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LEP energy calibration above the W pair production threshold

The LEP Energy Working Group

Abstract

The normalisation of the LEP beam energy above the WW threshold is described. The beam energy is measured very precisely by resonant depolarisation at lower energies. This technique has been optimised to allow measurements at several beam energies up to 55 GeV. Magnetic measurements using NMR probes in selected dipole magnets, and a flux loop which measures the field in all the bending dipoles, are then used to extrapolate the energy scale to the energy at which collisions occur. The beam energy is used to calibrate the measurement of the W boson mass, and has been determined with a precision of 25 MeV for the data taken in 1997. This is well below the total experimental uncertainty on the W mass measured at LEP.

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1 Introduction

The large electron-positron (LEP) collider, has been running above the W pair production threshold since 1996, when the LEP2 programme began. The beam energy at LEP2 is used to set the absolute energy scale for the measurement of the W boson mass. The aim is for a total experimental uncertainty on the W mass of around 30 MeV. Since $\Delta M_{\mathrm{W}}/M_{\mathrm{W}} = \Delta E_{\mathrm{beam}}/E_{\mathrm{beam}}$, this sets a target of 10 to 15 MeV for the beam energy uncertainty. This contrasts with LEP1, where the Z mass and width were measured with uncertainties of about 2 MeV, corresponding to beam energy uncertainties of typically 1 MeV.

At LEP1, the average energy around the LEP ring at a particular moment in time was measured very precisely using resonant depolarisation (RDP)[1]. Unfortunately, due to broadening resonances, there is insufficient build up of transverse polarisation at LEP2 energies to be able to calibrate the beam energy using RDP in such a direct way. Measurements of the magnetic fields bending the beam must therefore be used to extend the calibration to top energy. This becomes the dominant systematic uncertainty in the beam energy measurement.

The RDP technique measures the spin tune, ν, which is proportional to the beam energy and to the total magnetic field in the vertical plane:

$$\nu \propto E_{\mathrm{beam}} \propto \oint B \cdot \mathrm{d}\ell \tag{1}$$

The extrapolation to physics energy depends on a comparison of the measured RDP energies, E_{pol}, with the magnetic fields measured by NMR probes in the LEP dipoles. The uncertainty is inferred from the agreement between the NMR fields and the total bending field measured by the flux loop. At present, the method proceeds as follows:

- The NMR's are calibrated with respect to RDP measurements at low energy.
- The energy for physics running is inferred from a linear extrapolation of this calibration. The NMR's record the field for all RDP measurements and for physics running.
- The NMR's sample a small fraction of the field while the flux loop measures nearly all the bending field, and the two can be cross-calibrated over the full range from RDP to physics energies. This provides the consistency check for the measurement and leads to the dominant systematic uncertainty.

Having fixed the overall normalisation, the energy must be then corrected for variations as a function of time, due to earth tides, temperature effects, leakage currents from neighbourhood electric trains and so on. There are also corrections to relate the average energy to the collision energy at each interaction point, in particular due to the exact accelerating RF configuration. All of these effects have been rather well understood at LEP1 [1], and contribute a total additional uncertainty below 10 MeV at LEP2. The energies for each interaction point are calculated at least every 15 minutes, and these values are provided to the LEP experiments.

In the following section, the data samples and magnetic measurements are described. The energy model is outlined in section 3. The derivation of the energy scale is then described in sections 4 and 5. More information on time dependent corrections is given in section 6, and on IP specific corrections in section 7. The systematic uncertainties for the whole analysis are reviewed in section 8.

2 Data samples

2.1 Luminosity delivered by LEP2

LEP has delivered about 10pb^{-1} at each of two centre-of-mass energies, 161 and 172 GeV, in 1996, and over 50pb^{-1} at a centre-of-mass energy of around 183 GeV in 1997. Combining the data from all four LEP experiments, these data give a measurement of the W mass with a precision of about 90 MeV.

2.2 Polarisation measurements

The successful RDP experiments in 1996 and 1997 are listed in table 1. The first fill with two energies in the same fill was at the end of 1996. The need for more RDP measurements motivated the k-modulation programme and the development of a dedicated polarisation optics (the 60/60 optics) for use in 1997 [2]. This resulted in 5 fills with more than one energy point, and 2 fills with 4 energy points, which allow a linearity check. The lever arm over which tests can be made increased from 5 to 14 GeV between 1996 and 1997, and the maximum RDP calibrated energy increased from 50 to 55 GeV. Although a 2% level of polarisation was observed at 60 GeV, this was insufficient to allow an energy measurement.

Date	Fill	41 GeV	44 GeV	50 GeV	55 GeV	Optics
19/08/96	3599			yes		90/60
31/10/96	3702		yes			90/60
03/11/96	3719		yes	yes		90/60
17/08/97	4000		yes			90/60
06/09/97	4121		yes	yes		60/60
30/09/97	4237		yes	yes		60/60
02/10/97	4242	yes	yes	yes	yes	60/60
10/10/97	4274		yes			90/60
11/10/97	4279	yes	yes	yes	yes	60/60
29/10/97	4372	yes	yes			60/60

Table 1: Fills with successful polarisation measurements in 1996 and 1997. Successful measurements are marked “yes”.

2.3 Magnetic measurements

The LEP dipole fields are monitored in two ways. There are 16 NMR probes positioned inside some selected main bend dipoles, see figure 1. Each octant has at least one NMR, and octants 1 and 5 have NMR's in several adjacent dipoles (and in one instance two probes in the same dipole). The NMR's measure the local magnetic field with a precision of $\sim 1 \times 10^{-6}$, and can be read out essentially continuously (typically every 30 seconds), but each probe only samples the field in a small region of one out of 3200 dipoles. The probes are read out during normal physics running, and also during RDP and flux-loop measurements.

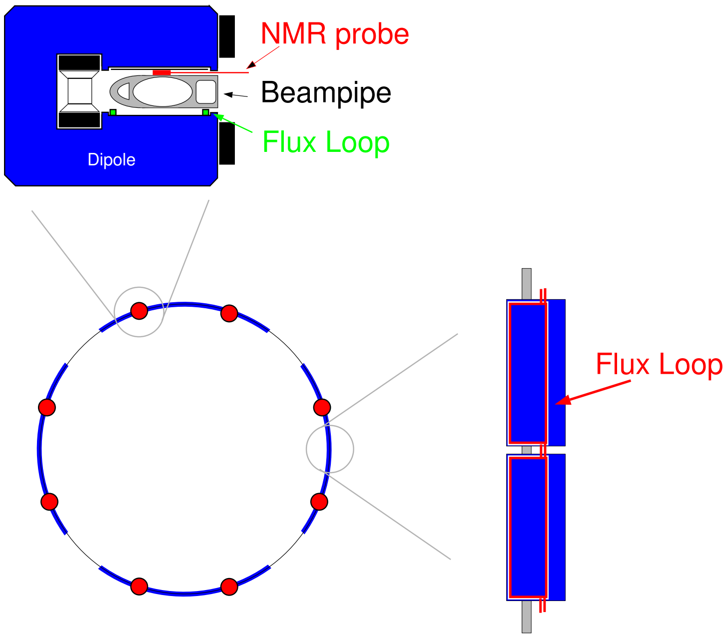


Figure 1: The two systems for monitoring the LEP magnetic field.

The flux loop is also shown schematically in figure 1. In contrast to the NMR probes, the flux loop measures 96.5% of the total bending field, including 98% of the main bend dipole field. The loop does not include the weak (10% of normal strength) dipoles at the ends of the arcs, the double-strength dipoles in the injection regions, or other magnets such as quadrupoles or horizontal orbit correctors.

The flux loop measures the change in field during a dedicated demagnetisation cycle, outside physics running. Only a handful of these measurements were made in 1997. These measurements included a pause at several intermediate field values, so as to allow time for the NMR's to lock and be read out. These special flux-loop measurements are referred to by an adjacent LEP fill number (one measurement in fills 4000, 4121 and 4206 and two measurements in fill 4434). This possibility to cross-calibrate the field measured by the flux loop and by the NMR probes is crucial to the analysis.

3 The energy model

The LEP beam energy is calculated as a function of time according to this formula [1]:

$$\begin{aligned} E_{\mathrm{beam}}(t) &= (E_{\mathrm{initial}} + \Delta E_{\mathrm{dipole}}(t)) \\ &\cdot (1 + C_{\mathrm{tide}}(t)) \cdot (1 + C_{\mathrm{orbit}}(fill)) \\ &\cdot (1 + C_{\mathrm{h.corr.}}(t)) \cdot (1 + C_{\mathrm{QFQD}}(t)) \end{aligned} \quad (2)$$

In comparison to previous years, the model to describe the dipole behaviour has been simplified since 1996, when all 16 NMR's were available. In 1995 there were two probes in the tunnel, and before then only in a reference magnet outside the LEP tunnel, but powered in series with the main dipoles. Previously, the initial field at the start of a physics fill E_{initial} was derived from comparisons with polarisation data, and the rise in a fill $\Delta E_{\text{dipole}}(t)$ was calculated from a description of the effects of parasitic electrical currents on the beam pipe caused by trains travelling in the neighbourhood, and for changes in the dipole field generated by temperature. These energies are now derived directly from the average over the larger sample of NMR's available [1]

Tide, orbit, horizontal orbit corrector and QFQD effects must also all be taken into account when comparing the energy measured at a particular time by polarisation with the magnetic field measured in the main dipoles by the NMR probes. These corrections are discussed in section 6, and in more detail in Ref. [1].

There are additional effects which cause a shift between $2 \times E_{\text{beam}}$ and the centre-of-mass energy at each interaction point. These are discussed in section 7.

4 Comparison of polarisation data with NMR measurements

The magnetic fields B^i_{NMR} measured by each NMR $i = 1, 16$, are converted into an equivalent energy by a two parameter fit to the energies measured by resonant depolarisation of the form

$$E^i_{\text{NMR}} = a^i + b^i B^i_{\text{NMR}}. \tag{3}$$

The RDP energies are compared to the energy measured by each NMR at the end of the LEP ramp, E_{initial} according to equation 2. The comparison is made in this way so that so that corrections due to coherent motion of the quadrupoles (eg. tides) and the average dipole rise $\Delta E_{\text{dipole}}(t)$ are properly taken into account in order to arrive at a calibration of the each individual probe. (The dipole rise term also requires calibrated NMR probes, so the procedure is iterative.) The residuals, $E_{\text{pol}} - E^i_{\text{NMR}}$, are examined for each NMR. Different probes show different behaviour, but this behaviour is reproduced from fill to fill. The residuals averaged over NMR's at each polarisation point are shown in figure 2(a), in which the errors are displayed as the rms$/\sqrt{N}$, where N is the number of NMR's functioning for the measurement (up to 16). This figure shows the average residuals with respect to the simultaneous fit to all polarisation fills in 1997, which was used to calibrate the NMR's. In figure 2(b), the residuals for the two fills with four RDP energies are shown, when the fit is made to each fill individually. The residuals show a reproducible small but statistically significant deviation from zero, with the 45 and 50 GeV points being a few MeV higher than those at 40 and 55 GeV. Despite some fill-to-fill scatter, this shape is present in all fills, not just the two fills with four RDP energies.

[1] E_{initial} and $\Delta E_{\text{dipole}}(t)$ are both averages over working NMR probes. However, the first gives equal weight to each probe, since each magnet behaves independently during the ramping of LEP, while the second gives equal weight to each octant, since temperature and parasitic current effects vary octant by octant (see Ref. [1]).

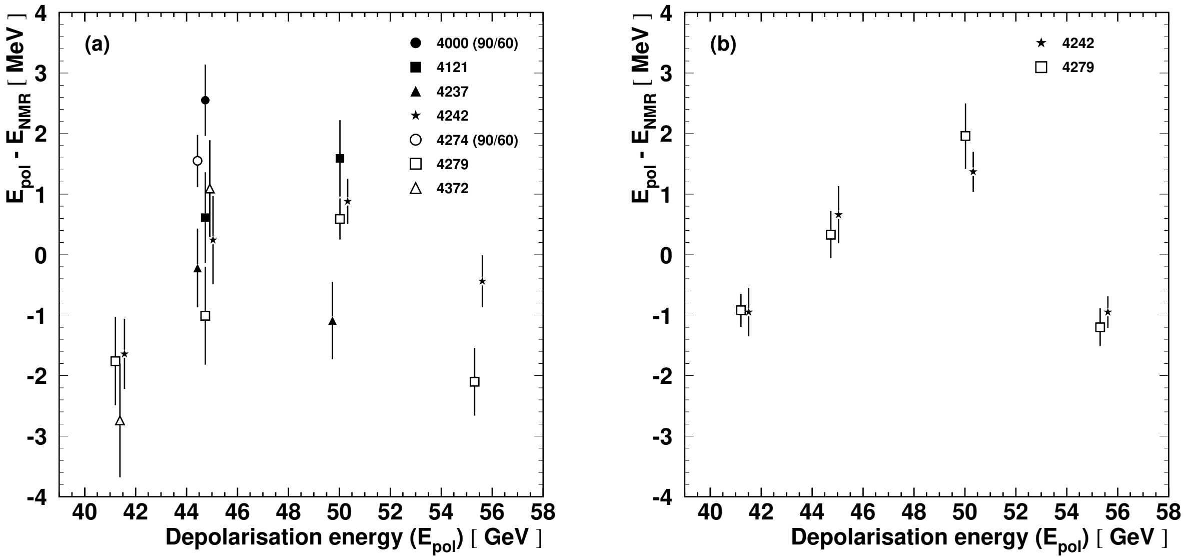


Figure 2: Residuals of the fit comparing RDP energies to the energies predicted by the model (a) for a simultaneous fit to all fills and (b) for individual fits to each four point fill.

4.1 Optics difference

The development of a dedicated polarisation optics implied that a cross-calibration of polarisation and physics optics was necessary. The horizontal orbit corrector settings and current differences in the vertically and horizontally focusing quadrulpoles cause some difference between the average energies of a few MeV. Simulations show that this energy difference can be of either sign, depending on the exact imperfections in the machine, and the difference is predicted to scale with the beam energy. The residual average beam energy difference between RDP measurements with the two optics is:

$$E(90/60) - E(60/60) = +2 \text{ MeV at } 45 \text{ GeV}. \tag{4}$$

We therefore assign a systematic uncertainty of 4 MeV to cover uncertainties due to this effect at physics energy.

4.2 Predicted energy in physics

The field measured by each NMR during physics running can be used to predict the beam energy from the fit to the RDP measurements: $E^i_{\text{NMR}} = a^i + b^i B^i_{\text{NMR}}$. The average magnetic field is calculated for each NMR over all of the physics running at a nominal beam energy of 91.5 GeV. The average physics energies derived from these average fields have an rms scatter over the NMR's of about 40 MeV, contributing $40/\sqrt{16} = 10$ MeV to the systematic uncertainty from the normalisation procedure.

In 1996, the RDP measurements were fitted to a line passing through the origin, $E_{\text{pol}} = p^i B^i_{\text{NMR}}$. If this is tried for the 1997 data, the rms scatter increases to 60 MeV.

Using different polarisation fills as input to the procedure gives some variation. Fill 4372 has the most unusual behaviour. This is partly because it only samples the two lower energy points, which have a different average slope to the four point fills. This fill also has the smallest number of functioning NMR probes, since it is at the end of the year. It therefore has little weight in the overall average. An overall uncertainty of 5 MeV is assigned to cover the range of central values derived using different combinations of polarisation fills.

4.3 Non-linearity test

The average residuals of the fit show a characteristic shape which is a measure of the non-linearity in the energy range 41–55 GeV. The amplitude of the deviations is larger than the statistical scatter over the NMR's. The linearity is best examined by making fits to the individual 4 point fills. If the errors are inflated to achieve a χ^2/dof of 1, then they imply an uncertainty at physics energy of 7 MeV. This uncertainty is strongly correlated with the slightly larger error derived in section 4.2, so we assign no additional uncertainty to cover this alternate evaluation.

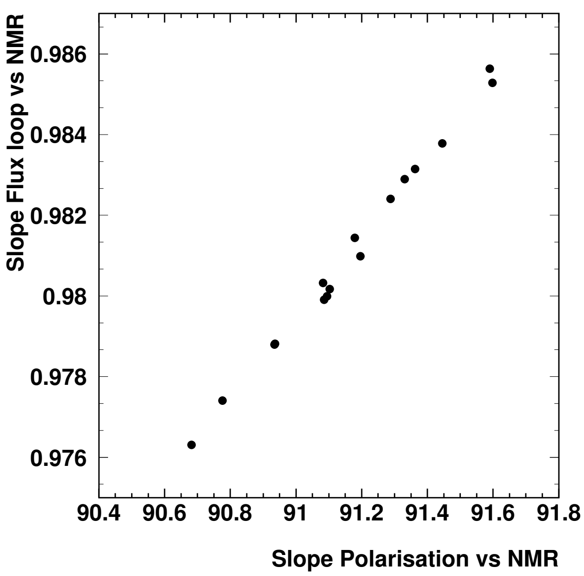
5 Cross calibration of NMR probes and total field measured by the flux loop.

The total change of field measured by the flux loop can also be fitted to a straight line as a function of NMR magnetic field for each probe:

$$B_{\mathrm{FL}} = c^i + d^i B^i_{\mathrm{NMR}} \tag{5}$$

From octant-to-octant and magnet-to-magnet the probes show different behaviour. The slopes are not constant with energy. However, the behaviour is reproduced from fill-to-fill.

As one would expect if the RDP measured energies, E_{pol}, are proportional to the total bending field measured by the flux loop, B_{FL}, there is a strong correlation between the



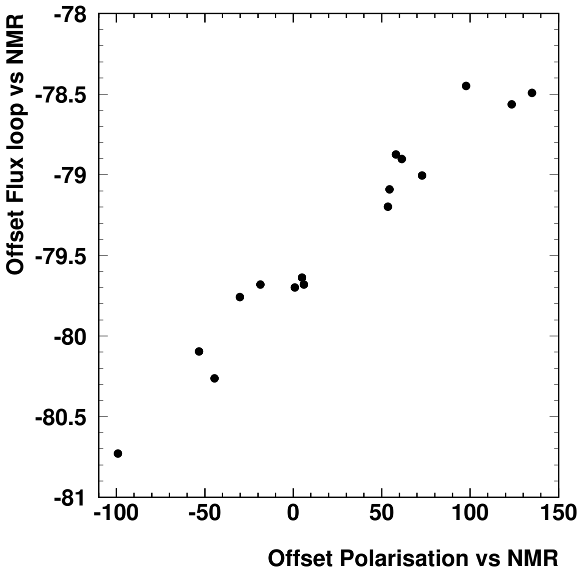


Figure 3: The slopes and offsets of equations 6 and 7 showing good correlation.

fitted parameters of the polarisation fit and the flux loop fit, when the NMR/flux-loop fit is restricted to the range corresponding to 40–55 GeV:

$$E_{\text{pol}} = a^i + b^i B^i_{\text{NMR}} \qquad \text{and} \tag{6}$$

$$B_{\text{FL}} = c'^i + d'^i B^i_{\text{NMR}} \qquad \text{fit restricted to 40–55 GeV.} \tag{7}$$

Having established this correlation, which is shown in figure 3, the flux loop can now be used to test the extrapolation method. From the fit in the 40–55 GeV region, the NMR's can be used to predict B_{FL} at physics energy, and this prediction can be compared with the flux loop measured B_{FL}. Any difference between the prediction and the direct measurement of the field can then be interpreted as a possible bias in the predicted energy.

An example of the fit to one flux-loop measurement is shown in figure 4, and the deviations at physics energy for all five flux loops are shown in figure 5. The bias at physics energy is up to 20 MeV, with an rms over the probes of 30–40 MeV, corresponding to an uncertainty of 10–20 MeV depending on the number of working probes. The bias tends to decrease during the year. This is partially understood as being due to the smaller sample of NMR's available in the latter part of the year, as illustrated in figure 5, but for the common probes the difference in bias between the first and last fills is still -23 ± 5 MeV. No cause for this change has been identified so far. The systematic effects that have been observed, for example a discrepancy of around 2 MeV if two very close by energy points are measured, are too small to explain the trend. Flux loop measurements at the beginning of the 1998 running period show a net bias of less than 10 MeV, with all NMR probes working.

The tests with the flux loop are not used to correct the NMR calibration from polarisation data, but allow an independent estimate of the precision of the method. The difference between the NMR and flux-loop measured field increase is taken to assign a systematic uncertainty of 20 MeV. This covers the maximum difference seen during the year. An average over the flux loop measurements would give a smaller estimate. Flux-loop measurements taken in the first part of the 1998 running with all 16 NMR probes working show a bias of 5 to 10 MeV.

A detailed comparison of NMR and flux loop data in the region of the RDP measurements shows a different non-linearity to the NMR–polarisation comparison. However, the average residuals to the flux loop fits are only a few MeV in this region (see figure 6), which is at the limit of the expected precision.

5.1 Bending field not measured by the flux loop

The flux loop is embedded in each of the main LEP bending dipoles, but only samples 98% of the total bending field of each dipole. The effective length of the flux loop varies during the ramp because the fraction of the fringe fields overlapping neighbouring dipoles varies. The saturation of the dipoles, expressed as the change in effective length, was measured before the LEP startup on a test stand for different magnet cycles. The correction between 45 and 90 GeV is of the order of 10^{-4}, corresponding to a 5 MeV uncertainty in the physics energy.

The weak ("10%") dipoles matching the LEP arcs to the straight sections contribute 0.2% to the total bending field. Assuming that their field is proportional to that of the main bends between RDP and physics energies to better than 1%, their contribution to any non-linearity in the extrapolation is around 1 MeV.

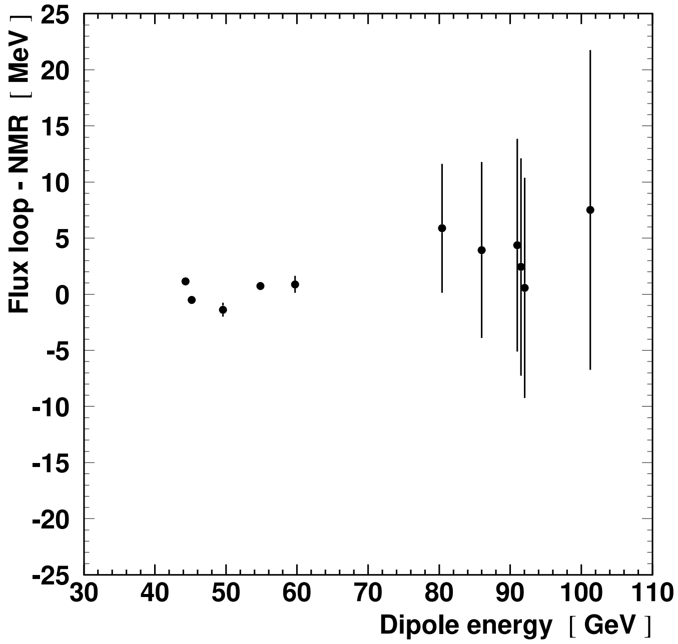


Figure 4: The difference between the energy predicted by the flux loop and by the NMR probes, as a function of equivalent beam energy. This is for the flux loop measurement in fill 4000, after a fit in the region of 40 to 55 GeV. The error bars show the rms scatter divided by the square root of the number of probes.

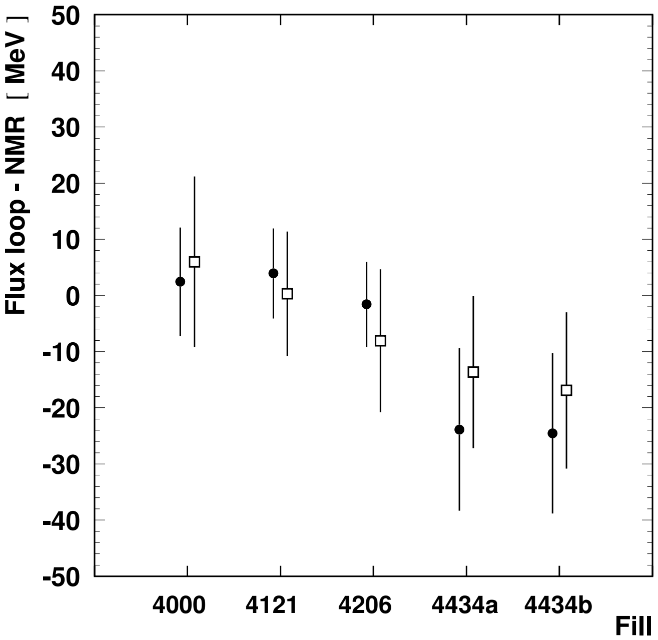


Figure 5: The difference in energy between flux loop and NMR probes at the equivalent of physics energy for each flux loop measurement. The solid points are for all working probes for each fill, while the empty points are for the common set of probes working for all fills.

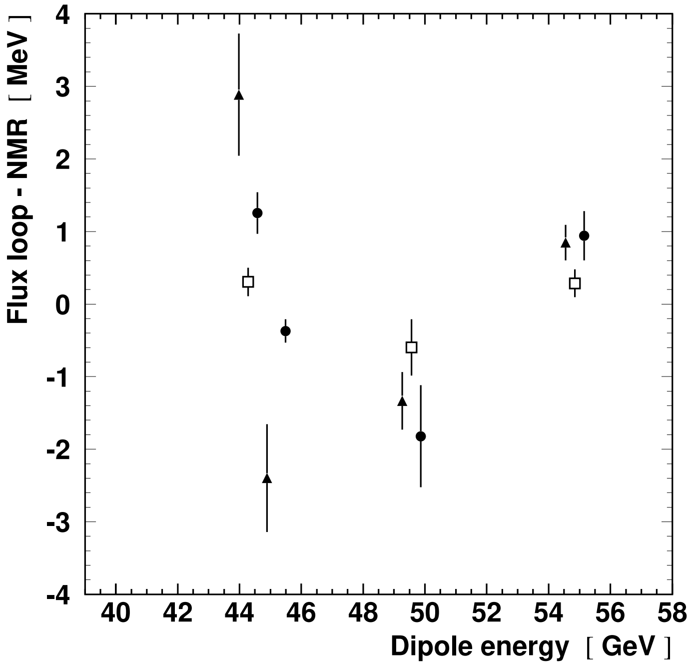


Figure 6: The residuals in the low energy region for flux loop measurements 4000,4121 and 4206, using the subset of NMR probes available at all points shown in all 3 fills. Too few NMR's are working for all low energy points to include fill 4434 here.

The bending field of the double strength dipoles in the injection region contributes 1.4% of the total $\oint B \cdot \mathrm{d}\ell$. Their bending field has been measured by additional NMR probes installed in the tunnel in 1998, and found to be proportional to the bending field of the main dipoles to rather better than 10^{-3}, which gives a negligible additional systematic uncertainty.

6 Time dependent effects

The beam energy during a fill is changed by several phenomena which are discussed in more detail in the following sections.

6.1 Central Frequency and Machine Circumference

For a circular accelerator like LEP the orbit passing on average through the centre of the quadrupoles is referred to as the *central orbit*, and the corresponding RF frequency setting is known as the *central RF frequency* f_c^{RF}. When the RF frequency f^{RF} does not coincide with f_c^{RF} the beam senses on average a dipole field in the quadrupoles, which causes a relative beam energy change ΔE of :

$$\frac{\Delta E}{E} = -\frac{1}{\alpha} \frac{f^{\mathrm{RF}} - f_c^{\mathrm{RF}}}{f^{\mathrm{RF}}} \tag{8}$$

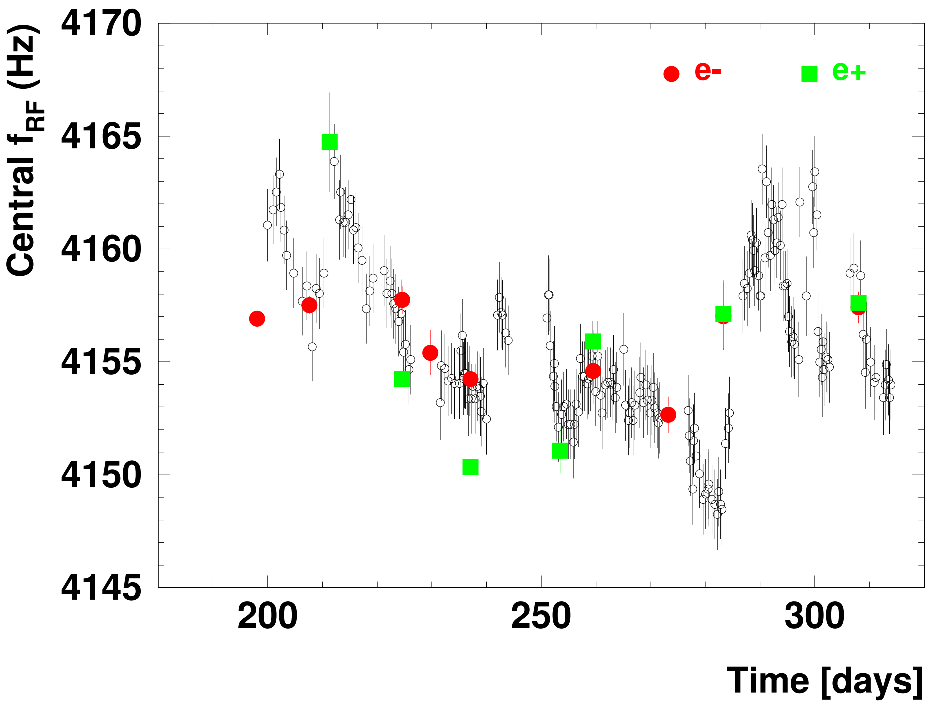


Figure 7: Evolution of the central frequency as a function of time for the 1997 LEP run. The solid points are actual f_c^{RF} measurements for e^- and e^+ while the open points are obtained from x_{arc}. On some occasions, the electrons and positron f_c^{RF} differ by up to 4 Hz. Note that the vertical scale shows a variation in the last four digits of the LEP RF frequency which is approximately 352 MHz.

where α is the momentum compaction factor. This depends on the optics used in LEP, and its value is $1.54 \cdot 10^{-4}$ for the 102/90 optics, $1.86 \cdot 10^{-4}$ for the 90/60 optics and $3.86 \cdot 10^{-4}$ for the 60/60 optics.

The central frequency is only measured on a few occasions during a year's running and requires non-colliding beams [3]. The monitoring of the central orbit and of the ring circumference relies on the measurement of the average horizontal beam position in the LEP arcs [4]. As the length of the beam orbit is constrained by the RF frequency, a change in machine circumference will be observed as a shift of the beam position relative to the beam position monitors (BPMs). Figure 7 shows the evolution of the central frequency determined through x_{arc} as well as the direct f_c^{RF} measurements. The x_{arc} points have been normalised to the electrons f_c^{RF} measurements. The occasional difference of about 4 Hz between the electrons and positron f_c^{RF} is not understood. A similar systematic effect has also been seen in 1998. Therefore a systematic error of $\pm$ 4 Hz is assigned to the central frequency. This error results in an uncertainty of 1.5 MeV in the predicted difference between of the energies measured by RDP with the 90/60 and 60/60 optics at 45 GeV beam energy. This is covered by the 2 MeV uncertainty already assigned from the measured difference.

A correction to the energy of each fill using the measured orbit offset in the LEP arcs is applied to track the change in f_c^{RF}.

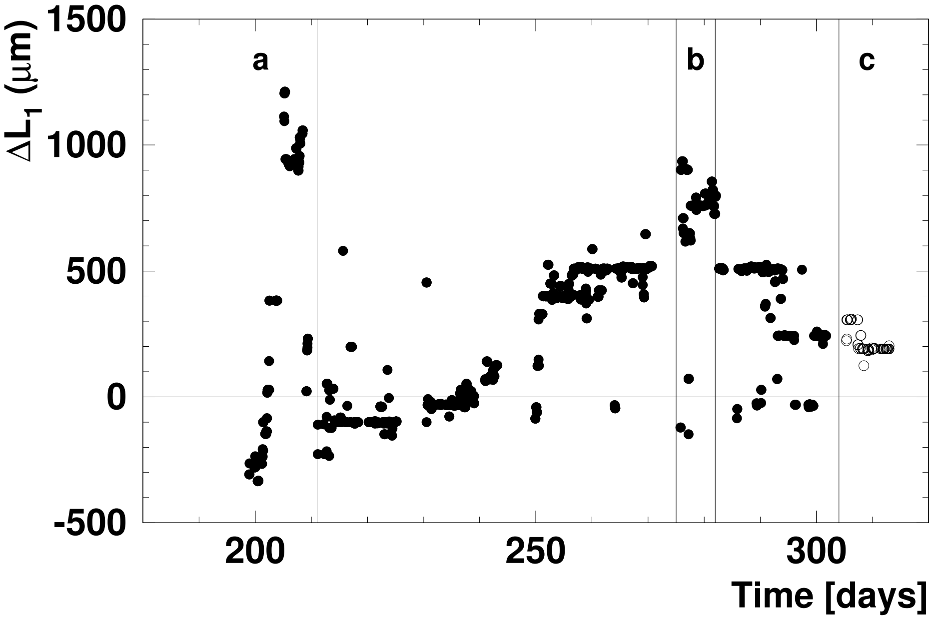


Figure 8: Evolution of the orbit lengthening ΔL_1 during the 1997 LEP run. In period (a) the beam energy is 45 GeV, in period (b) 65 and 68 GeV. In period (c) the 102/90 optics was tested at 91.5 GeV. All other data correspond to beam energies between 91 and 92 GeV with the 90/60 optics. For $\Delta L_1 = 500\ \mu m$, the energy shift is about 11 MeV at 91.5 GeV with the 90/60 optics.

6.2 Horizontal Corrector Effects

Horizontal correctors influence the beam energy either through a change of the integrated dipole field or through a change of the orbit length ΔL_1 [5]. In general the two effects could be mixed and cannot be easily disentangled, although simulations show that the orbit lengthening effect should dominate. For a given orbit and corrector settings, the predicted energy shifts can differ by 30% between the two models, which implies that a 30% error should be applied to the energy shifts predicted for the corrector settings.

In general the settings of the horizontal correctors change between optics and between physics and energy calibration energies. A correction for this effect must therefore be applied to interpolate beam energies from calibration to physics runs. For an orbit lengthening ΔL_1 the energy change is :

$$\frac{\Delta E}{E} = -\frac{\Delta L_1}{\alpha C} \tag{9}$$

where C is the LEP circumference. Figure 8 shows the evolution of ΔL_1 in physics for the 1997 LEP run. The size of the effect is somewhat larger than in previous years: for a large fraction of the run ΔE reaches approximately 11 MeV. The contributions of the horizontal correctors to the energies measured by RDP for the 90/60 and 60/60 differ by about 4 MeV at 45 GeV beam energy. Since this correction is applied in the model of the beam energy, it is naturally taken into account in the NMR calibrations.

Recent simulations predict that the orbit corrector settings should *not* influence the central frequency by more than 0.5 Hz, and were confirmed by measurements made during the 1998 LEP run. However, at LEP1, after the $x_{\rm arc}$ correction for the varying central frequency was applied for each fill, only relative changes from the orbit corrector settings were taken into account during a fill [1]. The typical size of these corrections was small, around 1 MeV. In view of the new simulations, a correction for $x_{\rm arc}$ and for the energy shifts due to the horizontal corrector configurations are now applied.

The average beam energy shift from the orbit corrector settings is 6 MeV for the high energy running in 1997, which is much larger than in previous years. The 30% model uncertainty would imply an error of 2 MeV. This is conservatively increased to half of the total correction, 3 MeV, in view of the evolving understanding of the interplay of central frequency and orbit corrector effects.

6.3 Earth tides

The model of earth tides is well understood from LEP1 [1]. It should be noted that the amplitude of the tide effect is proportional to energy, and so is larger at LEP2.

6.4 Effects from the main bend dipoles

The average of the NMR's is used to take into account changes is energy due to temperature and parasitic currents. Bend modulation [1] was not performed in 1996, but in 1997 was routinely carried out at the start of each fill from 5 August (fill 3948). The average rise in dipole field during a fill was therefore smaller in 1997 than in 1996: the total dipole rise effect on the average beam energy was only 3.5 MeV. From experience at LEP1, and the fact that the 16 NMR's give a good sampling of the whole ring, the uncertainty is expected to be less than 25% of the effect, so 1 MeV is assigned as the uncertainty due to the dipole rise.

For one or two fills, there was an error in the bend modulation sequence such that it terminated at the wrong excitation current, leading to a slightly different initial bending field. Also, a small drift in the dipole excitation current for the same nominal setting was observed during the year. These effects lead to an rms of 11 MeV for 148 fills with a nominal beam energy of 91.5 GeV. Interpreted as an error on the modelling this would give an uncertainty of sub 1 MeV. However there are a couple of fills where the initial energy was confused by the initial records having an energy from the end of the ramp, leading to an error on the mean of 2 MeV.

6.5 RF frequency shifts

For the first time in 1997, the RF frequency was routinely increased from the nominal value to change the horizontal damping partition number [6]. This is a useful technique at high energy, causing the beam to be squeezed more in the horizontal plane, and increasing the specific luminosity, whereas at lower energy, beam-beam effects prevent the horizontal beam size reduction. Less desirable side effects are that the central value of the beam energy decreases, the beam energy spread increases, and slightly more RF accelerating voltage is needed to keep the beam circulating. A frequency shift of +100 Hz gives an energy decrease of about 150 MeV. Occasionally, when an RF unit tripped off, the operators would temporarily decrease the RF frequency to keep the beam lifetime high, in which case the energy values were immediately recalculated instead of waiting the usual 15 minutes.

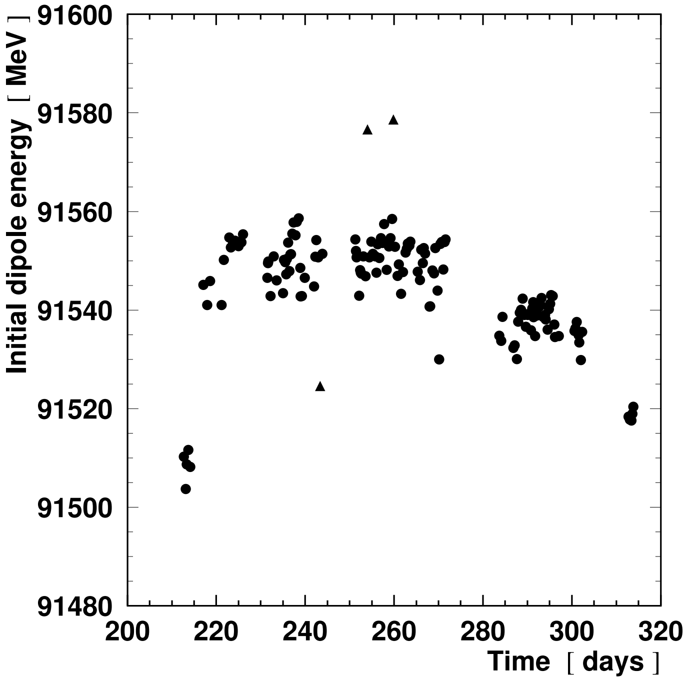


Figure 9: Initial dipole field for all 183 GeV fills. The fills marked with a triangle have anomalous initial currents in the main bend dipoles.

7 IP specific corrections

As at LEP1, corrections to the centre-of-mass energy arise from the non-uniformity of the RF power distribution around LEP and from possible offsets of the beam centroids during collisions in the presence of opposite-sign vertical dispersion[1].

7.1 Corrections to E_{CM} from the RF System

Since the beam energy loss due to synchrotron radiation is proportional to E^4_{beam}, operation of LEP2 requires a large amount of RF acceleration to maintain stable beam orbits. To provide this acceleration, new super-conducting (SC) RF cavities have been installed around all of the experiments in LEP. This implies that, contrary to LEP1, the exact anti-correlation of RF effects on the beam energy at IP4 and IP8 is no longer guaranteed, and that large local shifts in the beam energy can occur at any of the IPs. This also implies that the energy variation in the beams (the "sawtooth") as they circulate around LEP is quite large (see Figure 10.), which increases the sensitivity of the centre-of-mass energy to non-uniformities in the energy loss arising from differences in the local magnetic bend field, machine imperfections, etc.

The modelling of the energy corrections from the RF system is carried out by the iterative calculation of the stable RF phase angle ϕ_s which proceeds by setting the total energy gain of the beams as they travel around the machine equal to all of the known energy losses. The measured value of the synchrotron tune Q_s and the energy offsets between the beams as they enter and leave the experimental IPs are used to constrain energy variations due to overall RF voltage scale and RF phase errors. In particular, the

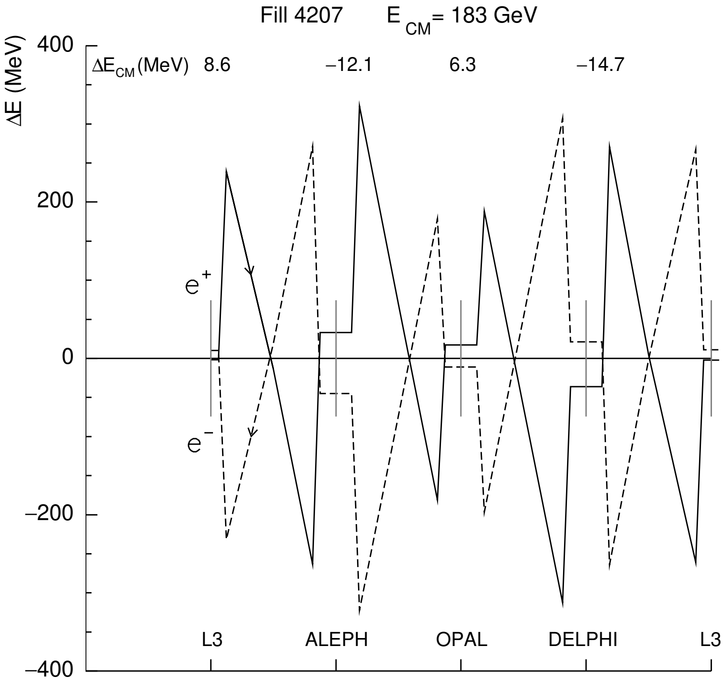


Figure 10: The evolution of the energy of each of the counter-rotating beams as they circulate in LEP, the so called *energy sawtooth*. The electron beam is represented by the left-going dotted line, the positron beam by the right-going solid line. The light grey lines mark the positions of the IPs. $\Delta E = 0$ denotes the average energy of LEP, i.e. that given by the rest of the energy model. The energy gains provided by the RF are clearly visible. This configuration represents a typical RF complement and centre-of-mass energy corrections for the 1997 running.

phase error at each IP is set using the size of the energy sawtooth measured by the LEP BOM system compared with the total RF voltage at that IP. This is a powerful constraint on potential phasing errors, as was seen in the LEP1 analysis.

The average corrections for the 1996 and 1997 running are shown in Table 2. The corrections for running below the WW threshold are typically smaller by a factor of two, with correspondingly smaller errors. The bunchlet-to-bunchlet variation in corrections is negligible.

As at LEP1, the errors on the energy corrections are evaluated by a comparison of those quantities (Q_s, the orbit sawtooth, and the longitudinal position of the interaction point) which can be calculated in the RF model and can be measured in LEP. In addition, uncertainties from the inputs to the model, such as the misalignments of the RF cavities and the effects of imperfections in the LEP lattice, must also be considered. Since many of the uncertainties on the RF corrections scale with the energy loss in LEP, however, the overall uncertainty due to RF effects is larger at LEP2 than at LEP1. Note that the errors are given below in terms of $E_{\rm beam}$, which is obtained by dividing the error on $E_{\rm CM}$ by two.

Comparison of the measured and calculated Q_s values reveals a discrepancy in the modelled and measured overall RF voltage seen by the beam. The difference is small, on the order of 4%, which can be explained by an overall scale error in the measured voltages or a net phase error in the RF system of a few degrees. An overall scale error changes

LEP IP	ΔE_{CM} (MeV) $E_{\mathrm{CM}} = 161$ GeV	$E_{\mathrm{CM}} = 172$ GeV	$E_{\mathrm{CM}} = 183$ GeV
2	19.7	18.8	8.1
4	–5.5	–5.8	–10.8
6	20.2	19.5	5.9
8	–9.3	–8.0	–13.3

Table 2: The average corrections (in MeV) to E_{CM} for each LEP IP and each energy point for the 1996 and 1997 RF models.

the energy corrections by a corresponding amount (*i.e.*, a 10 MeV correction acquires an error of 0.4 MeV, which is negligible), whereas phase errors can shift the energy by larger amounts at the IP closest to the error. For 1996, the overall error due to this mismatch was computed by assuming the entire phase error was localised to one side of an IP, and the largest shift taken as the error for all IPs (4 MeV E_{beam}). For 1997, the total energy gain was normalised so that Q_s was correct, and the phase errors for each IP were calculated using the orbit measurement of the local energy gain. This resulted in a smaller error of 1.5 MeV on E_{beam} from the voltage scale and phasing effects.

The positions of all of the RF cavities in LEP have been measured repeatedly using a beam-based alignment technique with a systematic precision of 1 mm and a 1 mm rms scatter over time. The systematic error on the energy corrections is evaluated by coherently moving the RF cavities by 2 mm away from (towards) the IPs, and observing the change in E_{CM}. This results in a 1.5 MeV error on E_{beam} for 1996 and 1997.

Recently, a study of the effects of imperfections in the LEP lattice on the energy loss of the beams at LEP2 has been performed [7]. Calculations of the centre-of-mass energy in an ensemble of machines with imperfections similar to those of LEP yields an rms spread of 2.5 MeV E_{beam} in the predicted energy at the IPs due to non-uniformities in the energy loss of the beams. These shifts only depend on the misalignments and non-uniformities of the magnetic elements in LEP, which are essentially unmeasureable, and is not contained in any of the other error sources.

In order to keep the error estimate as conservative as possible, the error on the energy corrections from the RF should be considered 100% correlated between IPs and energy points. The total error from RF effects for each energy point is given in Table 3.

7.2 Opposite sign vertical dispersion

In the bunch train configuration, beam offsets at the collision point can cause a shift in the centre-of-mass energy due to opposite sign dispersion [1]. The change in energy is evaluated from the calculated dispersion and the measured beam offsets from beam-beam deflections scans.

The dispersions have been calculated using MAD for all the configurations in 1996 and 1997. No dedicated measurements of dispersion were made in 1996, while in 1997 the measured values agree with the prediction to within about 50%. This is the largest cause of uncertainty in the possible error correction. Beam offsets were controlled to within a few microns by beam-beam deflection scans. The resulting luminosity weighted correction to the centre-of-mass energies are typically 1 to 2 MeV, with an error of about 2 MeV. No corrections have been applied for this effect, and an uncertainty of 2 MeV has been assigned.

8 Summary of systematic uncertainties

The contributions from each source of uncertainty described above are summarised in table 3.

Source	Error [MeV]
Extrapolation from NMR–polarisation	
NMR rms/sqrt(N) at physics energy	10
Different E_{pol} fills	5
Flux-loop test of extrapolation	
NMR flux-loop difference at physics energy	20
Field not measured by flux loop	5
Polarisation systematic	1
e^+e^- energy difference	2
Optics difference	4
Corrector effects	3
Tide	1
Initial dipole energy	2
Dipole rise modelling	1
IP specific corrections ($\sigma(E_{\mathrm{CM}})/2$)	
RF model	4
Dispersion	2
Total	25

Table 3: Summary of contributions to the beam energy uncertainty.

The first groups describe the uncertainty in the normalisation derived from NMR-polarisation comparisons, NMR-flux-loop tests and the part of the bending field not measured by the flux-loop. These extrapolation uncertainties dominate the analysis.

The following errors concern the polarisation measurement, specifically its intrinsic precision (which is less than 1 MeV), the possible difference in energy between electrons and positrons, and the difference between optics.

None of the additional uncertainties from time variations in a fill, and IP specific corrections contribute an uncertainty greater than 5 MeV.

8.1 Uncertainty for data taken in 1996

The analysis of the 1996 data was largely based on a single fill with RDP measurements at 45 and 50 GeV. The apparent consistency of the flux-loop and NMR data compared to RDP data was about 2 MeV over this 5 GeV interval, i.e. a relative error 4×10^{-4}, which using a naive linear extrapolation would give an uncertainty of 13.5 (15) MeV at 81.5 (86) GeV. These errors were inflated to 27 (30) MeV before the 1997 data were available, since there was no test of reproducibility from fill to fill, there was no check of the non-linearity possible from a fill with two energy points, and the field outside the flux loop had not been studied. Although more information is available in 1997, we retain this larger uncertainty for the 1996 data, partly motivated by the sparsity of RDP measurements in that year, and also by the observation that the single parameter fit that was used lead to a larger scatter of predicted energies in physics when applied to the 1997 data.

The uncertainties for the two 1996 data samples can be assumed to be fully correlated. However, the extrapolation uncertainty for the 1997 data is somewhat better known. Since the energy difference between the maximum RDP energy and the physics energy is nearly the same in the two years, it can be assumed that the 25 MeV uncertainty of 1997 data is common to the 1996 data.

8.2 Lower energy data taken in 1996 and 1997

During 1996 and 1997, LEP also operated at the Z resonance, to provide data samples for calibrating the four experiments, and at intermediate centre-of-mass energies, 130–136 GeV, to investigate effects seen at the end of 1995 at “LEP 1.5”. The dominant errors on the beam energy are from the extrapolation uncertainty, and scale with the difference between physics energy and RDP energy. The optics difference scales in the same way. Several other effects such as the tide correction are proportional to the beam energy. The dipole rise per fill depends in addition on whether bend modulation was carried out at the start of fill. The total beam energy uncertainties are found to be 6 MeV for Z running, and 14 MeV for LEP 1.5 running.

9 Conclusions and outlook

The method of energy calibration by magnetic extrapolation of resonant depolarisation measurements at lower energy has made substantial progress with the 1997 data. The huge improvements in establishing polarisation above the Z have allowed a robust application of the method, and the mutual consistency of the RDP, NMR and flux-loop data has been established at the 20 MeV level at physics energy, with a total systematic uncertainty in the beam energy of 25 MeV.

In future, a new apparatus will be available for measuring the beam energy. The LEP Spectrometer Project [8] will measure the bend angle of the LEP beams using beam pick ups with new electronics to measure the position to the order of a micron precision as they enter or leave a special dipole in the LEP lattice whose bending field has been surveyed with high precision. A first phase of the spectrometer is already in place for the 1998 running, with the aim of checking the mechanical and thermal stability of the position measurement. In 1999, the new magnet will be installed, and the aim is to use this new, independent method to measure the beam energy to 10 MeV at high energy.

It should be possible to propagate any improvement in the beam energy determination back to previous years by correcting the extrapolation and correspondingly reducing the uncertainty.

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