Nonlinearities in the Response of Beam Position Monitors

R.Assmann*, B.Dehning*, J.Matheson*, J.Prochnow†

*CERN, Geneva, Switzerland †RWTH, Aachen, Germany

Abstract. At the LEP e+/e- collider at CERN, Geneva, a Spectrometer is used to determine the beam energy with a relative accuracy of 10^{-4}.

The Spectrometer measures the change in bending angle in a dipole magnet, the beam trajectory being obtained using beam position monitors (BPMs), which must have an accuracy close to $1\,\mu$m in order to achieve the desired precision.

The BPMs used feature an aluminium block with an elliptical aperture and capacitive pickup electrodes. The response depends on the electrode geometry and also on the shape of the monitor aperture. In addition, the size of the beam itself contributes if the beam is off-centre.

The beam size varies according to the beta and dispersion functions at the Spectrometer, so that each BPM may exhibit a systematic shift of the measured beam position. We have investigated the implications of such shifts on the performance of the Spectrometer.

We present analytical results, a computer model of the BPM response, and comparison with measurements. The model suggests strategies such as beam-based alignment to minimise the systematic effects arising from the BPMs.

INTRODUCTION

The Large Electron-Positron (LEP) collider at CERN is used to study decays of W bosons, an energy of 100 GeV per beam having been surpassed during 1999. The goal for the beam energy calibration is an uncertainty below 15 MeV per beam.

Up to 60 GeV, resonant spin depolarisation is used to calibrate the beam energy [1]; however, as the beam energy is increased further, a polarisation level sufficient for the measurement cannot be achieved. For this reason, the beam energy is calculated from the bending magnetic field of LEP at physics energies.

Measurement of the bending field is carried out by means of nuclear magnetic resonance (NMR) probes [2]. However, the accuracy is limited by the fact that only part of the total bending field is covered. Hence the LEP Spectrometer [3] was proposed with the intention of being an alternative method of energy determination.

The Spectrometer consists of a steel dipole magnet equipped with reference NMR probes. The integral field of the magnet has been determined as a function of the reference probe readings with a relative accuracy of 3×10^{-5} [4]. At each side of the magnet, there are three BPM stations for the reconstruction of the beam bending angle (Fig. 1). As the energy of the beams is ramped, the magnet current is also ramped, to keep the bending angle constant. The BPMs are used to monitor any changes in the angle. The Spectrometer is calibrated at low energy against resonant depolarisation, avoiding the need for an absolute angle measurement. The target accuracy is $\delta E/E < 1\times10^{-4}$, which sets a limit on the tolerable BPM random error of $1\,\mu$m.

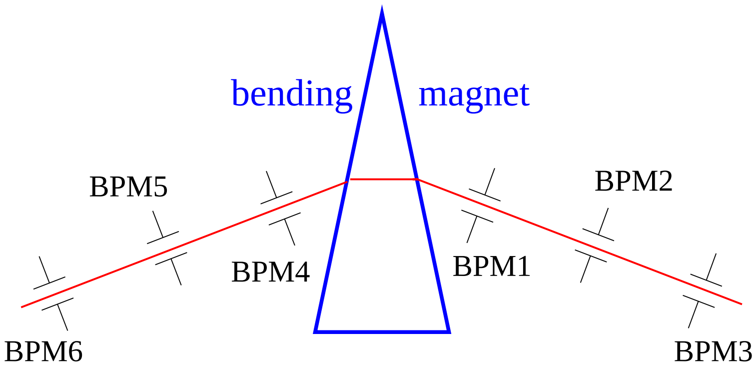

FIGURE 1. Layout of the Spectrometer

Each BPM consists of four stainless steel button electrodes mounted in an aluminium block. The block aperture is elliptical, with a horizontal axis of 131 mm and a vertical axis of 70 mm. The pickup electronics were custom manufactured for the Spectrometer application, with a specification of $\sim 1\mu m$ resolution [5].

It has been shown previously that the beam size affects the response of BPMs in a circular beam pipe [6]. The charge induced at such a pipe depends on the electric field normal to the pipe surface; the charge distribution must be such that the conducting pipe is rendered an equipotential surface. Hence, the BPM response depends on the shape of the pipe as well as the characteristics of the pickup electrodes.

We have re-derived analytically [7] the BPM response for a round pipe, finding a difference between our result and that given in [6]. The analysis suggested a method for numerical calculation of the BPM response, which we have applied to the case of circular and elliptical pipes. The model of the elliptical BPM allows conclusions to be drawn regarding effects on the Spectrometer arising from nonlinearities in pickup response. In addition, the work suggests possible methods for beam-based alignment of the Spectrometer BPMs and for the measurement of beam size.

BPM RESPONSE TO A GAUSSIAN BEAM

Analytical Results For a Circular Aperture

The full analytical derivation is given elsewhere [7]. The method of image charge was used, the beam being represented as many infinite line charges with Gaussian distributions in x and y. For a line charge within a pipe radius a, an image charge may be defined such that, under the influence of the two charges, all points on the pipe have the same potential. For one such line charge at the position (r, ϕ) and its image at $(a^2/r, \phi)$, the field normal to the pipe surface at an angle θ to the x-axis is given by:

$$E_{total} = E_\lambda \frac{a^2 - r^2}{a^2 + r^2 - 2ar\cos(\theta - \phi)} \tag{1}$$

where $E_\lambda = \lambda / 2\pi\epsilon_0 a$ is the field due to an infinite line charge with charge per unit length λ. This expression may be expanded in a power series so that, for a beam which is Gaussian in both transverse directions, we have:

$$\begin{aligned} E &= \frac{E_\lambda}{2\pi\sigma_x\sigma_y} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[1 + 2\sum_{k=1}^{\infty}\left(\frac{r}{a}\right)^k \cos k(\theta - \phi)\right] \\ &\quad \times \exp - \left[\frac{(x-\bar{x})^2}{2\sigma_x^2}\right] \exp - \left[\frac{(y-\bar{y})^2}{2\sigma_y^2}\right] dx\ dy \end{aligned} \tag{2}$$

By writing the series explicitly, then writing $\cos\phi = x/r$ and $\sin\phi = y/r$ and using standard trigonometric identities, the integration results in:

$$\begin{aligned} E \approx E_\lambda\{1 + 2[&\frac{\bar{x}}{a}\cos\theta + \frac{\bar{y}}{a}\sin\theta + \left(\frac{{\sigma_x}^2 - {\sigma_y}^2}{a^2} + \frac{\bar{x}^2 - \bar{y}^2}{a^2}\right)\cos 2\theta \\ &+ \frac{2\bar{x}\bar{y}}{a^2}\sin 2\theta + \left[3\left(\frac{\sigma_x^2 - \sigma_y^2}{a^2}\right) + \left(\frac{\bar{x}^2 - 3\bar{y}^2}{a^2}\right)\right]\frac{\bar{x}}{a} cos 3\theta \\ &+ \left[3\left(\frac{\sigma_x^2 - \sigma_y^2}{a^2}\right) + \left(\frac{3\bar{x}^2 - \bar{y}^2}{a^2}\right)\right]\frac{\bar{y}}{a} sin 3\theta\,]\} \end{aligned} \tag{3}$$

Taking the BPM buttons to be points at their centres, sustituting for θ in the above expression gives the four button signals. These may then be combined using a suitable algorithm to give x and y outputs. The algorithm used by our electronics is:

$$\Delta_X = \frac{(S1 - S3) - (S2 - S4)}{(S1 + S2 + S3 + S4)} \qquad \Delta_Y = \frac{(S1 - S3) + (S2 - S4)}{(S1 + S2 + S3 + S4)} \tag{4}$$

where $S1$ to $S4$ denote the button signals in order of increasing angle. Substituting for the signals of buttons at angles $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ from the x-axis we have:

$$\Delta_X = \sqrt{2}\,\frac{x}{a}\left[1 - \left(3\frac{\sigma_x^2 - \sigma_y^2}{a^2} + \frac{x^2 - 3y^2}{a^2}\right)\right] \tag{5}$$

$$\Delta_Y = \sqrt{2}\,\frac{y}{a}\left[1 - \left(3\frac{\sigma_y^2 - \sigma_x^2}{a^2} + \frac{y^2 - 3x^2}{a^2}\right)\right] \tag{6}$$

from which it can be seen that the response of the system to a single line charge is itself nonlinear, tailing off at large displacements, and that the beam size becomes important for an off-centre beam. These analytical results were confirmed by a numerical model.

Numerical Results for an Elliptical Aperture

The conformal mapping $W = \arcsin(Z/k)$ maps an ellipse in the complex plane Z to the geometry of a parallel-plate capacitor in the complex plane W [8]. A line charge placed within the ellipse in Z appears within the capacitor in W, where the image charges in W may be found by considering multiple reflections in the plates. Each image charge may then be transformed back into Z. The above procedure

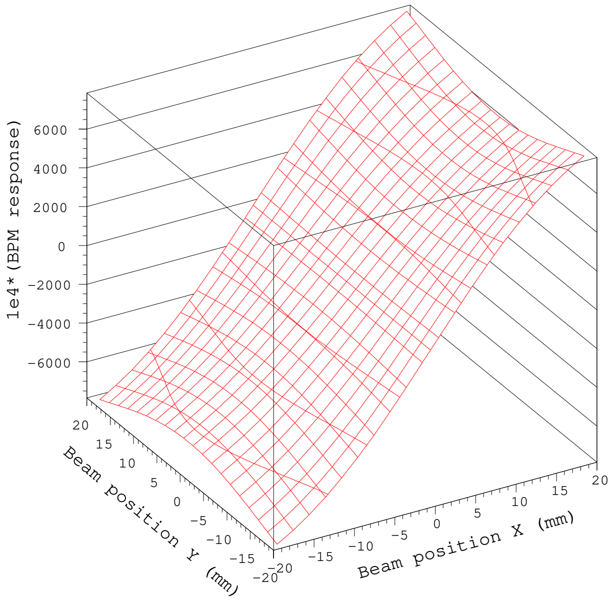

FIGURE 2. Response of an elliptical BPM: x output

was implemented in FORTRAN code and the CERN library function RNORMX used to generate 5000 charges within the beam. For each line charge, the image charge positions are calculated and used to obtain the field at any point on the pipe. The fields due to all such line charges are then added up.

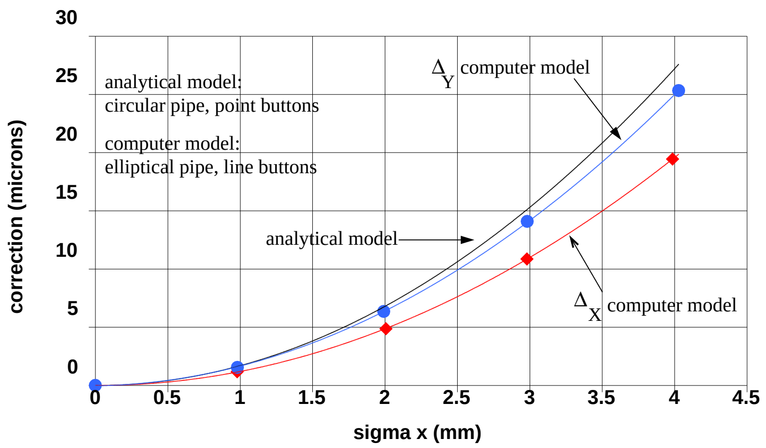

FIGURE 3. Corrections due to beam size for a circular and an elliptical vacuum chamber

During the development of the LEP BPMs, their response was characterised using an antenna mounted on an $x - y$ stage [9]. The antenna was positioned at points on a grid of spacing 5 mm, scanning in x and y, and the BPM button signals measured at each point. This scenario was simulated using the numerical model and the results compared with data [10]. The BPM output Δ_X from the model is shown in Fig. 2. The model was found to agree with data to ~5 %, treating the buttons as points. A more sophisticated model, representing them by a straight strip of length equal to their diameter, yielded ~ 2 % agreement.

The effect of changing beam width on BPM response is shown in Fig. 3. The beam was taken to lie at a position of (1 mm,0) and the x correction calculated as a function of varying σ_x. The y correction was calculated for a beam at (0,1 mm). The results from the simulation are represented by the plot symbols. The elliptical BPM is less sensitive to beam size in x than in y, whilst the larger buttons lead to a lower overall sensitivity to beam size, as may also be shown from the analytical model [11].

THE EFFECT OF THE NONLINEARITIES ON THE SPECTROMETER

Effect on the Energy Determination

During the ramp from 45 GeV to 100 GeV, the beam size changes; in addition, the beam size varies from BPM to BPM as the beta functions and dispersion change. Even if the BPMs were perfectly aligned, tracks not passing through there centres would be reconstructed with an incorrect angle. However, the BPMs may also be offset with respect to each other; in this case if the pickups were linear, an error would not be introduced on the change in the bending angle. It is again the change

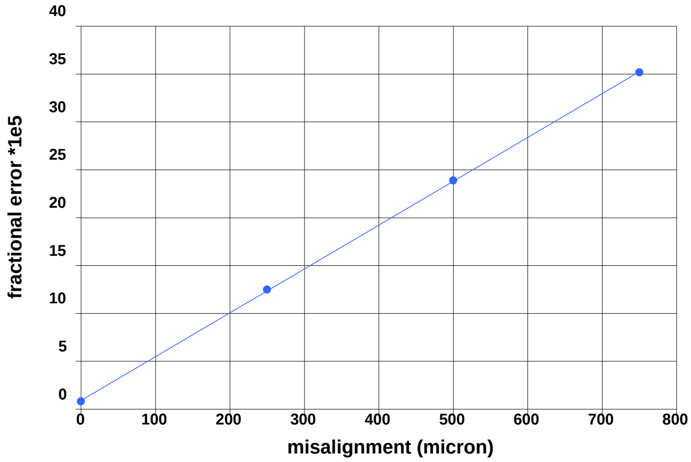

FIGURE 4. Fractional error on the bending angle due to BPM misalignment

in beam size during the ramp that introduces such an error, shown as a function of BPM offset in Fig. 4.

For the simulation, it was assumed that the innermost BPMs were displaced away from the centre of the LEP ring and the outermost BPMs displaced towards the centre by an equal amount. This is a worst case scenario. The alignment errors of the BPM blocks are expected to be randomly distributed with an RMS of $\sim$100 μm, so that a realistic value for the correlated error is 50 μm. The beam was assumed to lie at -100μm from the pickup centres at 45.0 GeV, drifting to +100μm after the ramp to 94.5 GeV.

From Fig. 4, it can be seen that, as long as the BPMs are aligned to the design tolerance, the systematic error due to BPM offsets will be acceptably small.

Using the Nonlinearities for Alignment

Inspection of (6) reveals the possibility of developing a beam-based alignment technique for the Spectrometer. We require a stable beam which has been aligned adequately to the standard LEP BPMs at either side of the Spectrometer. Then, if we are able to move each BPM in turn in x and y, a plot of Δ_Y (for some large value of y) versus distance moved will yield a parabola with minimum at the point where the BPM is centred on the beam. In reality, we expect some random beam movement during the procedure, which can be suppressed by replacing Δ_Y with the triplet residual for a Spectrometer arm; e.g., if we move BPM2, this is defined as: $T_Y{=}(\Delta_Y^{BPM1} + \Delta_Y^{BPM3})/2 - \Delta_Y^{BPM2}$ and analogously for T_X. However, if the horizontal axis of the BPM is not exactly parallel to the direction of motion, the movement in x will affect the beam position y in the moving BPM. This could be dealt with by performing two scans, one at y and one at $-y$. Simulations have shown that subtracting one parabola from the other results in a parabola with the

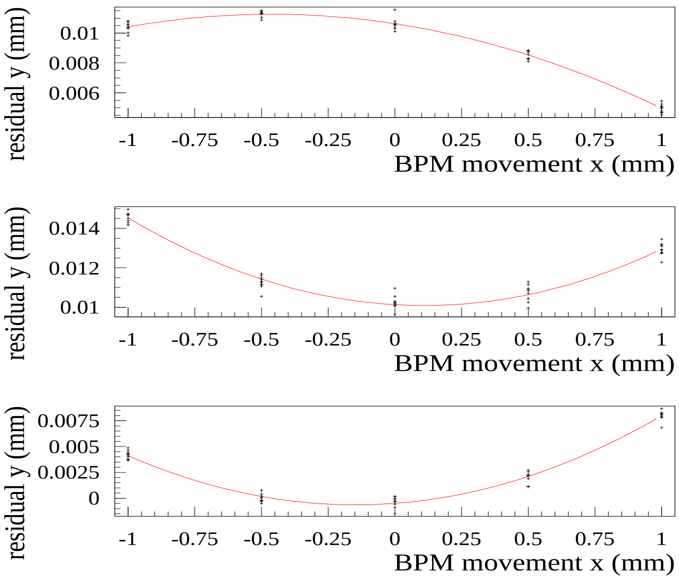

FIGURE 5. Beam-based alignment of the BPMs

correct centre, even if the BPM is tilted.

Fig. 5 shows results from a simulation assuming 10 μm beam drifts between measurements, the random error on the BPM output being taken as 0.3 μm; the BPM had a tilt of 2 mrad and began with an offset of 175 μm. The BPMs in the Spectrometer are movable in x, but not y, so that a beam movement from $y = 2$ mm to $y =$-2 mm was assumed; the error on this beam movement was taken to be $\sim$50 μm in x and y. The stationary points of the upper two parabolae are shifted by the BPM tilt, but their sum predicts a movement of -148 μm to centre the BPM on the beam. This represents an improvement in the accuracy obtainable compared to the conventional LEP alignment.

The BPM nonlinearities also allow a measurement of beam size; if the BPM is movable in x and y, a scan in x at a large value of y will yield a parabolic plot of T_Y vs. x, the offset of which in T_Y contains the term $\sigma_x^2 - \sigma_y^2$. This information is lost if the BPM cannot be moved in y, as the $\sigma_x^2 - \sigma_y^2$ term tends to cancel from BPM to BPM. However, from (5), T_X measured as a function of BPM movement in x contains the same information, if the behaviour of the electronics is adequate to enable its retrieval. We hope to evaluate the feasiblity of measuring beam size in this way during the year 2000 LEP running period. We also note that the BPM geometry proposed for LHC is significantly more sensitive to beam size than that at LEP, so that the BPM nonlinearities might allow accurate emittance measurements at LHC.

CONCLUSIONS

We have shown analytically and by simulations that the response of the Spectrometer BPMs is nonlinear and also depends on beam size, our numerical model being found to agree with antenna measurements. The nonlinearities make it important that the BPMs are well aligned; however, simulations show that measurement of the nonlinearities could allow more accurate alignment than standard techniques. In addition, a measurement of beam size and hence emittance is possible using movable BPMs. It is planned to acquire experimental data during the year 2000 LEP running period.

REFERENCES

1. L.Arnaudon et al., *Accurate determination of the LEP beam energy by resonant depolarization*, *Z. Phys. C66*, 45-62 (1995)
2. A.Blondel et al., *Evaluation of the LEP centre-of-mass energy above the W-pair production threshold*, CERN-SL/98-073 (1998)
3. See for example M.Placidi in: *Proceedings of the 8th LEP Performance Workshop*, Chamonix CERN-SL-98-006-DI (1998)
4. F.Roncarolo, *Diploma Thesis*, Politecnico di Milano (2000)
5. Bergoz Instrumentation, 01630 St. Genis Pouilly, France
6. R.H.Miller et al., *Nonintercepting Emittance Monitor*, 12th. Int. Conf. on High Energy Accelerators, Batavia, Illinois (1983)
7. J.Matheson, *Nonlinearities in the Response of BPMs for the LEP Spectrometer*, to be published
8. Z.Nehari, *Conformal Mapping*, McGraw-Hill (1952)
9. J.Borer, C.Bovet, D.Cocq, *Algorithmes de Linearisation des Pick-Ups*, LEP Note 582 (1989)
10. J.Borer, D.Cocq, *private communication*
11. R.Assmann et al., *Beam Size Dependence of Beam position Measurements and its use for Emittance Determination*, CERN-SL-99-799 OP, to be published