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QCD corrections to $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ measurements at LEPI.

Internal Note

The LEP Heavy Flavor Working Group[1]

Abstract

The measurement of the forward-backward asymmetries of heavy quarks provides one of the most precise determinations of $\sin^2\theta_{\rm eff}^{\rm lept}$ in Z decays. The asymmetries are significantly affected by QCD effects mostly due to hard gluon radiation. In this note the QCD corrections for $A_{\rm FB}^{\rm b\bar{b}}$ and $A_{\rm FB}^{\rm c\bar{c}}$ are discussed. The interplay between the experimental techniques used to measure the asymmetries and the QCD effects are studied in detail, resulting in recommendation on how the QCD corrections to the asymmetries should be estimated.

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1 Introduction

The forward-backward asymmetry of $e^+e^- \to Z \to b\bar{b}$ measured at LEP provides one of the most precise values of $\sin^2\theta^{\rm lept}_{\rm eff}$. The precision achieved today on the electroweak parameters and in particular on $\sin^2\theta^{\rm lept}_{\rm eff}$is such that interesting constraints on the Higgs mass can be extracted [1].

QCD corrections play an important role when interpreting the measured asymmetries in terms of $\sin^2\theta^{\rm lept}_{\rm eff}$. Until winter 1996, the QCD corrections used within the LEP electroweak working group followed the prescriptions given in [5]. There the experimental biases to the QCD corrections were considered only in the errors on the correction. The contribution of the QCD corrections to the total error on $\sin^2\theta^{\rm lept}_{\rm eff}$with such an approach was not negligible and, as was shown in [6], experimental biases are important and can be estimated.

The present note attempts to give a more complete picture of the QCD corrections and gives new recommendations for the presentation of $A^{\rm b\bar{b}}_{\rm FB}$ and $A^{\rm c\bar{c}}_{\rm FB}$measurements. After a short introduction, analytical calculations of the QCD corrections to the heavy quark asymmetries are presented. A method is proposed to combine these theoretical calculations with experimental effects estimated using Monte Carlo models. The sources of these experimental biases are studied in some detail. Finally the QCD corrections and their uncertainties, available for most of the LEP measurements, are presented.

2 QCD corrections

The forward-backward asymmetries are defined relative to the angle between the incoming and the outgoing fermion in the reaction $e^+e^- \to q\bar{q}$. While the incoming fermion direction is very well known, the direction of the outgoing fermion is not measurable. It is approximated in the experimental measurements by the thrust direction.

In Figure 1, some of the topologies in lowest QCD order which influence the asymmetry are shown. In particular, the emission of a hard gluon into one hemisphere, with the quark and anti-quark recoiling into the other hemisphere, "flips" the direction of the quark. Such gluon emission can destroy any information about the asymmetry of the event.

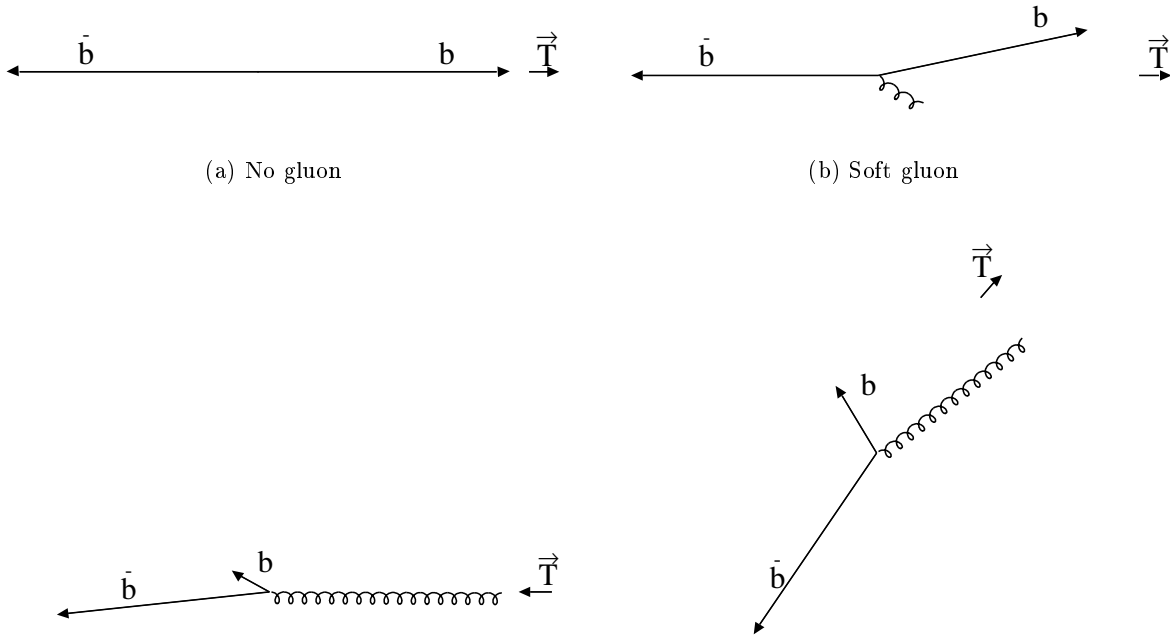
In the Standard Model, the differential cross-section $\mathrm{d}\sigma/\mathrm{d}\cos\theta$ in its most general form is given by [7]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\,\sigma_U + \frac{3}{4}\sin^2\theta\,\sigma_L + \frac{3}{4}\cos\theta\,\sigma_F \ , \qquad (1)$$

where $\sigma_{U,L}$ are the unpolarised and longitudinally polarised cross-sections and σ_F is the difference between the right- and left-handed polarised cross-section. The angle θ is measured between the incoming and the outgoing fermion. This cross-section describes the decay of a spin one boson into two spin 1/2 fermions. At Born level if the fermions were massless no longitudinal component would be present. For massive fermions like heavy quarks discussed here a small longitudinal component is expected. In the presence of gluon radiation in the final state, as illustrated in Figure 1, the relative contributions of the different cross-sections are modified; in particular, the longitudinal cross-section increases and is no longer negligible.

Eq. 1 can be rewritten as

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{3}{6+2a}(1+a\cos^2\theta) + A^{\rm q\bar{q}}_{\rm FB}\cos\theta \qquad (2)$$



(a) No gluon

(b) Soft gluon

(c) Hard gluon

(d) Thrust forward, quark backward

Figure 1: Pictorial representations of the different topologies which can influence the asymmetry. (a) No gluon radiation: thrust direction and quark direction are equal; (b) "Soft" gluon radiation: The thrust direction is a good estimator of the primary $b(\bar{b})$ direction, and the charge of the primary quark is unambiguously related to the charge of the tagged particle in the hemisphere; (c) "Hard" gluon radiation: the thrust and the quark directions are sensitive to such gluon radiation, as their direction is flipped to the hemisphere opposite to the gluon, it can be noticed that no charge information is contained in the hemisphere of the gluon; (d) The thrust direction, $\vec{T}$, being defined such that $\vec{T}.\vec{p} > 0$ with $\vec{p}$ corresponding to the quark direction, the event can still be a forward event when using the thrust direction even if the quark has been flipped into the backward hemisphere.

where :

the forward-backward asymmetry is defined by

$$A_{\rm FB}^{\rm q\bar{q}} = \frac{3}{4}\frac{\sigma_F}{\sigma_U + \sigma_L} \tag{3}$$

the shape coefficient is

$$a = \frac{\sigma_U - 2\sigma_L}{\sigma_U + 2\sigma_L} \tag{4}$$

the total cross-section is

$$\sigma = \sigma_U + \sigma_L \tag{5}$$

The bare asymmetry $A_{\rm FB}^{0,\,\rm q\bar{q}}$determined using the direction of the primary quark before gluon radiation, is related to $A_{\rm FB}^{\rm q\bar{q}}$by :

$$A_{\rm FB}^{\rm q\bar{q}} = (1 - C_q) A_{\rm FB}^{0,\,\rm q\bar{q}} \tag{6}$$

This correction coefficient C_q when $A_{\rm FB}^{\rm q\bar{q}}$corresponds to a measured asymmetry, includes not only contribution from pure theoretical calculations but also any experimental biases affecting the QCD correction.

3 Theoretical estimation of the QCD corrections

A number of analytic calculations exist which predict the QCD effects for the forward-backward asymmetries [7, 8]. In most cases the correction quoted is that between the asymmetry measured using the primary quark direction and the asymmetry measured using the quark direction at the end of the perturbative phase ("after gluon radiation"). In [8] the results are given in order α_s^2 for massless quarks. In [7] first order QCD corrections are also quoted relative to the thrust direction, calculated using all partons before hadronisation. These last calculations include mass corrections. The thrust being in practice the observable used to define the event direction, these results are used as the basis of the correction. Nevertheless to reduce the effect from the renormalisation scale in the first order QCD correction and to estimate higher order effects, the second order QCD corrections given in [8] are also used.

The method used to obtain the values and uncertainties of the theoretical estimates of the QCD corrections is as follows:

- The correction is based on the first order QCD correction using the parton-level thrust direction from [7] , with $\alpha_s\left(M_z\right) = 0.118 \pm 0.007$ [14]. (For completeness results using the primary quark direction are also quoted in this note.)
- Two masses for the quarks are considered in the first order calculations [7]: the pole mass ($m_b = 4.5$ GeV and $m_c = 1.5$ GeV); and the $\overline{MS}$ mass at scale M_Z ($m_b = 3$ GeV and $m_c = 0.7$ GeV). These give two different predictions. For the correction itself, the value midway between the two predictions will be used with half of the difference taken as the systematic uncertainty.
- The second order QCD calculations from [8] updated with $m_{\rm t} = 175 \pm 6$ GeV [6] are used to fix the scale of the first order QCD corrections. It can be seen in Table 1 that the differences between the second and first order calculations are small. As the calculations are not complete (quark masses are neglected and no result based on the parton thrust direction is available), a correction half of the value of that predicted for massless quarks [8] is used and a systematic uncertainty of the same size assigned. This estimate should be compared with the first order correction, which are reduced by $\sim -20\%$ by the mass and thrust effects.
- In the second order term, α_s has been varied by ± 0.02 to take into account of scale uncertainties.
- The contribution to the second order QCD corrections to $A_{\rm FB}^{\rm b\bar{b}}$ ($A_{\rm FB}^{\rm c\bar{c}}$) coming from gluon splitting in $b\bar{b}$ ($c\bar{c}$) events is not considered. The present measurements correct at the experimental level for these contributions.

The theoretical correction obtained can be written as

$$A_{\rm FB}^{\rm q\bar{q}} = \left(1 - \delta_{QCD}\right) A_{\rm FB}^{0,\,\rm q\bar{q}} = \left(1 - \frac{\alpha_s(M_Z)}{\pi} c_1 - \left(\frac{\alpha_s}{\pi}\right)^2 c_2\right) A_{\rm FB}^{0,\,\rm q\bar{q}} \qquad (7)$$

Event direction		$b\bar{b}$ events			$c\bar{c}$ events		
	mass in GeV	m_b=0	m_b=3	m_b=4.5	m_c=0	m_c=0.7	m_c=1.5
quark	a [7]	.906	.907	.901	.906	.908	.909
	c_1 [7]	1.	0.86	0.80	1.	0.96	0.93
	c_2	1.9±0.4 [6,8]	0.95		4.6±0.4 [6,8]	2.3	
	δ_{QCD}^{quark}	0.0325±0.0030			0.0387±0.0049		
thrust	a [9]	.952	-	-	.952	-	-
	c_1 [7]	0.89	0.81	0.77	0.89	0.88	0.86
	c_2	-	0.95		-	2.3	
	$\delta_{QCD}^{parton,T}$	0.0310±0.0026			0.0359±0.0044		

Table 1: The values of a from [7] and [9] and c_1 from [7] quoted for different mass hypotheses and two different definitions of the event direction (quark or thrust). For c_2 , the second order QCD correction, a value corresponding to $m_q = 0$ and the quark direction from [8] is quoted. All the other numbers in this table are estimated following the description given in the text.

Error source	δ_{QCD}^{quark}		$\delta_{QCD}^{parton,T}$	
	$b\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{c}$
Theoretical error on m_b or m_c (on c_1)	0.0011	0.0006	0.0008	0.0004
$\alpha_s(M_Z)$ (0.118±0.007) (on first order)	0.0022	0.0029	0.0018	0.0019
Second order theoretical error (on c_2)	0.0006	0.0006	0.0006	0.0006
Second order renorm. scale (α_s 0.1:0.14)	0.0009	0.0022	0.0009	0.0022
Thrust and mass effect (on c_2)	0.0013	0.0032	0.0013	0.0032
Total error	0.0030	0.0049	0.0026	0.0044

Table 2: Following the description given in the text, the different sources of errors on the QCD corrections are quoted depending if the quark (δ_{QCD}^{quark}) or the thrust ($\delta_{QCD}^{parton,T}$) directions is used. The total error corresponds to the numbers quoted in Table 1.

A summary of the different values c_1 and c_2 quoted in [7, 8] is given in Table 1 with the resulting QCD corrections. Two sets of theoretical corrections are quoted, corresponding to the two definitions of the events direction :

- δ_{QCD}^{quark} for the primary quark direction after gluon radiation
- $\delta_{QCD}^{parton,T}$ for the parton thrust direction

The different contributions to the errors on δ_{QCD}^{quark} and $\delta_{QCD}^{parton,T}$ are listed in Table 2. It should be noted that:

- the use of the thrust instead of the quark direction slightly reduces the QCD correction (-5 – -10 %) as illustrated in Figure 1d;
- the QCD corrections decrease when the mass of the quark considered increases as would be expected from Galilei's law of inertia. When the quark direction is used, the QCD correction goes down by 20% for $m_b = 4.5$ GeV compared to a massless case. This mass effect is less pronounced when the thrust axis is used.

As will be discussed Section 5, the fact than a (see equation 2) is different from unity has non-negligible effect on most of the experimental asymmetry measurements performed at LEP and cannot

be neglected when the effect of the QCD corrections are studied. The estimates of a at the quark level quoted Table 1 have been calculated using the formalism in [7] using the various mass assumptions. In the next section (Tables 3 and 4), the theoretical expectations quoted for a at the quark level correspond to the value midway between those obtained assuming the current and the $\overline{MS}$ quark masses, as given in Table 1. The estimates used for a at the thrust level and quoted in Table 1 have been calculated using the formalism in [9] which assumes massless quarks. The estimated uncertainties on these predictions of a are given in Tables 3 and 4. They include the uncertainties in the renormalisation scale in the first order, estimated by varying α_s between 0.10 and 0.14; they also include the uncertainties in the quark masses, with the thrust level mass uncertainties taken to be in the same proportion to the correction as those at quark level [7]. The total uncertainties are found to be 0.015 at quark level and 0.009 at the thrust level.

4 Effects of the hadronisation and decay from simulation

The results quoted in the previous section are not directly usable, since in practice the experimental thrust is computed from stable particles observed in the detector and not from partons before hadronisation. The only way to assess the change in the QCD correction between these two steps is by Monte Carlo simulation. In this section, the study is limited to the ideal case were all final particles are detected. In the next section results corresponding to the experimental conditions are given.

The results obtained in Section 2 can be compared with different simulations using three different definitions of the event direction :

- δ_{QCD}^{quark}, calculated using the direction of the primary quark after gluon radiation.
- $\delta_{QCD}^{parton,T}$, calculated using the direction of the thrust axis including partons after gluon radiation, with the sign of the thrust direction given by the direction of the quark.
- $\delta_{QCD}^{hadron,T}$, calculated using the direction of the thrust axis after hadronisation, including all stable particles (including neutrinos), the thrust direction being signed by comparison with the direction of the weakly decaying hadron containing the quark.

The values of δ_{QCD}^{quark} and $\delta_{QCD}^{parton,T}$ obtained by analytical calculations have been presented in the previous section and can be directly compared to the results obtained with Monte Carlo simulations. In Table 3 for the $b\bar{b}$ events and Table 4 for the $c\bar{c}$ events the results of the analytical calculations are compared with the results obtained using a "pure" JETSET 7.4 Monte Carlo [11] or different version of tuned generators by ALEPH , DELPHI and OPAL. The tuned generators include an up-to-date description of the production and branching ratio of b and c hadrons together with an improved description of the global observables.

Also shown are results for the shape parameter a. The theoretical numbers on the quark level given are the average of the results for the massive quarks quoted in Table 1. No theoretical results are available on the hadron level.

In general it can be seen that:

- the agreement between calculation and Monte Carlo for δ_{QCD}^{quark} is poor (between 5% and 0.6% depending of the Monte Carlo);

Source	a			δ_{QCD} in %		
	b quark	thrust	hadron	b quark	thrust	hadron
Theory	.904±.015	.952 ±.009	-	3.25±0.30	3.10±0.26	-
Pure JETSET 7.408	.893±.003	.954±.003	.955±.003	3.95±0.02	3.17±0.02	2.94±0.02
ALEPH All MC †	.895±.003	.958±.003	.983±.003	3.84±0.09	3.07±0.04	2.79±0.08
ALEPH 1992 MC	.890	.954	.976	4.13	3.34	3.01
ALEPH 1993 MC	.880	.947	.967	3.71	3.00	2.78
ALEPH 1994 MC	.899	.961	.985	3.80	3.01	2.75
DELPHI 1994 MC [10] †	.88	.95	.95	4.38 ±0.03	3.43±0.03	3.06±0.03
OPAL JETSET [13] †	.892	.954	.953	4.02±0.18	3.04±0.19	3.02±0.20
OPAL HERWIG [13]	.895	.954	.988	4.11±0.14	2.86±0.13	2.75±0.14

Table 3: Different values of a (see equation 2) and δ_{QCD}(see equation 6) for $e^+e^- \to b\bar{b}$ events. The first line corresponds to the theoretical estimate presented Section 2. The line quoted as "pure JETSET 7.408" is obtained with the JETSET generator with default tuning. All other lines correspond to results obtained with generators tuned by each LEP experiments. The line quoted with a † are used Table 5 to estimate the reference values $\delta_{QCD,ref}^{hadron,T}$. Within one experiment, several different generators have been used (JETSET or HERWIG [12] in the case of OPAL) , or even different tunes have been attempted (one per year in the case of ALEPH, the full sample being quoted as "All").

Source	a			δ_{QCD} in %		
	c quark	thrust	hadron	c quark	thrust	hadron
Theory	.909±.015	.952 ±.009	-	3.87±0.49	3.59±0.44	-
Pure JETSET 7.408	.853 ± .005	.948±.005	.950 ± .005	5.49±0.04	3.95±0.04	3.51±0.04
DELPHI 1994 MC [10]†	0.85	.95	.95	5.74±0.03	4.12±0.03	3.65±0.03
OPAL JETSET [13]†	.86±0.01	.96	.96	4.95±0.18	4.03±0.18	3.80±0.19
OPAL HERWIG [13]	.872±.002	.973±.002	.975±.002	4.75±0.17	3.18±0.16	3.26±0.17

Table 4: Different values of a (see equation 2) and δ_{QCD}(see equation 6) for $e^+e^- \to c\bar{c}$ events. The first line corresponds to the theoretical estimates presented Section 2. The line quoted as "pure JETSET 7.408" is obtained with the JETSET generator with default tuning. All other lines are calculated using generators tuned by each LEP experiments. The line quoted with a † are used Table 5 to estimated the reference values $\delta_{QCD,ref}^{hadron,T}$.

- there is a much better agreement for $\delta_{QCD}^{parton,T}$(20% or higher);
- for $e^+e^- \to b\bar{b}$ the shape parameter a is well reproduced by the simulation. This is not the case for $e^+e^- \to c\bar{c}$ at the quark level.

Qualitatively this pattern is expected since the parton shower Monte Carlo radiate many more gluons than a fixed $O(\alpha_s^2)$ calculation. As the thrust is used in the experimental measurement of $A_{\rm FB}^{\rm b\bar{b}}$ and $A_{\rm FB}^{\rm c\bar{c}}$, the correct description by the simulation of $\delta_{QCD}^{parton,T}$and a when using the thrust direction encourages the use of the Monte Carlo prediction to study and estimate biases in the QCD corrections.

The estimates of $\delta_{QCD}^{hadron,T}$, only accessible using Monte Carlo models, show significant difference compared to $\delta_{QCD}^{parton,T}$. The source of these differences is the hadronisation itself, the decay of b and c hadrons having a smaller effect. It should be noticed that the hadronisation decreases the correction

which can only happen by reconnecting the quark after the shower with quarks and gluons that are closer to the original quark direction. The exact size of this non-perturbative effect is not known. It should be noted that in Tables 3 and 4 the predictions of the different generators for $\delta_{QCD}^{hadron,T}$ show some variation. With the same generator but with different tunings, even within the same experiment, δ_{QCD}^{quark} and $\delta_{QCD}^{parton,T}$ can have also non negligible variation (up to 13%). An example of the differences between the simulations, and a possible source of some of the variations observed is the chosen cut off in energy for the gluon emission before starting the hadronisation.

To account for the effect of hadronisation on the QCD corrections, an average of the differences between $\delta_{QCD}^{parton,T}$ and $\delta_{QCD}^{hadron,T}$ based on the JETSET estimates quoted Tables 3 and 4 is used. Adding these averages to the values of $\delta_{QCD}^{parton,T}$ from Table 1 defines the reference values of $\delta_{QCD,ref}^{hadron,T}$. The uncertainties on $\delta_{QCD,ref}^{hadron,T}$ are those on $\delta_{QCD}^{parton,T}$ from Table 2 with an extra contribution corresponding to the difference between $\delta_{QCD}^{parton,T}$ and $\delta_{QCD,ref}^{hadron,T}$. Table 5 summarise the value of $\delta_{QCD,ref}^{hadron,T}$ obtained.

Value	$b\bar{b}$ events	$c\bar{c}$ events
$\left\langle \delta_{QCD}^{parton,T} - \delta_{QCD}^{hadron,T} \right\rangle$	0.0022	0.0035
$\delta_{QCD,ref}^{hadron,T}$ in %	2.88 $\pm$ 0.34	3.24 $\pm$ 0.56

Table 5: Estimated value of $\delta_{QCD,ref}^{hadron,T} = \delta_{QCD}^{parton,T} - \left\langle \delta_{QCD}^{parton,T} - \delta_{QCD}^{hadron,T} \right\rangle$. The average value of $\left\langle \delta_{QCD}^{parton,T} - \delta_{QCD}^{hadron,T} \right\rangle$ is obtained using the lines quoted with a † in Tables 3 and 4

5 Experimental biases

For the measured asymmetries, three main effects can modify the QCD corrections at the hadron level ($= \delta_{QCD,ref}^{hadron,T}$). These effects are :

- the experimental event axis reconstruction ,
- the bias in the selected event topology due to any kind of acceptance cut or weight ,
- the fitting method and the event shape hypothesis as a function of the polar angle.

Event axis reconstruction
The experimentally reconstructed thrust differs from the thrust axis based on all final-state particles because of unseen neutrinos, and because of the acceptance and resolution of the detectors in energy and angle. For example, excluding the neutrinos in the thrust reconstruction using simulated final-state particles reduces the QCD corrections to $A_{\rm FB}^{\rm b\bar{b}}$ by $\sim .002$

Selection bias
Event selection can directly influence the size of the QCD corrections. If some criteria applied to select the events has a sensitivity to the strength of the gluon emission, the size of the QCD corrections will be changed. Hard gluon production results in softer momentum spectrum for the b and c produced and for the associated decay products. In $A_{\rm FB}^{\rm b\bar{b}}$ or $A_{\rm FB}^{\rm c\bar{c}}$ analysis based on leptons, the typical cut of 3 GeV on the lepton momentum rejects a large fraction of these events. In Figure 2 an example of the change in the QCD correction observed as a function of the lepton momentum and transverse

momentum is shown.

While a direct rejection of events with a hard gluon will decrease the QCD effects, changes in the QCD corrections are also expected if the weight of an event in the asymmetry measurement is a function of the gluon activity itself. A good example of such an effect can be seen in asymmetry measurements based on the charge flow techniques. In such analyses, the hard gluon emission has a tendency to reduce the average charge separation between the hemispheres. Since the charge separation in these measurements is determined from the data, a large part of the QCD corrections is implicitly done in the analysis, and any further effect is significantly reduced. The effect of the hard gluon emission in these analyses is not so much to change the asymmetry but to reduce the statistical power of the measurement.

Fitting method

To take into account the angular acceptance and efficiency of the detectors, the experimental asymmetries are not extracted using a global counting of the forward and backward events but by doing a fit based on equation 2. In general, the asymmetry measurements take 1 for the a coefficient in equation 2 and do not consider the changes in shape induced by the QCD corrections. The shape effect therefore introduces an experimental bias, which can be absorbed into the QCD correction. It has been estimated by Monte Carlo that for a simple χ^2 fit on a $b\bar{b}$ sample with no angular acceptance cut that the QCD corrections were reduced by $\sim .004$ when a was set to 1 instead to its nominal value.

For each measurement the influence of the experimental cuts and measurement techniques on the QCD corrections has to be estimated. In practice, each LEP experiment has calculated the relative change of the QCD corrections using Monte Carlo simulation (see Tables 3 and 4), and has derived a QCD bias s_q, which is the ratio of the corrections at the measurement level (C_{q}) to the corrections at the hadron level ($\delta_{QCD}^{hadron,T}$) and given in Table 6. On average the effects from the QCD corrections are reduced significantly compared to the theoretical expectation.

6 Conclusion

The final QCD corrections to a given forward-backward asymmetry measurement can be expressed as

$$A_{\mathrm{FB}}^{\mathrm{q\bar{q}}} = \left(1 - s_q \times \delta_{\mathrm{QCD},ref}^{\mathrm{hadron},T}\right) A_{\mathrm{FB}}^{0,\,\mathrm{q\bar{q}}} = (1 - C_q)\, A_{\mathrm{FB}}^{0,\,\mathrm{q\bar{q}}} \tag{8}$$

where

$\delta_{\mathrm{QCD},ref}^{\mathrm{hadron},T}$, introduced in Section 4, corresponds to the QCD corrections when the event direction is defined by the thrust direction including all final particles. It is dominated by the calculations available at the parton level but it also takes into account the hadronisation and hadron decay chain effects estimated by Monte Carlo models.

s_q , introduced in Section 5, corresponds to the experimental bias to the QCD correction. It must be evaluated using Monte Carlo simulations for each analysis.

The C_q values used for the different measurements considered in [1] for the 1997 winter conferences are summarised in Table 7. The errors to be propagated to those on $A_{\mathrm{FB}}^{0,\,\mathrm{q\bar{q}}}$, arising from this correction, are of two kinds:

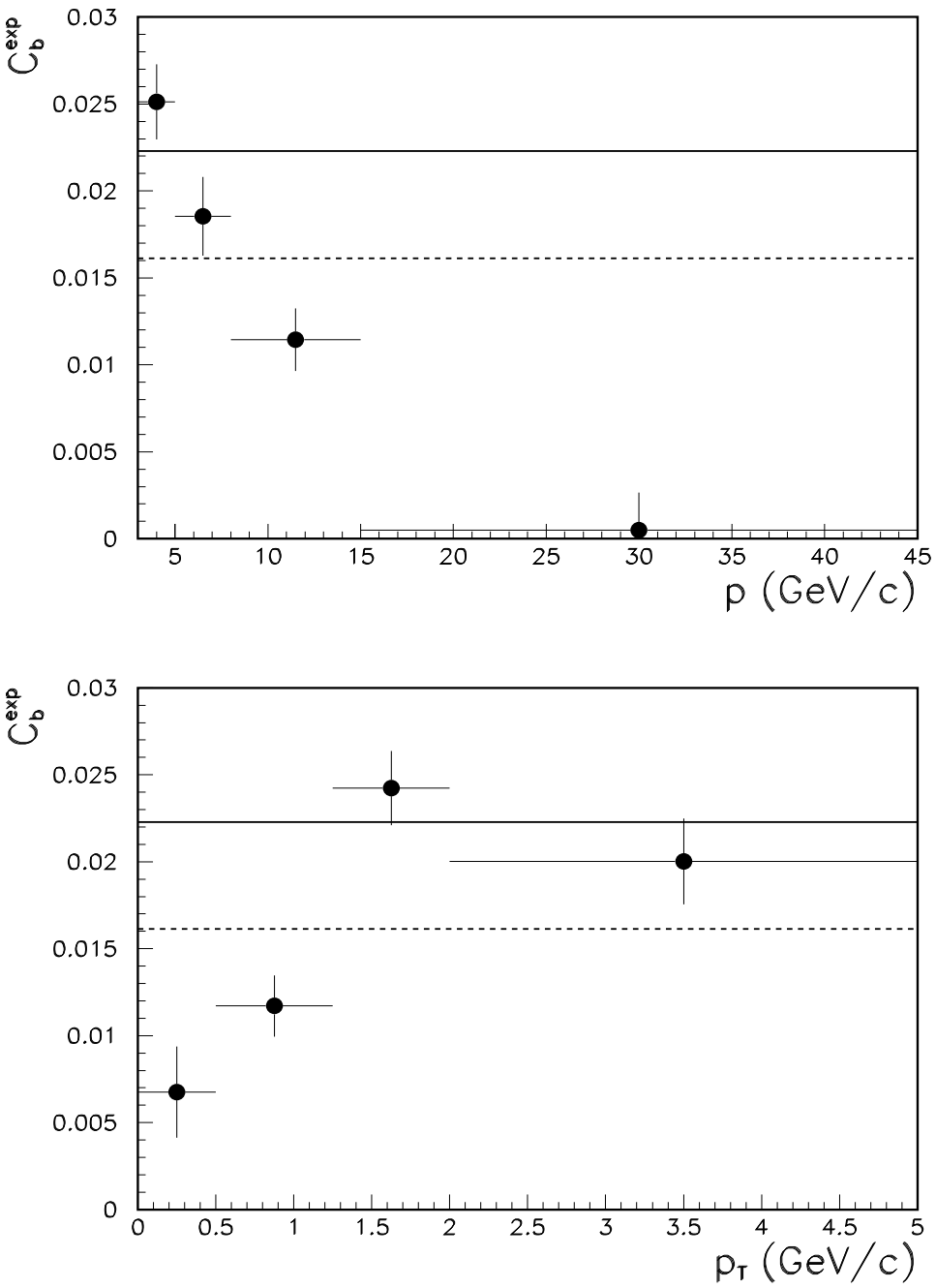


Figure 2: QCD correction factors C_{b}estimated using ALEPH Monte Carlo for the $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ analysis using lepton. The QCD corrections are presented as a function of the momentum (p) and transverse momentum ($p_\perp$) of the lepton candidate. All the candidates ($b \to l$,$b \to c \to l$, ...) are used here. The dashed line correspond to the average correction over all the kinematic range (with only the identification cut of $p > 3$ GeV applied), while the solid line corresponds to the average correction for the candidates used in the analysis ($p_\perp > 1.25$ GeV). The trend as a function of the momentum as explained in the text comes from the anti-correlation between the QCD corrections and the b hadron energy. The trend as a function of the transverse momentum is connected to the interplay between the gluon radiation and the performance of the jet algorithm. In events with hard gluon emission this can occasionally lead to higher values of the reconstructed $p_\perp$.

Experiment	Lepton analysis		D analysis		Jet Charge
	s_b	s_c	s_b	s_c	s_b
ALEPH	.74±.07	.6±.3 †	-	.3±.3 †	-
DELPHI	.52±.06	.19±.11	.46±.14	.44±.13	.24±.46
L3	.6±.2†	.6±.3 †	-	-	-
OPAL	.69±.13	.37±.08	.29±.13	-.06±.09	.36±.32

Table 6: Value of the scaling ($s_q = C_{\rm q}/\delta_{QCD}^{hadron,T}$) of the QCD corrections found for different LEP asymmetry measurements. It should be noticed that the quantity quoted with a † are just extrapolation from the other numbers.

Exp.	Lepton analysis		D analysis		Jet Charge
	C_b in %	C_c in %	C_b in %	C_c in %	C_b in %
ALEPH	2.13±.25±.20 [15]	1.94±.34±.97 [16]†	-	0.97±.17±.97 [16]†	-
DELPHI	1.50±.18±.17 [21]	0.62±.11±.36 [21]	1.32±.16±.40 [21]	1.43±.25±.42 [21]	0.69±.08±1.32 [21]
L3	1.73±.20±.58 [17]†	1.94±.34±.97 [17]†	-	-	-
OPAL	1.99±.23±.37 [18]	1.20±.21±.26 [18]	0.84±.10±.37 [19]	-.19±.03±.29 [19]	1.04±.12±.92 [20]

Table 7: Value of the QCD corrections ($A_{\rm FB}^{\rm q\bar{q}} = (1 - C_q)\, A_{\rm FB}^{0,\,\rm q\bar{q}}$) for the different LEP measurements used in [1] . The first error quoted is correlated between the different measurements and comes from the error on $\delta_{QCD,ref}^{hadron,T}$, the second error is dominated by the statistical precision with which the bias has been determined for each measurement. References to the asymmetry measurements associated to all these QCD corrections are also given. It should be noticed that the quantity quoted with a † are just extrapolation from the other numbers.

- the systematic errors on $\delta_{QCD,ref}^{hadron,T}$ quoted in Table 5 which are correlated between the different LEP measurements;
- the errors on s_q quoted Table 6, dominated by the Monte Carlo statistics used to estimate them, which are uncorrelated among the different measurements.

As the error on $\delta_{QCD,ref}^{hadron,T}$ is scaled by the experimental bias s_q, only a small fraction of the correlated error will propagate through to the LEP average of $A_{\rm FB}^{0,\,\rm q\bar{q}}$. In the results presented in [1], the total correlated errors at the level of the systematics were 0.00060 and 0.00120 for $A_{\rm FB}^{0,\,\rm b\bar{b}}$and $A_{\rm FB}^{0,\,\rm c\bar{c}}$ respectively. The contribution of the QCD corrections to these errors was only ~ 0.00017 and ~ 0.00018.

The spread of the different QCD corrections, as will be noted from Table 7, indicates clearly that the knowledge of the experimental bias for each asymmetry measurement is mandatory in order to extract $A_{\rm FB}^{0,\,\rm b\bar{b}}$ and $A_{\rm FB}^{0,\,\rm c\bar{c}}$ correctly.

References

[1] The LEP collaborations ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group, and the SLD Heavy Flavor Group, A combination of preliminary LEP electroweak Measurements and constraints on the Standard Model
6th December 1996, CERN-PPE/96-183
7th April 1997, LEPEWWG/97-01 , ALEPH 97-032 PHYSIC 97-027 , DELPHI 97-30 PHYS 683 , L3 Note 2071 , OPAL Technical Note TN 472 , SLD Physics Note 61.

[2] OPAL Collaboration, G. Alexander *et al.*, Z. Phys.**C70** (1996) 357;
OPAL Collaboration, *Updated measurement of the heavy quark forward-backward asymmetries and the average B mixing using leptons in multihadronic events*, OPAL physics note PN-226 (1996).

[3] OPAL Collaboration, K. Ackerstaff *et al.*, *Measurement of the b quark forward-backward asymmetry around the Z peak using jet charge and vertex charge*, CERN-PPE/97-06, submitted to Z. Phys..

[4] OPAL Collaboration, G. Alexander *et al.*, CERN PPE/96-101, accepted by Z. Phys..

[5] Nucl. Instrum. Methods**A 378** (1996) 101.

[6] LEP Heavy Flavor electroweak working group, *Presentation of LEP Electroweak Heavy Flavor Results for Summer 1996 Conferences*, LEPHF 96-01.
(http://www.cern.ch/LEPEWWG/heavy/)

[7] A. Djouadi, J. H. Kühn, P. Zerwas, Z. Phys.**C 46** (1990) 411;
P.Zerwas, B.Lampe, Z. Phys.**C 67** (1995) 123.

[8] G. Altarelli, B. Lampe, Nucl. Phys. **B 391** (1993) 3.

[9] G.A. Schuler, PhD thesis, Univ. of Mainz, 1986, Internal Report DESY T-87-01;
B. Lampe, G. Kramer, G. Schierholz and J. Willrodt, Phys. Lett. **B 79** (1978)249,
errata Phys. Lett. **B 80** (1979) 433.

[10] DELPHI Collaboration, P. Abreu *et al.*, Z. Phys.**C73** (1996) 11

[11] T. Sjöstrand, Comp. Phys. Comm. **82** (1994) 74.

[12] G. Marchesini *et al.*, Comp. Phys. Comm. **67** (1992) 465.

[13] OPAL Collaboration, G. Alexander *et al.*, Z. Phys.**C 69** (1996) 543.

[14] B.R. Webber, Proc. of the XXVII International Conference on High Energy Physics, Glasgow, UK, edited by P. J. Bussey (1994) 213.

[15] ALEPH Coll., D. Buskulic *et al.*, Phys. Lett.**B384** (1996) 414

[16] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. **C62** (1994) 1;
ALEPH Collaboration, D. Buskulic *et al.*, *The Forward–Backward Asymmetry for Charm Quarks at the Z pole: an Update*, Contributed Paper to EPS-HEP-95 Brussels, **eps0634**.
This note may be found at http://alephwww.cern.ch/ALPUB/oldconf/HEP95/HEP95.html.

[17] L3 Collaboration, O. Adriani *et al.*, Phys. Lett. **B292** (1992) 454;
L3 Collaboration, M. Acciarri *et al.*, Phys. Lett. **B335** (1994) 542;
L3 Collaboration, *L3 Results on* $A_{\rm FB}^{\rm b\bar{b}}$, $A_{\rm FB}^{\rm c\bar{c}}$ *and* χ *for the Glasgow Conference,* L3 Note 1624;

[18] OPAL Collaboration, G. Alexander *et al.*, Z. Phys. **C70** (1996) 357;
OPAL Collaboration, R. Akers *et al.*, *Updated Measurement of the Heavy Quark Forward-Backward Asymmetries and Average B Mixing Using Leptons in Multihadronic Events*, OPAL Physics Note PN226 contributed paper to ICHEP96, Warsaw, 25-31 July 1996 **PA05-007**
OPAL Collaboration, R. Akers *et al.*, *QCD corrections to the bottom and charm forward-backward asymmetries*, OPAL physics note PN284

[19] OPAL Collaboration, CERN-PPE/96-101 24 July 1996.

[20] OPAL Collaboration, R. Akers *et al.*, Z. Phys. **C67** (1995) 365.
OPAL Collaboration, *Measurement of the b quark forward-backward asymmetry around the Z0 peak using jet charge and vertex charge*, CERN PPE/97-06.

[21] DELPHI Collaboration, P.Abreu *et al.*, Z. Phys **C65** (1995) 569;
DELPHI Collaboration, P.Abreu *et al.*, Z. Phys **C66** (1995) 341;
DELPHI Collaboration, *Measurement of the Forward-Backward Asymmetries of* $e^+e^- \rightarrow Z \rightarrow b\overline{b}$ *and* $e^+e^- \rightarrow Z \rightarrow c\overline{c}$, DELPHI 95-87 PHYS 522.
This and all other Delphi notes are available at http://wwwcn.cern.ch/~pubxx/www/delsec/delnote/.