A Nonlinearity in Q_s

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- Synchrotron oscillation revisited
  - ➜ linear $\rightarrow$ nonlinear regime
- Why this could be an issue for Y2K data
  - ➜ Impact on energy determination with Q_s
  - ➜ Numerical check with Monte Carlo

Longitudinal Oscillations

- RF voltage: $V_{RF}(\psi) = V_0 \sin\psi$

- Stable phase: $\sin\psi_s = U_0/e$

- Small amplitude oscillation around ψ_s:

$$\ddot{\psi} + \Omega^2 \psi = 0$$

with

$$\Omega^2 = \omega_{rev}^2 \left(\frac{\alpha_c h}{2\pi E_0}\right) e \frac{dV}{d\psi}(\psi_s)$$

- Synchrotron tune Q_s:

$$Q_s^2 = \left(\frac{\Omega}{\omega_{rev}}\right)^2 = \left(\frac{\alpha_c h}{2\pi E_0}\right) e V_0 \cos\psi_s$$

$$= \left(\frac{\alpha_c h}{2\pi E_0}\right) \sqrt{e^2 V_0^2 - U_0^2}$$

Q_s at Large Amplitudes

- Driving force: $Q_s^2 \;\propto\; \cos\psi_s \;=\; F$ (tangent!)

  ➜ for large amplitudes a better approximation of the driving force is needed....

- Change in driving force $\Delta F \propto F'' = -\cos\psi_s$

- Example:

$$\langle F \rangle = \frac{1}{2}\,[\cos(\psi_s - \Delta\psi) + \cos(\psi_s + \Delta\psi)]$$

and

$$\Delta F = \frac{1}{2}\,[\cos(\psi_s + \Delta\psi) - 2\cos\psi_s + \cos(\psi_s - \Delta\psi)]$$
$$\propto \cos''\psi_s = -\cos\psi_s$$

$$\triangleright \quad \Delta Q_s^2 \propto -Q_s^2$$

- Frequency shift due to nonlinearity:

$$\boxed{\Delta Q_s \propto -Q_s}$$

Impact on Energy Determination

- In 2000 the longitudinal oscillation was excited for the Q_s MDs
(no excitation in 1998 and 1999!)

- A frequency shift $\propto -Q_s$ changes the the voltage calibration factor g in the calibration datasets (40 - 60 GeV) if not taken into account

- If analysis assumes a g constant in energy
➜ determined energy of HE dataset too low!

⊳ Check with a Monte Carlo

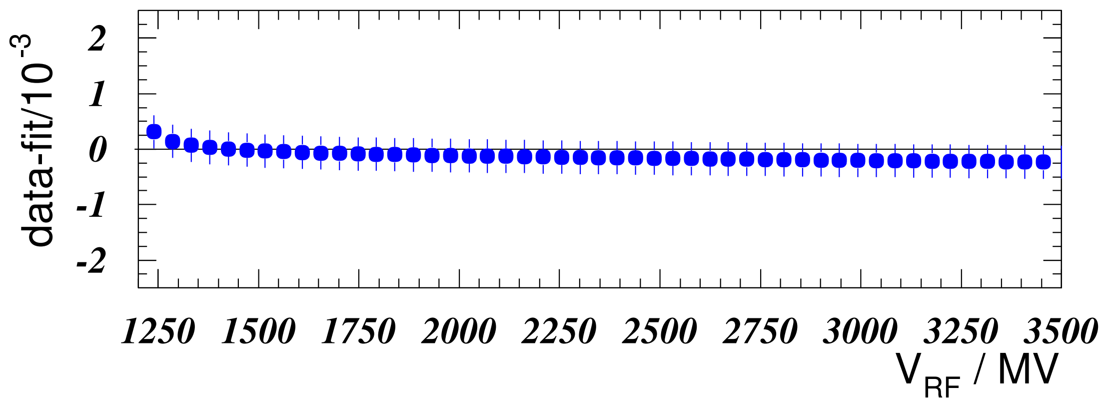
Test with Monte Carlo

With $Q_s = Q_s + f_x Q_S$ and $f_x = -\,5\ 10^{-3}$

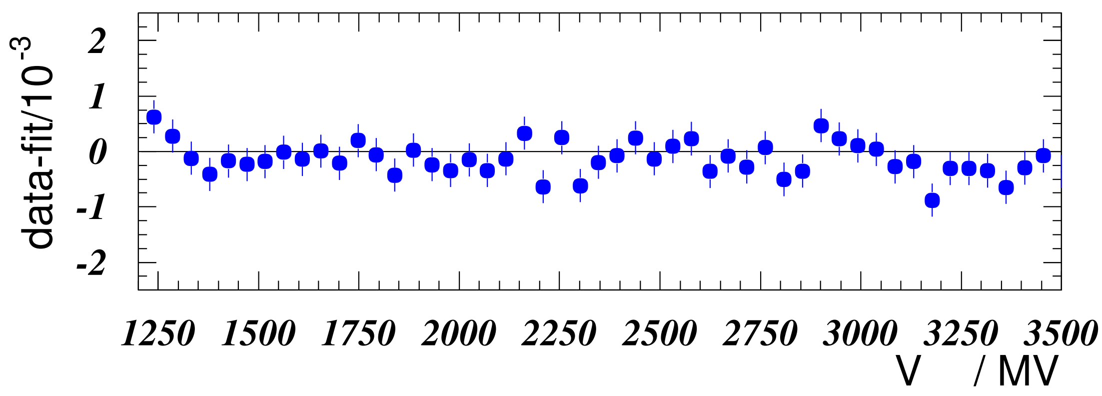
- g seems to have slope of 10^{-4} / GeV
- energy too low by $\approx$ 170 MeV at 80 GeV

- Difference between "data" and fit:

MC with shift

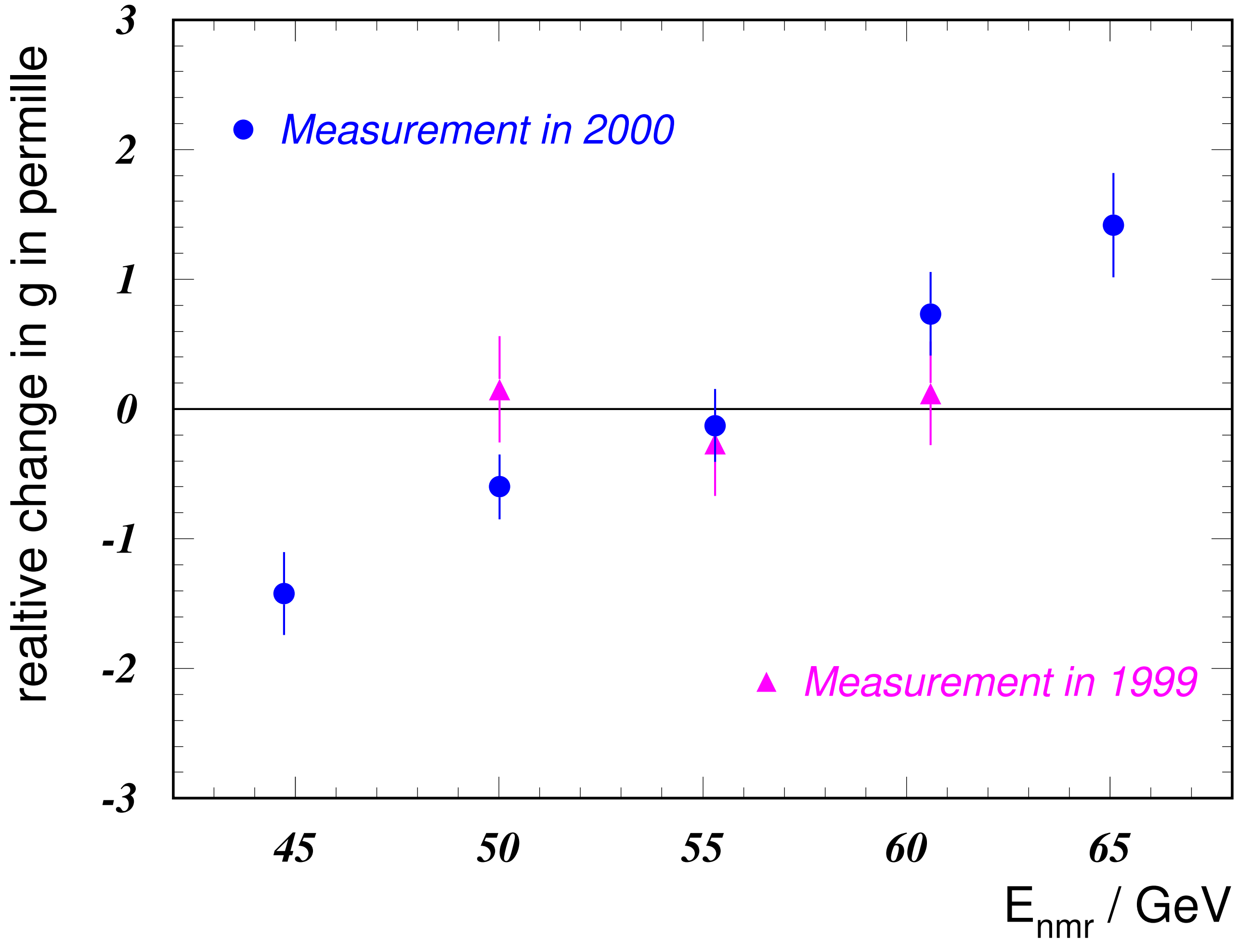


MC with shift and noise



Observation

- Measured voltage calibration factor:



Q_s and RF Voltage

- Synchrotron tune Q_s depends on
  - total RF voltage
  - beam energy
- Measurements of Q_s as function of V_{RF}: