EUROPEAN LABORATORY FOR PARTICLE PHYSICS

CERN - SL DIVISION

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Measurement of Tidal Deformations of the LEP Ring with Closed Orbits

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Abstract

Terrestrial tides due to the sun and the moon deform the LEP ring and lead to measurable effects on beam energies and on closed orbits. In this note the closed orbit measurements in the LEP arcs are used to measure directly the tidal deformation of the LEP ring. A change in circumference of $-4.1 \pm 0.2\ \mu$m is found for a relative increase in the local gravity of 10^{-9}, in good agreement with measurements from energy calibration by resonant depolarization and geological observations.

Geneva, Switzerland

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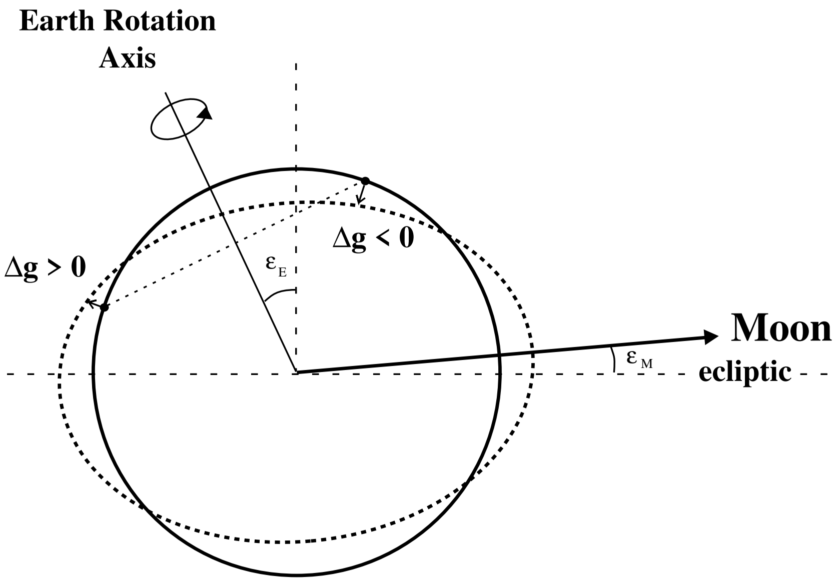

Figure 1: Tidal deformation of the Earth surface due to the presence of the moon. The change in gravity Δg is indicated for an observer at the latitude about 45° North. No sun tides have been drawn. The inclination of the Earth rotational axis ϵ_E with respect to the plane of the ecliptic is 23.5°. The inclination of the lunar orbital plane ϵ_M is 5.1°.

1 Introduction

The influence of terrestrial tides on LEP was discovered in 1991 when the beam energy calibrations were performed for the first time using the technique of resonant depolarization [1, 2]. More studies performed during the 1992 and 1993 LEP runs demonstrated that the beam energy variations were in good agreement with the predictions of tide models [3, 4]. The change in circumference induced by the tides was also observed directly on the closed orbits [5, 6, 7], but up to now accurate measurements of the effects of tides on LEP have only been made using resonant depolarization. The analysis of this data requires however a good understanding of all other parameters which influence the behaviour of the beam energy during LEP fills. Although the experiments used to determine the sensitivity of the energy to tides seemed to be rather well understood, the discovery of a continuous beam energy rise of about 5 MeV in the first 12 hours after the magnet ramp is a strong motivation to find other observables to cross-check the understanding of the tides. For this reason the closed orbits are used in this note to measure accurately the change in circumference of the LEP ring due to tides.

2 The Tides and LEP

Tides are caused by the $1/r^2$ dependence of the gravitational attraction of the sun and the moon. The presence of a celestial body leads to a quadrupolar deformation of the surface of the Earth and to a change in radius ΔR

$$\Delta R \sim \frac{M}{a^3} P_2(cos\theta) = \frac{M}{2a^3}(3cos^2\theta - 1) \qquad (1)$$

where θ is the angle between the vertical and the celestial body and a is the distance to the body of mass M. P_2 is the 2^{nd} order Legendre polynomial. Figure 1 shows a sketch of the

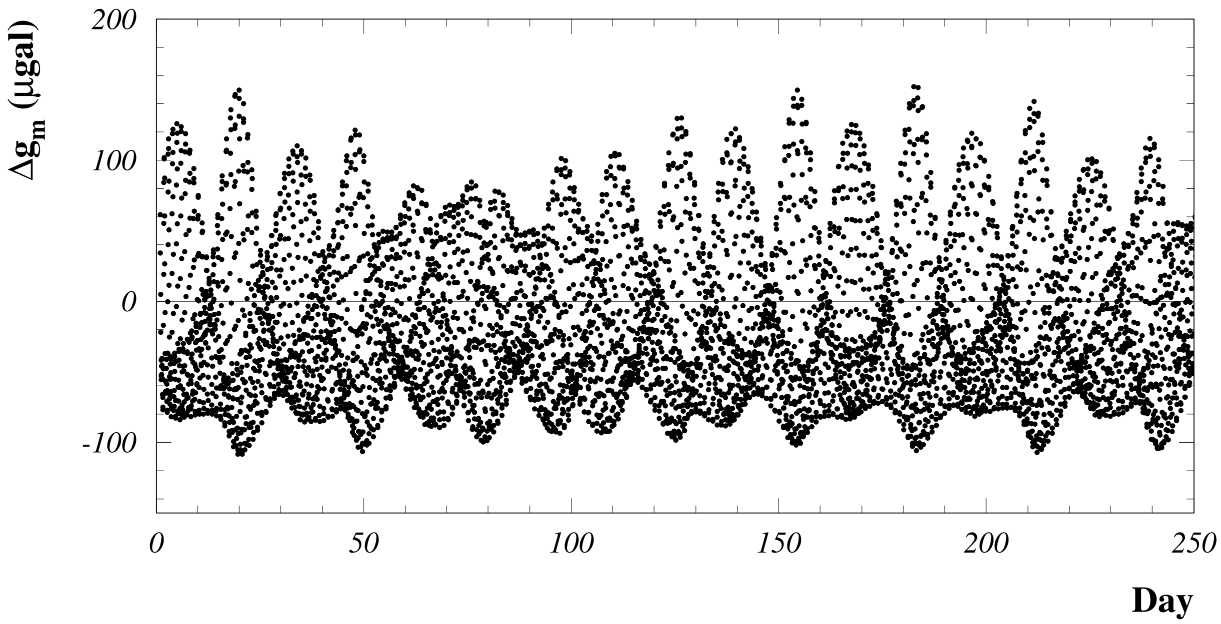

Figure 2: Correction to the local gravity Δg_m predicted by the CTE tide model for LEP (1 μgal = 10^{-8} m s^{-2}). The time scale starts on January 1^{st} 1996. The largest amplitudes occur at periods of full moon and new moon which alternate every 14 days. The first high tide on January 5 *th* corresponds to full moon. The modulation of the amplitudes is due to the eccentricity of the lunar orbit and the seasonal variation of the angle between the vertical and the sun.

lunar tide bulge. Sun tides are about 50% weaker than moon tides. Under the influence of the tidal forces the earth surface moves vertically by about 25 cm in the area around Geneva. The associated horizontal strain ϵ modifies the LEP circumference C by up to 1 mm (peak-to-peak).

Since in LEP the particles are ultra-relativistic, their orbit length is determined by the frequency of the RF system which is normally kept constant. As a consequence of the ring deformation, the beam is forced to change it position in the quadrupoles and the additional bending field leads to a change in energy ΔE :

$$\frac{\Delta E}{E} = -\frac{1}{\alpha}\frac{\Delta C}{C} = -\frac{\epsilon}{\alpha} \tag{2}$$

where α is the momentum compaction factor. For the $90^\circ/60^\circ$ phase advance LEP optics used between 1993 and 1995, $\alpha = 1.86 \cdot 10^{-4}$.

The influence of the tides on the beam energy could be described with a CTE (Cartwright-Taylor-Edden) model which includes the 505 main harmonics of the tides [8]. A computer program based on such a tide model, where some of the parameters have been adjusted with actual gravity measurements, is available to estimate the tides at a given time [9]. This program actually gives the *correction* Δg_m to be applied to the measured local gravity to obtain the mean gravity. The true change in gravity Δg has therefore the opposite sign : $\Delta g = -\Delta g_m$. Δg_m will often be used in this document to be consistent with the definitions used in reference [4] and by the LEP Energy Working Group. Figure 2 shows the predicted tide corrections for 1996. The coefficient κ_T relating the energy change to Δg_m has been measured in the various energy calibration experiments [4] :

$$\frac{\Delta E}{E} = \kappa_T \Delta g_m \qquad\qquad \kappa_T = (-8.6 \pm 0.8) \cdot 10^{-7}/\mu\,\mathrm{gal} \tag{3}$$

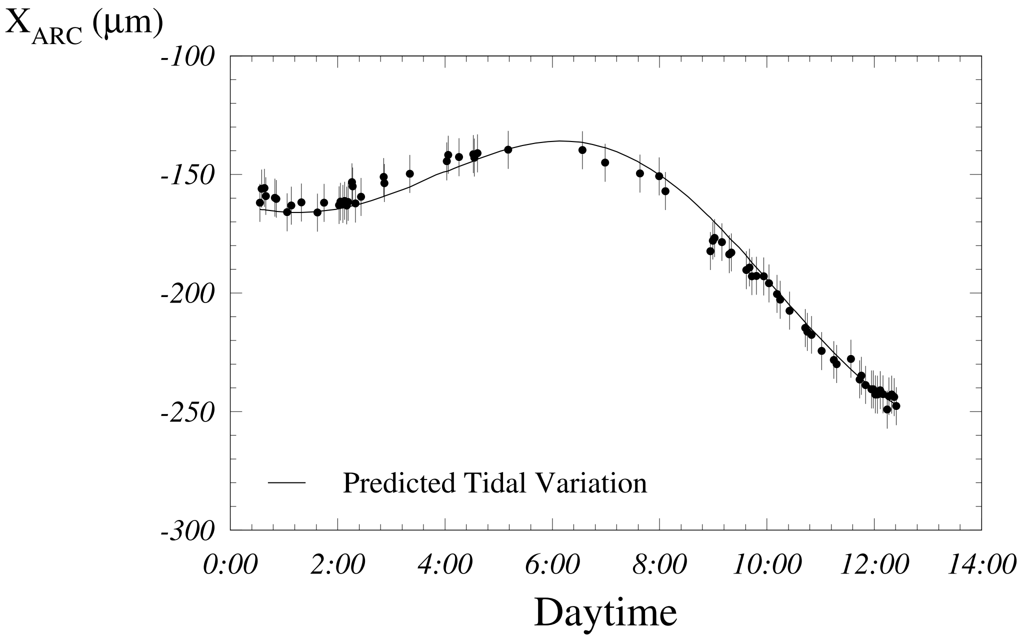

Figure 3: Example of the evolution of X_{ARC} during a LEP fill. The data points follow the variation expected from the CTE tide model and the parameters (κ_T, ω_T) measured by resonant depolarization.

Δg_m is usually expressed in gal (1 gal = 1 cm s^{-2}). At high tides $|\Delta g|$ reaches about 140 μgal in the area around LEP (Figure 2). The change in circumference is :

$$\frac{\Delta C}{C} = -\alpha \, \kappa_T \, \Delta g_m \tag{4}$$

Using the average acceleration of gravity $g = 9.80$ m s^{-2}, we obtain the fraction of gravity change λ_T that couples into horizontal strain :

$$\epsilon = \frac{\Delta C}{C} = \lambda_T \, \frac{\Delta g}{g} \qquad\qquad \lambda_T = \alpha \, \kappa_T \, g = -0.16 \pm 0.2 \tag{5}$$

This value of λ_T is in good agreement with geological observations [8]. The maximum strain ϵ is $\approx \pm 2.2 \cdot 10^{-8}$. This is by all standards a very tiny deformation.

The horizontal movement of the beam relative to the quadrupoles can be observed directly with the Beam Orbit Measurement system (BOM). To enhance the sensitivity and reduce the systematic errors, the horizontal beam position is averaged over the BPMs (Beam Position Monitors) located in the regular arc FODO cells between PU.QD20 and PU.QD48 [6, 7] :

$$X_{ARC} = \frac{1}{N} \sum_{j \in ARC} X_j \tag{6}$$

Up to $N = 240$ BPMs can be used for the average. The horizontal dispersion D_x is ideally identical at all the selected BPMs. X_{ARC} varies with the tides according to :

$$\Delta X_{ARC} = D_x \frac{\Delta E}{E} = \omega_T \, \Delta g_m \qquad\qquad \omega_T = D_x \, \kappa_T \tag{7}$$

Figure 3 shows an example of a tidal distortion of the ring observed with the BOM system. The circumference change can be calculated from ΔX_{ARC} using the relation

$$\Delta C = -\frac{C \, \alpha}{D_x} \, \Delta X_{ARC} \tag{8}$$

leading to

$$\Delta C = \Omega_T \, \Delta g_m \qquad\qquad \Omega_T = -\frac{C \, \alpha \, \omega_T}{D_x} \tag{9}$$

The negative sign in the two preceding equations has to be inserted to take into account that X_{ARC} measures the movement of the beam *relative* to the BPMs. But it is in fact the BPMs which move with the ring. Equation 9 is of course identical to Equation 5.

The analysis of the tides with the closed orbits and with resonant depolarization have different problems and advantages. The measurements by resonant depolarization are delicate because the LEP beam energy is sensitive to other parameters that are subject to change during the experiments. The corresponding corrections must be well understood and carefully removed from the data. The data sample is also very limited. The BOM measurements can suffer from orbit corrections and bad monitor readings. These problems tend to be randomly distributed in time which is not the case for the corrections to the beam energy data. The orbit data sample is also very large since any closed orbit acquired while LEP is in coast can be used for the analysis. Both techniques are of course sensitive to ring deformation that have a different origin than tides [7].

3 Analysis of the Closed Orbits and Results

This study is based on the same tide model that is used to describe the beam energy variations. A data sample of about 1700 closed orbits acquired during physics fills in 1994 and 1995 has been used for the analysis. The sample of ARC BPMs is scanned for channel which show frequent problems or a suspicious behaviour. Such BPMs (about 40 out of the 240) are completely removed from the analysis.

In a first step, for each orbit j belonging to fill i, the measured position $X_{ARC,ij}$ is corrected for the tides

$$X^c_{ARC,ij} = X_{ARC,ij} - \omega_T \Delta g_m \tag{10}$$

and the average tide-corrected position is calculated for the n_i orbits of each fill i :

$$\overline{X}^c_{ARC,i} = \frac{1}{n_i} \sum_{j=1}^{j=n_i} X^c_{ARC,ij} \tag{11}$$

The coefficient ω_T is adjusted by minimising the sum of the residuals $\delta_{ARC,ij}$ around the mean position for all orbits :

$$\sum_{i,j} \delta^2_{ARC,ij} = \sum_{i,j} \left(X^c_{ARC,ij} - \overline{X}^c_{ARC,i} \right)^2 \tag{12}$$

Slow long term variations of the LEP circumference do not affect the minimisation since $\delta_{ARC,ij}$ is only sensitive to changes that happen on the time scale of a physics fill. The distributions of the residuals is shown after optimisation of ω_T in Figure 4. The RMS spreads are 7.8 and 5.4 μm for 1994 and 1995 respectively. The difference is probably due to an improvement of the resolution of the BPM ADCs in 1995 [10]. To extract κ_T from ω_T, the horizontal dispersion D_x must be measured using a change in RF frequency :

$$\Delta X_{ARC} = -\frac{D_x}{\alpha f_{RF}} \, \Delta f_{RF} = S_{RF} \, \Delta f_{RF} \tag{13}$$

Figure 5 shows a measurement of S_{RF}. α is obtained from energy calibrations by resonant depolarization. Results for S_{RF}, α and D_x are given in Table 1 for 1994 and 1995. The

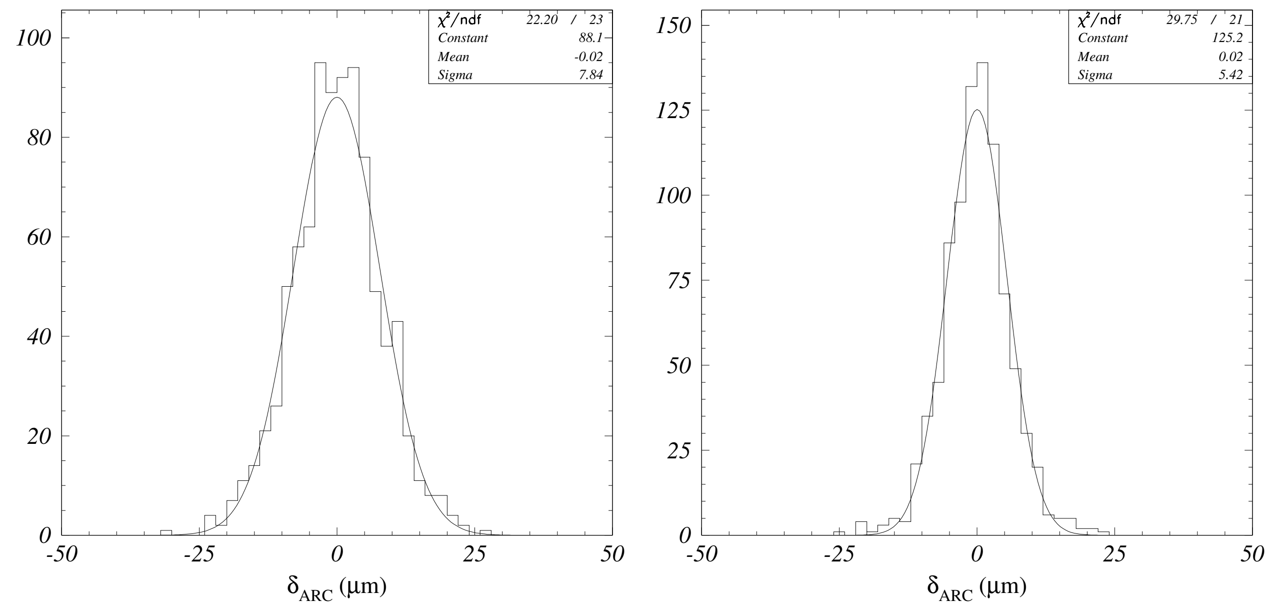

Figure 4: Distribution of the residuals $\delta_{ARC,ij}$ after optimisation of the tide coefficient for the 1994 (left) and the 1995 (right) data samples.

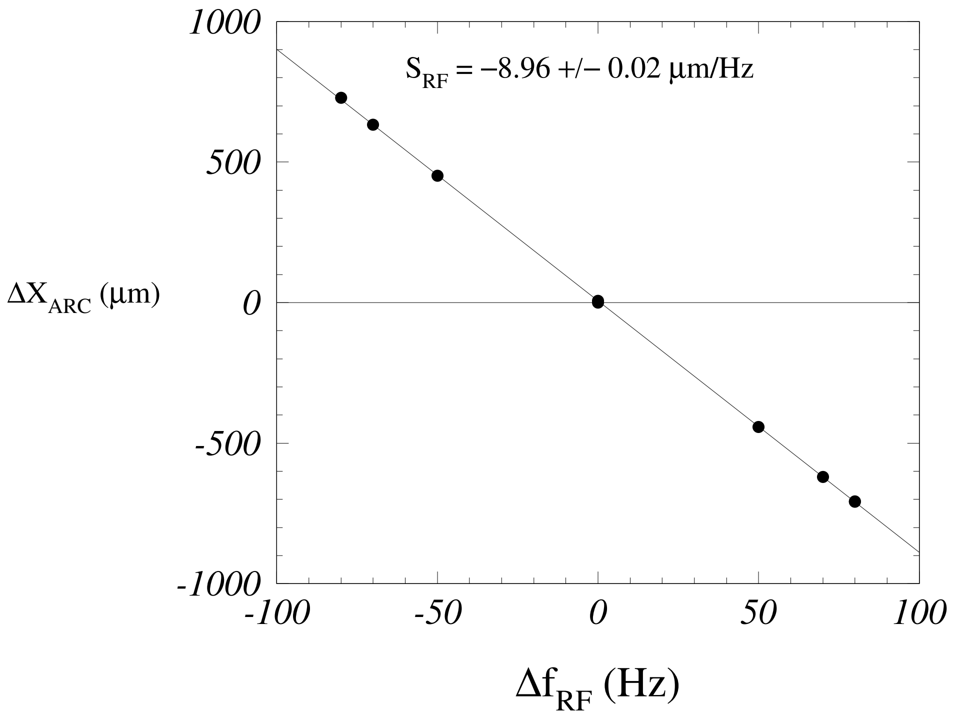

Figure 5: Example of a measurement of the dependence of X_{ARC} on the RF frequency in 1995. The linearity is excellent and the RMS scatter of ΔX_{ARC} around the fitted line is smaller than 2μm.

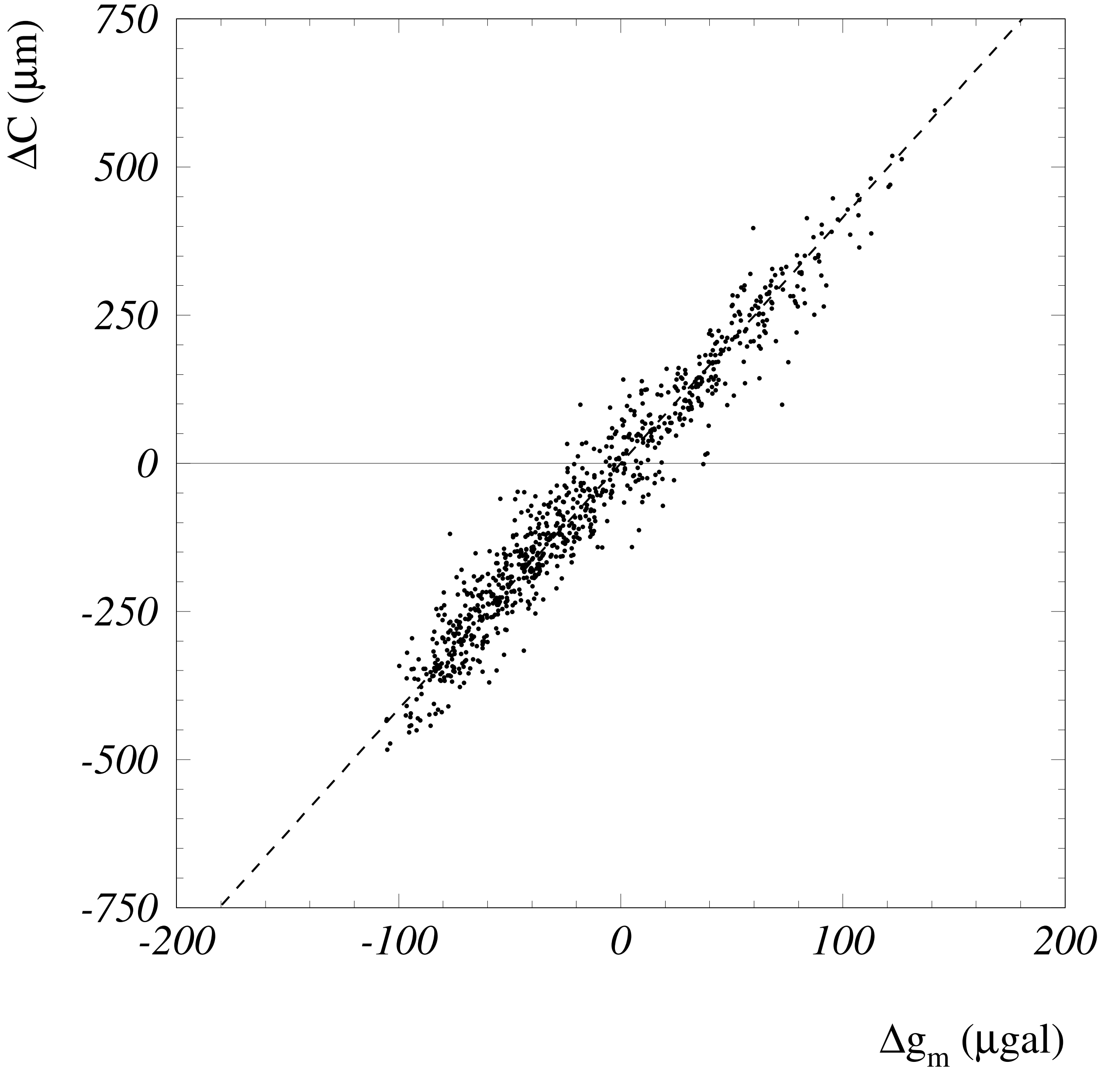

Figure 6: Correlation between the change in circumference ΔC measured with the BOM system and the tidal gravity correction Δg_m predicted by the CTE tide model. The slope is 4.1 ± 0.2 μm/μgal.

Table 1: Comparison of measured and calculated values of S_{RF}, α and D_x for the 90°/60° phase advance optics of 1994 and 1995. The results of the MAD program for the k05p46 and l05p46 optics are identical.

	S_{RF} (μm/Hz)	α (10^{-4})	D_x (mm)
1994	-9.06 ± 0.03	1.836 ± 0.040	586 ± 13
1995	-8.95 ± 0.01	1.864 ± 0.014	588 ± 5
MAD	-9.13	1.856	597

Table 2: This table compares the BOM results for ω_T, Ω_T and κ_T with the measurements from resonant depolarization. The BOM measurements are dominated by systematic errors.

Data Set	ω_T (μm/μgal)	Ω_T (μm/μgal)	κ_T ($10^{-7}/\mu$gal)
BOM 1994	-0.48 ± 0.03	4.0 ± 0.2	-8.2 ± 0.4
BOM 1995	-0.49 ± 0.03	4.1 ± 0.2	-8.3 ± 0.4
BOM Average	-0.49 ± 0.02	4.1 ± 0.2	-8.3 ± 0.3
Res. Depolarization	-0.51 ± 0.05	4.3 ± 0.4	-8.6 ± 0.8

data agrees with calculations using the MAD [11] program for the 90°/60° phase advance lattices within about 2%. We will use this difference as an estimate for the systematic error on D_x and on the average BPM calibration. With the parameters of Table 1, we are able to extract the coefficients κ_T and Ω_T from ω_T. The results are given in Table 2. The LEP circumference changes by -4.1 ± 0.2 μm for a change in gravity Δg of 1 μgal ($\Delta g/g = 10^{-9}$). The BOM measurements are dominated by the systematics errors due to the tails in the residuals distributions. These errors are independent for 1994 and 1995. The systematic error of 2% on D_x and S_{RF} has been assumed to be correlated for 1994 and 1995. The measurements made with the closed orbits agree well with the results from resonant depolarization.

Figure 6 shows the measured change in circumference ΔC due to the tides as a function of the tidal gravity variations predicted by the CTE tide model. ΔC has been calculated with the help of Equation 8 from the difference $X_{ARC,ij} - \overline{X}^c_{ARC,i}$.

4 Conclusion

A sample of closed orbits has been used for a quantitative determination of the deformation of LEP due to terrestrial tides. The measured decrease of the LEP circumference of 4.1 ± 0.2 μm for a relative increase in the local gravity of 10^{-9} is in good agreement with previous measurements by resonant depolarization and with geological observations.

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