Beam Breakup Considerations in the Design of Multiple Off-Axis Gaps in an Induction Accelerator Cell for *SLIA*

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Measurements of the transverse impedances of the SLIA prototype cell were performed using the bead pull technique. These measurements were compared to a computer model (BBUS) of the prototype cell. With appropriate damping, the results look very encouraging for the SLIA accelerator.

INTRODUCTION

The *SLIA POCE* experiment involves a spiral line accelerator, requiring that the accelerator cells have multiple off-axis gaps. This generates a host of potential problems with beam breakup which have several unique properties. These are off-axis excitation and gap to gap coupling. To mitigate these properties, a coaxial shielded gap was developed (see figure 1), which uncouples the gaps and symmeterizes the fields within each gap. This is because the coaxial structure cuts off nonsymmetrical fields below its TE_{11} cutoff frequency. The idea is to have the most important dipole modes and in particular the accelerating and beam loading waveforms below this symmetrical-forcing cutoff frequency. To the degree that this is true, the beam appears to be in an on-axis accelerating gap. There are typically two to five dipole modes between the coax and beam pipe cutoff frequencies. These modes must be damped to reduce the beam breakup growth rate to an acceptable level, depending on the number of gaps and the beam current being accelerated.

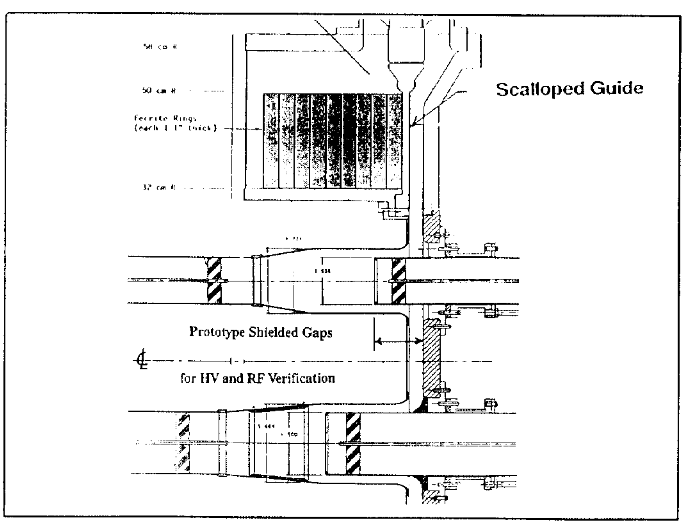

Figure 1----Prototype Cell Geometry

The beam breakup growth rate is given by

$$\mathbf{A_r = A_0\, e^{\Gamma} / Sqrt(16\, \pi\, \Gamma)} \text{ [1], where}$$
$$\Gamma = N_{gap}\, R(\mathrm{Ohm/cm})\, I(\mathrm{kA}) / (300\, B_z(\mathrm{kG})),$$

N_{gap} is the number of acceleration gaps, I is the electron beam current, B_z is the solenoidal focusing field, t_p is the electron beam pulse width and f is the mode frequency. $\mathbf{A_r}$ and $\mathbf{A_0}$ are the final and initial beam centroid amplitudes at the mode frequency.

The method used to determine the R and R/Q's for the *SLIA* prototype acceleration cell was a bead pull measurement, measuring the frequency shifts due to a dielectric "bead". The frequency is down shifted by an amount proportional to the square of the electric field at the bead position. By moving the "bead" across the accelerating gap, the magnitude of the electric field versus position within the gap can be determined. Thus, R/Q can then be derived.

The transit time factor is needed to correlate to the electric fields "seen" by the beam passing the gap at the speed of light to the static probe measurements. The transit time factor was arrived at by integrating the fields generated by a computer simulation(*BBUS*)[2] across the accelerating gap. Also, the "bead" measures total electric field magnitude and not its direction.

In order to address the computer model sensitivity to how the radial waveguide portion of the cell is terminated, four different choices for the model termination were used. This is because the computer model ends at the top of the radial waveguide and approximates the ferrite core absorption of RF by a factor times the free space wave impedance, with complete absorption at Zs = 1 to complete reflection at Zs = 0 or infinity. On the whole, the changes in the model termination have only a few percent effect on the R/Q determinations from the bead pull data. The data analysis uses the average value of these

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four computer calculations to derive the transverse impedance from the measured frequency shifts.

The goal of this effort is to find a design with R values less than 30 ohm/cm.

R/Q DERIVATION

Starting with the basic definitions of R/Q and the Slater Perturbation Theorem, we will derive the final formula for R/Q in ohm/cm for a measured frequency shift. The basic definition of R/Q for a dipole mode is

$$R/Q_{dipole} = [\int dA_z/dx\, e^{-i\omega t}\, dz]^2 / (2\omega U) \cdot (\omega/c) \quad (\text{ohm/cm}).^{[3]} \qquad \textit{(equation 1)}$$

For comparison, for longitudinal modes, R/Q is $R/Q_{long} = [\int E_z\, e^{-i\omega t}\, dz]^2 / (2\omega U)$ (ohm).

In both cases, the integrals are defined on axis from minus infinity to infinity. U is the stored energy. These integrals can be shown to be independent of the path of integration as long the path is parallel to the axis, within the pipe radius and at the speed of light (t=z/c). Taking the path integral along the pipe wall, these integrals can be truncated to just across the gap, for the relevant fields go to zero at the walls.

The basic principle of the "Slater Perturbation Theorem" is that in a standing wave, the energy is constantly oscillating between all magnetic and all electric, which means that the electric and magnetic field energies are strictly equal. If either of these field energies is modified by a probe, there will be a frequency shift to bring back the balance. The frequency shift is

$$\delta f / f = k \int (\mu H^2 - \varepsilon E^2)\, d\tau / (4\, U), ^{[4]} \qquad \textit{(equation 2)}$$

where the fields are those before the perturbation, the integral is over the probe volume. The factor k is a geometrical factor which relates how the field energy is redistributed around the bead.

Starting with the definition of R/Q, equation 1, and using

$$\int dA_z/dz\, e^{-i\omega t}\, dz \,\Big|_{axis} = (c/\omega b) \int E_z\, e^{-i\omega t}\, dz \,\Big|_{gap} \qquad \textit{(equation 3)}$$

where b is the pipe radius, and writing

$$\int E_z e^{-i\omega t} dz = \frac{\int E_z e^{-i\omega t} dz}{\int E_z\, dz} \; \frac{\int E_z\, dz}{\int |E| dz} \; \frac{\int |E| dz}{\text{gap}}\ \text{gap}$$

or

$$= \text{tf} \; * <Ez/Et> * <|E|> * \text{gap} \qquad \textit{(equation 4)}$$

where we identify the first ratio as the transit time factor, tf, and the second factor as <Ez/Et>, the average direction of the electric field to the axis, and the third as the average magnitude of the electric field over the gap.

Doing the volume integral over the "bead", which is a plastic ring just under the beam pipe radius, in equation 2, we have $\delta f / f = -k\varepsilon E^2 * \upsilon/2 / (4U)$. The factor of one half is for the cosine dependence of the field over the perturbing ring volume. υ is the ring volume.

Putting it all together, we have from equations 1-4 that R/Q is

$$R/Q = 240\,\pi / (k\, \varepsilon/\varepsilon_0) * \text{gap}^2 / (\upsilon/2) * [\, c/\omega b\,]^2 * [\text{tf} * <Ez/Et> * <\text{sqrt}(-\delta f/f)>]^2 \text{ ohm/cm} \qquad \textit{(equation 5)}$$

where <sqrt(-δf/f)> is an experimental quantity and tf and <Ez/Et> are code generated.

BEAD PULL MEASUREMENTS

The k factor is determined by fitting four measured modes in a pillbox cavity to the analytic R/Q values for those modes. The resulting k factor was then used for the other experimental geometries. The R/Q's of the pillbox were matched to an accuracy of (-7%,1%,1%,8%), and a standard deviation of 5%.

The bead pull measurements on the prototype cell were done at a series of anode pipe insertion lengths, ranging from 10.6 cm to 18 cm (see Figure 1). The most thoroughly studied geometry is the 18 cm anode insertion length. All of the results are for the upper geometry with a 4.5 cm beam pipe, 6.0 cm outer coax radius, 2.7 cm radial gap, scalloped guide, outer coax length between taper and roll up of 15.9 cm, a 2 cm roll up, 10 cm taper and the gap center line is 15 cm off the core center line. The lower gap shows the positions for the ferrite-epoxy ring and cone dampers as shaded areas near the taper and coax corner. The cone and ring are similarly mountable on the upper gap.

The average measured R/Q's for the 1 Ghz mode with a 18 cm anode insertion is 1.224 with a range of +15% -17%. The average computer model R/Q value was 1.777 with a range of +17% -20%. The experimental reproducibility has a standard deviation of 13%.. With the addition of the damping ring and cone, only an upper limit on the value of Q was obtained, allowing a prediction based on the average R/Q value of the previous bead pulls. This results R equal to about 22 ohm/cm.

The 1.8 Ghz mode for the same 18 cm anode insertion results in experimental R/Q's of .428 with a range of +20% - 25% versus the computer model of .356 +15% - 13%. With the ring and cone dampers, again only an upper limit on the Q measurement could be done, with a predicted R of 21 ohm/cm.

The Q values were measured for the 18 cm anode insertion for the cases of no ring or cone, ring only, cone only and both ring and cone, respectively, all without applied magnetic field. The Q's start at about 60 and end up somewhere less than 10 and 3, for the main modes. However, the Q's of the two main modes go up to 17 and 48 respectively in the presence of a 5.5 kG magnetic field over the ring and cone. These are the values of Q used above for predicting the resulting R values in the cases where the Q's are too low to do bead pull measurements.

Similarly, the bead pull results for anode insertion lengths of 16 cm, 14 cm, 13 cm, 12 cm, and 10.6 cm are shown in figure 2. These are the R/Q's for mode 1 at about 1 Ghz, the so called "coax mode".

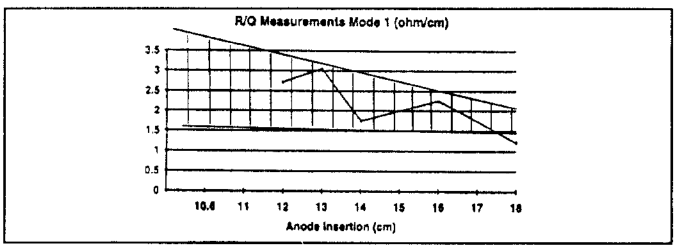

Figure 2----Mode 1 "Coax Mode" at 1.0Ghz

The standard deviation of the differences between the computer model (*BBUS*) and bead pull R/Q's for mode 1 over all anode lengths is 30%. The experimental standard deviation of mode 1 of the 18 cm case over 12 repetitions is 13%. The differences between the model and the bead pull measurement R/Q's are at about the two sigma level relative to the experimental reproducibility over all the anode lengths. The band in figure 2 is the range of computer model predictions. The points are the average bead pull R/Q's for each length.

The bead pull appears to have a stronger length dependence than the code, but the effect is at the margin of uncertainty. This effect may be due to the taper which is not modeled by the *BBUS* code. The bead pull R/Q's are within or just outside the range of values predicted by the different terminations of the radial waveguide in the *BBUS* model.

Figure 3 shows the R/Q's for mode 2 at about 1.8 Ghz, the so called "trapped mode".

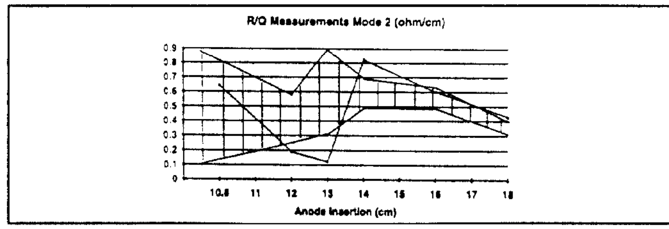

Figure 3----Mode 2 "Trapped Mode" at 1.8 Ghz

The CR-124 ferrite-epoxy, that was used to make the ring and cone, was also subjected to direct electron beam strikes of multiple pulses at 1000 A/cm, 1Mev and 100 ns with only some surface discoloration.

SUMMARY

The R/Q's measured are reasonably close to the computer model in most cases. The R can be reduced to below the design limit of 30 ohm/cm if the mode Q's can be damped to the range of less than about 23 and 70, for the main modes. With the use of the ring and cone dampers, Q's less than 17 and 48, respectively, should be achieved. Thus, the BBU problem for the difficult off-axis shielded gap geometry of the *SLIA* accelerator should be within design tolerances, even for 150 gaps. In particular, the 18 cm anode insertion choice should have transverse impedances of about 22 and 21 ohms/cm, respectively, for the main modes. For comparison, the *ATA* accelerator at Livermore has about a 12 ohm/cm transverse impedance on a 6.7 cm pipe, which roughly scaled to the *SLIA* 4.5 cm pipe would be equivalent to 27 ohm/cm.

ACKNOWLEDGMENTS

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[1]V.K.Neil,L.S.Hall and R. Cooper, "Futher Theoretical Studies of the Beam Breakup Instability, UCRL-81167, May 25,1978.

[2]Tom Genoni of Mission Reasearch in Albuquerque, N.M. developed this code. It is an "analytic" model that approximates the geometry with square corners.

[3]R.J.Briggs,D.L.Birx,G.J.Caporaso and V.K.Neil, Particle Accelerators,1985, Vol. 18, pp. 41-62. (Equ.2.1.1).

[4]Ginzton,"Microwave Measurements", McGraw-Hill, 1957, p. 439, eq.10.25.