frequencies ω_n. The variation of the revolution frequencies around their mean value ω_0 is proportional the the deviation of the particle momenta p from their mean p_0:

$$\frac{\delta\omega}{\omega_0} = \eta \frac{\delta p}{p_0} \qquad (3)$$

with the frequency dispersion

$$\eta = \gamma^{-2} - \alpha_p \qquad (4)$$

Here, γ is the relativistic Lorentz factor, and α_p is the momentum compaction factor of the ring lattice. α_p is related to the transition γ_t by $\alpha_p = \gamma_t^{-2}$. Once η is known, it is possible to infer from the beam current frequency spectrum the momentum width of the beam. It is sufficient to measure the spectrum at one given harmonic. This is one of the main applications of Schottky diagnosis. If the momentum distribution is bounded by a momentum deviation $\pm\delta p_{\max}$, then there is a harmonic $k_{\max}$ above which the correspondence $\Omega(\delta p)$ ceases to be unique, i.e. where the Schottky harmonics begin to overlap:

$$k_{\max} > \frac{1}{2} \left| \eta \frac{\delta p_{\max}}{p_0} \right|^{-1} \qquad (5)$$

The width of the frequency distribution is proportional to the harmonic number.

Signal detectors are linear devices which respond to the beam with a voltage

$$u(t) = \frac{qeZ_L}{2} \sum_{n=1}^{N} \omega_n \sum_{k=-\infty}^{\infty} S(x_{kn}, y_{kn}, t) * \delta(t - t_n - kT_n) \qquad (6)$$

Here, Z_L the line impedance. S is called the sensitivity of the detector. One should note that the position (x, y) changes from revolution to revolution if the particle performs betatron oscillations around the closed orbit. The signal from a coasting depends on the variation of S over the range of betatron amplitudes. The position of the nth particle at the kth revolution is given by its coordinates x and y. $*$ denotes a convolution in the time domain, i.e. $f(t)*g(t) = \int d\tau\; f(\tau)\, g(t-\tau)$. The form of the pulse train $S(\ldots, t)$ determines the frequency response of the device. Let us first look at the signal from a longitudinal pick-up.

1.2 *Longitudinal Signal of a Coasting Beam*

We assume that the signal is independent of position, because the sensitivity is constant over the range of betatron amplitudes at the detector: $S(x, y, \ldots) \approx S_0$. Then the signal spectrum is directly proportional to the current spectrum. Let the momentum distribution $\Psi\,(\delta p/p_0)$ be normalized to N. In the case of non-overlapping Schottky bands at the frequency $\Omega(\delta p) = k\omega_0(1 + \eta\delta p/p_0)$ the power spectrum is

$$P(\Omega) = \frac{(qef_0)^2}{4Z_L}\left|\tilde{S}_0(\Omega)\right|^2 \frac{\Psi\,(\delta p/p_0)}{|\eta k|} \tag{7}$$

i.e. the spectrum has the following properties:

- The total power at each harmonic is proportional to N. Once the sensitivity at Ω has been gauged, e.g. by comparison with a beam current transformer, the total power is therefore a good measure for the beam current. This is particularly helpful at low intensity, if the current transformer response is well below the noise limit.
- It is proportional to the square of the charge state. Highly charged heavy ions are therefore easy to be seen in a Schottky spectrum.
- The width of the signal is proportional to the harmonic number k, the power density is inversersely proportional to the absolute value of both k and η.

Fig. 1 shows the longitudinal Schottky spectrum of a hypothetical Gaussian momentum distribution with an extremely large $\eta\sigma = 2\%$, bounded at $\delta\omega_{\max}/\omega_0 = \pm 5\%$. This is an unusually broad spectrum drawn only for illustrated purposes in order to show the properties discussed above in an overview.

1.3 *Transverse Signal of a Coasting Beam*

Detectors for transverse signals are usually take the difference signal from a pair of opposite electrodes. The sensitivity is then approximately linear around the center of the beam pipe. For example a horizontal pick-up can be characterized by a sensitivity

$$\tilde{S}(x, y, \Omega) \approx x\; \left.\frac{\partial \tilde{S}(x, y, \Omega)}{\partial x}\right|_{x=y=0} := xS'(\Omega) \tag{8}$$

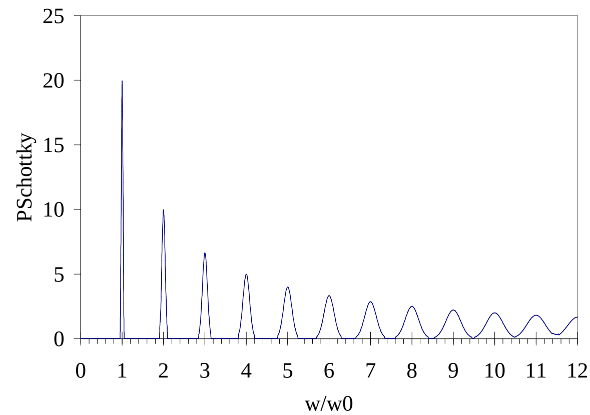

Figure 1: Hypothetical Schottky spectrum of bounded Gaussian distributed beam

Imagine a beam performing betatron oscillations around the electrical center $x = 0$ of the detector,

$$x_{kn} = A_n \sin 2\pi Q_x k + \mu_n \tag{9}$$

A_n is the betatron amplitude related to the one-particle emittance $\epsilon_{x,n}$ and the horizontal beta function β_x at the detector via $A_n = \sqrt{\epsilon_{x,n}\beta_x}$. μ_n is the random betatron phase. According to Eq. 6, the detector output changes sign in the rhythm of the betatron oscillations. The modulation produces sidebands around the revolution harmonics

$$\begin{aligned} \omega_{k,\pm} &= (k \pm Q_x)\,\omega \\ &= \omega_0\,(k \pm Q_{x0})\left(1 + \eta\frac{\delta p}{p_0}\right) \pm Q_{x0}\xi_x\frac{\delta p}{p_0} \\ &+ O\left(\frac{\delta p}{p_0}\right)^2 \end{aligned} \tag{10}$$

If the effect of the chromaticity ξ_x can be neglected at sufficiently high harmonics $k \gg Q_{x0}\xi_x/\eta$, the power spectrum is (assuming that left and right sidebands do not overlap)

$$P(\omega_{k,\pm}) = \langle\epsilon_x\rangle\,\beta_x\frac{(qef_0)^2}{16Z_L}\left|S'(\Omega)\right|^2\frac{\Psi\,(\delta p/p_0)}{|\eta k|} \tag{11}$$

where $< \epsilon_x >$ is the mean emittance at the given momentum deviation. Therefore the measurement of transverse Schottky power spectra allows

- to infer the linear chromaticity at low harmonics,
- to infer the fractional part of the Q value,
- to deduce the mean emittance, even as a function of off-momentum, if the power spectral density is properly gauged.

1.4 *Longitudinal Signal of a Bunched Beam*

A bunched beam at constant energy (no acceleration or deceleration) gives rise to coherent lines which are not the subject of this paper. However, there is also a random (Schottky) contribution in the signal. A bunch exists because it is stabilized by synchrotron oscillations. These can be characterized by an amplitude and a random phase. The synchrotron oscillations have the frequency ω_s and give rise to a series of sidebands around the revolution harmonics at frequencies $k\omega + j\omega_s$. Although the synchrotron frequency becomes amplitude-dependent at large amplitudes, the small-amplitude synchrotron oscillation frequency can often be measured. It is given by

$$\omega_s = \frac{\omega_0}{\beta}\sqrt{\frac{\eta h q e V}{2\pi m \gamma c^2}} \qquad (12)$$

The synchrotron satellites can therefore be used to determine the effective accelerating voltage V seen by the beam, including effects like transit time etc.

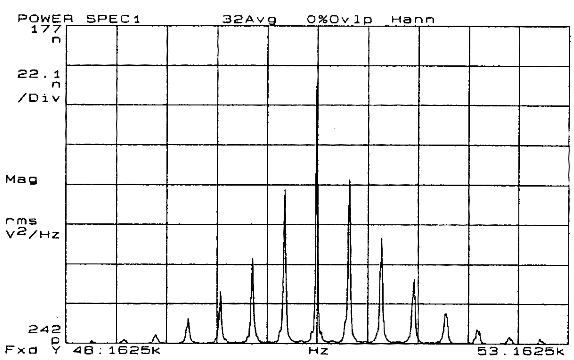

Figure 2: Bunched beam Schottky spectrum with synchrotron sidebands

2 SIGNAL SUPPRESSION

The interpretation of Schottky spectra is straightforward unless signal suppression enters into the game. Signal suppression is caused by collective effects inside the beam which effectively screen the signal from the detector. The effects are well-understood quantitatively [2] and can be analyzed by ways of a Vlasov equation formalism. In the case of the longitudinal spectrum, the screening is described by a dielectric function ϵ. it is sufficient to take Eq. 7 and perform the replacement

$$\Psi\left(\delta p/p_0\right) \mapsto \frac{\Psi\left(\delta p/p_0\right)}{\left|\epsilon(\Omega,k)\right|^2} \qquad (13)$$

A detailed discussion of the screening effect can be found in [2]. It becomes significant if the beam approaches the Keil-Schnell circle, i.e if

$$\frac{qeI}{|\eta|\beta^2\gamma m c^2\left(\delta p/p_0\right)^2}\left|\frac{Z_{\|}(\Omega)}{k}\right| \approx 1 \qquad (14)$$

The Keil-Schnell criterion is proportional to the beam current I, the beam impedance and inversely proportional to the square of the momentum width $\delta p/p_0$. Screening effects are often observed in machines with strong cooling and with non-relativistic beams that exhibit a large space charge impedance. A round beam of radius a in a round vacuum chamber of radius b has a purely imaginary space charge impedance

$$\mathrm{Im}\left.\frac{Z_{\|}}{k}\right|_{\text{(space charge)}} = -\frac{Z_0}{2\beta\gamma^2}\left(1+\ln\frac{b}{a}\right) \qquad (15)$$

where $Z_0 = 377\Omega$ is the wave impedance of the vacuum. In screened Schottky spectra, the total power is no more proportional to N. The spectral density in the center of the distribution is reduced and peaks appear on the left and right hands of the center which can be attribution to propagating and counter-propagating acoustic plasma waves. Below transition, the left-hand wave is more pronounced, as it becomes unstable above the stability limit. It is important to be aware of these effects even if one does not see the screened 'ears' in the Schottky spectra at first sight, as the screening destroys the possibility of measuring beam intensities as well as momentum width.

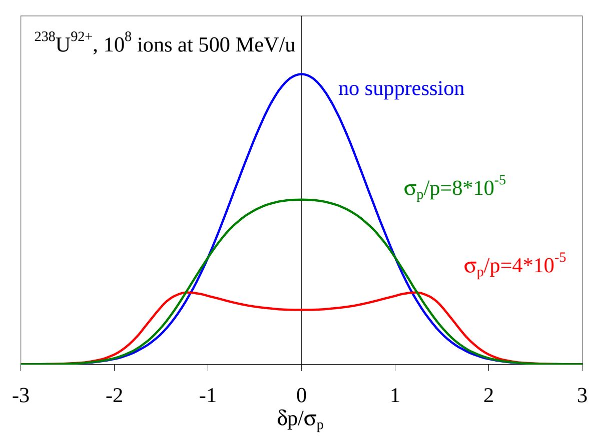

Figure 3: Calculated spectra of signal suppression due to space charge

3 DETECTORS

3.1 General considerations

Two main considerations dictate the choice and construction of a Schottky detector: bandwidth and sensitivity. Large bandwidth is desirable for all-purpose devices used not only for Schottky diagnosis but also for the detection of instabilities, the observation of coherent lines of bunched beam during acceleration or deceleration, and so on. On the other hand, high sensitivity is needed if one needs a high temporal resolution, wants to measure unstable beams, or if the product q^2N is so small that one has to cope with signal-to noise problems.

3.2 Broad-band devices

Useful broad-band devices are quarter-wave or capacitive Schottky detectors working at frequencies of up to 150 MHz. Typical harmonic numbers that occur in small rings with circumference between 100 m and 200 m are of the order of some ten. A completely different class of detector are the Schottky pick-ups used in stochastic cooling loops, which may work at frequencies up to 8 GHz. Here harmonic numbers of some 1000 come into play. The frequency curve of quarter-wave or capacitive devices is roughly sinusoidal,

$$S(\ldots, 2\pi f) \propto \sin\left(\frac{f}{f_c}\right) \qquad (16)$$

The center frequency f_c for quarter-wave devices is given by

$$f_c = \frac{c}{2\,(1+\beta^{-1})\,L} \qquad (17)$$

where L is the mechanical length of the pick-up plates.

3.3 Narrow-band devices

Narrow-band devices consist of high-Q cavities with critical coupling, i.e. the unloaded Q is twice as large as the Q with coupling.

3.4 Broad-band devices with lumped resonant circuits

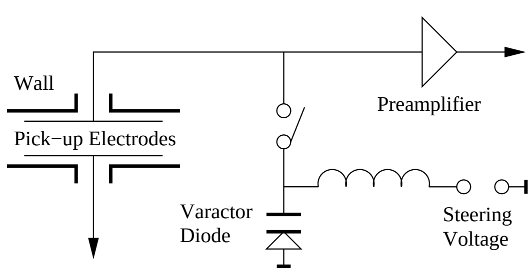

Figure 4: Broad-band pick-up with tunable exterior circuit

If the working frequency is not too high, a broad-band Schottky pick-up can be made resonant [3], if it is connected to a resonant circuit consisting of a cable and a tunable varactor diode. With such a simple circuit, the sensitivity can be enhanced by more than 6 dB. It is also possible to use the resonant cable at its second (or higher) harmonic.

3.5 Amplifiers

The signal to noise ratio is determined by the effective temperature of the pick-up and the noise number of the first preamplifier. A quantitative treatment of thermal noise problems is beyond the scope of this paper. Examples can be found, e.g in [3], [4],

4 SPECTRUM ANALYSIS

4.1 Analog Spectrum Analyzers

Analog spectrum analyzers are easy-to-use measurement devices which measure the spectral power inside a small band, the width of which is given by the resolution bandwidth (RBW). This process is repeated for N_p adjacent bands until one gets a spectrum of typically $N_p = 1000$ points. If the signal is random (as in Schottky signals), the power is not time-constant. It is effectively averaged by using a low-pass filter after the power detector with a bandwidth which is called the video bandwidth (VBW). With random signals, it makes sense to choose $VBW = RBW/10$. It is useful to combine the effect of the video filtering with digital power averaging of many consecutive spectra. The measurement time T_S per spectrum is given by the Fourier limit of the RBW or the VBW, hence it is of the order $T_S \approx 2N_p/\min(RBW, VBW)$.

4.2 Digital Systems

The analog spectrum analyzers are getting replaced more and more by their digital relatives. In digital spectrum analyzers, the rf frequency band to be measured is converted into the low-frequency range using a stable reference frequency and a single sideband or image reject mixer. The signal is sampled at the sampling frequency f_s, and converted to digital. With a sampling rate of up to 10 MHz, 18 bit ADC's are nowadays available. This digital signal is Fourier analyzed. Even with the sampling rate mentioned above, real-time Fourier analysis is now available with fast digital signal processors (DSP's). Low pass-filtering (in order to avoid aliasing) and windowing in the time domain (in order to avoid spurious spectral sidelobes) are standard procedures discussed in many textbooks on digital signal analysis. If a spectral resolution Δf is required, the necessary record length is $2.56/\Delta f$. The factor 2.56 is larger than the Nyquist factor 2 because of the necessity of windowing. Note that the requirements on Δf are inversely proportional to the harmonic number because the width of Schottky spectra increases proportional to frequency. If spectral resolution is a critical issue, one should chose the highest possible revolution harmonic where signal to noise problems are still tolerable. The sampling rate f_s, on the other hand, determines the maximum usable bandwidth W that is observed simultaneously: $W = f_s/2.56$. The factor 2.56 expresses the necessity to keep the maximum band frequency away from aliasing frequencies. Spectral averaging over $N_{\rm avg}$ averages is needed because of the random nature of the Schottky signals. The estimated error of the power in a given line is simply $N_{\rm avg}^{-1/2}$. This has always to be taken into account in the quantitative analysis of digital Schottky spectra. The optimum signal to noise ratio depends on the signal to noise ratio of the rf signal behind the first preamplifier. It should never be deteriorated by unnecessary low rf levels in long cables, by low-quality IRM's, or unstable reference signals.

5 EXAMPLES

5.1 Cooling diagnosis

An important application of Schottky diagnosis is the observation of beam cooling (electron cooling, stochastic cooling, laser cooling). An instructive example is shown in figure 5. It shows vertical Schottky spectra taken during stochastic cooling of a uranium beam using a commercial Tektronix 3066 spectrum analyzer. The signal was taken directly from the stochastic cooling signal using a directional coupler.The lowest spectrum is displayed at the bottom, the time between successive curves is 80 ms. Each curve is an average of 250 non-overlapping single frames taken during 320 μs. The Fourier transform of each frame yields a spectrum of 641 frequency points in a bandwidth of 2 MHz. The sampling rate was 5 MHz with a resolution of 12 bits. As the beam was not well centered with respect to the pick-up,the longitudinal part of the spectrum is also seen. Because of the longitudinal cooling, the width decreases, but the area remains stable because there was no particle loss. The width of the vertical sidebands decreases simultaneously, of course, but the area of the vertical sidebands decreases, as well. This can be used for the diagnosis of vertical cooling. However, these curves have not been published, because the stochastic cooling loop was not opened for the measurement. The closed loop may lead to signal suppression. This could be the reason why there is a slight asymmetry in the upper spectra.

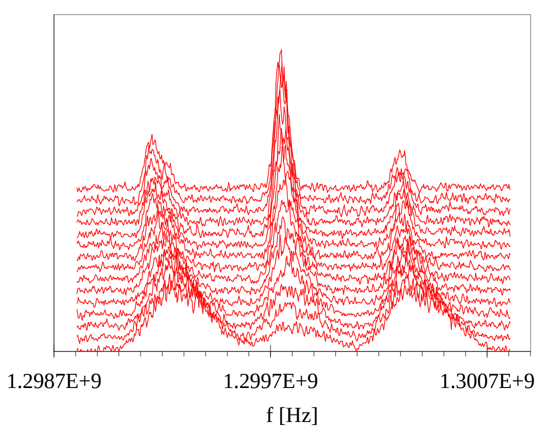

Figure 5: Waterfall diagram of vertical Schottky spectra at high harmonic taken during stochastic cooling of a uranium beam ($7 * 10^5$ fully stripped ions)

5.2 Nuclear Spectroscopy

An application to nuclear spectroscopy is displayed in figure 6. If there are different nuclear species stored in a ring like the ESR at GSI, the frequencies of the centroids are determined by their mass to charge ratio A/q:

$$\frac{\delta f}{f} = -\alpha_p \frac{\delta(A/q)}{A/q} \qquad (18)$$

With electron cooled beams and good power supplies, extremely sharp Schottky spectra can be measured, which can even be used to infer nuclear masses. The figure shows a spectrum measured in a band of 300 Hz with a resonant probe as discussed in section 3.4. The spectra were taken at 3 kHz span, and 200 spectral averages were used in the off-line analysis. The time between subsequent spectra is 96 s. Shown are secondary ions produced by shooting a lead beam on a target at 800 MeV/u. The secondary ions shown in the figure were measured at 400 MeV/u. The most prominent line in the middle is due to fully stripped $^{206}\,Tl^{81+}$. This ion has an isomeric state with a half life in the rest frame of 3.74 min, which is populated, as well. The half life seen in the laboratory is longer due to Lorentz time dilatation. The frequency difference seen between the two $^{206}\,Tl^{81+}$ lines shows nicely how a difference in excitation energy is turned into a difference in rest mass. The third line on the right is due to the beta decay of $^{206}\,Tl^{81+}$ into $^{206}\,Pb^{81+}$. This is a so-called bound beta decay because the decay electron is captured in the electronic K-shell of the daughter nucleus. Time-resolved measurements as these allow to infer decay rates. The measurement also shows a small drift of the revolution frequency during the measurement. It is of the order of 10^{-6} and can be attribution to the dipole magnet power supplies or the electron cooler voltage. A small spurious line on the low-frequency part of the spectrum does not show this variation. The total number of particles in the spectrum is less than 1000. Single ions have been identified in Schottky spectra like these.

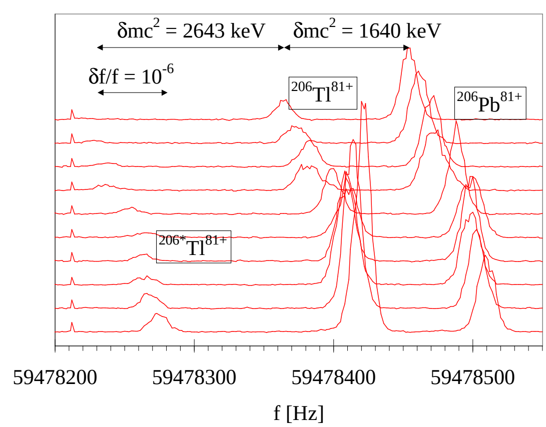

Figure 6: Spectra showing decay and decay products with three nuclear species

6 REFERENCES

[1] D. Boussard, CERN Report 95-06, 749-782

[2] S. Cocher, I. Hofmann, Part. Acc. 34, 189 (1990)

[3] U. Schaaf, PhD Thesis, GSI Report 91-22

[4] C. Gonzalez, F. Pedersen, Proc. PAC 1999, 474-476