nag_log_gamma (s14abc)

1. Purpose

nag_log_gamma (s14abc) returns a value for the logarithm of the Gamma function, $\ln \Gamma(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_log_gamma(double x, NagError *fail)
```

3. Description

This function evaluates $\ln \Gamma(x)$, $x > 0$. It is based on Chebyshev expansions.

If x is too large there is a danger of setting overflow so the function evaluation must fail.

4. Parameters

x

Input: the argument x of the function.
Constraint: $\mathbf{x} > 0.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to 0.0: $\mathbf{x} = \langle value \rangle$.
For zero and negative values the function is undefined, the function returns zero.

NE_REAL_ARG_GT

On entry, **x** must not be greater than $\langle value \rangle$: $\mathbf{x} = \langle value \rangle$.
x is too large, the function would overflow, the function returns the value of the function at the largest permissible argument.

6. Further Comments

6.1. Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively, and E be the absolute error in the result. If δ is somewhat larger than the relative ***machine precision***, then $E \simeq |x \times \psi(x)|\,\delta$ and $\epsilon \simeq |(x \times \psi(x))/(\ln \Gamma(x))|\,\delta$ where $\psi(x)$ is the digamma function $\Gamma'(x)/\Gamma(x)$.

Except near $x = 1$ or 2 relative error is attenuated by the function or at least is not greatly amplified.

For large x, $\epsilon \simeq (1 + 1/\ln x)\,\delta$ and for small x, $\epsilon \simeq \delta/\ln x$.

The function $\ln \Gamma(x)$ has zeros at $x = 1$ and 2 and hence relative accuracy is not maintainable near those points. However, absolute accuracy can still be provided near those zeros.

If, however, δ is of the order of the ***machine precision***, then rounding errors in the function's internal arithmetic may result in errors which are slightly larger than those predicted by the equalities. It should be noted that even in areas where strong attenuation of errors is predicted the relative precision is bounded by the effective ***machine precision***.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 6 p 255.

7. See Also

nag_gamma (s14aac)
nag_incomplete_gamma (s14bac)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_log_gamma(s14abc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s14abc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s14abc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s14abc Example Program Data
             1.0
             1.25
             1.5
             1.75
             2.0
             5.0
             10.0
             20.0
             1000.0
```

8.3. Program Results

```
s14abc Example Program Results
     x           y
   1.000e+00   0.000e+00
   1.250e+00  -9.827e-02
   1.500e+00  -1.208e-01
   1.750e+00  -8.440e-02
   2.000e+00   0.000e+00
   5.000e+00   3.178e+00
   1.000e+01   1.280e+01
   2.000e+01   3.934e+01
   1.000e+03   5.905e+03
```