Nonlinear Response of Orbit Monitors

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Abstract

The LEP Spectrometer is used to determine beam energy by measuring the bending angle of the beam in a dipole magnet, using six beam position monitors (BPMs), which must have an accuracy close to 1 μm. The BPMs feature an Al block with an elliptical aperture and four capacitive pickup electrodes; their response depends on the pickup geometry, the aperture shape and the size of the beam. The beam size varies from BPM to BPM, which may give shifts of the measured position. We have investigated the implications of such shifts on the Spectrometer performance. We summarise our current understanding of the BPM behaviour using both a computer model of their response and measurements.

1 INTRODUCTION

The goal for the beam energy calibration at LEP is an uncertainty below 15 MeV per beam. Up to 60 GeV, resonant spin depolarisation (RDP) is used for calibration [1], whilst higher energies are deduced from the bending field [2]. Only part of the bending field is sampled, so that the LEP Spectrometer was proposed to give an alternative method.

The Spectrometer consists of a dipole magnet with three BPM stations at each side, to measure the bending angle. It is calibrated at low energy against RDP, avoiding the need for an absolute angle measurement. The target accuracy sets a limit on the tolerable BPM random error of 1 μm. Each BPM consists of four button electrodes mounted in an Al block with an elliptical aperture.

It has been shown that the beam size affects the response of BPMs in a circular beam pipe [3]. Since the pipe is an equipotential, the BPM response depends on the aperture shape as well as the pickup geometry.

We have re-derived analytically [4] the BPM response for a round pipe, finding a difference between our result and that given in [3]. We have used a numerical model for both circular and elliptical pipes, confirming our analytical solution and drawing conclusions about the effect on the Spectrometer from BPM nonlinearities. In addition, the work suggests possible methods for beam-based alignment of the BPMs and for beam size measurement.

2 BPM RESPONSE TO A GAUSSIAN BEAM

Analytical Results For a Circular Aperture

The full analytical derivation is given elsewhere [4]. The beam was represented as many infinite line charges with Gaussian distributions in x and y. For a line charge within a pipe radius a, an image charge may be defined such that, under the influence of the two charges, the pipe is an equipotential. For one such line charge at the position (r, ϕ) and its image at $(a^2/r, \phi)$, the field normal to the pipe surface at an angle θ to the x-axis is given by:

$$E_{total} \; = \; E_\lambda \frac{a^2 \; - \; r^2}{a^2 + r^2 - 2ar\cos(\theta - \phi)}$$

where $E_\lambda \; = \; \lambda/2\pi\epsilon_0 a$ is the field due to an infinite line charge with charge per unit length λ. This expression may be expanded in a power series so that, for a beam which is Gaussian in both transverse directions, we have:

$$E \; = \; \frac{E_\lambda}{2\pi\sigma_x\sigma_y} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[1 + 2\sum_{k=1}^{\infty}\left(\frac{r}{a}\right)^k \cos k(\theta-\phi)\right] \times \; \exp - \left[\frac{(x-\bar{x})^2}{2\sigma_x^2}\right] \exp - \left[\frac{(y-\bar{y})^2}{2\sigma_y^2}\right] dx\, dy$$

By writing the series explicitly, then writing $\cos\phi = x/r$ and $\sin\phi = y/r$ and using standard trigonometric identities, the integration results in $E \approx E_\lambda[1+2(E_2+E_4+E_6)]$, where the series has been truncated at the sextupole term and:

$$\begin{aligned}
E_2 &= \frac{\bar{x}}{a}\cos\theta + \frac{\bar{y}}{a}\sin\theta \\
E_4 &= \left(\frac{{\sigma_x}^2 - {\sigma_y}^2}{a^2} + \frac{\bar{x}^2 - \bar{y}^2}{a^2}\right)\cos 2\theta + \frac{2\bar{x}\bar{y}}{a^2}\sin 2\theta \\
E_6 &= \left[3\left(\frac{\sigma_x^2 - \sigma_y^2}{a^2}\right) + \left(\frac{\bar{x}^2 - 3\bar{y}^2}{a^2}\right)\right]\frac{\bar{x}}{a} cos3\theta \\
&+ \left[3\left(\frac{\sigma_x^2 - \sigma_y^2}{a^2}\right) + \left(\frac{3\bar{x}^2 - \bar{y}^2}{a^2}\right)\right]\frac{\bar{y}}{a} sin3\theta
\end{aligned}$$

Treating the BPM buttons as points, sustituting for θ gives the button signals, which may then be combined to give x and y outputs. The algorithm used by our electronics is:

$$X_{BPM} = [(S_{\pi/4} - S_{5\pi/4}) - (S_{3\pi/4} - S_{7\pi/4})]/T$$
$$Y_{BPM} = [(S_{\pi/4} - S_{5\pi/4}) + (S_{3\pi/4} - S_{7\pi/4})]/T$$

where the S are the button signals and T is their total; thus:

$$X_{BPM} = \sqrt{2}\,\frac{x}{a}\left[1 - \left(3\frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{x^2 - 3y^2}{a^2}\right)\right]$$

$$Y_{BPM} = \sqrt{2}\,\frac{y}{a}\left[1 - \left(3\frac{\sigma_y^2 - \sigma_x^2}{a^2} + \frac{y^2 - 3x^2}{a^2}\right)\right]$$

which predicts that the BPM response to a single line charge is nonlinear, tailing off at large displacements, and that the beam size becomes important for an off-centre beam. These analytical results were confirmed by a numerical model. An example of our analytical and numerical results is given in (Fig. 1), where we include a comparison with [3].

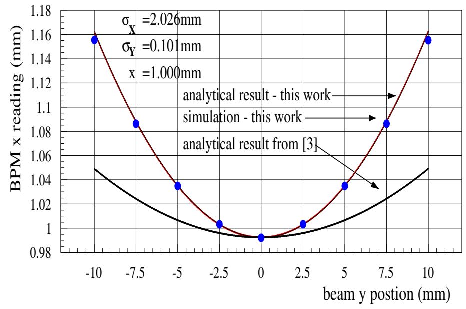

Figure 1: Comparison between analytical and numerical results

Numerical Results for an Elliptical Aperture

The conformal mapping $W = \arcsin{(Z/k)}$ maps an ellipse in the complex plane Z to the geometry of a parallel-plate capacitor in the complex plane W [5]. A line charge placed within the ellipse in Z appears within the capacitor in W, where the image charges in W may be found by considering multiple reflections in the plates. Each image charge may then be transformed back into Z. The above

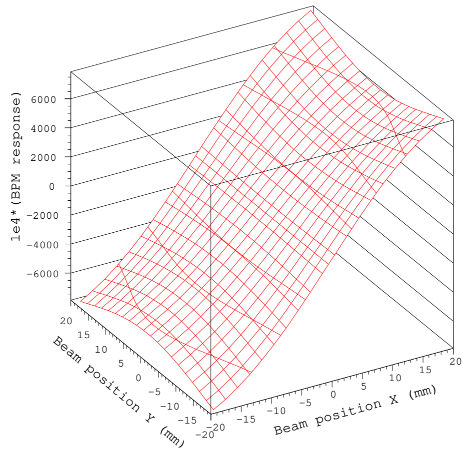

Figure 2: Response of an elliptical BPM: x output

procedure was implemented using the CERN library function RNORMX to generate 5000 charges within the beam. For each line charge, the image charge positions are calculated and used to obtain the field at any point on the pipe. The fields due to all such line charges are then added up.

The response of LEP BPMs has been characterised using an antenna mounted on an $x - y$ stage [6] and scanned in x and y. This scenario was simulated using the numerical model and the results compared with data [7]. The BPM output X_{BPM} from the model is shown in Fig. 2. The

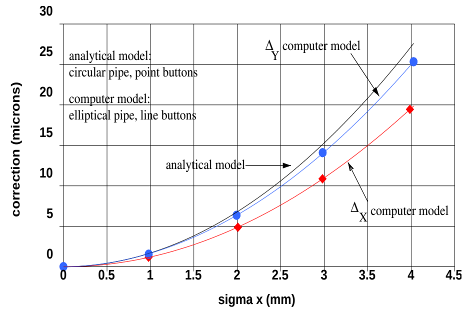

Figure 3: Corrections due to beam size for a circular and an elliptical vacuum chamber

model was found to agree with data to $\sim$5 %; representing the buttons by straight strips of length equal to their diameter, improved the agreement to ~ 2 %.

The effect of changing beam width on BPM response is shown in Fig. 3. The beam was taken to lie at a position of (1 mm,0) and the x correction calculated as a function of varying σ_x. The y correction was calculated for a beam at (0,1 mm). The results from the simulation are represented by the plot symbols. The elliptical BPM is less sensitive to beam size in x than in y, whilst larger buttons lead to a lower overall sensitivity to beam size, as may also be shown from the analytical model.

3 EFFECT OF THE NONLINEARITIES ON THE SPECTROMETER

Effect on the Energy Determination

During the LEP ramp, the beam size changes; it also varies from BPM to BPM with the beta functions and dispersion. Even for perfectly aligned BPMs, off-centre tracks would be reconstructed with an angle error. The BPMs may also

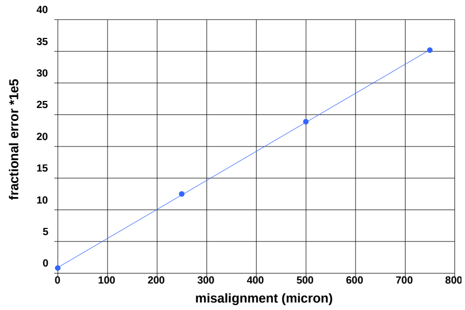

Figure 4: Fractional error on the bending angle due to BPM misalignment

be offset with respect to each other; if the pickups were linear, an error would not be introduced on the change in the bending angle. Again the change in beam size during the ramp introduces such an error, shown as a function of BPM offset in Fig. 4.

For this simulation it was assumed that the innermost BPMs were displaced away from, and the outermost BPMs towards, the centre of LEP by an equal amount; i.e. a worst case scenario. The alignment errors of the BPM blocks are expected to be randomly distributed with an RMS of $\sim$100 μm, so that a realistic value for the correlated error is 50 μm. The beam was assumed to lie at -100μm from the pickup centres at 45.0 GeV, drifting to +100μm after the ramp to 94.5 GeV.

Fig. 4 shows that, as long as the BPMs are aligned to the design tolerance, the systematic error due to BPM offsets will be acceptably small.

Using the Nonlinearities for Alignment

The analytical results for X_{BPM} and Y_{BPM} reveal the possibility of developing a beam-based alignment technique for the Spectrometer. If we are able to move each BPM in turn in x, a plot of Y_{BPM} (for some large value of y) versus distance moved will yield a parabola with minimum at the point where the BPM is centred on the beam. We

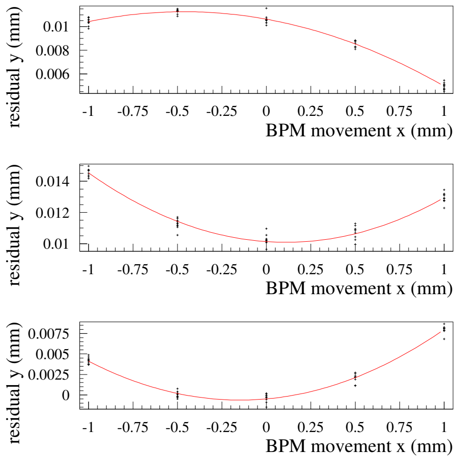

Figure 5: Beam-based alignment of the BPMs

expect some random beam movement during the procedure, which can be suppressed by replacing Y_{BPM} with the triplet residual for a Spectrometer arm; this is defined as: R_Y=$(Y_{BPM1}+Y_{BPM3})/2 - Y_{BPM2}$ and analogously for R_X. However, if the horizontal axis of the BPM is not exactly parallel to the direction of motion, the movement in x will affect the beam position y in the moving BPM. This could be dealt with by performing two scans, one at y and one at $-y$; subtracting one parabola from the other results in a parabola with the correct centre, even if the BPM is tilted.

Fig. 5 shows simulation results assuming 10 μm beam drifts between measurements, the random error on the BPM being taken as 0.3 μm; the BPM had a tilt of 2 mrad and began with an offset of 175 μm. The BPMs in the Spectrometer are movable in x, but not y, so that a beam movement from $y = 2$ mm to $y =$ -2 mm was assumed. The stationary points of the upper two parabolae are shifted by the BPM tilt, but their sum predicts a movement of -148 μm to centre the BPM on the beam. This represents an improvement in the accuracy obtainable compared to the conventional LEP alignment.

The BPM nonlinearities also allow a measurement of beam size; if the BPM is movable in x and y, a scan in x at a large value of y will yield a parabolic plot of R_Y vs. x, the offset of which in R_Y contains the term $\sigma_x^2 - \sigma_y^2$. This information is lost if the BPM cannot be moved in y, as the $\sigma_x^2 - \sigma_y^2$ term tends to cancel from BPM to BPM. However, R_X measured as a function of BPM movement in x contains the same information, if the behaviour of the electronics is adequate to enable its retrieval. We hope to evaluate the feasiblity of measuring beam size in this way during the year 2000 LEP running period. We also note that the BPM geometry proposed for LHC is significantly more sensitive to beam size than that at LEP, so that the BPM nonlinearities might allow accurate emittance measurements at LHC.

4 CONCLUSIONS

We have shown analytically and by simulations that the response of the Spectrometer BPMs is nonlinear and depends on beam size. The results from our analytical model differ from those in [3] and have been confirmed by the simulations for a circular pipe; the numerical model for an elliptical pipe was found to agree with antenna measurements. The nonlinearities make it important that the Spectrometer BPMs are well aligned. Simulations show that measurement of the nonlinearities using movable BPMs could allow more accurate alignment than standard techniques, if a sufficiently large movement is available. In addition, a measurement of beam size and hence emittance may be possible. It is planned to acquire experimental data during the year 2000 LEP running period.

5 REFERENCES

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