nag_tanh (s10aac)

1. Purpose

nag_tanh (s10aac) returns a value for the hyperbolic tangent, $\tanh x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_tanh(double x)
```

3. Description

The function calculates an approximate value for the hyperbolic tangent of its argument, $\tanh x$.

For $|x| \leq 1$ the function is based on a Chebyshev expansion.

For $1 < |x| < E_1$ (where E_1 is a machine-dependent constant),

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

For $|x| \geq E_1$, $\tanh x = \operatorname{sign} x$ to within the representation accuracy of the machine and so this approximation is used.

4. Parameters

x

Input: the argument x of the function.

5. Error Indications and Warnings

None.

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in the argument and the result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{2x}{\sinh 2x} \delta \right|.$$

That is, a relative error in the argument, x, is amplified by a factor approximately $2x/\sinh 2x$ in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.), but if δ is due simply to the round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation round-off.

It should be noted that this factor is always less than or equal to 1.0 and away from $x = 0$ the accuracy will eventually be limited entirely by the ***machine precision***.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.5 p 83.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_tanh(s10aac) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s10aac Example Program Results\n");
  Vprintf("           x       y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s10aac(x);
      Vprintf("%12.1f%12.5f\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s10aac Example Program Data
           -20.0
            -5.0
             0.5
             5.0
```

8.3. Program Results

```
s10aac Example Program Results
           x       y
       -20.0    -1.00000
        -5.0    -0.99991
         0.5     0.46212
         5.0     0.99991
```