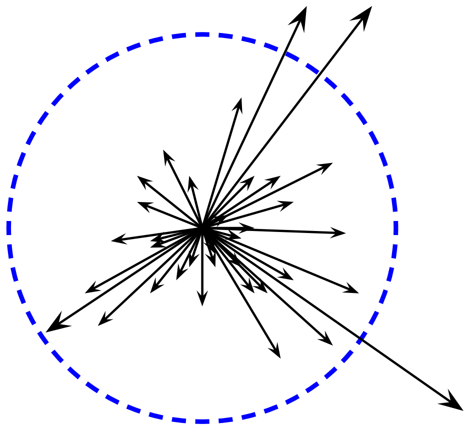
Scaling violations of quark and gluon jet fragmentation functions at e^+e^- annihilations

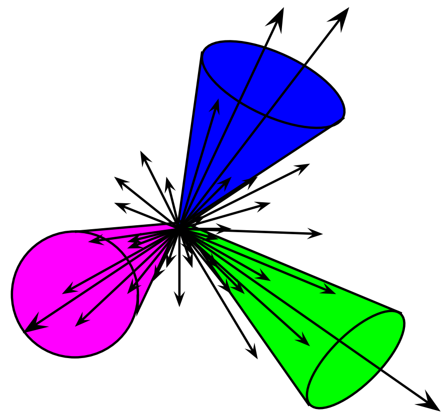
Marek Taševský

Universitaire Instelling Antwerpen

OPAL Collaboration, CERN

$e^+e^- \to Z^0 \to q\bar{q}(g)$





2 hemispheres

3 jets found by a jet algorithm

Jet properties defined by an inclusive sum over the hemisphere

↓

No jet finder

↓

UNBIASED JETS

SCALE $= \sqrt{s}/2$

Jet properties defined by particles assigned to a jet

↓

Jet finder dependence

↓

BIASED JETS

SCALE $= Q_{jet}$

Unbiased jets are used in theory calculations.

The measured fragmentation function is defined here as

$$\frac{\mathbf{1}}{\mathbf{N}_{\mathrm{jet}}(\mathrm{scale})} \frac{\mathrm{d}\mathbf{N}_{\mathbf{p}}(\mathbf{x}_{\mathrm{E}}, \mathrm{scale})}{\mathrm{d}\mathbf{x}_{\mathrm{E}}}$$

number of charged non-identified particles in bins of $x_{\mathrm{E}} = \frac{E_{\mathrm{part}}}{E_{\mathrm{jet}}}$ and scale normalized to number of jets in bins of scale. E_{jet} = energy of the jet to which the particle with energy E_{part} is assigned.

In total, 7 types of frag.functions were measured:

UNBIASED JETS		**BIASED JETS**
udscb	← ARE →	**udscb**
udsc	← THEY →	**udsc**
b	← MUTUALLY →	**b**
	CONSISTENT ?	**gluon**

If there is a consistency, then:

- Q_{jet} **scale is an appropriate scale for hadron production in 3-jet events.**
- **Comparison of measured biased jets with theory makes sense.**

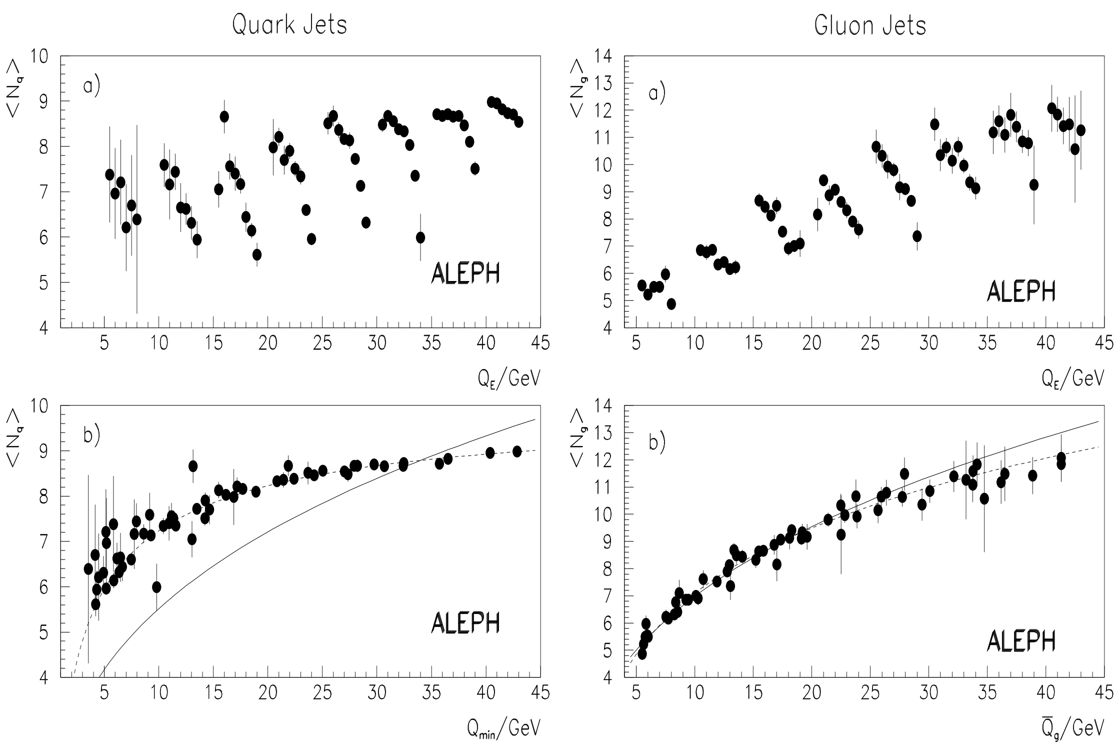
Scale for biased jets

$\mathbf{Q}_{\rm jet}$ scale used introduced by ALEPH → Z.Phys.C76(1997)191

$$\mathbf{Q_{jet} = E_{jet} \sin\frac{\theta}{2}}$$

θ = angle between the jet with $E_{\rm jet}$ and the closest jet.

Each of the eight bands correspond to jets with the same energy but with a different angle to the nearest jet. The jet multiplicity depends on the event topology, not just the jet energy.



$Q_{\rm jet}$ scale reduces the jet energy and topology dependences compared to the scale $E_{\rm jet}$.

Event Selection

- **OPAL DATA:**

LEP1 (1993–1995): $\sqrt{s} = 91.2$ GeV, $\mathcal{L}$ =130 pb^{-1}
LEP2 (1997–2000): $\sqrt{s}$ =183-209 GeV, $\mathcal{L}$ =690 pb^{1}

- **Standard hadronic event selection applied**

- Reduction of ISR bg in LEP2 data:

$$\sqrt{s} - \sqrt{s'} < 10 \;\; (20^*) \text{ GeV}$$

- Reduction of 4-fermion bg (WW,ZZ→4f) in LEP2 data:

Event weight $W_{\text{qcd}} > -0.5$

- **3-jet event selection:**

Durham (Cone, Cambridge for syst. studies) jet alg. forced to find 3 jets

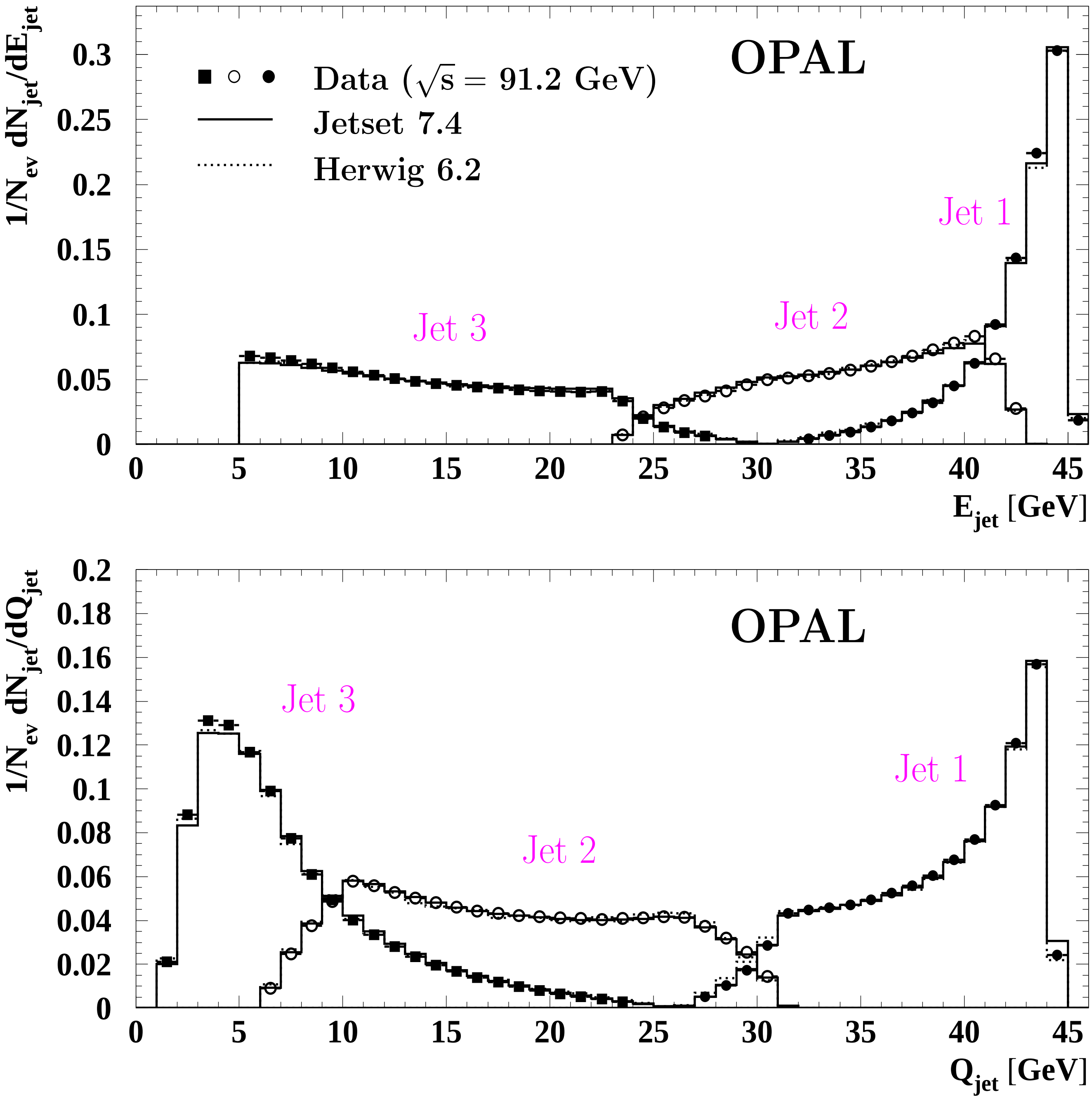
- Particle multiplicity per jet ≥ 2
- Sum of inter-jet angles $\geq 358°$
- Polar jet angle $|\cos\theta_{\text{jet}}| \leq 0.90$ (0.95^*)
- Corrected jet energy ≥ 5 GeV
- Inter-jet angle $\geq 30°$

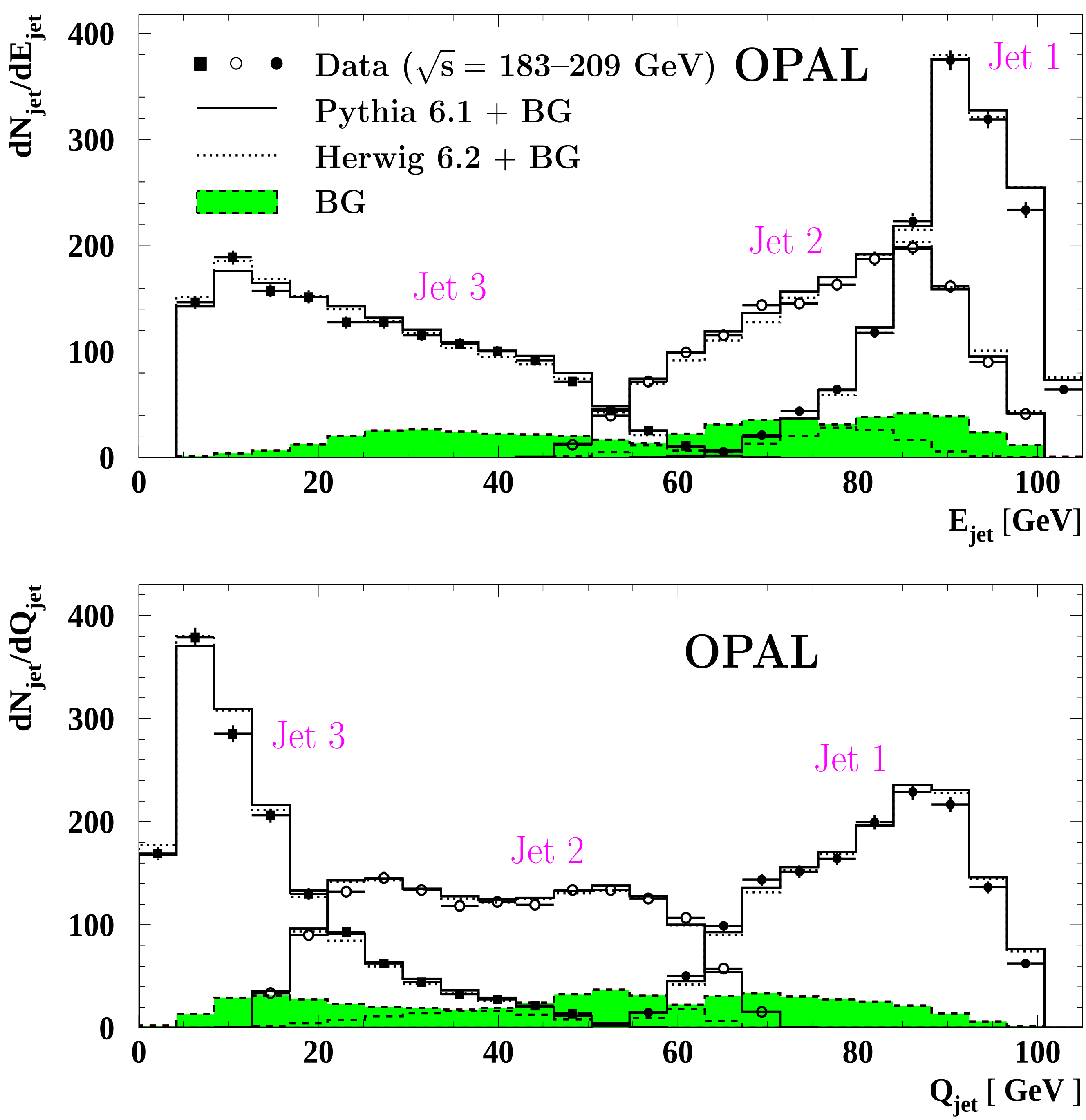
$$E_i^{\text{calc}} = \frac{\sin\theta_{jk}\sqrt{s}}{\sin\theta_{ij}+\sin\theta_{jk}+\sin\theta_{ik}}$$

Energy-momentum conserv. + planar massless kinem.

Jets ordered in energy: Jet 1 = the most energetic jet.

*= used in LEP2 3-jet analysis


OPAL
Data ($\sqrt{s}$ = 91.2 GeV)
Jetset 7.4
Herwig 6.2
Jet 1
Jet 2
Jet 3
$1/N_{ev}\ dN_{jet}/dE_{jet}$
E_{jet} [GeV]
OPAL
Jet 1
Jet 2
Jet 3
$1/N_{ev}\ dN_{jet}/dQ_{jet}$
Q_{jet} [GeV]



Data ($\sqrt{s}$ = 183–209 GeV)
OPAL
Pythia 6.1 + BG
Herwig 6.2 + BG
BG
Jet 1
Jet 2
Jet 3
dN_{jet}/dE_{jet}
E_{jet} [GeV]
OPAL
Jet 3
Jet 2
Jet 1
dN_{jet}/dQ_{jet}
Q_{jet} [GeV]

Correction procedure

1.step: bin-by-bin subtraction of 4f-background from LEP2 data using GRC4F MC model

2.step: unfolding of detector level jets in data and MC to the level of pure quark and gluon jets using purity matrices obtained from MC information

a) B-TAG method for biased and unbiased jets
- based on neural network
- output value from the neural net, VNN, serves to separate udsc, b and gluon jets

b) Energy-ordering method for biased gluon jets
- separates between udscb and gluon jets
- alternative to B-TAG

3.step: bin-by-bin correction for detector and ISR effects

B-TAG for biased jets

Any of three jets is used to extract frag.function.

- Define jets containing sec.vtces with **VNN>a** as *b-tag jets*.
- Define jets containing sec.vtces (if found) with **VNN<b** as *anti-tag jets*.

→ The b-tag jet and gluon jet samples are taken from events with one or two b-tag jets and at least one anti-tag jet.

→ If one b-tag and two anti-tag jets found, the lower energy anti-tag jet enters the gluon jet sample.

→ The udsc jet sample is formed by all three jets in events with no b-tag jet.

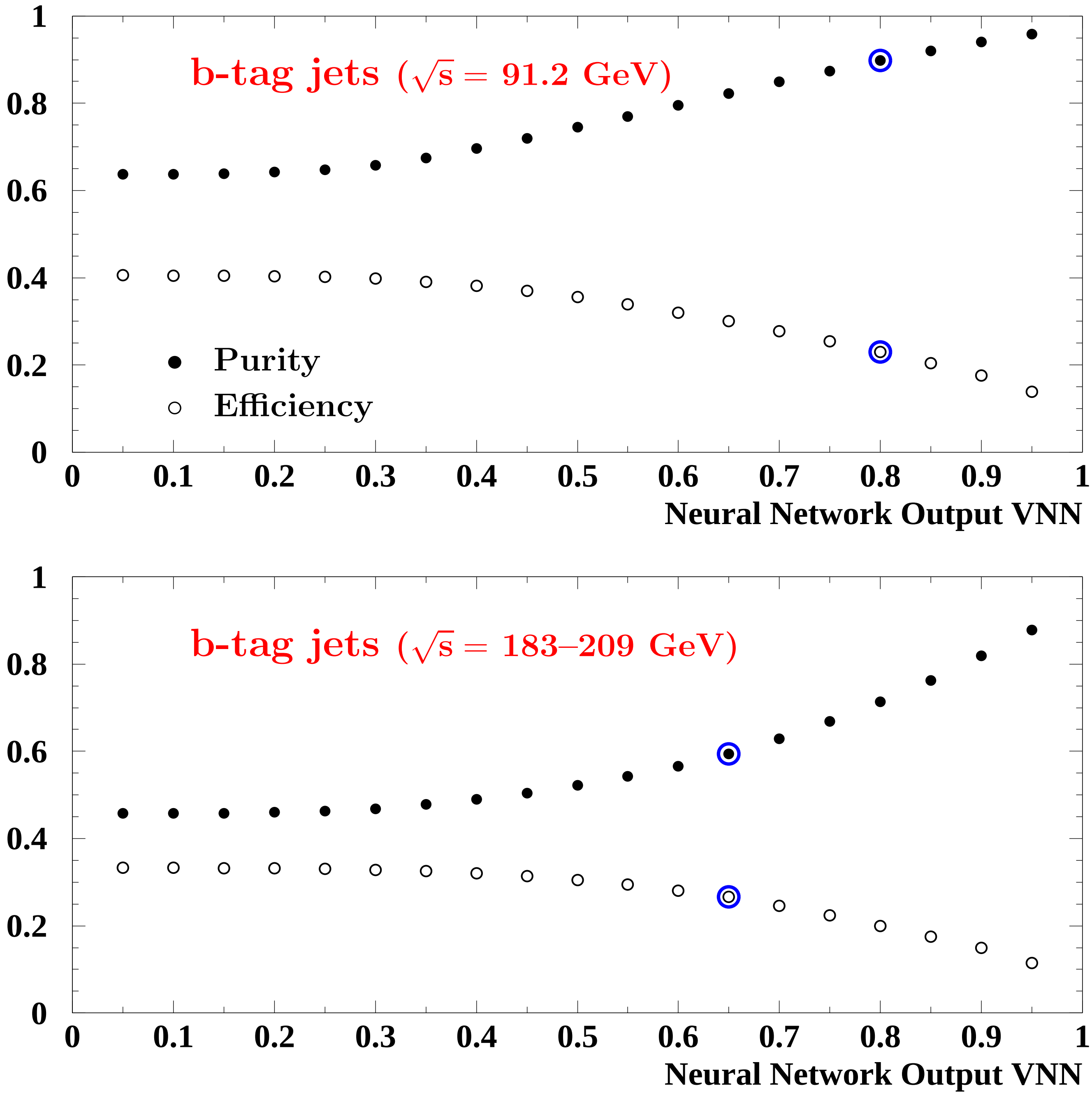
udsc sample: 3 jets per event
b-tag sample: 1-2 jets per event
gluon sample: 1 jet per event

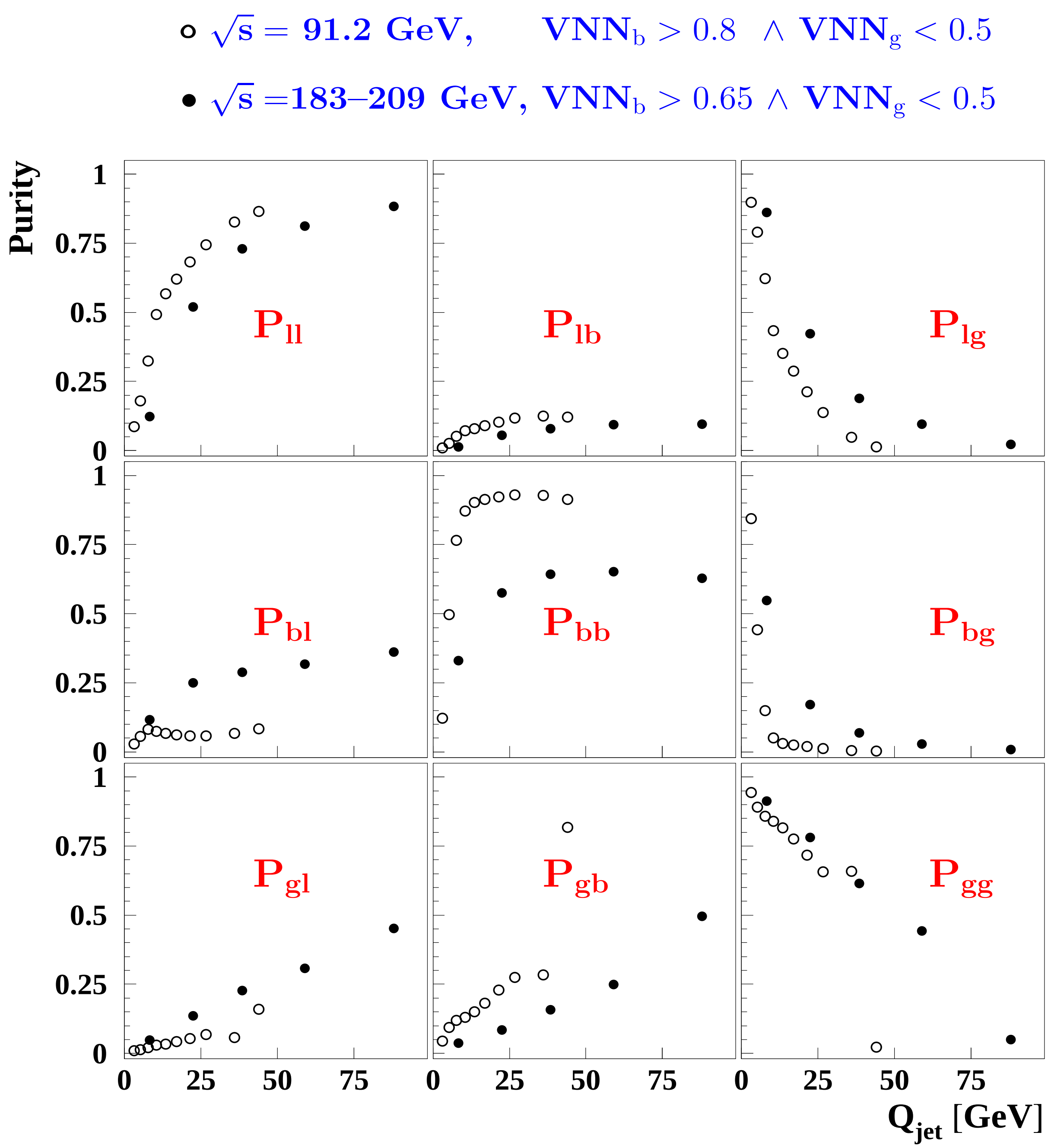
To obtain pure udsc, b or gluon jets, one has to solve

$$\begin{pmatrix} D_{\rm l} \\ D_{\rm b} \\ D_{\rm g} \end{pmatrix}^{\rm uncor} (x_{\rm E}, Q) = \begin{pmatrix} P_{\rm ll} & P_{\rm lb} & P_{\rm lg} \\ P_{\rm bl} & P_{\rm bb} & P_{\rm bg} \\ P_{\rm gl} & P_{\rm gb} & P_{\rm gg} \end{pmatrix} (Q) \begin{pmatrix} D_{\rm l} \\ D_{\rm b} \\ D_{\rm g} \end{pmatrix}^{\rm pure} (x_{\rm E}, Q)$$

E.g. $P_{\rm lb}$ = probability that a jet from the udsc jet sample comes from a b-quark.

Overall $P_{\rm bb}$ ($P_{\rm gg}$) is 90% (84%) for LEP1 data and 60% (80%) for LEP2 data.


b-tag jets ($\sqrt{s}$ = 91.2 GeV)
Purity
Efficiency
Neural Network Output VNN
b-tag jets ($\sqrt{s}$ = 183–209 GeV)
Neural Network Output VNN
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
0.9



○ $\sqrt{s}$ = 91.2 GeV, $VNN_b > 0.8 \wedge VNN_g < 0.5$
● $\sqrt{s}$ =183–209 GeV, $VNN_b > 0.65 \wedge VNN_g < 0.5$
Purity
1
0.75
0.5
0.25
0
P_{ll}
P_{lb}
P_{lg}
P_{bl}
P_{bb}
P_{bg}
P_{gl}
P_{gb}
P_{gg}
25
50
75
Q_{jet} [GeV]

Energy-ordering for biased jets

Based on QCD prediction that in 3-jet event the Jet 3 most likely comes from a gluon.

→ quark jet sample formed by jets 2

→ gluon jet sample formed by jets 3

Unfolding to the level of pure quark and gluon jets:

$$\begin{pmatrix} D_2 \\ D_3 \end{pmatrix}^{\text{uncor}} (x_{\mathrm{E}}, Q) \;=\; \begin{pmatrix} P_{2\mathrm{q}} & P_{2\mathrm{g}} \\ P_{3\mathrm{q}} & P_{3\mathrm{g}} \end{pmatrix} (Q) \begin{pmatrix} D_{\mathrm{q}} \\ D_{\mathrm{g}} \end{pmatrix}^{\text{pure}} (x_{\mathrm{E}}, Q)$$

E.g. $P_{2\mathrm{q}}$ = probability that a jet 2 comes from a quark.

B-TAG for unbiased jets

→ If two sec.vertices with VNN>a found in an event, both hemispheres enter the b-tag sample.

→ In remaining events, both hemi's enter the udsc sample.

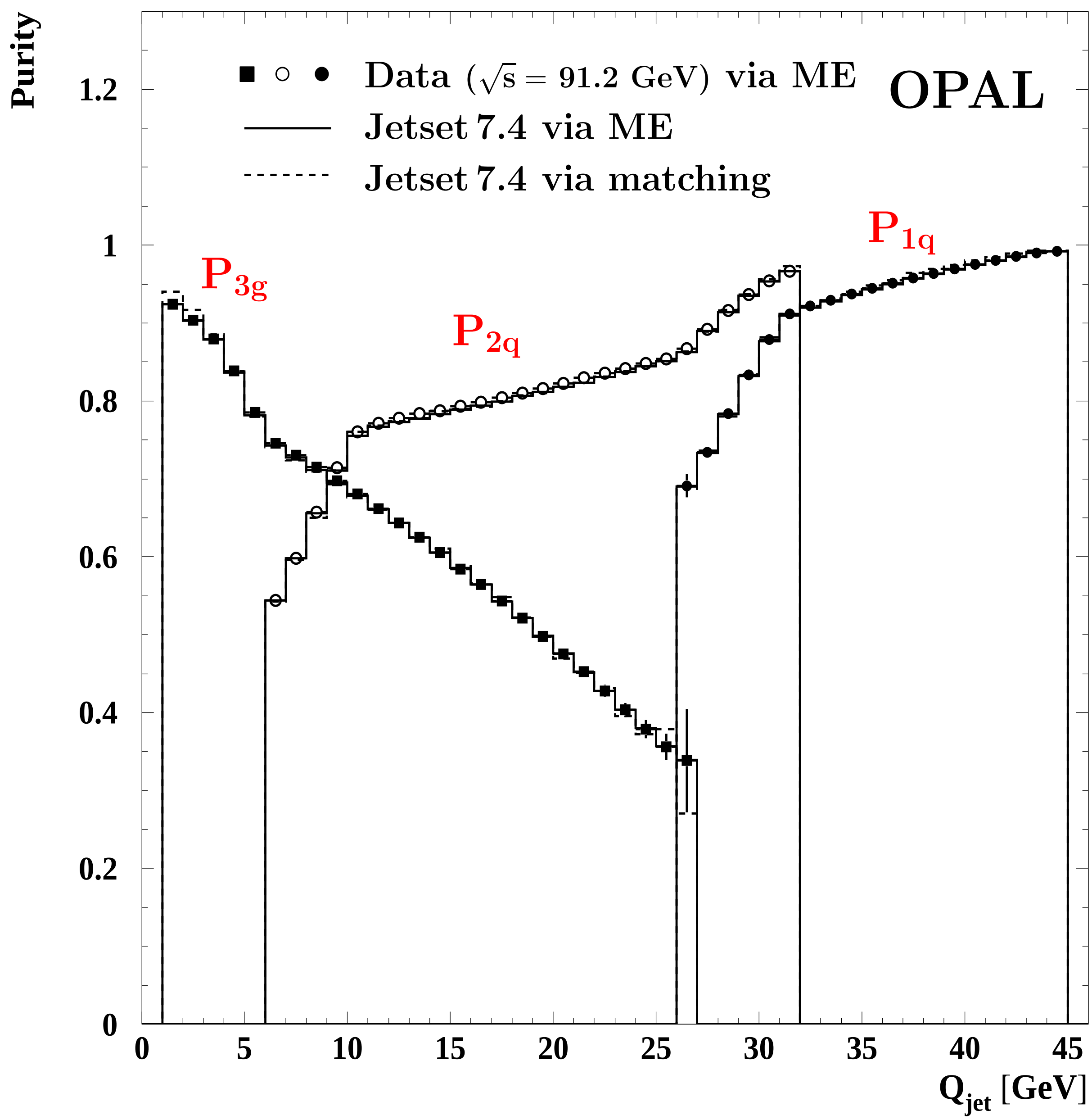
Unfolding to the level of pure udsc and b-quark hemispheres:

$$\begin{pmatrix} D_l \\ D_b \end{pmatrix}^{\text{uncor}} (x_{\mathrm{E}}, Q) \;=\; \begin{pmatrix} P_{\mathrm{ll}} & P_{\mathrm{lb}} \\ P_{\mathrm{bl}} & P_{\mathrm{bb}} \end{pmatrix} (Q) \begin{pmatrix} D_{\mathrm{l}} \\ D_{\mathrm{b}} \end{pmatrix}^{\text{pure}} (x_{\mathrm{E}}, Q)$$

E.g. P_{bb} = prob. that a b-tag hemi comes from a b-quark.

Overall P_{bb} (P_{ll}) is 99.7% (79%) for LEP1 data and 75% (89%) for LEP2 data.

Energy-Ordering



- Applicable only in the overlap region of jets 2 and 3

Event Statistics for Data

UNBIASED JET ANALYSIS (INCL. HADR. EVENTS):

Selection	LEP1	LEP2	BG(LEP2)
Hadronic events	2 387 227	10 866	11%
udsc hemisph.	4 740 774	20 146	11%
b-tag hemisph.	33 680	1 586	5%

BIASED JET ANALYSIS (3-JET EVENTS):

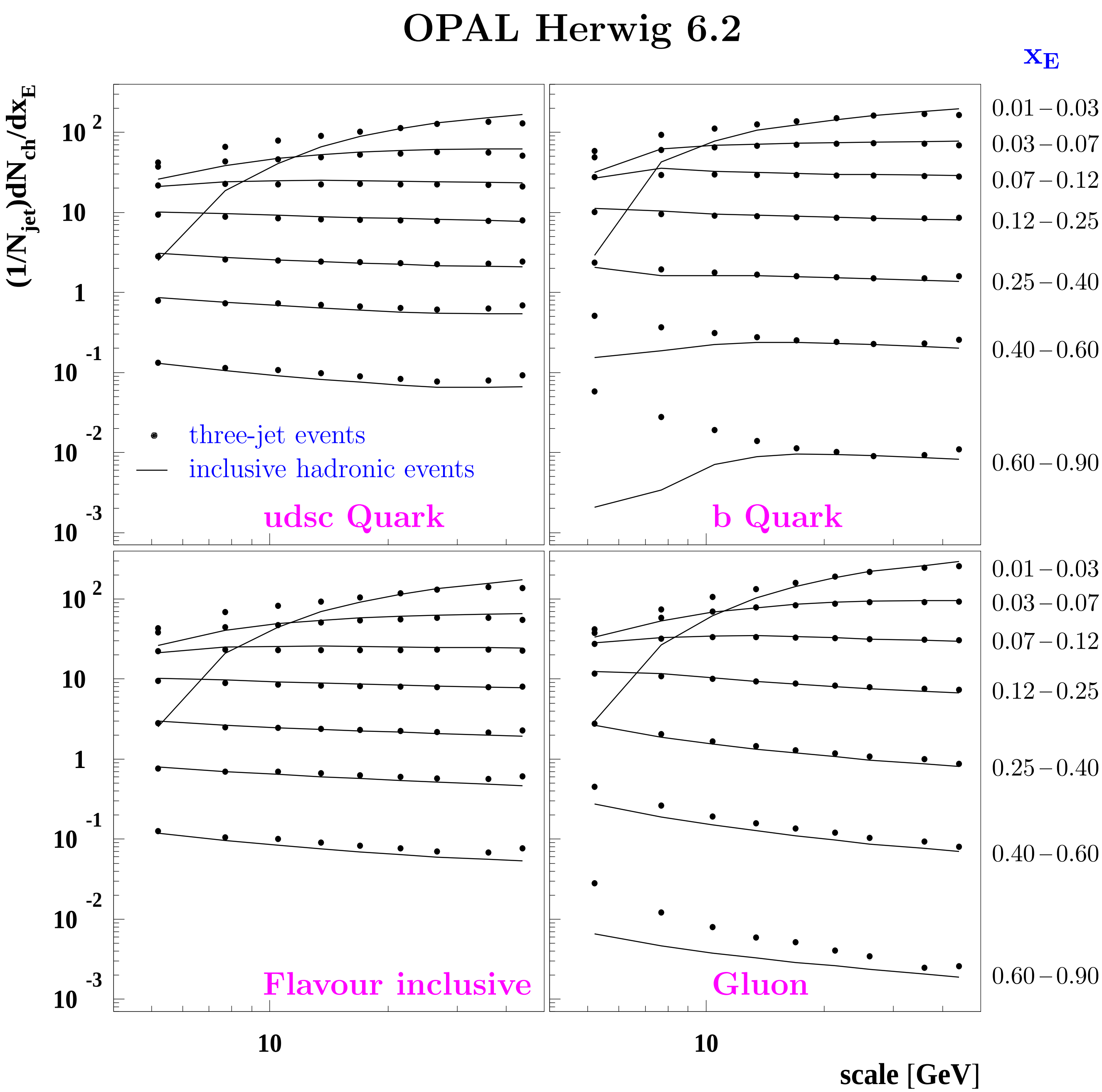
Selection	LEP1	LEP2	BG(LEP2)
Hadronic events	2 387 227	12 653	14%
three-jet events	965 513	6 177	16%
udsc jets	2 675 679	16 344	16%
b-tag jets	83 549	820	9%
Gluon jets	73 620	729	9%

Systematics studies

Sources:

1. Jetset/Pythia $\rightarrow$ Herwig
2. $|\cos\theta_{\rm part}| \leq 0.95 \rightarrow |\cos\theta_{\rm part}| \leq 0.70$
3. track selection: $d_0 \leq 5$ cm $\rightarrow$ $d_0 \leq 2$ cm
   $z_0 \leq 25$cm $\rightarrow$ $z_0 \leq 10$ cm
   $\mathrm{nhits}_{cj} \geq 40 \rightarrow \mathrm{nhits}_{cj} \geq 80$
4. Durham $\rightarrow$ Cone or Cambridge jet alg.
5. b-tag efficiency $\rightarrow$ b-tag efficiency $\pm 10\%$
6. VNN$\geq 0.8 \rightarrow$ VNN $\geq 0.5; 0.95$
7. jet selection: $\mathrm{N}_{\rm part}/\mathrm{jet} \geq 2 \rightarrow \mathrm{N}_{\rm part}/\mathrm{jet} \geq 4$
   $\sum_{i=1}^{3} \alpha_i \geq 358^\circ \rightarrow \sum_{i=1}^{3} \alpha_i \geq 356^\circ; 359^\circ$
   $\alpha_{1,2,3} \geq 30^\circ \rightarrow \alpha_{1,2,3} \geq 25^\circ; 35^\circ$
   $E_{\rm jet}^{\rm calc} \geq 5$ GeV $\rightarrow E_{\rm jet}^{\rm calc} \geq 3; 7$ GeV
8. Purity uncertainty: flavour $\rightarrow$ non-flavour assignment
9. LEP2 ISR bgr: $\sqrt{s'}$ (inv.mass) $\rightarrow \sqrt{s'}$ (en.bal.)
10. LEP2 4-f bgr: $\mathrm{W}_{\rm qcd} \geq -0.5 \rightarrow \mathrm{W}_{\rm qcd} \geq -0.8; 0.0$
    Bgr. $\rightarrow$ Bgr.$\pm$ 5%

MC study of bias



- All FFs at low x_E with low scales: hadron mass effect
- b-FF at high x_E with low scales: b-quark mass effect
- All FFs in last scale bin and gluon-FF at $x_\mathrm{E} > 0.4$: **BIAS**

NLO calculations

1. **Kniehl, Kramer, Pötter** (KKP)
   [Nucl.Phys.B582 (2000) 514]
2. **Kretzer** (Kr)
   [Phys. Rev. D62 (2000) 054001]
3. **Bourhis, Fontannaz, Guillet, Werlen** (BFGW)
   [hep-ph 0009101]

→ They provide NLO predictions of

$$\frac{1}{\sigma_{\rm tot}} \frac{{\rm d}\sigma^{(e^+e^- \to \gamma/Z \to hX)}}{{\rm d}x_{\rm E}}$$

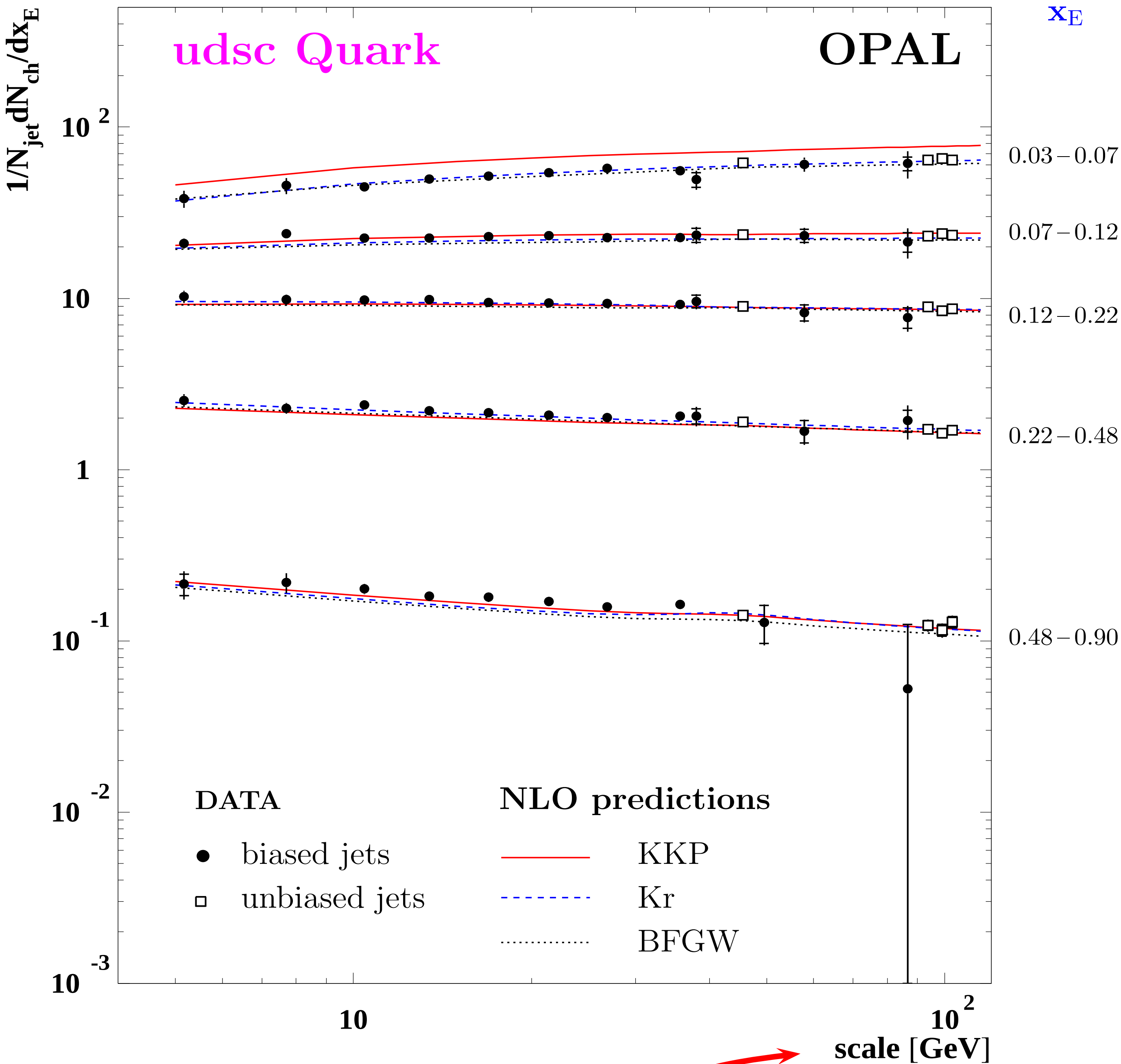
based on unbiased jet definition

→ α_s accuracy of hard subprocess $\sigma^{(e^+e^- \to q\bar{q})}$
→ α_s^2 accuracy of splitting functions
NLO corrections to $\sigma^{(e^+e^- \to q\bar{q}g)}$ not known yet but they will depend on a jet finder used

Assumption

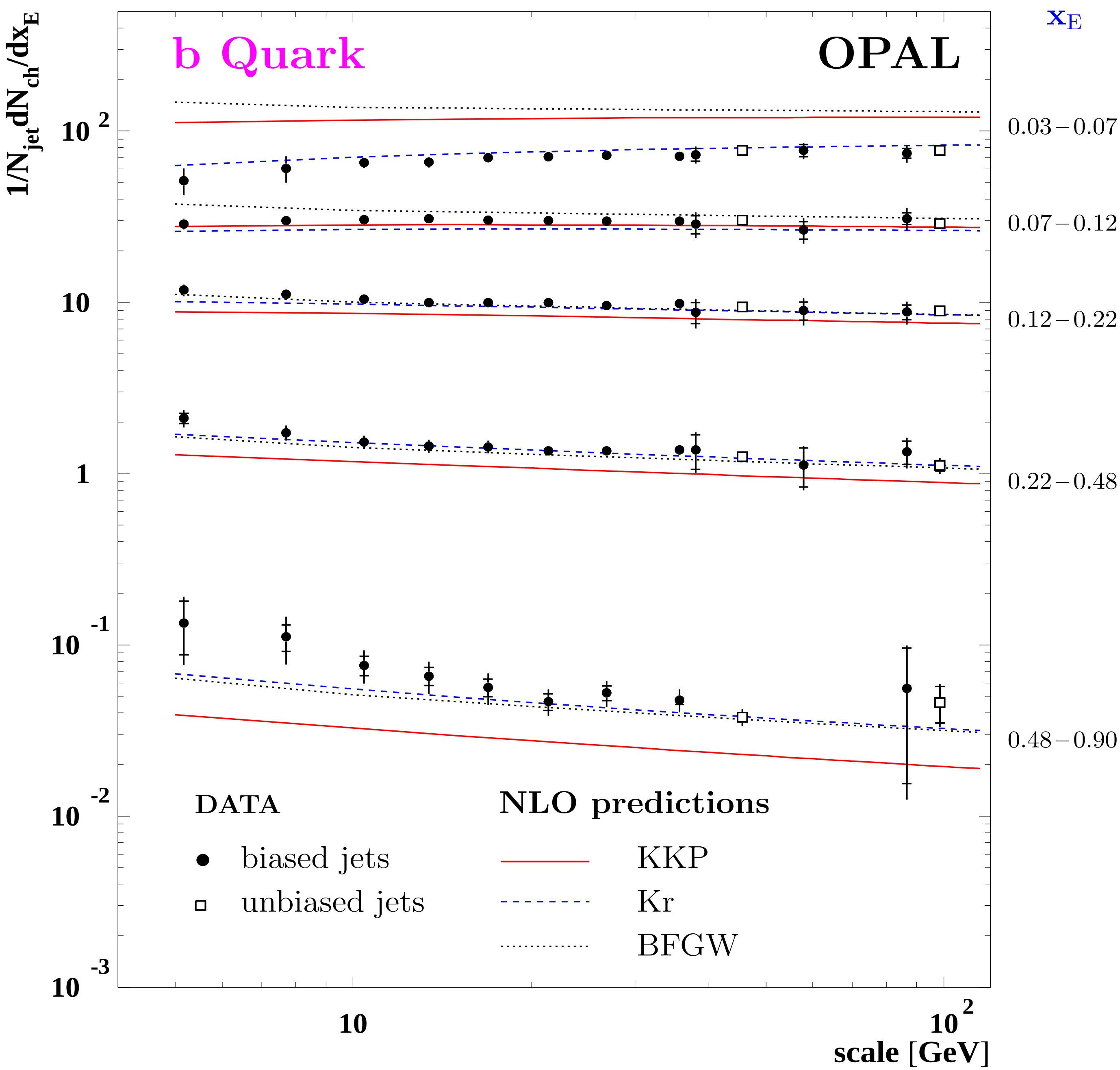
Biased jet results consistent with unbiased jet results
⇓
NLO corrections to 3-jet processes small
⇓
Comparison of biased jet results to theory meaningful

The three groups use $\mu_{\rm r} = \mu_{\rm f} =$ hard scale Q but differ in
! choice of data sets used in fits - definition of the scale Q - fit ranges - prescription for number of active flavours - treatment of heavy quarks and gluons. !

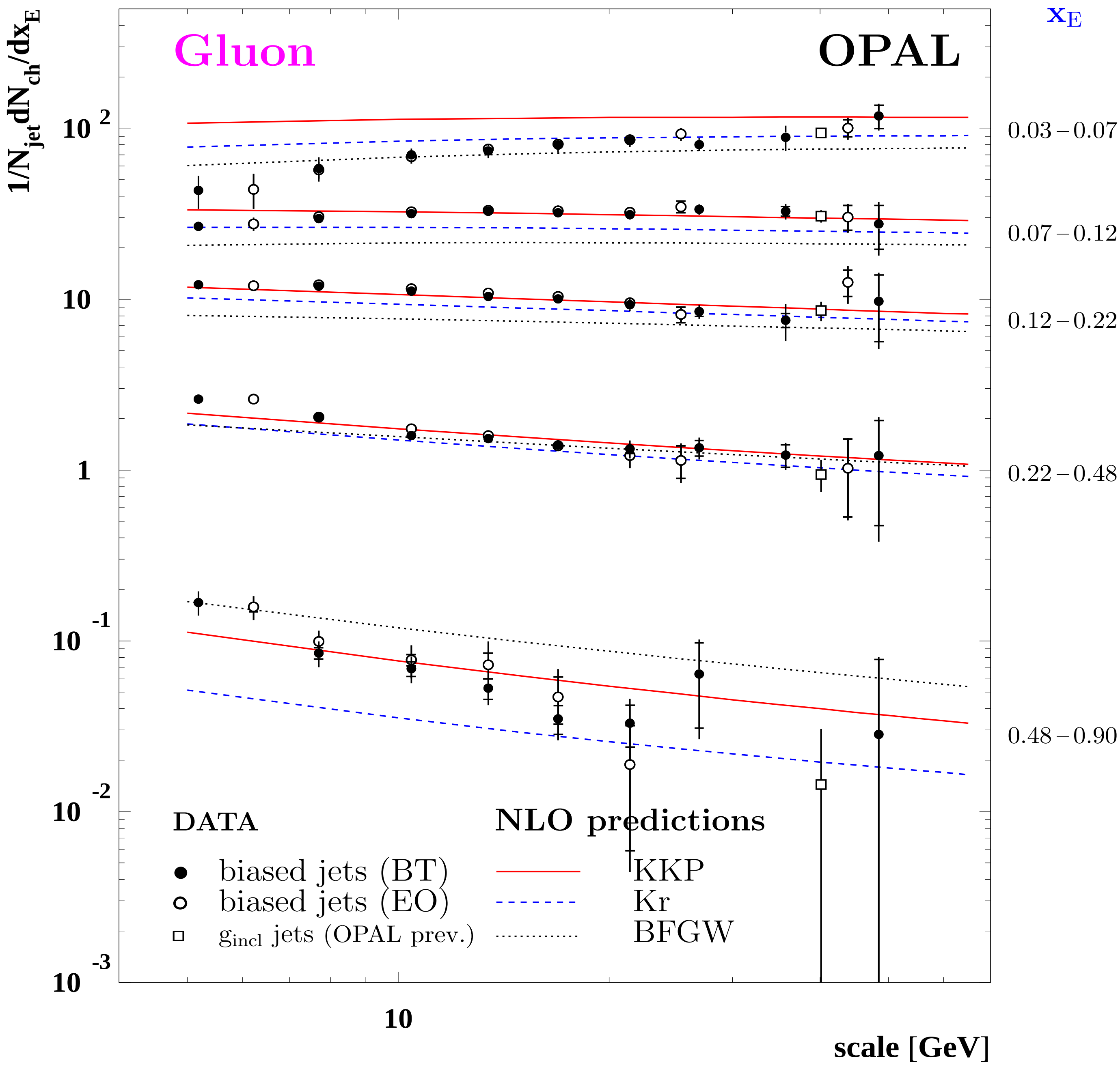


- Consistency between biased and unbiased jet results

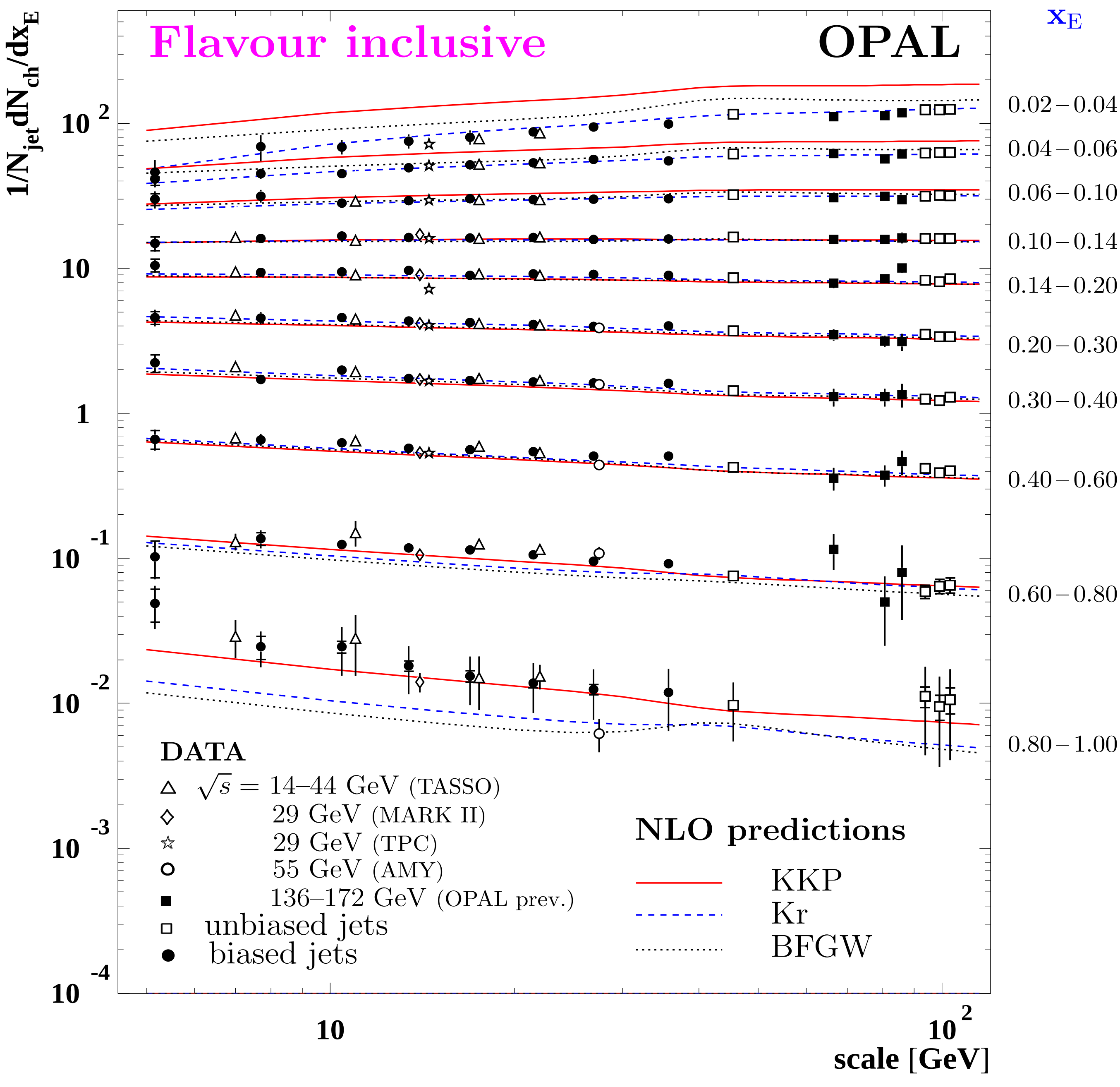
$\sqrt{s}/2$ for unbiased jets, Q_{jet} for biased jets



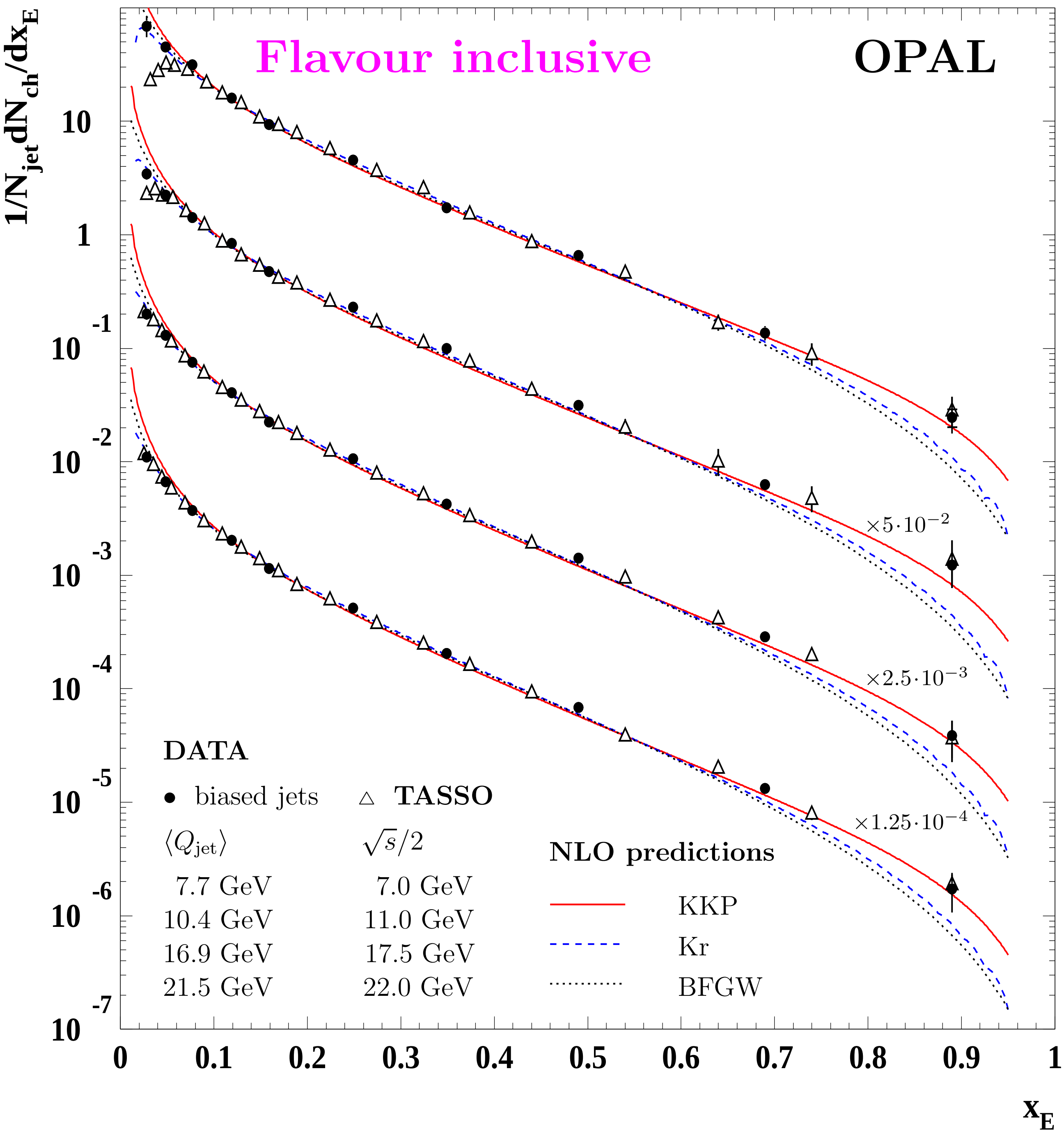
- Consistency between biased and unbiased jet results
- Large spread of the NLO predictions



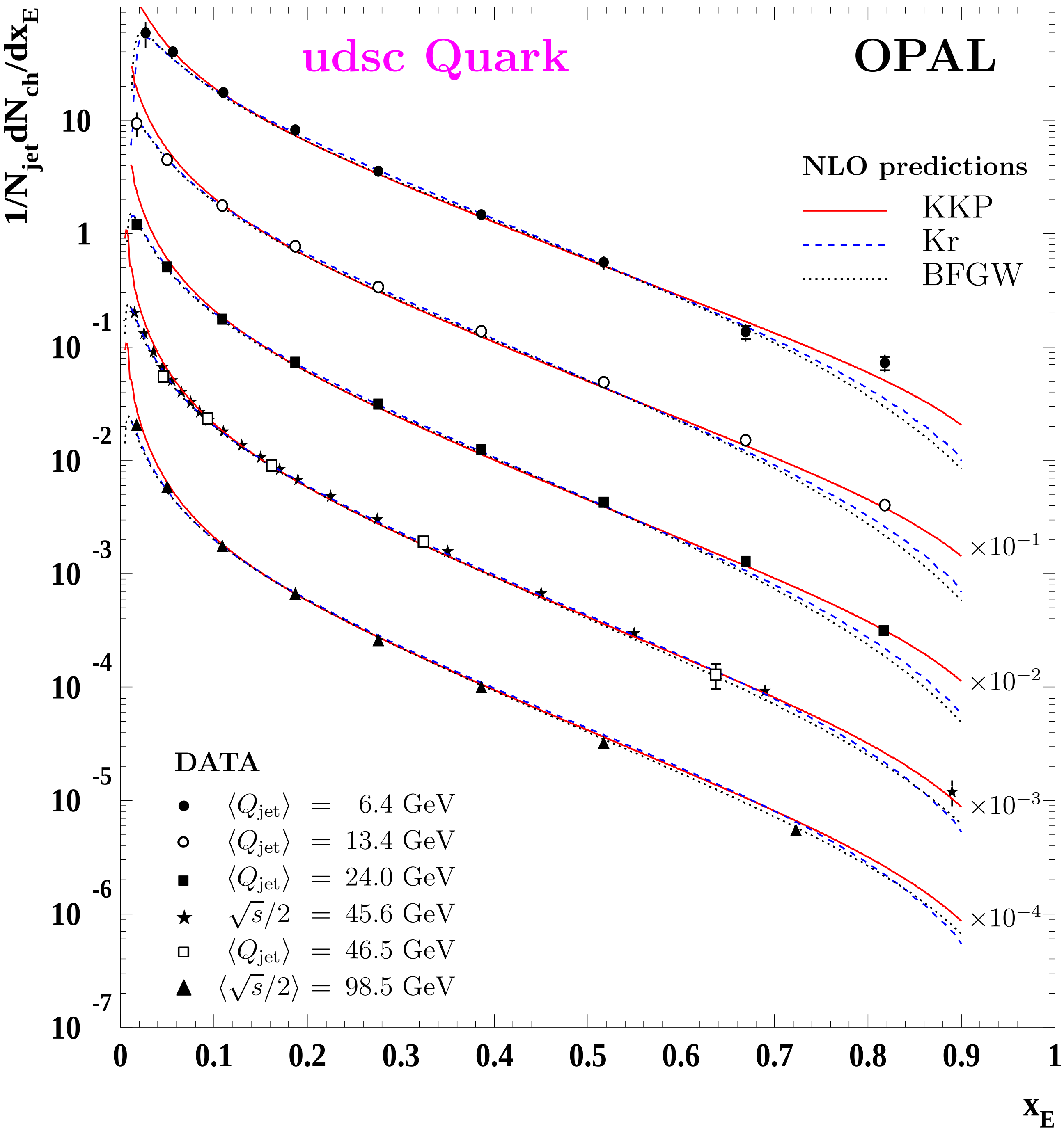
- Consistency between biased and unbiased jet results
- Large spread of the NLO predictions



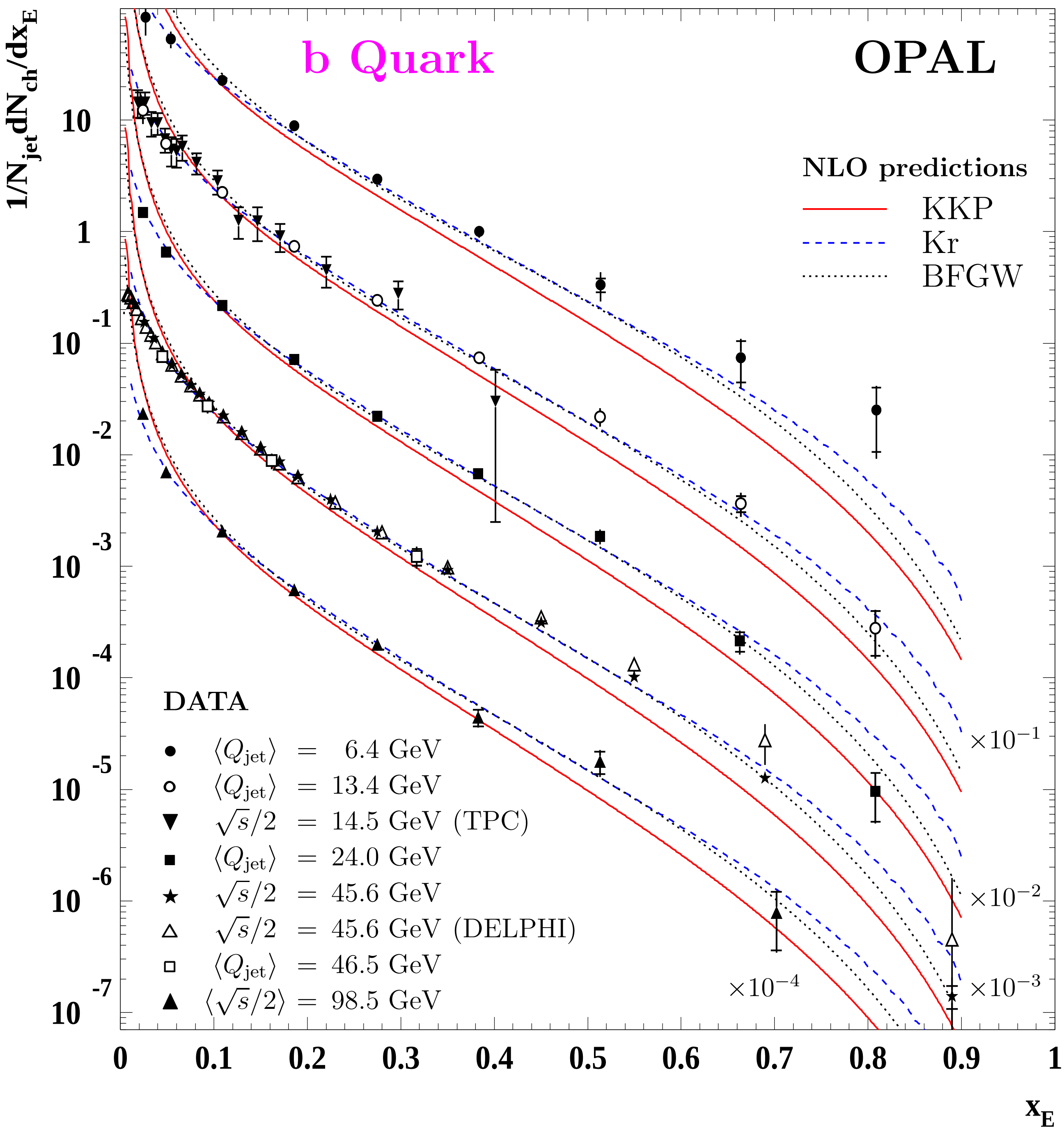
- Results from measured biased and unbiased jets consistent with published unbiased jet results



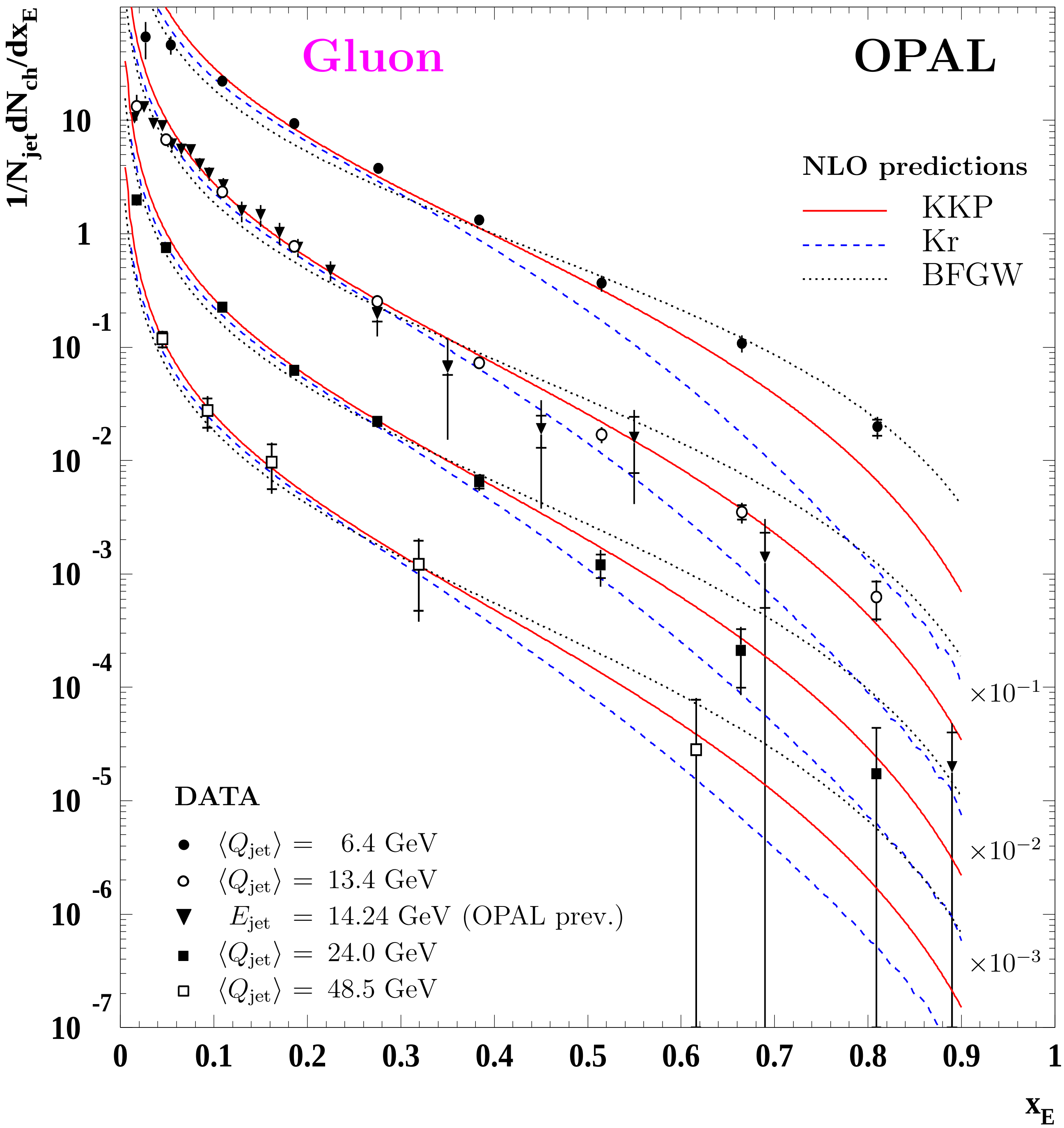
- Low x_E with low scale: hadron mass effect in unbiased jets



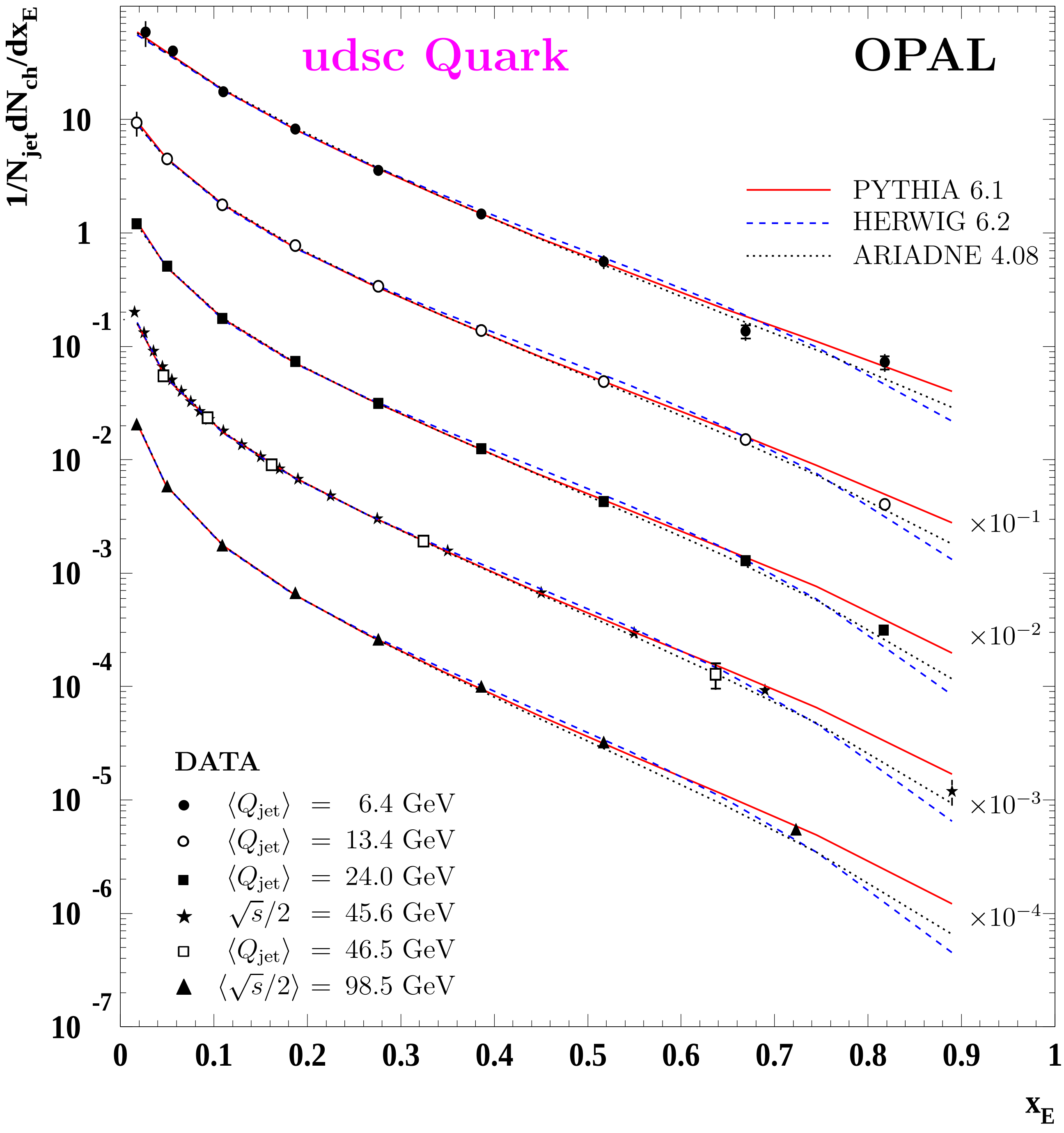
- Differences between NLO predictions at very low and very high $x_{\rm E}$



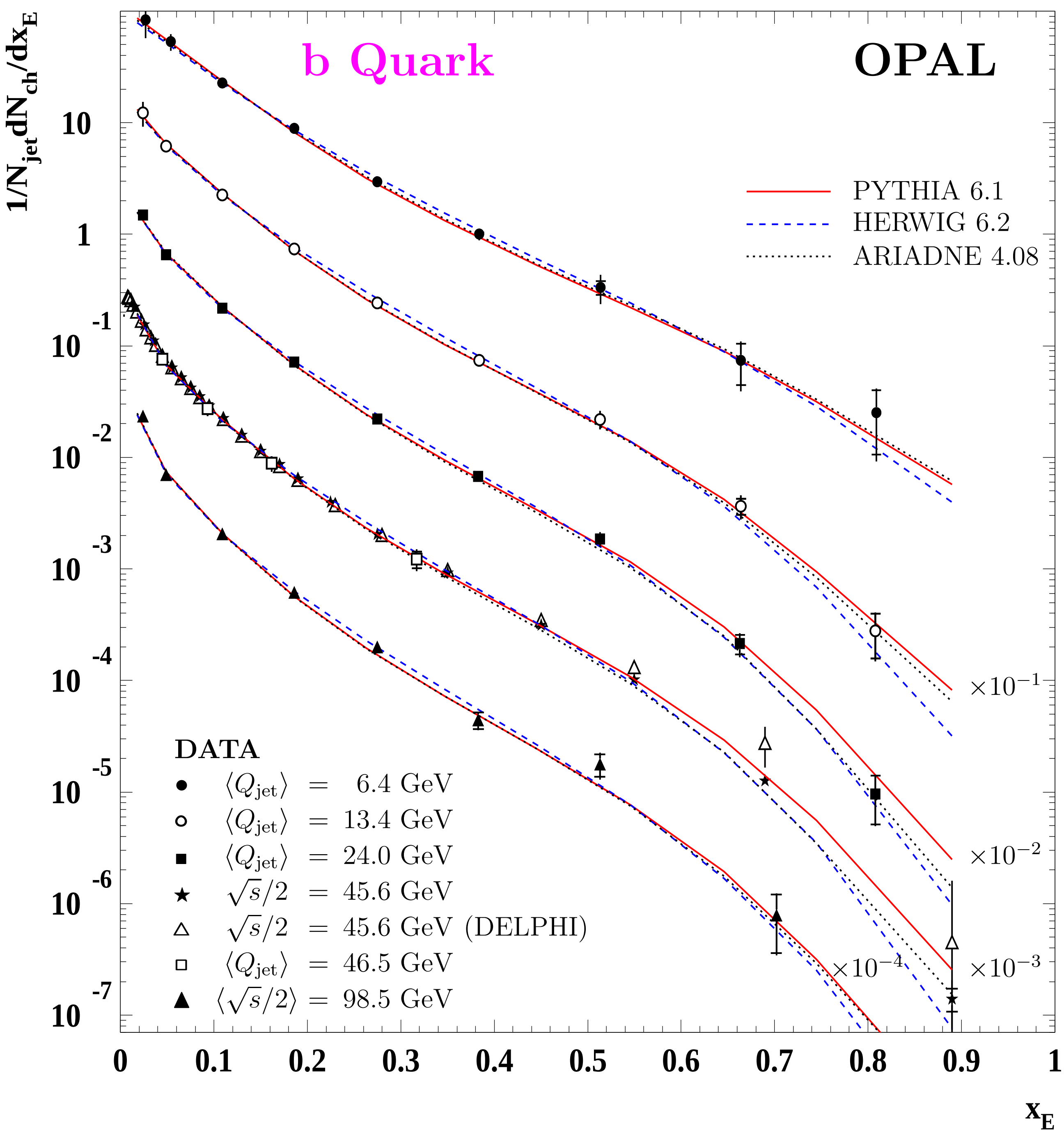
- Differences between NLO predictions at very low and very high x_{E}
- Data agree with TPC and DELPHI unbiased jet results



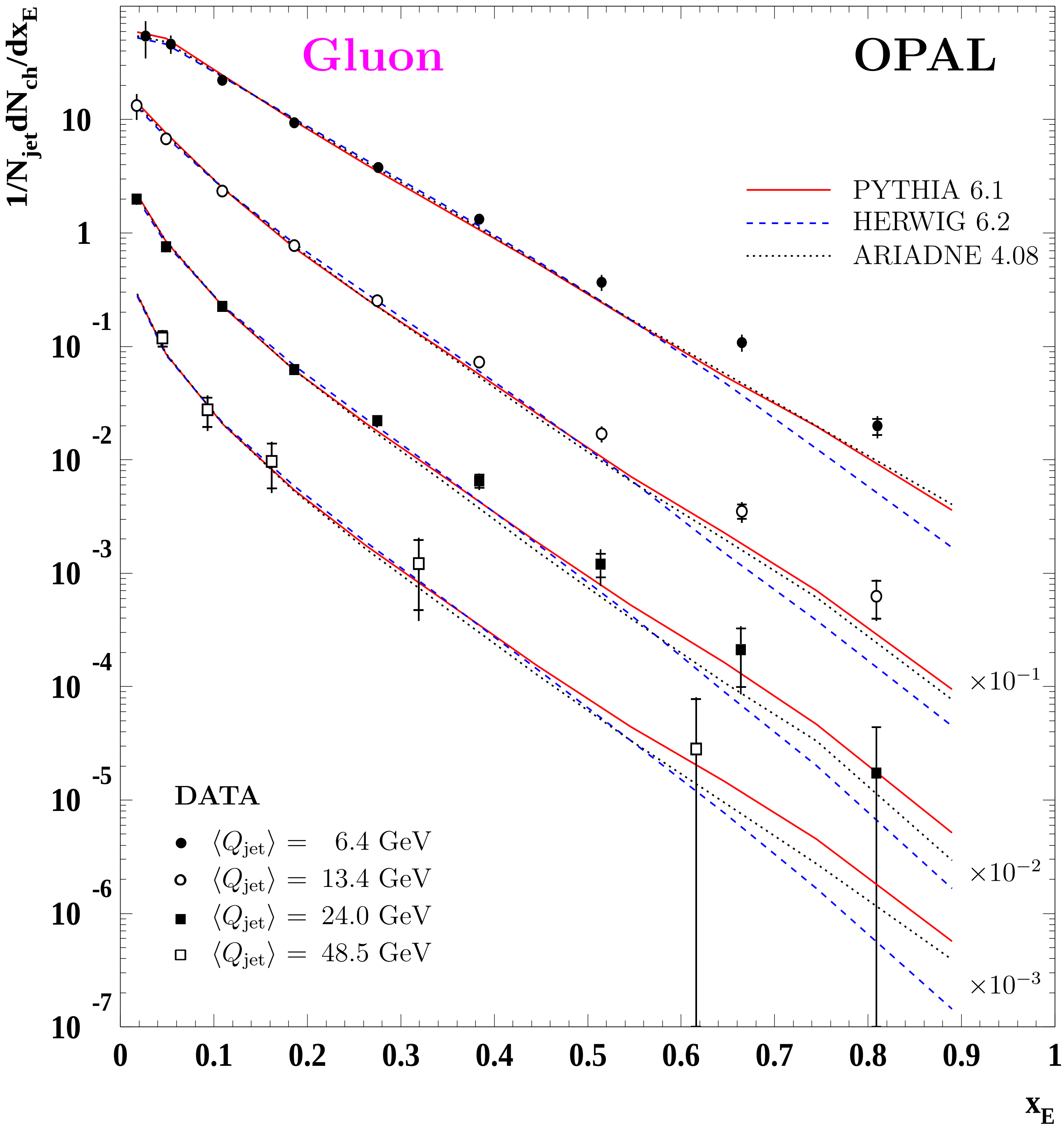
- Differences between NLO predictions at very low and very high x_{E}
- Data agree with OPAL boost algorithm results



- Very good description by all three MC models
  - measured (un)biased jets compared to MC (un)biased jets
  - OPAL tune for LEP1 data still good for LEP2 data



- Very good description by all three MC models
  - measured (un)biased jets compared to MC (un)biased jets
  - OPAL tune for LEP1 data still good for LEP2 data



- Good description by all three MC models except for small scales and high x_{E}

Charged particle multiplicities

$\sqrt{s}$ [GeV]	$\langle n_{ch}^{incl} \rangle$
91.2	$20.93 \pm 0.01 \pm 0.23$
183–189	$26.80 \pm 0.24 \pm 0.46$
192–202	$27.68 \pm 0.26 \pm 0.50$
204–209	$27.75 \pm 0.29 \pm 0.67$

$\sqrt{s}$ [GeV]	$\langle n_{ch}^{udsc} \rangle$
91.2	$20.32 \pm 0.03 \pm 0.27$
183–189	$26.43 \pm 0.26 \pm 0.81$
192–202	$27.38 \pm 0.31 \pm 0.85$
204–209	$26.87 \pm 0.32 \pm 0.99$

$\sqrt{s}$ [GeV]	$\langle n_{ch}^{b} \rangle$
91.2	$23.28 \pm 0.09 \pm 0.70$
183–209	$30.01 \pm 0.53 \pm 0.82$

The results are found in agreement with the previous measurements and with predictions of PYTHIA 6.1, HERWIG 6.2 and ARIADNE 4.08.

Conclusions

1. 7 types of FFs measured:

	biased jets	**unbiased jets**
	$Q_{\rm jet}$ [GeV]	$\sqrt{s}/2$ [GeV]
udscb	4–42.0	45.6; 91.5–104.5
udsc	4–104.5	45.6; 91.5–104.5
b	4–104.5	45.6; 91.5–104.5
gluon	4–70.0	

2. The results found to be consistent with all published results. Consistency between biased and unbiased jet results suggests that $Q_{\rm jet}$ is an appropriate choice of scale in events with a general 3-jet topology. This consistency also justifies the comparison with NLO predictions.

3. The scaling violation of the gluon jet FFs is observed to be stronger than that of the quark jet FFs.

4. The NLO calculations describe well the udsc jet FFs, but much worse the b- and gluon jet FFs.

5. The data compared to different fragmentation models. Pythia, Herwig and Ariadne describe the data well except for the range of high $x_{\rm E}$ with small scale for the gluon jet FFs.

6. Charged particle multiplicities in udscb, udsc and b events measured and found to be in agreement with previous measurements and with predictions of the three MC models.