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INSTABILITY GROWTH RATE CALCULATIONS FOR HIGH ENERGY STORAGE RINGS

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Summary

Tolerances in the manufacture of rf cavities for high energy electron-positron storage rings lead to a statistical distribution of higher mode resonant frequencies. This distribution and its associated fluctuations are considered in calculations of the growth rate of instabilities. It is found that in some cases the fluctuations can lead to large growth rates and must be accounted for when designing higher mode damping probes. Calculations for the Cornell 50 GeV electron-positron storage ring are used as illustrations.

Introduction

The electron-positron storage rings now being proposed (CESR II and LEP) have large numbers of rf accelerating structures. The higher modes of these structures can lead to beam instabilities. In CESR II where superconducting accelerating cavities are used; the unloaded Q's of the higher modes are of order 10^9. The loaded Q's of these modes must be significantly lower than this value to insure that the beam will be stable. Statistical effects play an important role when estimating the values of the loaded Q's.

The frequency shift for a coherent transverse beam mode m is[1]

$$\Delta\omega_m = \frac{1}{1+m}\,\frac{j}{2\nu\omega_0}\,\frac{c^2}{V}\,\frac{I}{\sqrt{2\pi}\,\sigma k}\,\frac{\omega_R}{\omega_0}\left(\frac{Z(\omega_R)}{n}\right)_{eff}$$

The symbols are defined in Appendix A. $(Z/n)_{eff}$ is the effective deflecting impedance given by

$$\left(\frac{Z(\omega_R)}{n}\right)_{eff} = \frac{\sum_{p=-\infty}^{\infty} Z_\perp(\omega_R,\omega_p)\,h_m(\omega_p-\omega_\xi)}{\sum_{p=-\infty}^{\infty} h_m(\omega_p-\omega_\xi)}$$

where $Z_\perp(\omega_R,\omega)$ is the deflecting impedance at angular frequency ω for a mode with resonant frequency ω_R.

$$Z_\perp(\omega_R\omega)=R_{ts}\frac{\left(\frac{\omega}{\omega_R}+j\,Q\left(1-\left(\frac{\omega}{\omega_R}\right)^2\right)\right)}{\left(\frac{\omega}{\omega_R}\right)^2+Q^2\left(1-\left(\frac{\omega}{\omega_R}\right)^2\right)^2}$$

and

$$R_{ts} = \left(\frac{Z''T^2}{Q}\right) Q\,L\,\frac{c}{\omega_R}.$$

The shunt impedance of the cavity higher mode $(Z''T^2/Q)$ can be measured or calculated and includes transit time effects. The summations in $(Z/n)_{eff}$ are performed using the results of Zotter[2]. The imaginary part of $\Delta\omega_m$ gives the growth rate of the mode; the mode is anti-damped for $Im(\Delta\omega_m)<0$.

In the examples which follow only the lowest frequency deflecting passband of the prototype CESR II cavity will be considered. The measured properties of this passband as well as other parameters which are used in the examples are given in Table I.

Spread in Resonant Frequency

When the storage ring has many accelerating structures, the resonant frequency of any particular higher mode will have a statistical spread because of manufacturing tolerances. Representing the spread by a Gaussian with standard deviation σ_R the effective impedance has a mean value

$$\overline{\left(\frac{Z}{n}\right)}_{eff} = \frac{1}{(\sqrt{2\pi})^3\sigma_R}\int_{-\infty}^{\infty} d\omega_R\left(\frac{Z(\omega_R)}{n}\right)_{eff}\exp\left(-\frac{(\omega_R-\omega_{Ro})^2}{2(2\pi\sigma_R)^2}\right)$$

which should be multiplied by the number of independent accelerating structures, N, to give the total deflecting impedance. The above integral is difficult to do numerically, and it is easier to estimate $\overline{(Z/n)}_{eff}$ by lowering the effective Q of the resonance and using the analytical expression for a single resonance.[4]

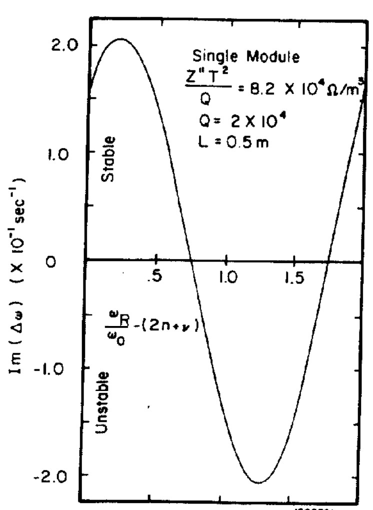

Fig. 1: $Im(\Delta\omega)$ (the growth rate) for the m=0 mode. A single module and only one member of the TM_{210} passband are considered.

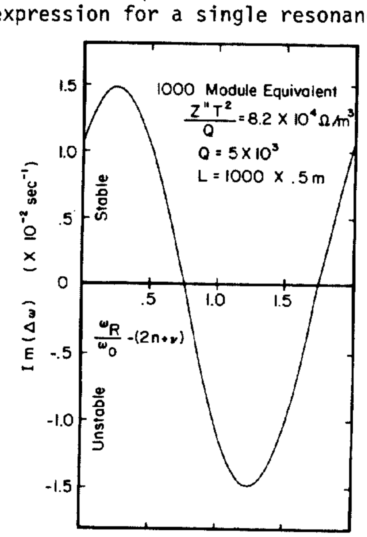

Fig. 2: Growth rate for the m=0 mode and 1000 modules. The reduced Q accounts approximately for the spread in resonant frequencies of the different modules.

Fig. 1 and 2 show the results for one of the higher modes in CESR II. Fig. 1 gives the growth rate for a single module. The growth rate is a sensitive function of

$$\varepsilon = \omega_R/\omega_0 - (n+\nu)$$

where n is an integer. For ε near zero(one) the impedance peaks near a positive (negative) frequency line in the power spectrum. Since the positive frequency lines lead to damping while the negative frequency ones lead to anti-damping, the motion is stable in the first case and unstable in the second.

When the spread in resonant frequencies is taken into account by lowering the Q from 20000 to 5000, the result of Fig. 2 is obtained. Even though 1000 modules are now being considered, the growth rate is one order of magnitude smaller! Lowering the Q has spread the resonance impedance over more lines (both positive and negative frequency) in the power spectrum. This leads to cancellation between damping and anti-damping contributions and therefore a reduced growth rate. The magnitude of the reduction is strongly dependent on the Q.

In any statistical problem with a finite number of elements (accelerating modules in this case) there will be departures from a smooth distribution due to statistical fluctuations. The rms spread in the growth rate is

$$\frac{1}{1+m}\,\frac{1}{2\nu\omega_0}\,\frac{c^2}{V}\,\frac{I}{\sqrt{2\pi}\,\sigma k}\,\frac{\omega_R}{\omega_0}\,\sigma_{z/n}$$

where

$$\sigma_{z/n} = \left(\frac{N}{(\sqrt{2\pi})^3\sigma_R}\int_{-\infty}^{\infty} d\omega_R \exp\left(-\frac{(\omega_R-\omega_{Ro})^2}{2(2\pi\sigma_R)^2}\right) \times\left(\left(\frac{Z(\omega_R)}{n}\right)_{eff} - \overline{\left(\frac{Z}{n}\right)_{eff}}\right)^2\right)^{1/2}$$

For a large σ_R(low Q in the multi-module equivalent) this spread can be much greater than the mean growth rate.

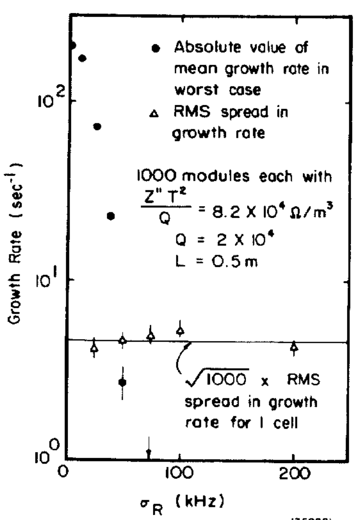

Fig. 3: The mean value and rms spread in the growth rate evaluated in the worst case for different spreads in the resonant frequency of the higher mode.

Fig. 3 shows the mean and rms spread in the growth rate for the cavity mode considered in Fig. 1. In making the calculations for the figure the worst case $\varepsilon = 1.28$ in Fig. 1 is considered. For a spread in resonant frequencies greater than approximately 45 kHz, the rms spread in the growth rate is greater than the worst case mean. As should be expected, this rms spread is the square root of the number of modules times the spread for a single cell (the rms spread of the growth rate shown in Fig. 1).

For sufficiently large σ_R's considering only the mean growth rate and not the possible spread in its value would be a serious mistake. The "average" accelerator would be stable, but there would be a significant probability that any real accelerator would be unstable. In addition, the actual distribution of resonant frequencies is sure to change in time as temperatures and tuning plunger positions change. Without accounting for the rms spread in growth rates, a stable accelerator may become unstable.

Monte Carlo Simulation

A Monte Carlo technique has been used to estimate the necessary values of loaded Q's. To date only the TM_{210} passband has been considered. Within the passband the growth rate is a strong function of only $\varepsilon = (\omega_R/\omega_0 - (2n+\nu))$ where the integer n is chosen to keep ε between 0 and 2. For each Monte Carlo trial the differences $\varepsilon_1-\varepsilon_2,..,\varepsilon_1-\varepsilon_5$ and the mean value of ε_1 are selected randomly and then assumed to be the same for each of the 1000 accelerating modules. The notation ε_i is used to denote ε for the ith member of of passband. For each accelerating module ε_1 is chosen randomly where it is assumed to be Gaussian distributed with the mean value selected above and an rms determined by σ_R.

The growth rates for the m=0 and m=1 transverse modes are then calculated for each member of the passband. Summing over the passband and the modules leads to the net m=0 and m=1 growth rates. The accelerator is considered stable if both of these modes have growth rates less than the radiation damping rate. The entire process is repeated for a number of Monte Carlo trials sufficient to obtain the desired accuracy.

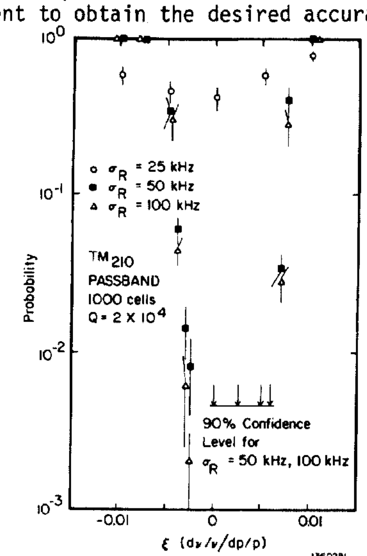

Fig. 4: Probability that CESR II will be unstable vs. chromaticity. The Monte Carlo calculation is described in the text.

Fig. 4 shows the results for a loaded Q of 2×10^4. At large negative chromaticity the m=0 mode is unstable while for large positive chromaticity the m=1 mode is unstable. For σ_R = 25 kHz the probability the accelerator will be stable is low for all values of the chromaticity. For frequency spreads of 50 kHz and 100 kHz the stable region extends over $0.0 \leq \xi \leq 0.006$. Changing the frequency spread from 50 kHz to 100 kHz does not affect the size of the stable region in chromaticity. This is consistent with the rms spread in growth rates being the dominant consideration for these values of σ_R (see Fig. 3).

It is also important to note that the region of chromaticity for stable operation is very limited. This places severe constraints on the lattice design unless feedback can be used to increase the size of the stable operating region.

Acknowledgements

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Table I. Lowest frequency deflecting passband and parameters used in the figures

TM_{210} Passband (Ref. 3)

f_R (GHz)	$\frac{Z"T^2}{Q}$ (Ω/m^3)
2.18	1.5×10^3
2.18	1.9×10^4
2.20	8.2×10^4
2.22	5.2×10^4
2.23	4.0×10^3

Parameters

E = 20 GeV
ν = 52.25
ξ = 0.00
I = 3.7 mA, 2 bunches
σ = 1.2 cm
f_o = 54.66 kHz
α = 3.7×10^{-4}

References

1. F. Sacherer, CERN 77-13, 198 (1977).
2. B. Zotter, ISR-TH/80-03 (1980). If one defines σ as the rms of the beam charge pulse, the appropriate σ to use in the formulae of Zotter's paper is $\sqrt{2}\sigma$.
3. J. Kirchgessner, P. Kneisel, H. Padamsee, J. Peters, D. Proch, R. Sundelin and M. Tigner, CLNS 80/462 (1980).
4. See for ex. A. Hofmann and J.R. Maidment, LEP-168 (1979).

Appendix A: Symbols used in the text

m:	Transverse mode number
ν:	Tune
f_o, ω_o:	Rotation frequency and angular frequency
c:	Speed of light
σ:	Bunch length (see Ref. 2)
f_R, ω_R:	Higher mode frequency and angular frequency
Q:	Quality factor
L:	Length of accelerating module
$Z"T^2/Q$:	Shunt impedance
n,p:	Integers
k:	Number of bunches
I:	Average current
V,E:	Equivalent beam voltage, beam energy
ξ, ω_ξ:	Chromaticity, chromaticity frequency shift
σ_R:	rms spread in resonant frequency of higher mode
ω_{Ro}:	Mean resonant frequency of higher mode
α:	Momentum compaction
h_m:	Mode m power spectrum