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Determination of the LEP Energy Spread Using Experimental Constraints

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Abstract

The luminosity weighted Center-of-Mass energy spread $\sigma_{E_{CM}}$ is determined for the 1993, 1994 and 1995 running periods of LEP. Two methods to evaluate the beam energy spread $\sigma_{E_{beam}}$ are compared. The direct calculation requires knowledge of the magnetic configuration of LEP and of the damping partition number. Alternatively, $\sigma_{E_{beam}}$ can be extracted from the bunch length as measured in the LEP experiments, in ALEPH for this note, if one knows the incoherent synchrotron oscillation tune $Q_s^{inc.}$. In 1995, the value of $Q_s^{inc.}$ was compared with the coherent one, $Q_s^{coh.}$, which is measured in the LEP RF system, by measuring the Q_s side-bands of the depolarizing resonance. The two evaluations of $\sigma_{E_{beam}}$ disagree by about 2%, which is taken as systematic error. The bunch-train related dispersion effects are then taken into account for each experiment to evaluate $\sigma_{E_{CM}}$ for 1995. The final precision is 2.5%, four times better than the previous evaluations.

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Contents

1 Introduction

An accurate determination of Γ_Z and the hadronic cross section requires a precise determination of the LEP center-of-mass energy spread, $\sigma_{E_{CM}}$. The bias on the measured cross section due to $\sigma_{E_{CM}}$, depends upon its second derivative with respect to the energy :

$$\delta\sigma = -0.5 \ \times \ \frac{d^2\sigma}{dE^2} \ \times \ \sigma^2_{E_{CM}} \tag{1}$$

where $\delta\sigma$ is the induced change in the cross section, which also leads to a change in the measured value of Γ_Z. A spread of 55 MeV requires a correction to the measured Z width of about -4 MeV [1]. This correction is essentially the same for all LEP experiments and the corresponding error fully correlated. Because the line shape scan is symmetric, the effect on M_Z is negligible.

For the 93 scan, the best estimate of the nominal center-of-mass energy spread (about 55 MeV) was given with a 10% uncertainty [2, 3]. The resulting error of 1.0 MeV on Γ_Z [1], was the single largest systematic error on Γ_Z. This estimate was obtained following the original idea of Bob Jacobsen [4] by a measurement of the bunch length at IP 4 [2] and 6 [3]. Relating the bunch length to energy spread requires knowledge of the incoherent synchrotron tune $Q_s^{inc.}$, while the measurement by the RF system gives the coherent one, $Q_s^{coh.}$. Uncertainty in the relation between $Q_s^{inc.}$ and $Q_s^{coh.}$ is the main source of systematic error.

The study presented here follows the same line and the result is extended to 1994 and the 1995 scan. The precision is improved by a detailed comparison of the calculation of the beam energy spread $\sigma_{E_{beam}}$ from the magnetic configuration of LEP with the value obtained from the bunch length measurement. Furthermore, the relation between $Q_s^{inc.}$ and $Q_s^{coh.}$ could be measured from the position of the Q_s side-band of the depolarizing resonance during one of the LEP energy calibrations with polarisation.

This paper is organised as follows. In section 2, the two methods are described from the theoretical point of view, with emphasis on the important parameters. In section 3 the way these parameters are obtained and their variation with time and among themselves are described. In section 3.5 the measurement of $Q_s^{inc.}$ from the Q_s side-band of the depolarizing resonance is reported. In section 4, the measurements of the bunch length are compared with the theoretical predictions as a function of several LEP parameters, and the observed disagreement is discussed. Finally, in section 5, the dispersion effects due to the bunch-train operation in 1995 are accounted for and the center-of-mass energy spread is given for each energy point and for each of the years 1993, 1994 and 1995.

2 Energy spread calculations

The RMS beam energy spread $\sigma_{E_{beam}}$ can be calculated for electrons in equilibrium between the radiation losses in the ring and the energy gain in the cavities by the well known formula [5]:

$$
\begin{aligned}
\sigma^2_{E_{beam}} &= \frac{55}{32\sqrt{3}}\frac{\hbar}{m_e c}\left(\frac{E_0}{m_e c^2}\right)^2\left(\frac{1}{J_e}\right)\frac{I_3}{I_2} && (2)\\
\text{where } I_2 &= \int_0^{2\pi R} G(s)^2\, ds, && (3)\\
I_3 &= \int_0^{2\pi R} |G(s)|^3\, ds, && (4)\\
G(s) &= \frac{eB_y(s)}{p_0 c} && (5)
\end{aligned}
$$

Here $B_y(s)$ is the local vertical bending magnetic field, and $m_e c^2, p_0 c, E_0$, are the electron mass, momentum and energy.

The damping partition number J_e relevant for energy spread is related to that relevant for the horizontal emittance J_x, as $J_e = 3 - J_x$. They can depart from the nominal values $J_x = 1$, $J_e = 2$ if the beam is not on the central orbit. As the running of 1993 to 1995 was always performed on central frequency, these variations are due to changes in LEP circumference, as due to tides or slower ground motion. In addition, the Pretzel mode of operation implies excursions around the central orbit and a slight modification of J_x.

The largest source of variation in the beam energy spread comes from the excitation of the emittance wigglers. Their purpose is to increase the horizontal beam size when the beam-beam forces would otherwise blow-up the beams in the vertical plane. Since the field is much more intense in the wigglers than in the rest of the machine, their contribution to I_3 and I_2 are very different, leading to a large change in $\sigma_{E_{beam}}$. These calculations were coded and parametrised by John Jowett [6].

In order to evaluate $\sigma_{E_{beam}}$, on therefore needs to know

1. the field integrals I_2 and I_3 for the ring and for the wigglers, as a function of their excitation, which varies widely.
2. the damping partition number, and therefore the actual LEP circumference as a function of time. This can be extracted from measurements of the central frequency, and monitored using the observed position of the beam in the arcs, $< x_{\rm arcs} >$, available as from 1994.

There are sizeable uncertainties in all respects in these parameters. In particular, since they vary widely, their evaluation will be very sensitive to logging imperfections.

Another method to evaluate the beam energy spread $\sigma_{E_{beam}}$ is to relate it to the RMS longitudinal size of the interaction region in the experiments, which has the advantage of being continuously measured from the distribution of event vertices and automatically luminosity-weighted. The relation between $\sigma_{E_{beam}}$ and σ_z is the following [7]:

$$\sigma_{E_{beam}} = \frac{\sqrt{2}E_{beam}}{\alpha_c R_{\rm LEP}} \times Q_s \times \sigma_z \tag{6}$$

where : E_{beam} is the beam energy, α_c is the LEP momentum compaction factor ($\frac{\Delta E_{beam}}{E_{beam}} = \frac{\Delta L}{\alpha_c L}$, where L is the orbit length), $R_{\rm LEP}$ is the LEP bending radius, Q_s is the synchrotron tune ($Q_s = \frac{\Omega R_{\rm LEP}}{c}$, Ω being the angular frequency of the energy oscillations). The factor $\sqrt{2}$ relates the longitudinal size of the interaction region to that of the individual bunches.

The main uncertainty in this determination of $\sigma_{E_{beam}}$ comes from the definition of Q_s. Equation 6 is written in terms of the incoherent $Q_s^{inc.}$, e.g. that of individual particles, while the quantity that is regularly measured by the RF system, and available on logging files, is the response of the bunches as whole, namely the coherent one, $Q_s^{coh.}$.

The relation between $Q_s^{coh.}$ and $Q_s^{inc.}$, parametrised as

$$\frac{Q_s^{coh.}}{Q_s^{inc.}} = 1 - \kappa \frac{I}{I_0} \quad , \quad (I_0 \; = \; 300 \; \mu \; A) \tag{7}$$

(where I is the bunch current), was the major source of uncertainty in the method, until it was calibrated using the Q_s-sidebands of the depolarization resonance in 1995. This value of κ will be used to compare the results obtained by this method to the model [6], and assess systematic errors.

The center-of-mass energy spread $\sigma_{E_{CM}}$ is related to the beam energy spread $\sigma_{E_{beam}}$ by the simple formula:

$$\sigma_{E_{CM}} = \sqrt{2} \times \sigma_{E_{beam}} \times C \; , \; (0 \leq C \leq 1) \tag{8}$$

For colliding beams having no or identical energy dispersion (as in 1993 and 1994) the *monochromatization* factor C is approximately equal to 1 [8].

This is not the case for data taken in 1995 with bunch train operation mode, beams have opposite vertical energy dispersion leading to a more complicated relation between $\sigma_{E_{beam}}$ and $\sigma_{E_{CM}}$ [15]. This relation can be simplified to :

$$\sigma^2_{E_{CM}} = 2 \; \sigma^2_{E_{beam}} \left[1 + \left(\frac{D_y \sigma_\epsilon}{2\sigma_y} \right)^2 \right]^{-1} \tag{9}$$

where D_y is the vertical beam dispersion and σ_y the vertical beam size. σ_ϵ is the relative beam energy spread.

3 LEP parameters

3.1 Damping partition number

The prediction of equation 2 depends on the "damping partition number" J_e. In the flat machine with pretzel off and on central orbit, $J_e = 2$. J_e is related to the damping time constant of energy oscillations ($1/\alpha_e$) by the formula $\alpha_e = J_e\ \alpha_0$, where $\alpha_0 = \frac{P_\gamma}{2E_b}$, P_γ being the average radiated power. The damping of energy oscillations is related to the damping of betatron oscillations through the Robinson theorem [7] $J_e + J_x + J_y = 4$. When the beams are off central orbit, $J_e = 2 + \epsilon$, $J_y = 1$, $J_x = 1 - \epsilon$, $\epsilon \ll 1$ depends on the property of the guide field, such as those related to changes in the nominal orbit length, which happen in the horizontal plane as follows:

1. short time variations due to e.g. tidal effects,
2. longer term variations due to ground motion,
3. design variations such as that induced by the Pretzel bumps.

The first two effects are visible in the horizontal orbit [9] measurements $< x_{\rm arcs} >$. The tidal effect averages out to zero to a good precision as data taking samples the tides. It contributes nevertheless to fluctuations of up to $\pm 3.5\%$ to J_e, leading to $\pm 1.7\%$ variations on $\sigma_{E_{beam}}$. The second effect occurs due to e.g. the level of lake Geneva, water level in the Jura, etc... This was studied using the measurement of $< x_{\rm arcs} >$. Figure 1 shows the variation of J_e with time for the 1995 data; variations of more than 5% over that period are observed. These variations also influence as expected the beam energy. For each fill, $\sigma_{E_{beam}}$ has been corrected to take into account the variation of J_e with time [10]. With pretzels on, J_e may vary between 1.98 and 2.07 [6].

The mean value of J_e (averaged over a fill period) is 1.98 for '93, 2.00 for '94 and 1.97 for '95 with typical RMS of 0.02.

3.2 Wiggler current, beam currents and $Q_s^{coh.}$

Since these parameters are susceptible to very rapid changes, they are stored every 4 minutes on the LEP logging data base. An example for a limited period of time in 1994 is given in figure figure 2.

As expected beam and wiggler currents are highly correlated, since wigglers are used at operator's discretion to increase the horizontal beam size thus reducing vertical beam-beam effects when running at large current [11]. Not so obviously anticipated is the fact that the recorded value of $Q_s^{coh.}$ increases through the fills, being apparently linked to the current in the wigglers, or perhaps to beam current. This is also well seen when averages of these quantities over the full data sample are plotted against each other, figure figure 3. There is a slight variation of coherent Q_s with wiggler current (15.10^{-4}, figure 3 top) and a smaller variation of Q_s with bunch current (7.10^{-4}, figure 3 middle).

In order to carefully check the variation of $\sigma_{E_{beam}}$ with respect these param-

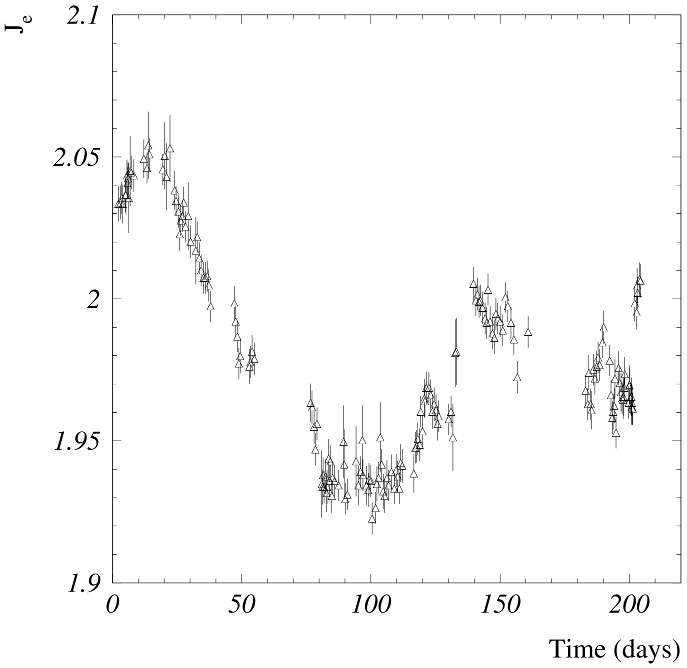

Figure 1: *J_e variation with time in 1995.*

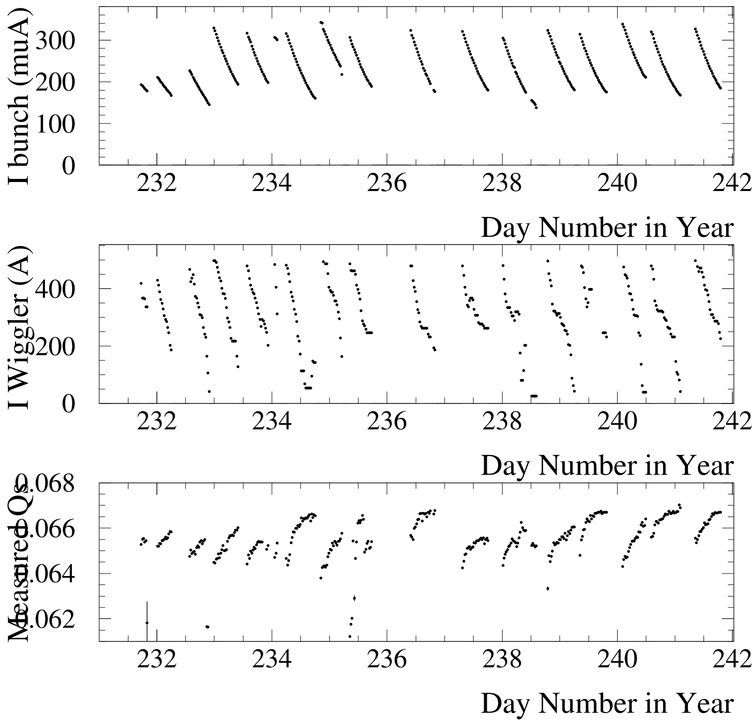

Figure 2: *94 Data : Bunch current (**top**), wiggler current (**middle**) and Q_s (**bottom**) variation with day number over a limited time period.*

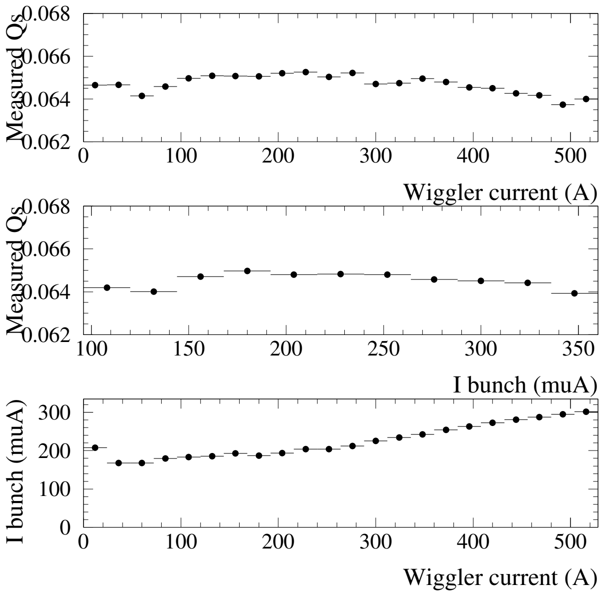

Figure 3: *94 Data :* **Top :** *Q_s variation with wiggler current,* **middle :** *Q_s variation with Bunch current,* **bottom :** *Bunch current variation with wiggler current.*

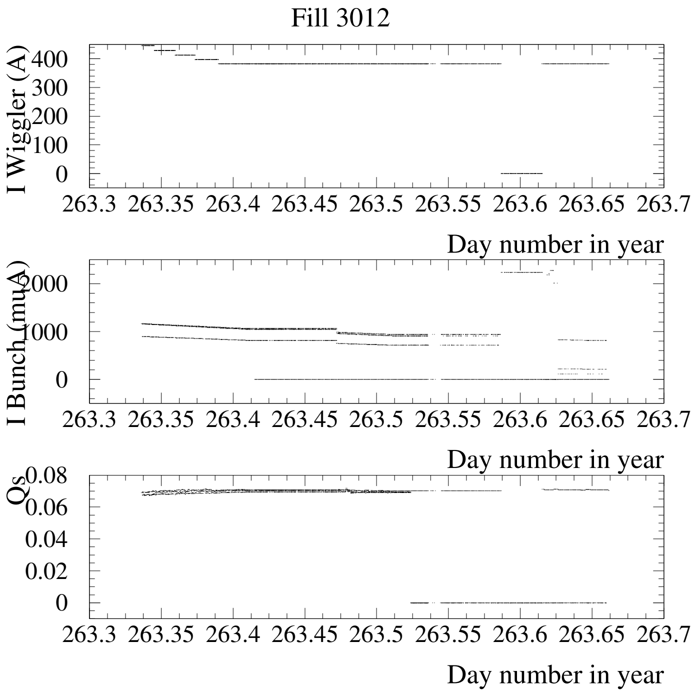

Figure 4: *Example of 'typical' recording of* LEP *parameters problem in 1995 for fill 3012:* **Top :** *Variation of wiggler current with time,* **Middle :** *Variation of bunch current with time,* **Bottom :** *Variation of $Q_s^{coh.}$ with time.*

eters, wrong or spurious records need to be eliminated. This is done by discarding events with unassociated $I_{wiggler}$, I_{bunch} or $Q_s^{coh.}$. Furthermore for '95 data, a delicate scanning of the recording and correction of those quantities have been done by the two last authors of [12]. An example of recording accident is shown in figure 4 within fill 3012. For some time intervals, the wiggler currents are not properly recorded (0 value), the same applies for some of the bunch current (which can even increase by step) and $Q_s^{coh.}$. Data corresponding to incorrect recording have been removed from the analysis as much as possible.

3.3 Longitudinal events vertex distribution

All data passing the ALEPH electro-weak selection criteria for 1993 (fills 1647 to 1937), 1994 (fills 2057 to 2435) and 1995 (fills 2859 to 3042) have been analysed.

For each hadronic event, the z vertex position is computed from the mean z_0 of *good* tracks [1)], in average 33 tracks are used to determine the event vertex.

ALEPH data are grouped in runs of typical time duration of 63, 38 and 93 minutes in 1993, 1994 and 1995 respectively. The corresponding average number of Z^0 per run was 550, 700 and 965. In order to merge data from several runs, the variation of the mean z vertex position from run to run, leading to an artificial increase of σ_z, has to be taken into account. This effect can be seen on Figure 5 where the mean z as a function of the run number is plotted. Within a fill variations up to 1.0 cm (in 95) are observed. Large variations of z within a run are related to variations of the RF conditions.

In the following, the average z position of the vertex within a run is subtracted from the individual event z vertex position. This reduces the RMS of the distribution. Figure 6 shows the variation of σ_z with run number for 94 data. Within a fill, σ_z decreases with time, the variation can be as large as 0.3 cm.

Figures 7, 8 and 9 show the distribution of σ_z, Q_s, wiggler and bunch current for 1993 (peak), 1994 and 1995 (peak) data respectively.

3.4 Variations of σ_z with wiggler current and Q_s

Figure 10 shows the variation of σ_z plotted with respect to $Q_s^{coh.}$ (left) and wiggler current (right). Their behaviours is consistent with the expectations [11], $1/Q_s$ and $a + bI^2_{wiggler}$ for variations of σ_z with Q_s and $I_{wiggler}$ respectively. σ_z does not perfectly vary like $1/Q_s$. In other words, $\sigma_z \times Q_s$ (e.g. $\sigma_{E_{beam}}$) is not constant; as will be shown, some scatter remains.

3.5 Experimental determination of $Q_s^{inc.}$

A determination of $Q_s^{inc.}$ was obtained by accurate determination of the Q_s side band of the depolarizing resonance during an energy calibration. The experiment was carried out on September 2 1995.

[1)] Selected tracks must have $|d_0| \leq 2cm$, $|z_0| \leq 5cm$, more than 4 TPC points and $|cos\theta| \leq 0.95$

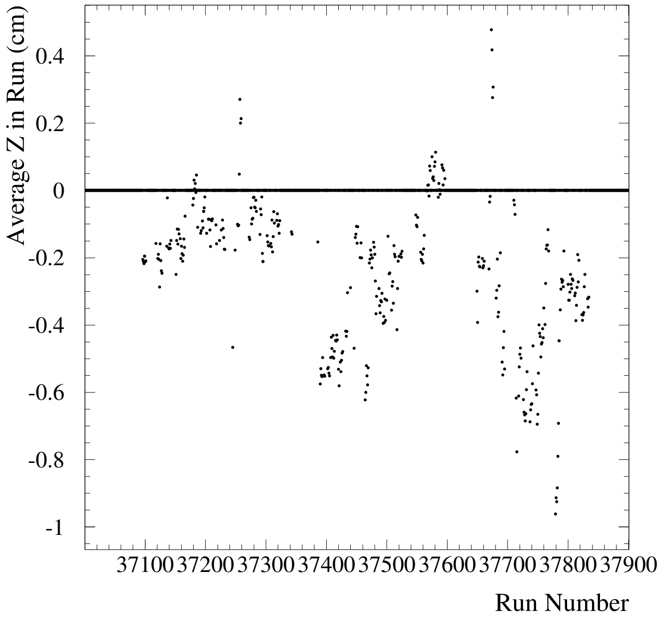

Figure 5: *'95 Data : average z event vertex versus run number,*

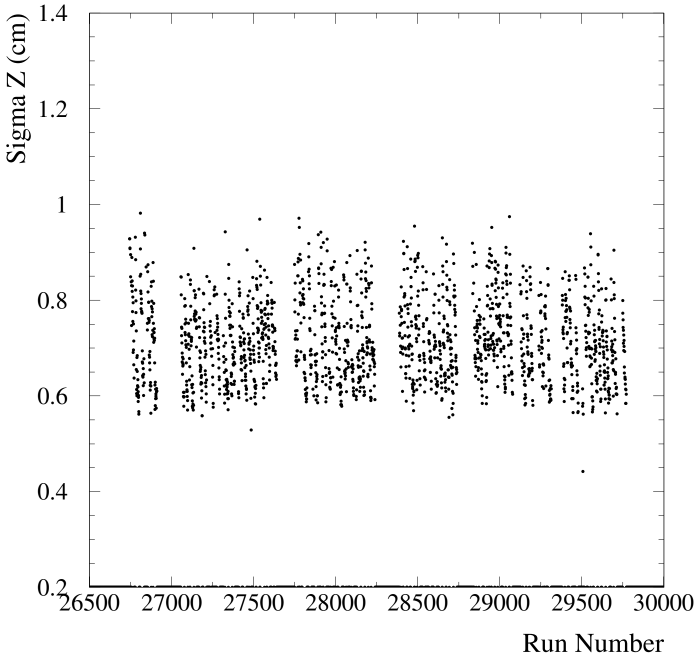

Figure 6: *'94 Data : σ_z variation with run number.*

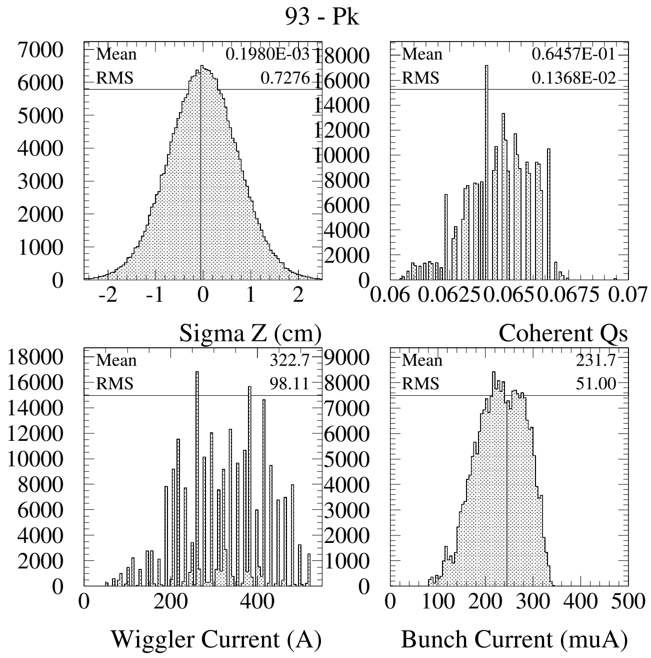

Figure 7: σ_z, $Q_s^{coh.}$, *Wiggler current and Bunch current distributions for 1993 peak data.*

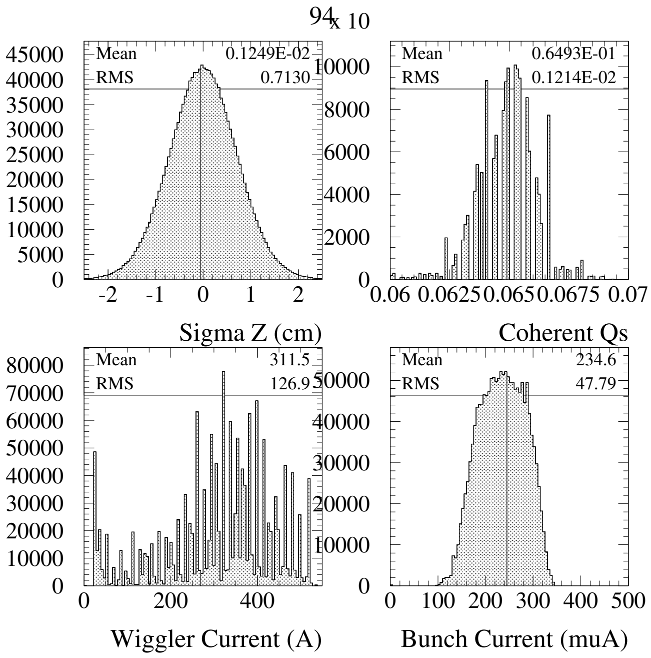

Figure 8: σ_z, $Q_s^{coh.}$, *Wiggler current and Bunch current distributions for 1994 data.*

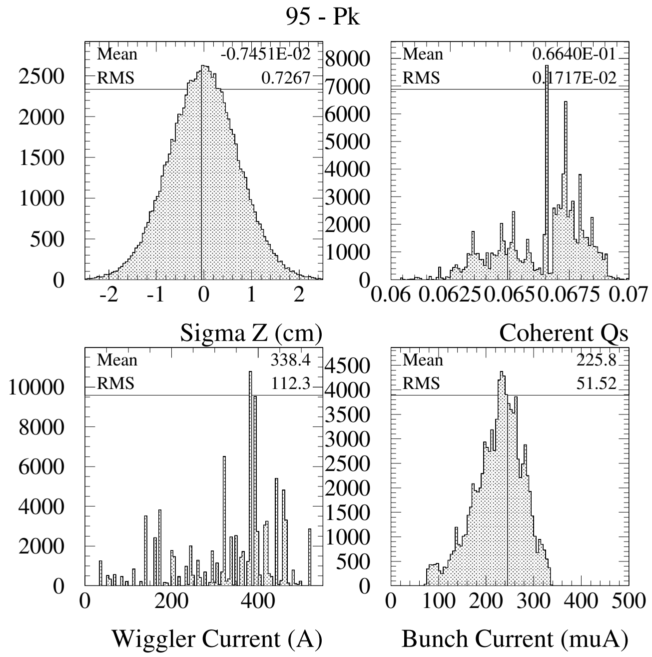

Figure 9: σ_z, $Q_s^{coh.}$, *Wiggler current and Bunch current distributions for 1995 peak data.*

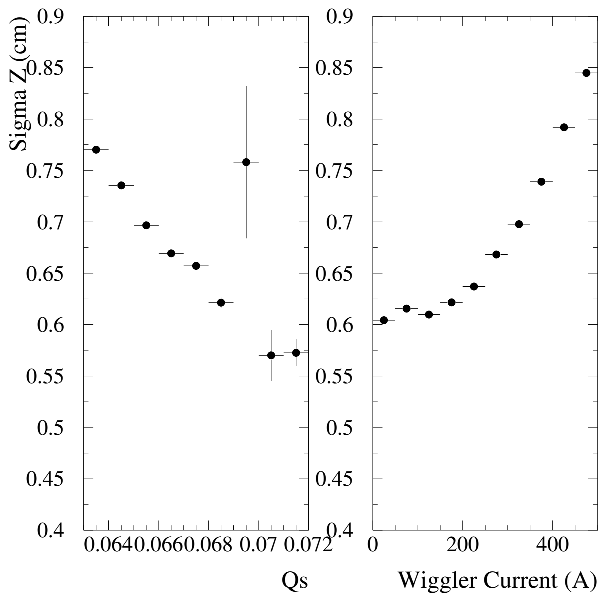

Figure 10: *'94 Data :* **Left :** *variation of* σ_z *with* $Q_s^{coh.}$, **right :** *variation of* σ_z *with wiggler current.*

The conditions of the experiment were as follows. The machine energy was "Peak + 2", three families of bunches ("a", "b" and "c") were circulating. Bunches of family "a" and "c" had typical bunch currents of 50 μA and 180 μA. After orbit trims, solenoid compensation etc, a polarisation level of around 15% had been achieved. In order to obtain high enough precision on the depolarizing resonance frequency, successive overlapping sweeps of full width $\delta\nu_s = 0.002$ were performed with an overlap of 0.001. The successive sweeps are shown on figure 11.

On that figure, time runs from bottom up, according to sweep number. The fractional part of the spin precession tune ν_s is indicated in the horizontal axis. In a first step (sweeps 1 to 6), a rough range for the depolarizing resonance is established by coarse sweeps of $\delta\nu_s = 0.012$. The degree of depolarization is indicated by the amount of black in the box corresponding to the sweep. A white box corresponds to no depolarization, and an entirely black box, to full depolarization. Two ranges are possible, $\nu_s = 0.448 - 0.458$ and $\nu_s = 0.480 - 0.478$. One is expected to be the main resonance, the other one the Q_s side band. A more precise evaluation of the frequency is then obtained with smaller sweeps $\delta\nu_s = 0.002$, in one of the possible ranges (sweeps 7 to 10). By varying the value of Q_s (sweeps 11-13) and the revolution frequency (sweeps 14-16), it is established that this line is the main resonance. Further improvement of the line frequency is obtained by overlapping sweeps (17,18). The other line is then scanned in small overlapping sweeps (19-22). Finally, the same operation is repeated for bunch family "c".

The depolarizing lines are shown in detail in figure 12. The interpretation of these lines is that the observed Q_s side band is in fact the mirror of the true one, e.g. the true frequency is $1 - \text{frac}\nu_s$ From these measurements, the frequency of the lines was determined. Not knowing the line shape precisely, the precision is limited to about the intrinsic RMS of a sweep width, e.g. $0.002/\sqrt{12} = 0.0006$. Having overlapping sweeps should allow a more precise determination, by taking the average of the individual measurements, weighted by e.g. $1 - \frac{P_{out}}{P_{in}}$. However, it is very clear from figure 12 that the line shape of the resonances is not very reproducible. In particular, the intensity of depolarization is quite variable from 50% to 120%. Assigning a variability of $\pm 35\%$ to each point, it was found by a small Monte-Carlo evaluation that a variability of ± 0.0005 on the line position would result. This was taken as the error on the line frequencies and propagated on the measurement of $Q_s^{inc.}$.

The measured value of $Q_s^{coh.}$ measured from the RF system was recorded on the LEP data base for each family of bunches at intervals of 4 minutes. However, this value is not very stable. The values at the time of the measurement of the Q_s side bands were considered. In the four time slots around the time of the measurement for bunch "a", the value of $Q_s^{coh.}$ fluctuated from 0.06164 to 0.06185 leading to $< Q_s^{coh.} >= 0.06175(10)$. Unfortunately, for bunch family "c" fluctuations from 0.0600 to 0.0610 occurred, leading to $< Q_s^{coh.} >= 0.06050(50)$.

Finally, the value of κ for the two experiments was derived using the bunch currents according to equation 7. The data are summarised in table 1. The measure-

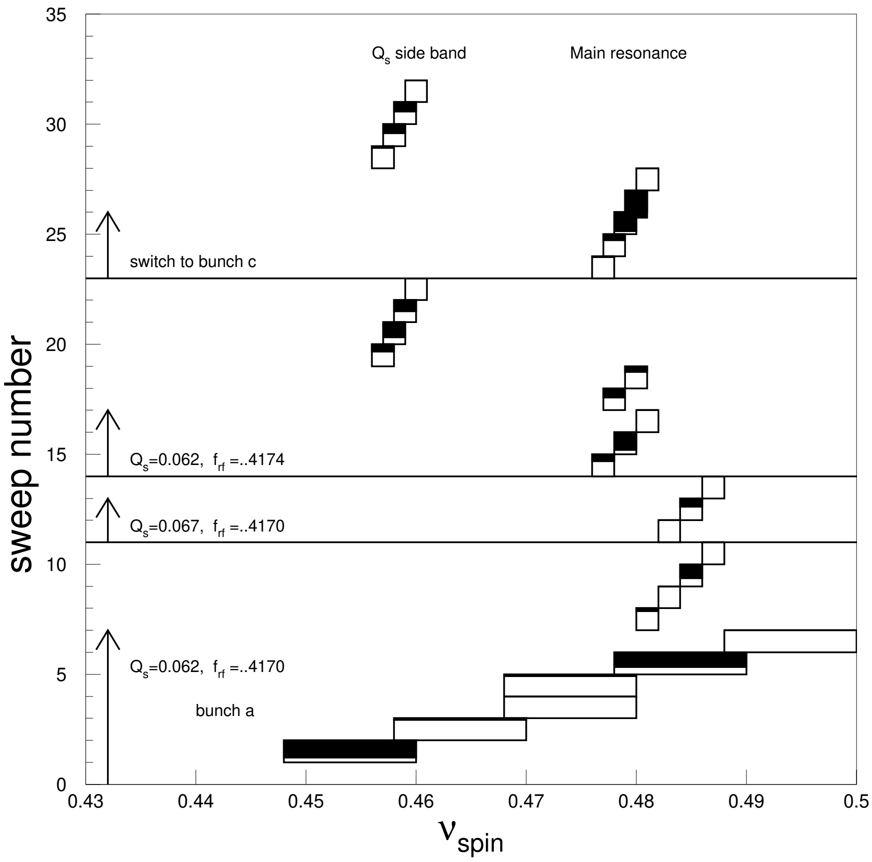

Figure 11: *Precision determination of Q_s side bands of the depolarization resonance on September 2 1995.*

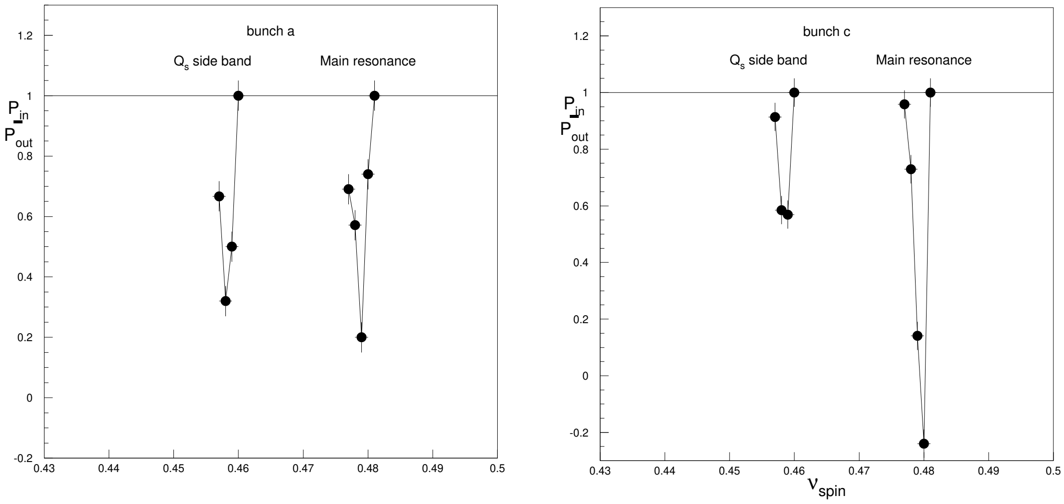

Figure 12: *Depolarizing lines. Left: bunch family "a"; Right: bunch family "c". The intensity of depolarization is represented by dividing the polarisation after the sweep P_{out} by the initial polarisation P_{in}.*

Bunch	Main res.	Q_s side-band	$Q_s^{inc.}$	$Q_s^{coh.}$	$\frac{Q_s^{coh.}}{Q_s^{inc.}}$	$I_b(\mu\mathrm{A})$	κ
"a"	0.4790(5)	0.5417(5)	0.0627(7)	0.0617(1)	0.984(12)	50	0.096(72)
"c"	0.4795(5)	0.5415(5)	0.0620(7)	0.0605(5)	0.976(14)	180	0.039(23)
average							0.045(22)

Table 1: *Determination of the factor κ.*

ments agree, although, naturally, a better precision is obtained for the measurement at higher current. The average of these two measurements yields

$$\kappa = 0.045 \pm 0.022 \tag{10}$$

3.6 Other LEP parameters

The other LEP parameters relevant for the present study are :

- the LEP momentum compaction factor α_c, measured to be $(1.86\pm0.02).10^{-4}$ [13]. In the analysis of the '93 scan [2], α_c was taken to be $1.9.10^{-4}$ (consequently $\sigma_{E_{beam}}$ was underestimated by 0.8 MeV).
- the LEP radius equal to 4243 meters.
- the beam energy taken to be 45625 MeV for peak data through this note.

Errors on the parameters values will be discussed in the last section of this note.

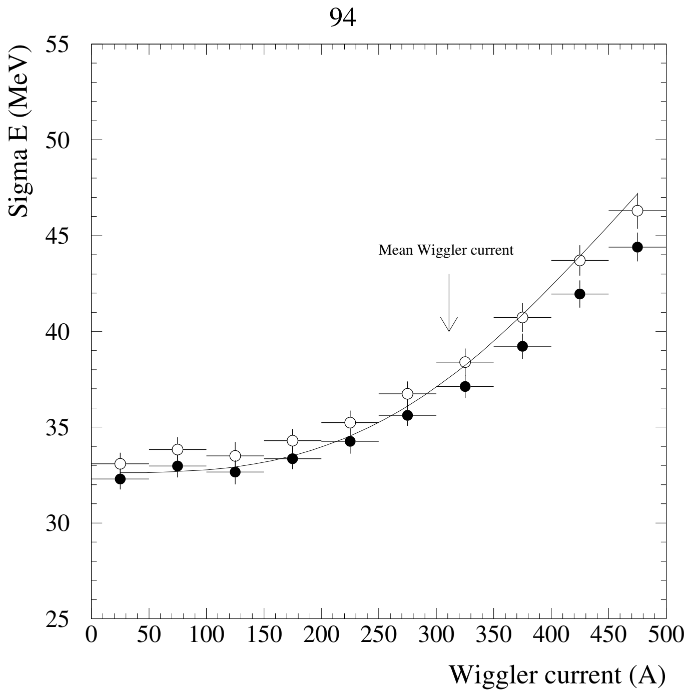

Figure 13: *1994 data : variation with wiggler currents of the experimental determination of* $\sigma_{E_{beam}}$ *from* σ_z. **black dots :** $\sigma_{E_{beam}}$ *computed using* $Q_s^{coh.}$, **open dots :** $\sigma_{E_{beam}}$ *computed using* $Q_s^{inc.}$ *with a* κ *value of* 0.045. *Also shown as a curve is the theoretical prediction from [6].*

4 Comparison of the two evaluations

The beam energy spread can now be evaluated in two different ways. First, by using equation 2 in the way coded by Jowett [6] one obtains what will be called from now on the "theoretical" prediction. Second by using formula 6 and 7, one can extract $\sigma_{E_{beam}}$ from σ_z to obtain the "experimental" value. The two evaluations are compared as a function of $I_{wiggler}$ on figure 13. From σ_z, the experimental $\sigma_{E_{beam}}$ has been computed either using directly $Q_s^{coh.}$ (black dots) without correction, or corrected to $Q_s^{inc.}$ with the measured κ value of 0.045 (open circles). Error bars are computed from the spread of σ_z and Q_s distributions. The mean wiggler current is $311A$ (see figure 8). As expected $\sigma_{E_{beam}}$ computed with $Q_s^{inc.}$ is larger than the one computed with $Q_s^{coh.}$. The difference of $\sigma_{E_{beam}}$ between the two possibilities is 1.4 MeV . Also shown on the same figure as a curve, the theoretical prediction. $\sigma_{E_{beam}}$ computed with $Q_s^{coh.}$ is always lower than the prediction. Furthermore it does not have the same behaviour at high wiggler currents. $\sigma_{E_{beam}}$ computed with $Q_s^{inc.}$is closer to the prediction at high current, and in particular for the average current. Nevertheless, it is very clear that the theoretical prediction has a somewhat different variation with wiggler current than the experimental determination.

A more quantitative evaluation of the level of agreement or disagreement of

Year-energy	$<\kappa>$	RMS	boxes	<entries /box>
93 peak-2	0.032 ± 0.004	0.071	406	170
93 peak	0.034 ± 0.002	0.051	459	474
93 peak+2	0.038 ± 0.003	0.057	374	260
94 peak	0.040 ± 0.001	0.032	747	826
95 peak-2	0.028 ± 0.003	0.086	651	96
95 peak	0.028 ± 0.003	0.074	471	192
95 peak+2	0.031 ± 0.003	0.079	683	152

Table 2: *Year and energy break-down of the comparison in terms of κ values of the experimental energy spread determination with the theoretical prediction.*

the two methods has been obtained as follows. Knowing $(Q_s^{coh.}, I_{bunch}, I_{wiggler}, J_e)$ the two methods can be used to extract $\sigma_{E_{beam}}$. The theoretical prediction will be noted $\sigma^{th}_{E_{beam}}$ the experimental determination using a zero value of κ, $\sigma^{\sigma_z,\kappa=0}_{E_{beam}}(t)$. The value of κ that would be necessary to make the two determinations agree is constructed:

$$\kappa(t) = \frac{I_0}{I(t)} \left(1 - \frac{\sigma^{\sigma_z,\kappa=0}_{E_{beam}}(t)}{\sigma^{th}_{E_{beam}}} \right) \tag{11}$$

The 3-dimensional space $(Q_s^{coh.}, I_{bunch}, I_{wiggler})$ was divided into boxes of reasonably stable values and sufficient amount of data. A value of κ was computed for each box. Figure 14 shows the distribution of the measured values of κ, weighted by the number of events in the box, obtained by this method, for 1994 data. Clearly κ is different from 0. Table 2 shows the mean values and RMS of the κ distribution for the three years of data taking. The RMS is consistent with the statistical precision of each sample (typically between 200 and 800 events). All values are compatible with the 1995 measured κ value of 0.045 ± 0.022. Furthermore, no difference is observed among the different energy points.

The variations of κ with beam current, wiggler current and $Q_s^{coh.}$ are shown on figures 15, 16 and 17. Error bars correspond to the statistical error on κ for each box, $\sigma/\sqrt{n}$, where n is the number of entries in each box. There is almost no variation of κ with I_{bunch}, which means that formula 7 is basically correct. In addition κ does not show much variation with respect to $Q_s^{coh.}$. However, a clear dependence with $I_{wiggler}$ is again observed. However, it remains within 1 sigma of the value of κ that was directly measured.

5 Beam energy spread

It was found in the previous sections that the two estimates of energy spread agree at the level of $\pm 2\%$, but not better, since a significant variation with wiggler

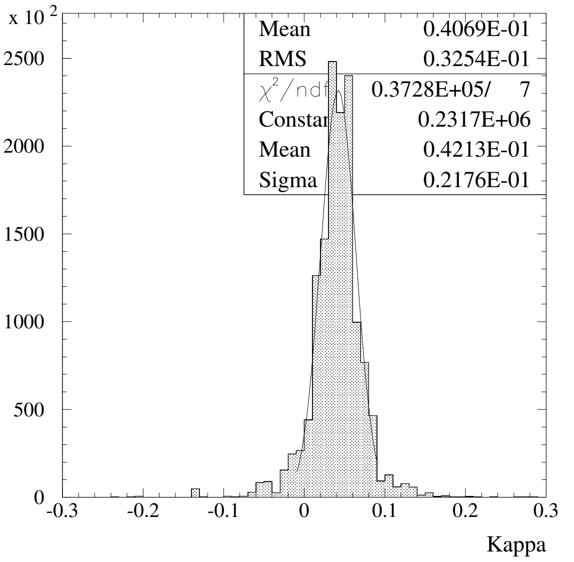

Figure 14: *Distribution of κ, derived by a comparison of $\sigma_{E_{beam}}$ to the prediction for 1994 data.*

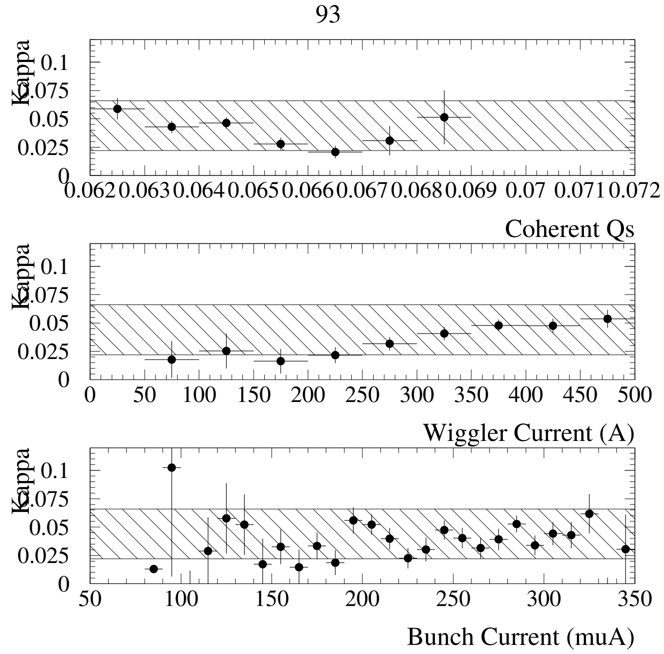

Figure 15: *93 Data :* **Top :** *variation of κ with $Q_s^{coh.}$*, **middle :** *variation of κ with wiggler current,* **bottom :** *variation of κ with bunch current. The hatched band indicates the allowed interval of κ obtained by direct measurement in 95.*

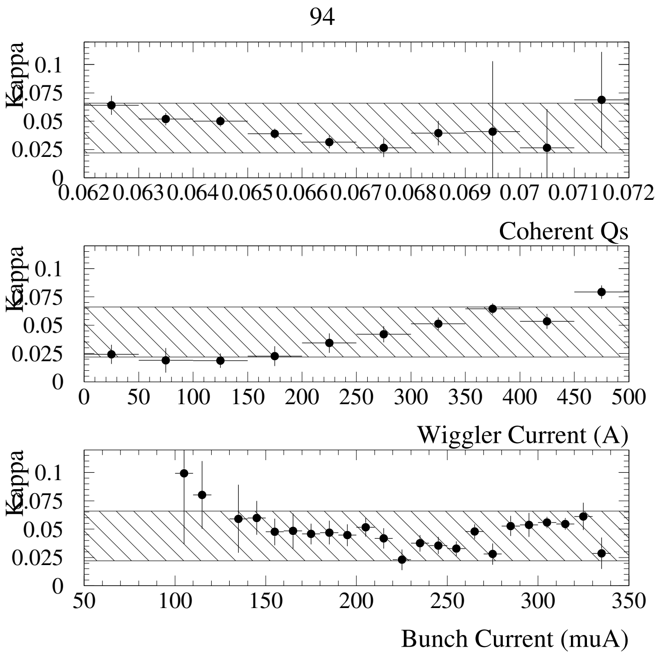

Figure 16: *94 Data :* **Top :** *variation of* κ *with* $Q_s^{coh.}$, **middle :** *variation of* κ *with wiggler current,* **bottom :** *variation of* κ *with bunch current. The hatched band indicates the allowed interval of* κ *obtained by direct measurement in 95.*

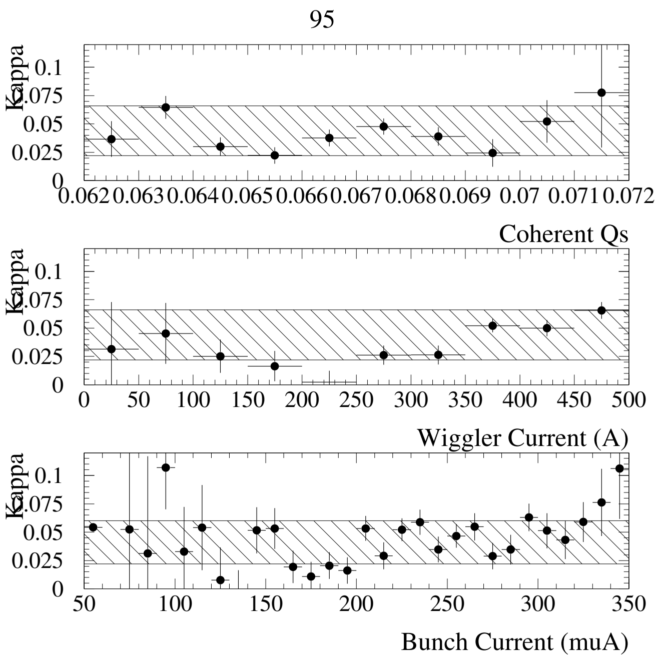

Figure 17: *95 Data :* **Top :** *variation of* κ *with* $Q_s^{coh.}$, **middle :** *variation of* κ *with wiggler current,* **bottom :** *variation of* κ *with bunch current. The hatched band indicates the allowed interval of* κ *obtained by direct measurement in 95.*

	93	94	95
peak - 2	38.64		39.92
peak	39.23	38.80	40.28
peak + 2	39.28		40.72

Table 3: *Beam energy spread in* MeV *for the different years as measured in* ALEPH .

excitation was found. This might point to a deficiency in the modelling of the magnetic field distribution in the wigglers, or in the rest of the machine, or even point to the fact that the relationship between $Q_s^{coh.}$ and $Q_s^{inc.}$ is more complicated than a simple beam current dependence.

Given the agreement between the value of κ determined by comparing to the $\sigma_{E_{beam}}$ to the prediction and the direct measurement within the errors, the measured κ value of 0.045 ± 0.022 is used for all data to evaluate the beam energy spread and the error. For each year, the beam energy spread is computed according to equation 6 using the RMS of the z distribution in ALEPH , the measured bunch current, $Q_s^{coh.}$ and $\kappa = 0.045$. The beam energy spread (luminosity weighted) measured at ALEPH for each year of data taking and each energy point is given in table 3. For '95 data, no difference between the different bunch families is observed. It is nice to see that the measured energy spread increases with energy, as it should.

5.1 Variation of $\sigma_{E_{beam}}$ with time

As expected from the large variation of σ_z within a fill, $\sigma_{E_{beam}}$ varies considerably on the same time scale, up to 15 MeV variations are observed (figure 18) . From fill to fill, $\sigma_{E_{beam}}$ varies too (figure 19), it has a sigma of 2.5 MeV .

6 LEP center-of-mass energy spread

In the presence of opposite sign energy dispersion between electrons and positrons, the LEP center-of-mass energy spread $\sigma_{E_{CM}}$ can be less than $\sqrt{2} \times \sigma_{E_{beam}}$ (see equation 9). The energy dispersion for the 93 and 94 pretzel running period has been evaluated [14], the reduction on the center-of-mass energy spread is estimated to be less than 0.2%. This corresponds to a change of 0.1 MeV on $\sigma_{E_{CM}}$, and is therefore neglected.

For '95 running, the reduction of the center-of-mass energy spread due to monochromatization cannot be neglected. This correction of a couple of percent is different for each I.P. The vertical beam size and dispersion have been measured [15] and are given, averaged for peak-2 and peak+2 energy points, in table 4 for the four different interaction points.

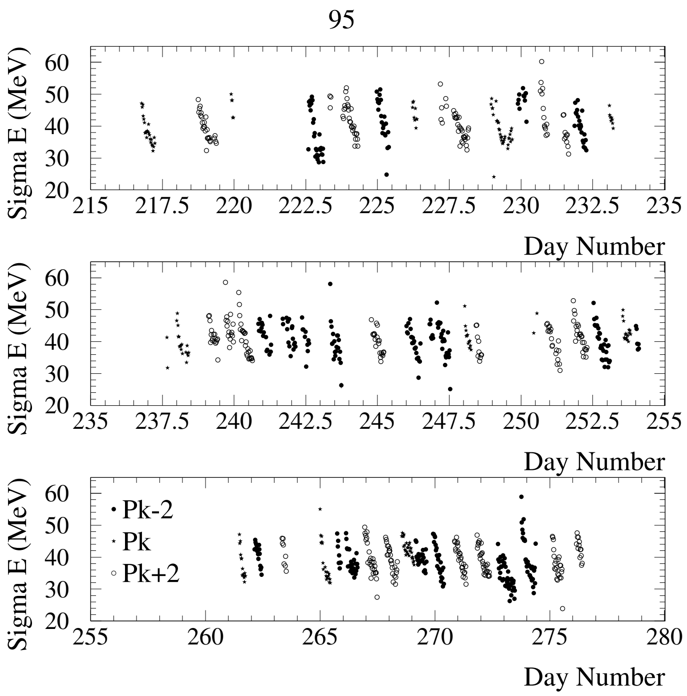

Figure 18: *1995 data :* $\sigma_{E_{beam}}$ *variation with time.*

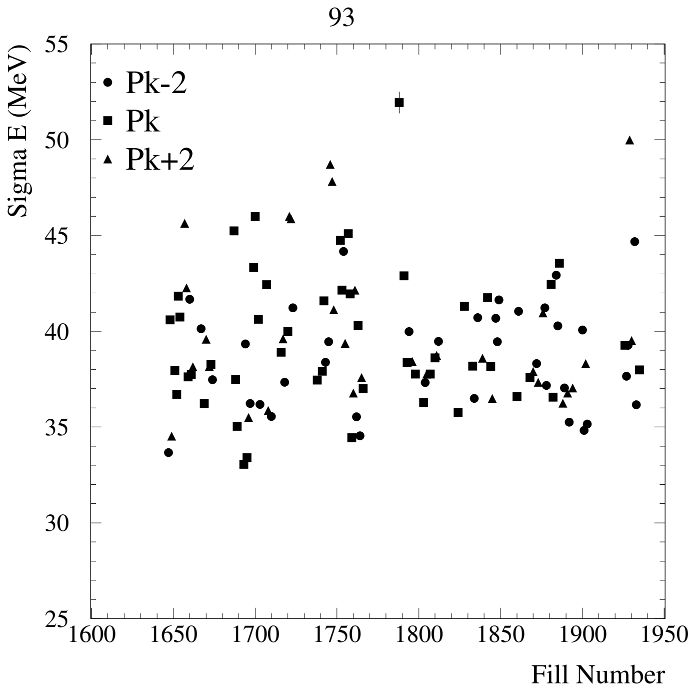

Figure 19: *1993 data :* $\sigma_{E_{beam}}$ *variation with fill number.*

	Beam size (μm)	Dispersion (μm)	Monochromatization factor
IP 2	3.9 ± 0.5	2000 ± 400	0.976 ± 0.011
IP 4	5.3 ± 0.5	-2000 ± 700	0.987 ± 0.010
IP 6	4.8 ± 0.5	1800 ± 800	0.987 ± 0.012
IP 8	4.1 ± 0.5	-1500 ± 700	0.987 ± 0.012

Table 4: *Vertical beam size and dispersion (microns) for the different Interactions Points. Together with the monochromatization factor.*

	93	94	95			
			IP2	IP4	IP6	IP8
peak - 2	54.6		55.0	55.7	55.7	55.7
peak	55.4	54.9	55.6	56.2	56.2	56.3
peak + 2	55.6		56.0	56.6	56.7	56.7

Table 5: *Center-of-mass energy spread in* MeV *for the different years and Interaction Points.*

The LEP center-of-mass energy spread is given in table 5 for the different years of running and for the different energy points.

6.1 Systematic error on the LEP center-of-mass energy spread

Several parameters contribute to the systematic error on the LEP center-of-mass energy spread.

LEP *momentum compaction factor*

The LEP momentum compaction factor α_c, has been measured [13] to be $(1.86 \pm 0.02).10^{-4}$. The 1% error on this parameter value, directly translates into a 1% error on $\sigma_{E_{beam}}$. That is 0.4 MeV error on $\sigma_{E_{beam}}$.

Difference between measured and computed Q_s

The difference between the measured and the computed $Q_s^{coh.}$ is -0.00034 for the '95 scan [12] and 0.0001 for the '93 scan [16]. It is a 10^{-3} effect at maximum (mean value of $Q_s = 0.065$), it is therefore neglected.

Beam energy variation

The beam energy has been taken to be 45625 MeV for peak data through this note. It is known that the beam energy varies within a fill and that some corrections have to be applied to this nominal beam energy [17]. However the corrections are of the order of a few MeV ($\leq$ 30MeV). It is a less than a 10^{-3} effect and it is neglected.

Systematics on the κ value

A change of κ by one standard deviation 0.022, changes the beam energy spread by 0.7 MeV .

Beam energy dispersion

For the '95 scan data, the *monochromatization* factor C is not equal to 1. The errors on the dispersion [15] can be translated into an error of the order of 0.01 on the *monochromatization* factor C, which leads to a 0.6 MeV error on $\sigma_{E_{CM}}$.

6.2 Summary

From the above discussion, the error on the LEP center-of-mass energy spread is 1.1 MeV for 1993 and 1994. For 1995 this error is increased to 1.3 MeV due to vertical beam energy dispersion. These numbers are fully correlated between years and experiments.

LEP center-of-mass energy variations within fills [17], increase the apparent energy spread. The RMS of the LEP center-of-mass energy distribution for each year and each energy point has to be added in quadrature to the number given in table 5. In previous analyses this term has been neglected as small compared to error on $\sigma_{E_{CM}}$.

7 Conclusion

The luminosity weighted beam energy spread has been estimated by two different ways, a theoretical evaluation based on modelling the magnetic field distribution in LEP , and a more experimental evaluation using the bunch length measured by ALEPH for the 1993, 1994 and 1995 electro-weak data. The two methods agree at the level of $\pm 2\%$, which is also the precision with which the relation between the coherent and incoherent synchrotron tunes is measured. Discrepancy at this level was found in the wiggler current dependence of the energy spread.

The luminosity weighted LEP center-of-mass energy spread $\sigma_{E_{CM}}$ is found to be between 55 and 57 MeV depending on the year and the interaction point. The systematic error on $\sigma_{E_{CM}}$ is 1.1 MeV for 1993 and 1994 and 1.3 MeV for 1995 data. This is more than a factor four improvement over the previously quoted 5 MeV uncertainty.

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