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Three target method for the lifetime measurement

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The extrapolation method of the $A_{2\pi}$ lifetime measurement which suggested in the DIRAC proposal has two week points:

1. To separate "Coulomb" and "non-Coulomb" pairs we simulate they in very rough approach. For "Coulomb" pairs we assume the Coulomb enhancement (weight) such as for the point-like $\pi^+\pi^-$ source. Still there is no formulas accounting dimensions and shape of the $\pi^+\pi^-$ source. In any case this formulas will include some model assumption about the $\pi^+\pi^-$ source.

2. To calculate the number of produced $A_{2\pi}$ we use the formula which connect it with the number of "Coulomb" pairs with the relative momentum $q < 2$ MeV/c. But accuracy of this formula is still unknown.

Let us consider another method which do not include these points. The main idea based on using of three different targets and formulas of this method are the same as for the subtraction method described in Appendix C of the DIRAC proposal.

Consider the total number of $\pi^+\pi^-$ pairs with the relative momentum $q < 2$ MeV/c obtained for the same number of prime proton-nucleus interactions (or the same flux of secondary particles) in measurements with three targets consisted of different materials and having equal thicknesses in radiation length units: N_1 for the Platinum target (the highest available Z), N_2 for target with an intermediate Z (Ti, Ni or Mo) and N_3 for the Berillium target. These pair numbers can be represented as

$$\begin{aligned} N_1 &= n_{A\,1} + N_{f\,1} \\ N_2 &= n_{A\,2} + N_{f\,2} \\ N_3 &= n_{A\,3} + N_{f\,3}. \end{aligned} \tag{1}$$

Here $n_{A\,1}$, $n_{A\,2}$ and $n_{A\,3}$ are the numbers of the atomic pairs and $N_{f\,1}$, $N_{f\,2}$ and $N_{f\,3}$ are the numbers of free pairs. The number of the atomic pairs is determined by the probabilities of $A_{2\pi}$ breakup which are different for the three targets (see Figure 1).

Consider the numbers of free pairs $N_{f\,1}$, $N_{f\,2}$ and $N_{f\,3}$. Pairs of $\pi^+\pi^-$-mesons are produced mainly in the interaction of an incident proton with a single nucleon of a target nucleus. So $N_{f\,1}$, $N_{f\,2}$ and $N_{f\,3}$ should be the same, taking into account also the same multiple scattering in these targets. Moreover the range of relative momentum $q < 2$ MeV/c is too small to include any difference in the pion pair production spectrum on different nuclei. So the only one assumption of this method is that the numbers of the pion pairs in the range of $q < 2$ MeV/c coming from any source, except the atom breakup, are the same:

$$N_{f\,1} = N_{f\,2} = N_{f\,3}\,. \tag{2}$$

In order to have the same efficiency of a pair registration, which could depend on the flux of secondary particles, the prime beam intensity in measurement with Berillium target should

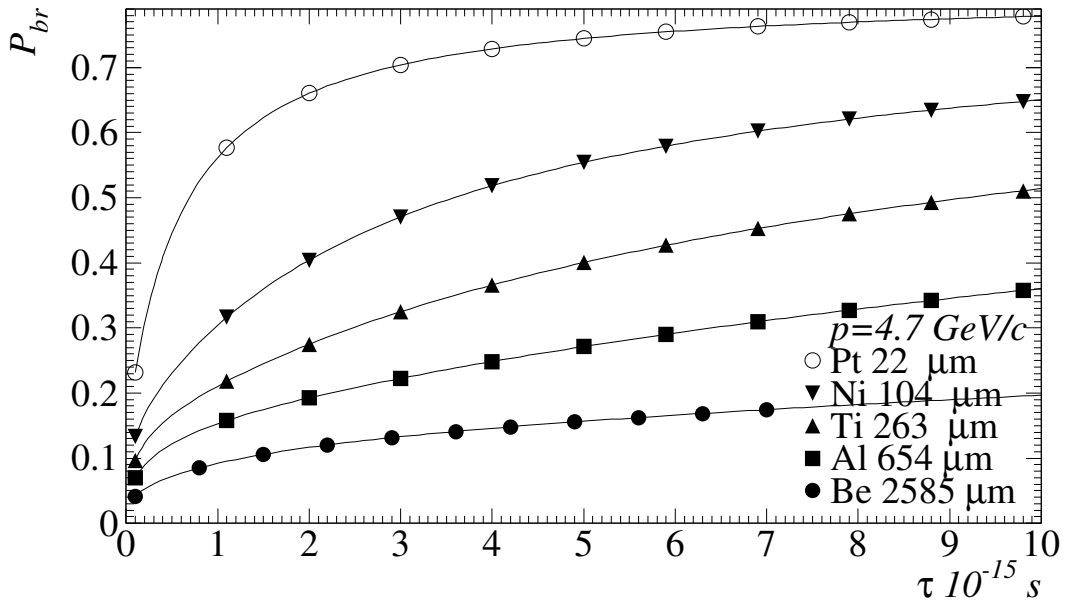

Figure 1: Probability of the $A_{2\pi}$ breakup as a function of the atom lifetime for different materials of targets having equal thicknesses in radiation length units.

be decrease in comparison with the Platinum target one to compensate a higher efficiency of Berillium for proton-nucleus interactions.

Let us consider the value

$$x \;=\; \frac{N_2 - N_3}{N_1 - N_3}. \tag{3}$$

Taking into account (1) and (2) we obtain

$$x \;=\; \frac{n_{A\,2} - n_{A\,3}}{n_{A\,1} - n_{A\,3}}\,. \tag{4}$$

The x value depends on the atomic pair numbers only and can be expressed via the probabilities of $A_{2\pi}$ breakup for the correspondent targets:

$$x \;=\; \frac{P_{\mathrm{br}\,2} - P_{\mathrm{br}\,3}}{P_{\mathrm{br}\,1} - P_{\mathrm{br}\,3}}\,. \tag{5}$$

The calculations have shown that x is a simple function of the $A_{2\pi}$ lifetime ($x = f(\tau)$) for fixed pair momentum (see Figure 2). Therefore the measurement of x value allows to determine the dimesoatom lifetime.

The expression for the standard deviation for x is

$$\sigma_x \;=\; \frac{1}{N_1 - N_3}\,\sqrt{\sigma_{N_2}^2 + x^2\sigma_{N_1}^2 + (1-x)^2\sigma_{N_3}^2}\,. \tag{6}$$

The correspondent error in the lifetime is

$$\sigma_\tau \;=\; \sigma_x/(dx/d\tau)\,. \tag{7}$$

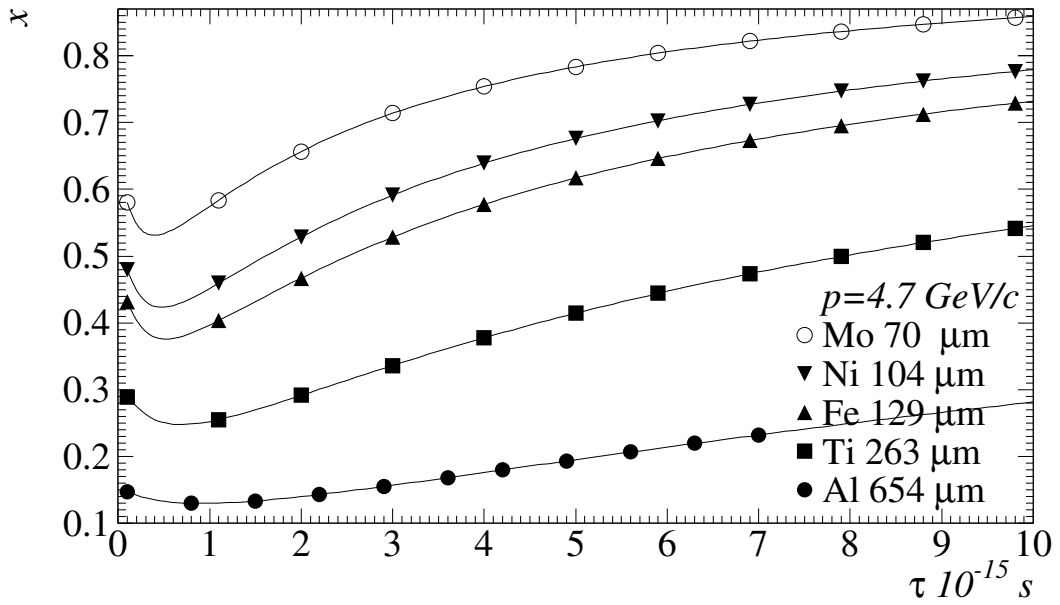

Figure 2: Values of x as a function of the atom lifetime for different materials of the second targets.

Using the procedure of the atom production simulation described in the proposal one can estimate the number of atoms to be produced in the three targets to obtain the $A_{2\pi}$ lifetime within required accuracy. In Figure 3 the number of $A_{2\pi}$, to be produced in all three targets to obtain the $A_{2\pi}$ lifetime within 10%, is shown as a function of the atom lifetime for the different materials of the second target. The ratio of accidental to real coincidences used in the simulation was equal to 2.0.

Some illustrative and final numbers are given in Table 1. Calculations were performed for the $A_{2\pi}$ lifetime $3.0 \cdot 10^{-15}$ s and momentum 4.7 GeV$/c$. The thicknesses of the Platinum and Berillium targets were 22.3 μm and 2585 μm corespontently, and the probabilities of atoms breakup in these targets are 0.704 and 0.133. For different materials of the second target with nucleus charge Z there are presented the probabilities of $A_{2\pi}$ breakup $P_{\mathrm{br}\,2}$; the value of x; the relative accuracy δx in measurement of x and the number of produced atoms in all three targets N_A, required to obtain the lifetime within 10%. The ratio $w_1 : w_2 : w_3$ between the numbers of produced $A_{2\pi}$ in each target provides the smallest value of N_A.

From Figure 3 one can conclude that the Fe target is the most suitable for the $A_{2\pi}$ lifetime measurement with the three target method for the range of the lifetime from 2.5 to $4.9 \cdot 10^{-15}$ s. But Iron has enough complicated dependence of destiny on temperature. So the Ni or Cu target could be used instead.

Foils from different materials for the three target method should have the value of the multiple scattering as close as possible in order to exclude big corrections based simulation of the multiple scattering.

The number of atoms to be produced in the Ni target will be enough to measure the lifetime with the extrapolation method also.

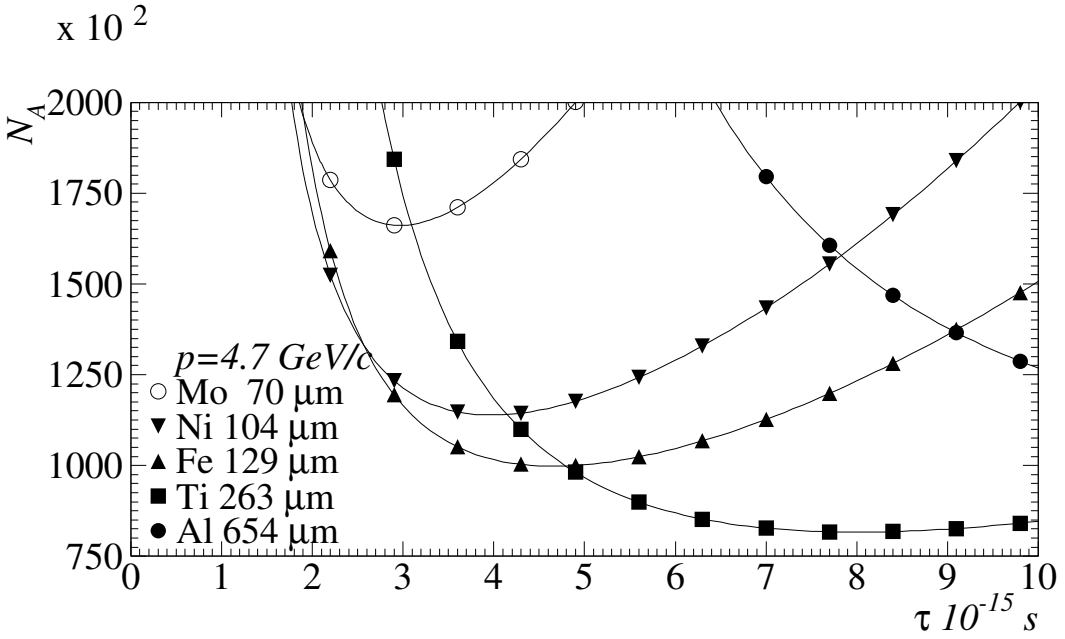

Figure 3: The number of $A_{2\pi}$ to be produced in the three target method to obtain the $A_{2\pi}$ lifetime within 10% as a function of atom lifetime for the different materials of the second target.

Table 1:

	Z	$P_{\mathrm{br}\,2}$	x	δx	$w_1 : w_2 : w_3$	N_A
Al	13	0.2227	0.1570	$3.5 \cdot 10^{-2}$	0.090 : 0.501 : 0.409	$9.1 \cdot 10^5$
Ti	22	0.3250	0.3362	$3.9 \cdot 10^{-2}$	0.186 : 0.501 : 0.313	$1.7 \cdot 10^5$
V	23	0.3720	0.4185	$3.6 \cdot 10^{-2}$	0.229 : 0.501 : 0.270	$1.3 \cdot 10^5$
Fe	26	0.4349	0.5285	$3.1 \cdot 10^{-2}$	0.284 : 0.501 : 0.215	$1.2 \cdot 10^5$
Ni	28	0.4707	0.5913	$2.8 \cdot 10^{-2}$	0.314 : 0.501 : 0.185	$1.2 \cdot 10^5$
Cu	29	0.4696	0.5894	$2.8 \cdot 10^{-2}$	0.313 : 0.501 : 0.186	$1.2 \cdot 10^5$
Mo	42	0.5407	0.7138	$2.0 \cdot 10^{-2}$	0.372 : 0.501 : 0.127	$1.7 \cdot 10^5$
Ta	73	0.6715	0.9429	$4.2 \cdot 10^{-3}$	0.475 : 0.500 : 0.025	$2.3 \cdot 10^6$
Re	75	0.6989	0.9909	$5.9 \cdot 10^{-4}$	0.496 : 0.500 : 0.004	$1.1 \cdot 10^8$