Jets and Multiplicities in e^+e^-

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- α_s from 4-jet rates
  (DELPHI [307])
- α_s, C_A and C_F from R_4, D_2 and 4-jet angular correlations
  (OPAL [12] [389])
- charged multiplicities in 3-jet events
  (DELPHI [315])
- helicity components of fragmentation functions
  (JADE [270], DELPHI [365])

Determination of α_s and it's running from $R_4(y_{\rm cut})$

$$R_4(y_{\rm cut}) = \frac{\sigma_{4-\rm jets}}{\sigma_{\rm tot}}(y_{\rm cut}),$$

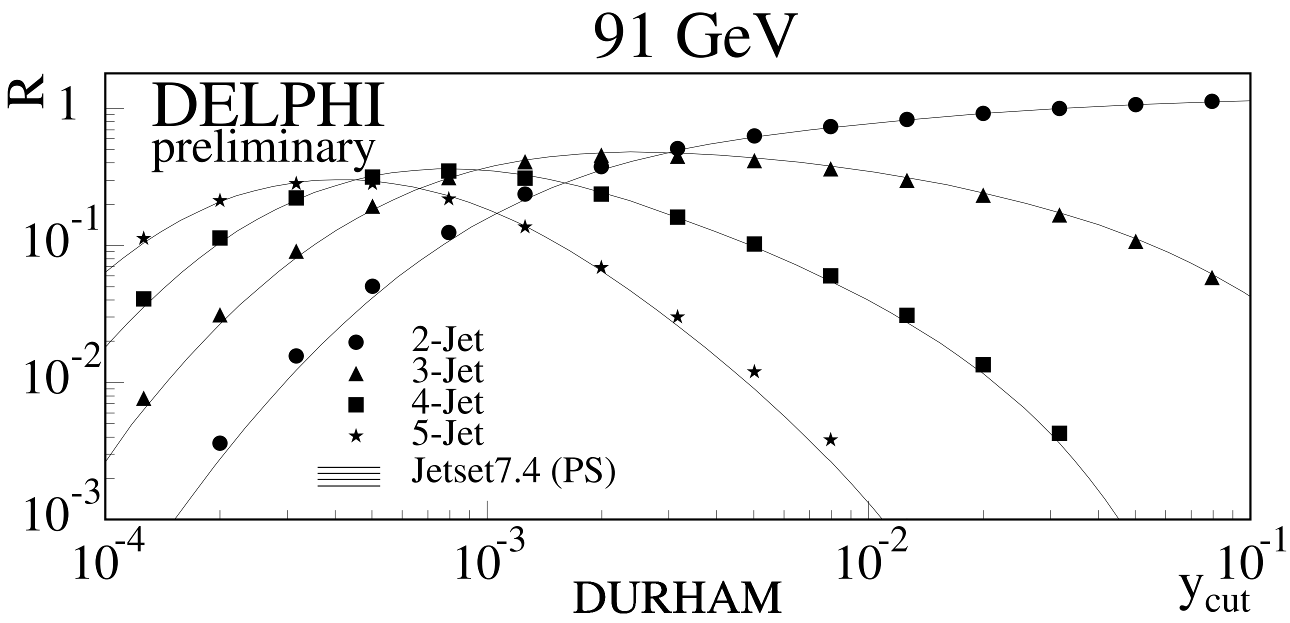
where a 4-jet event is defined as: $y_{34} > y_{\rm cut} > y_{45}$

DELPHI 2001-059:

2, 3, 4 and 5-jet rates at $\sqrt{s} = 91, 133, 189, 200$ and 207 GeV

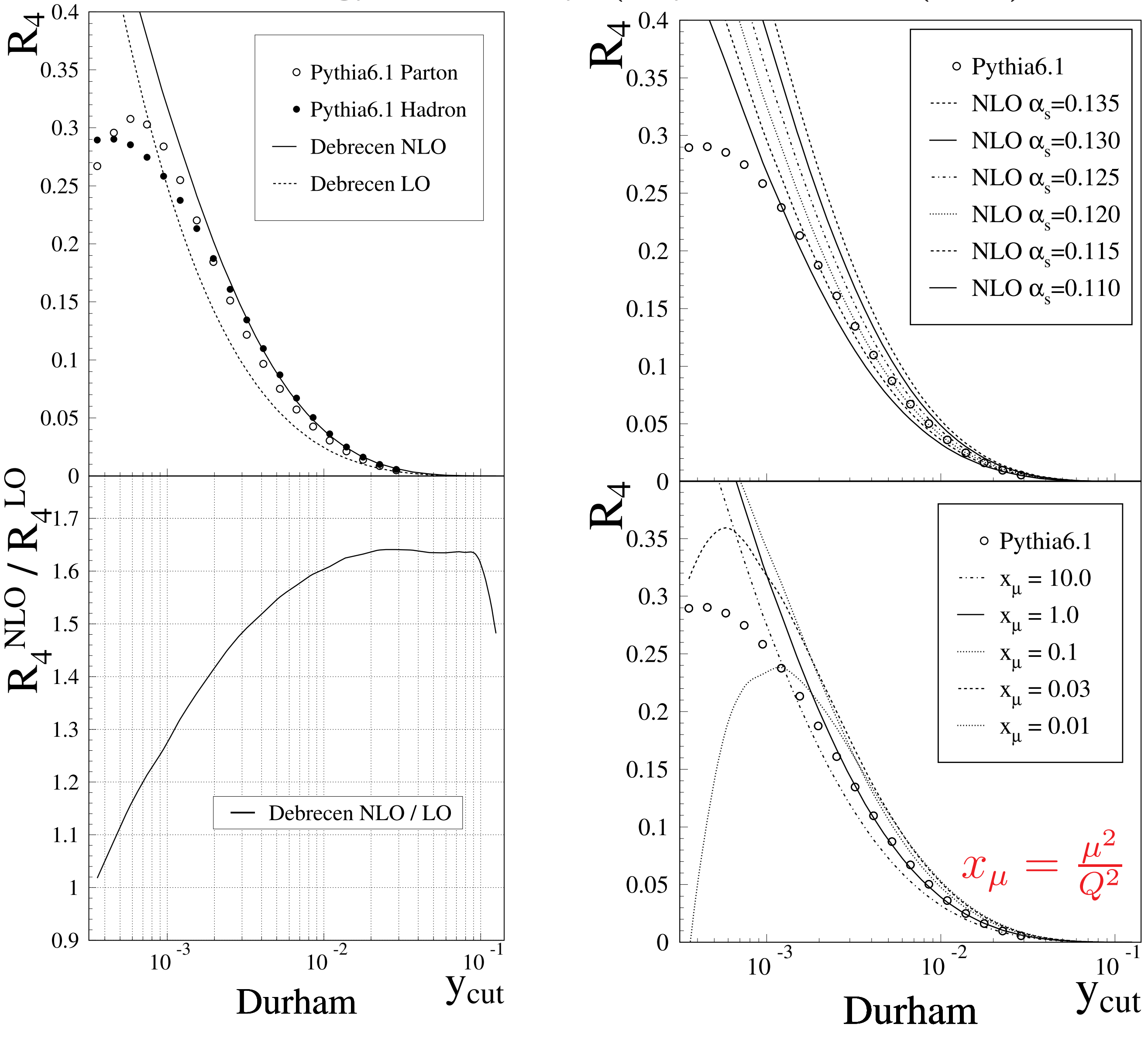
Using: Jade, **Durham** and Cambridge.

An example:



$$R_4(y) = B(y)\alpha_s^2(\mu) + \left[C(y) + 2B(y)\frac{\beta_0}{4\pi}\log\frac{\mu^2}{Q^2}\right]\alpha_s^3(\mu) + \mathcal{O}(\alpha_s^4)$$

DEBRECEN: Nagy & Trócsányi (Phys. Rev. D59 (1999) 014020)

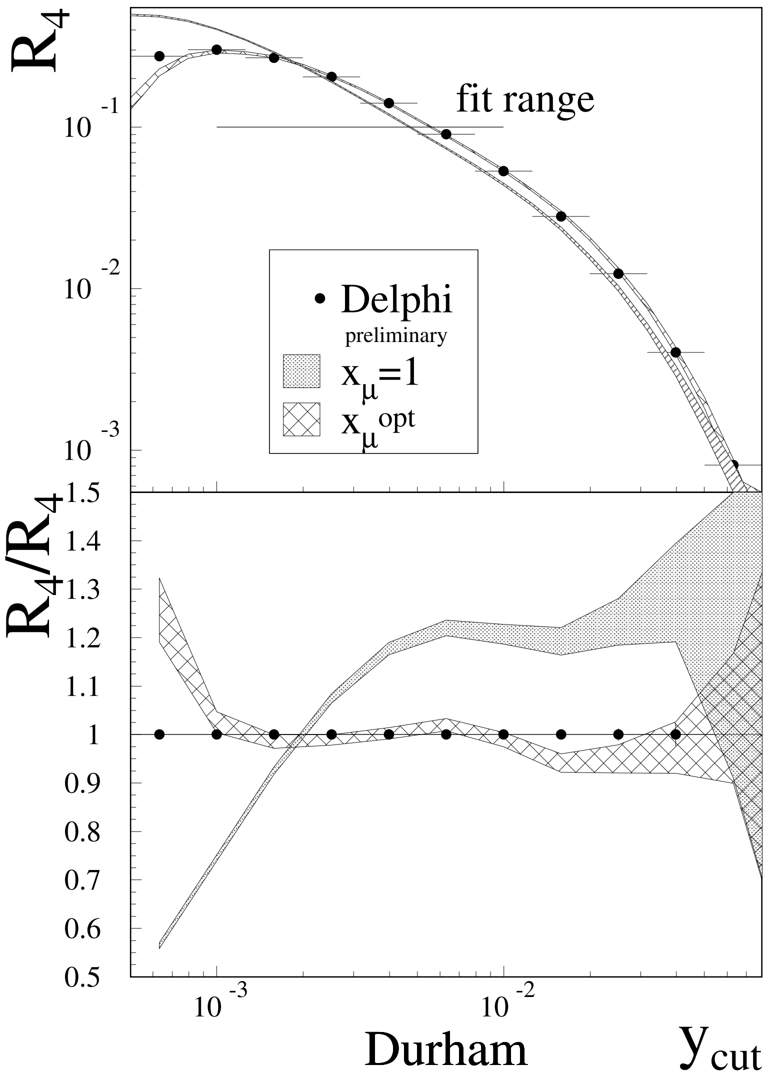


Indication of large missing higher orders:

ALEPH 2000-045: R_4 fitted to NLO+NLL

DELPHI this analysis: NLO + optimised scales

α_s from $R_4(y)$ at $\sqrt{s} = M_Z$



Fit procedure:

1) fit α_s & $x_\mu^{\rm opt}$ to $R_4(y)$

   $x_\mu^{\rm opt} = 0.015$

   (Cambridge: $x_\mu^{\rm opt} = 0.042$)

2) fix $x_\mu^{\rm opt}$ and fit α_s to

   to $R_4(y = 0.063)$

Scale uncertainty: ($x_\mu = \frac{\mu^2}{Q^2}$)
conventional: $\frac{1}{4} < x_\mu < 4$
Delphi: $\frac{1}{2}x_\mu^{\rm opt} < x_\mu < 2x_\mu^{\rm opt}$

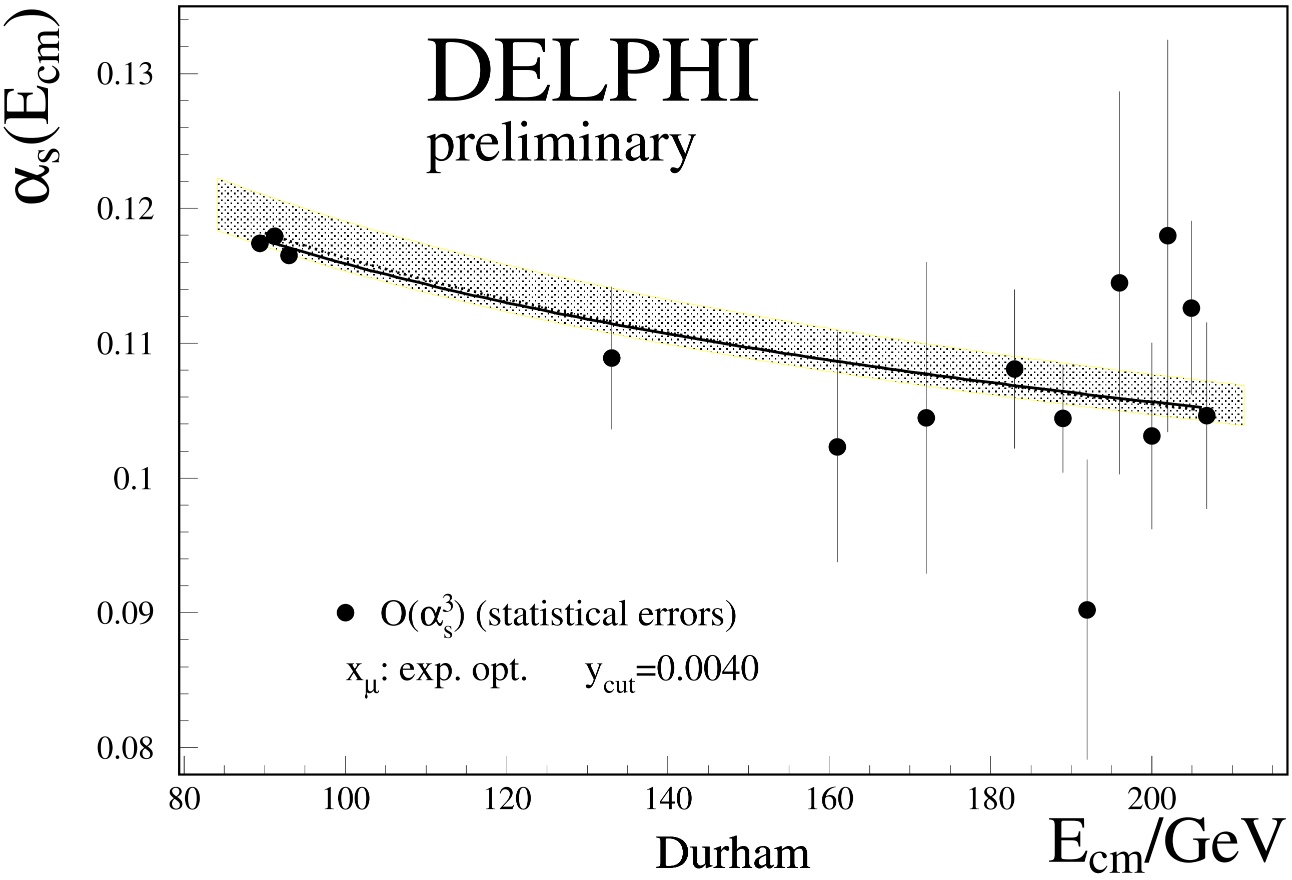
DELPHI preliminary:

$\boldsymbol{\alpha_s(M_Z) = 0.1178 \pm 0.0012(\mathrm{exp.}) \pm 0.0023(\mathrm{hadr.}) \pm 0.0014(\mathrm{scale})}$
(Cambridge: $0.1175 \pm 0.0010(\mathrm{exp.}) \pm 0.0017(\mathrm{hadr.}) \pm 0.0007(\mathrm{scale})$)

For comparison: **ALEPH 2000-045**: NLO + NLL
$\alpha_s(M_Z) = 0.1172 \pm 0.0003(\mathrm{exp.}) \pm 0.0008(\mathrm{had.}) \pm 0.0040(\mathrm{scale})$
NLO + NLL + $x_\mu^{\rm opt}$: $\alpha_s(M_Z) = 0.11760$, $x_\mu^{opt} = 0.67$

The running of α_s from 4-jet rates

DELPHI extracted α_s from R_4 at all LEP c.m-energies.



Good agreement with QCD expectation.

Simultaneous measurement of α_s, C_A and C_F from R_4, D_2 and 4-jet angular correlations.

(OPAL: CERN-EP-2001-001, Accepted by Eur. Phys. J. C)

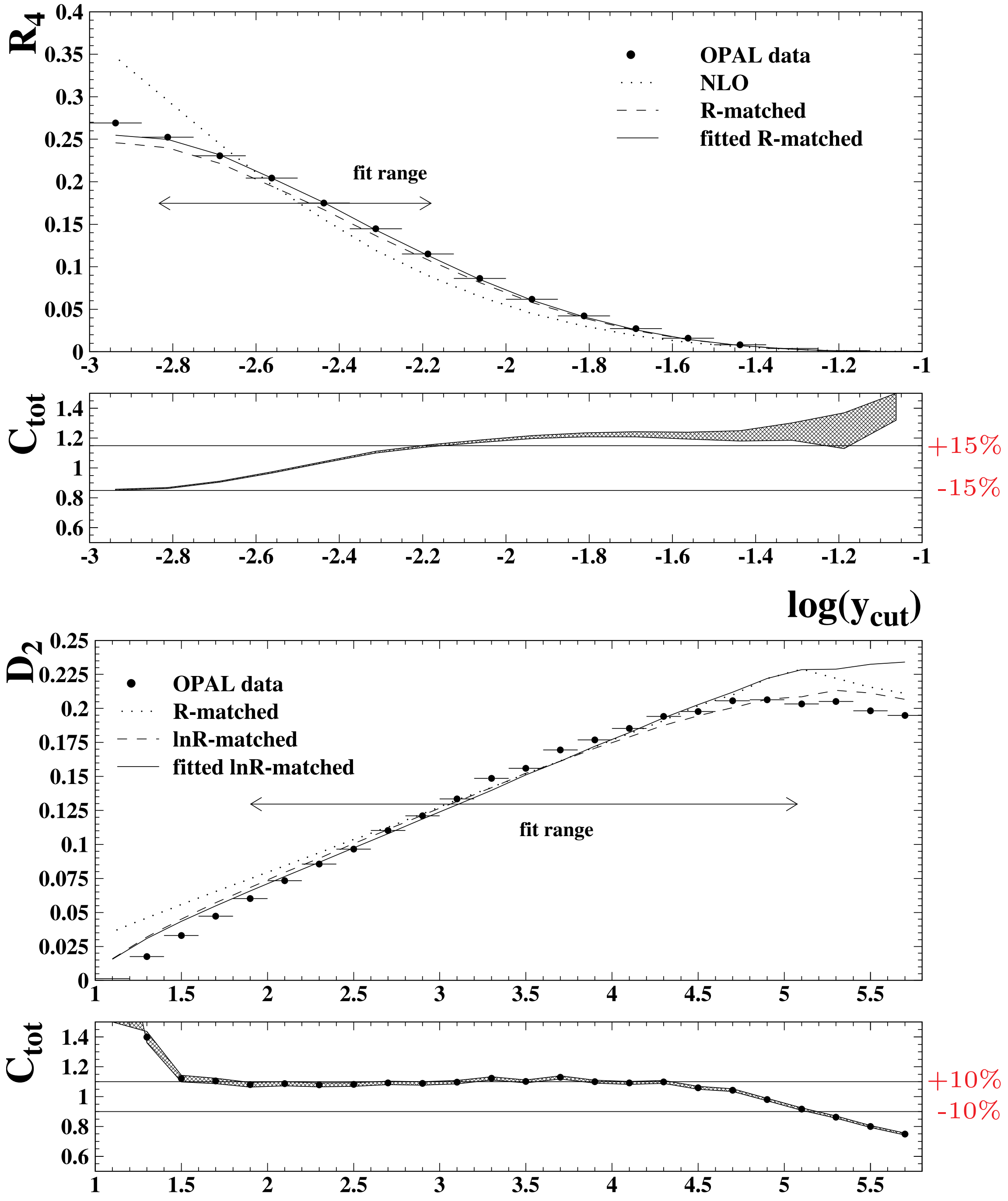
- $D_2(y_{23}) \equiv \frac{1}{\sigma_{\rm tot}} \frac{{\rm d}\sigma}{{\rm d}y_{23}}$,
- $R_4(y_{\rm cut}) \equiv \frac{\sigma_{4-{\rm jet}}(y_{\rm cut})}{\sigma_{\rm tot}}$,

and the 4-jet angular correlations ($E_{\rm jet\,1} > E_{\rm jet\,2} > E_{\rm jet\,3} > E_{\rm jet\,4}$):

- the Bengtsson-Zerwas angle:
  $\chi_{BZ} = \measuredangle[(\vec{p}_1 \times \vec{p}_2), (\vec{p}_3 \times \vec{p}_4)]$;
- the modified Nachtmann-Reiter angle:
  $\Theta_{NR} = \measuredangle[(\vec{p}_1 - \vec{p}_2), (\vec{p}_3 - \vec{p}_4)]$;
- the Körner-Schierholtz-Willrodt angle:
  $\Phi_{KSW} = \frac{1}{2}\left(\measuredangle[(\vec{p}_1 \times \vec{p}_4), (\vec{p}_2 \times \vec{p}_3)] + \measuredangle[(\vec{p}_1 \times \vec{p}_3), (\vec{p}_2 \times \vec{p}_4)]\right)$;
- the angle between the two lowest energy jets:
  $\alpha_{3,4} = \measuredangle[\vec{p}_3, \vec{p}_4]$.

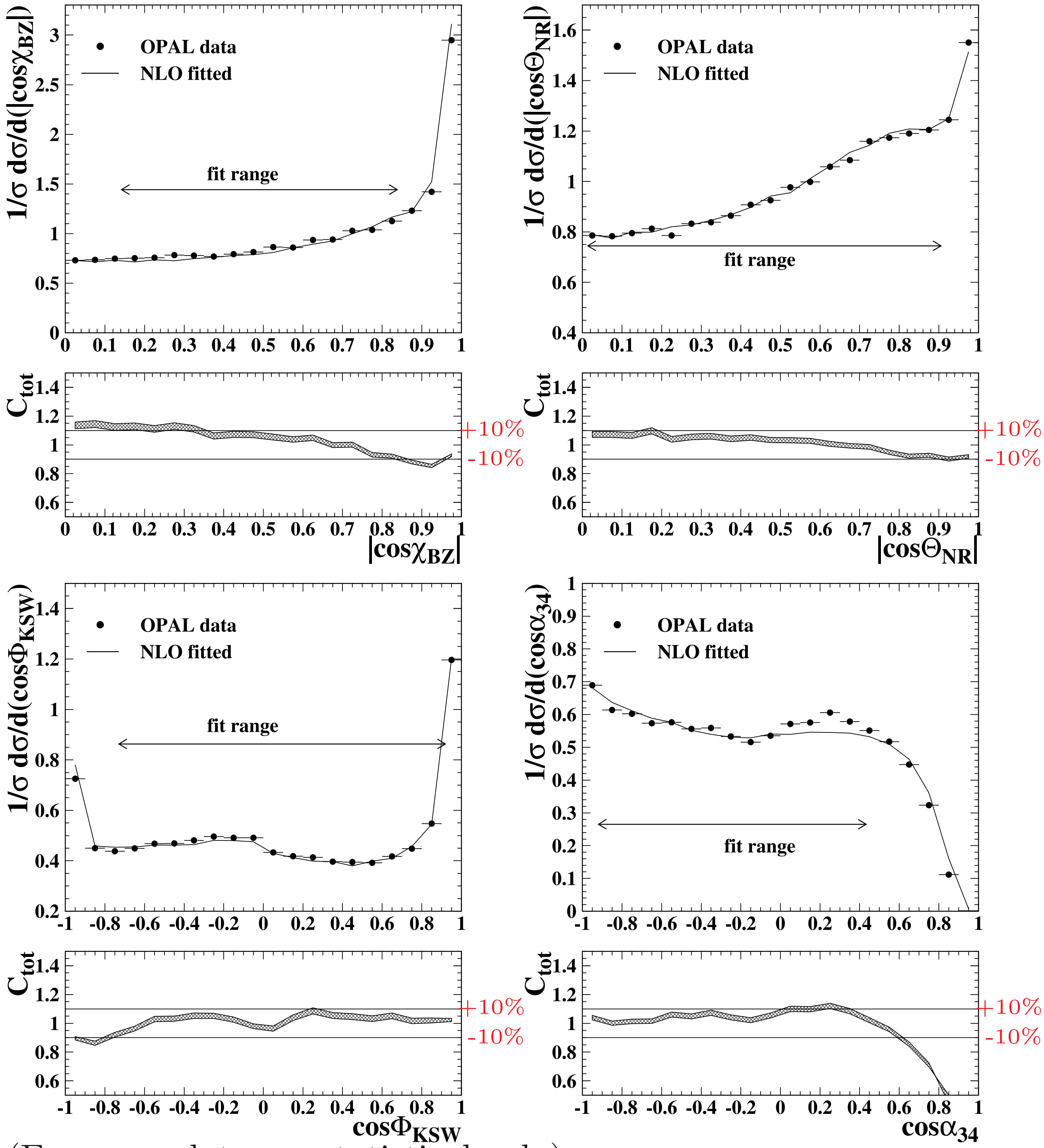
NEW: availability of NLO predictions for the angular correlations (Nagy & Trócsányi).

R_4 and D_2 fitted to NLO + NLL



(Errors on data are statistical only) $-\ln(y_{23})$

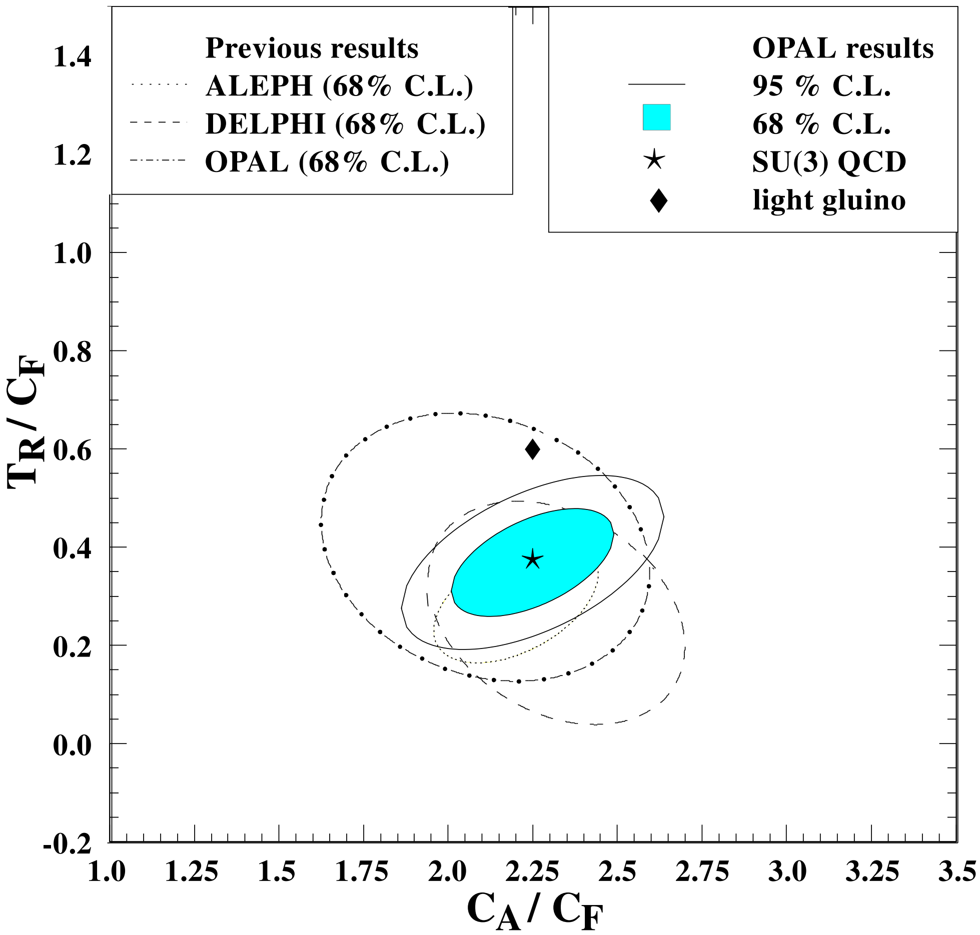
χ_{BZ}, Θ_{NR}, Φ_{KSW} and $\alpha_{3,4}$ fitted to NLO



(Errors on data are statistical only)

Results

$$\eta = \alpha_s C_F/2\pi = 0.0256 \pm 0.0003(\text{stat.}) \pm 0.0013(\text{syst.})$$
$$x = C_A/C_F = 2.25 \pm 0.08(\text{stat.}) \pm 0.14(\text{syst.})$$
$$y = T_R/C_F = 0.37 \pm 0.04(\text{stat.}) \pm 0.06(\text{syst.})$$



(note: ♦ = LO pQCD + light gluino + classical hadr. models)

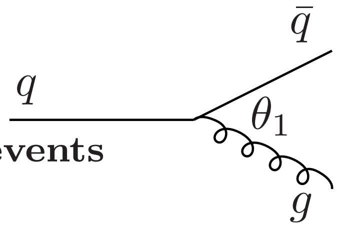
$$\boldsymbol{\alpha_s = 0.120 \pm 0.011(\text{stat.}) \pm 0.020(\text{syst.})}$$
$$\boldsymbol{C_A = 3.02 \pm 0.25(\text{stat.}) \pm 0.49(\text{syst.})}$$
$$\boldsymbol{C_F = 1.34 \pm 0.13(\text{stat.}) \pm 0.22(\text{syst.})}$$

Charged multiplicity in 3-jet events

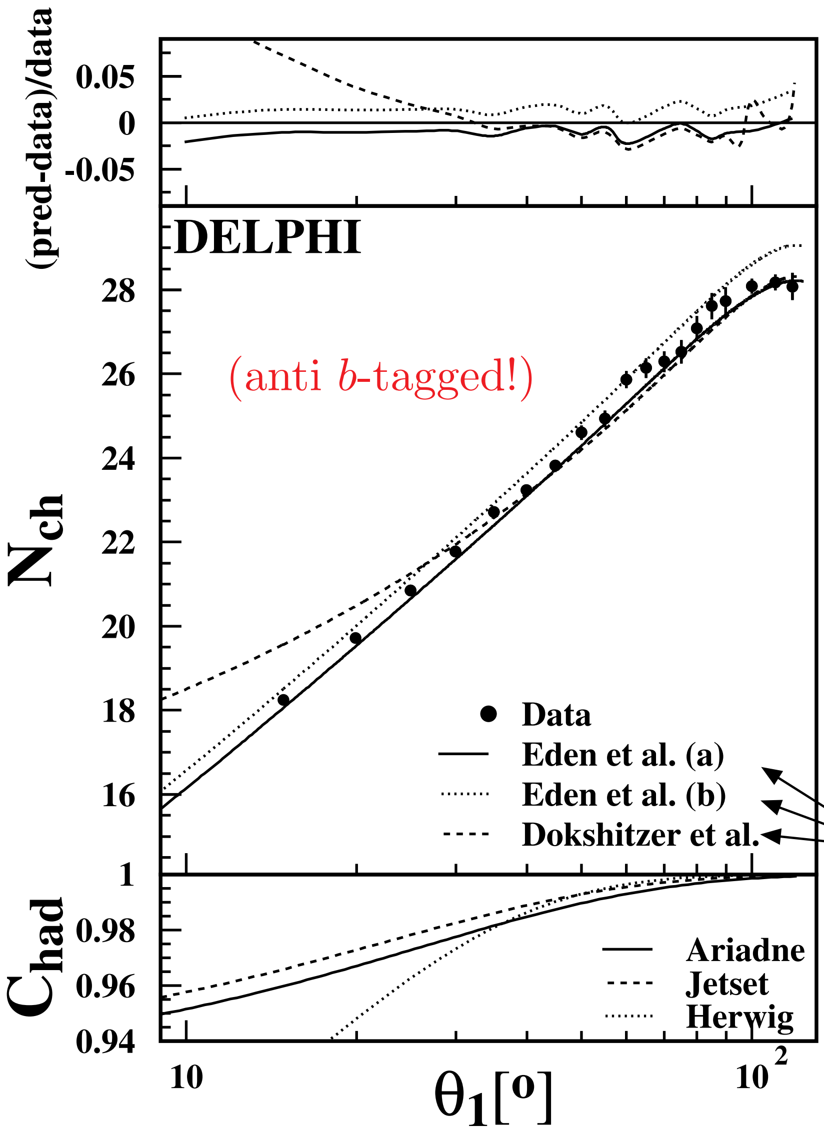
The difference $<\mathrm{N_g}>$ and $<\mathrm{N_q}>$ is related to C_A/C_F,

… but $<\mathrm{N_g}>$ is very hard to measure.

Suggested to measure N_{ch} in 3-jet events



DELPHI 2000-118: $\mathbf{dN_{q\bar{q}g}/d\theta_1}$ in symmetric events



predictions $\mathbf{N_{q\bar{q}g}}$:

$\mathbf{N_{q\bar{q}}(2E^*) + \frac{1}{2}N_{gg}(P_\perp)}$

(Dokshitzer, Khoze & Troyan)

($E^* = E_q$ in $q\bar{q}$ rest frame)

a: $\mathbf{N_{q\bar{q}}(L_{q\bar{q}}, k_{Lu}) + \frac{1}{2}N_{gg}(k_{Le})}$

b: $\mathbf{N_{q\bar{q}}(L, k_{Lu}) + \frac{1}{2}N_{gg}(k_{Lu})}$

(Eden, Gustafson & Khoze)

($L = \log\frac{s}{\Lambda^2}$, $L_{q\bar{q}} = \log\frac{s_{q\bar{q}}}{\Lambda^2}$,
$k_{Le} = \log\frac{s_{qg}s_{g\bar{q}}}{s_{q\bar{q}}\Lambda^2}$, $k_{Lu} = \log\frac{s_{qg}s_{g\bar{q}}}{s\Lambda^2}$)

based on the above prescriptions, on measured multiplicities.
and ..

Fair description by (a) and (b)

Extraction of C_A/C_F

Assuming (a) and (b) correctly describe $\mathrm{N}_{\mathrm{q\bar{q}g}}$:

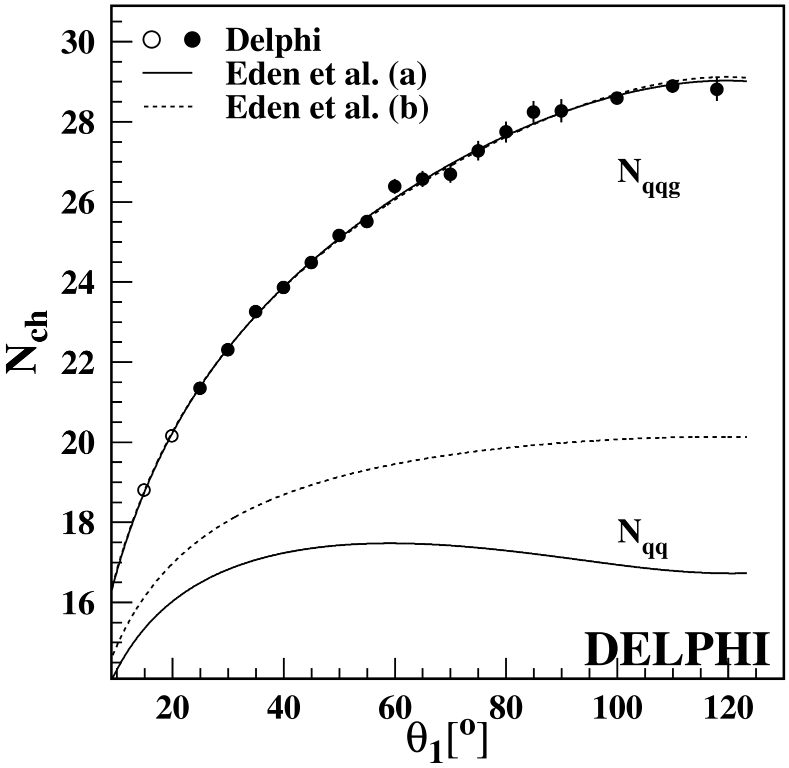
$$N_{q\bar{q}}(L, k_{Lu}) = N_{q\bar{q}}(k_{Lu}+c_q)+(L-k_{Lu}-c_q)\frac{\mathrm{d}N_{q\bar{q}}(L')}{\mathrm{d}L'}|_{L'=k_{Lu}+c_q},$$

$$\frac{\mathrm{dN_{gg}}(L')}{\mathrm{d}L'}|_{L'=L+\frac{11}{6}-\frac{3}{2}} = \frac{C_A}{C_F}\left(1-\frac{\alpha_0 c_r}{L}\right)\frac{\mathrm{dN_{q\bar{q}}}(L)}{\mathrm{d}L},$$

+ some assumptions on behaviour multiplicity ratio at low scales.

DELPHI extracts $\frac{C_A}{C_F}$ from a fit of (a) or (b) to $\mathrm{N}_{\mathrm{q\bar{q}g}}$

$\mathrm{dN_{q\bar{q}g}}/\mathrm{d}\theta_1$
(without b rejection, therefore offset fitted to (a) and (b))



Based on equation (a):

$$\frac{C_A}{C_F} = 2.221 \pm 0.032(\mathrm{stat}) \pm 0.047(\mathrm{exp}) \pm 0.058(\mathrm{had}) \pm 0.075(\mathrm{theo})$$

DELPHI 2001-067: Generalisation to all 3-jet events

Helicity components of fragmentation functions

Cross section: $e^+e^- \rightarrow hX$

$$\frac{\mathrm{d}^2\sigma^h}{\mathrm{d}x_p \mathrm{d}\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\frac{\mathrm{d}\sigma_T^h}{\mathrm{d}x_p} + \frac{3}{4}\sin^2\theta\frac{\mathrm{d}\sigma_L^h}{\mathrm{d}x_p} + \frac{3}{4}\cos\theta\frac{\mathrm{d}\sigma_A^h}{\mathrm{d}x_p},$$

where $x_p = \frac{2p}{\sqrt{s}}$, $\theta = \measuredangle(h, \text{electron beam})$.

Why interesting?

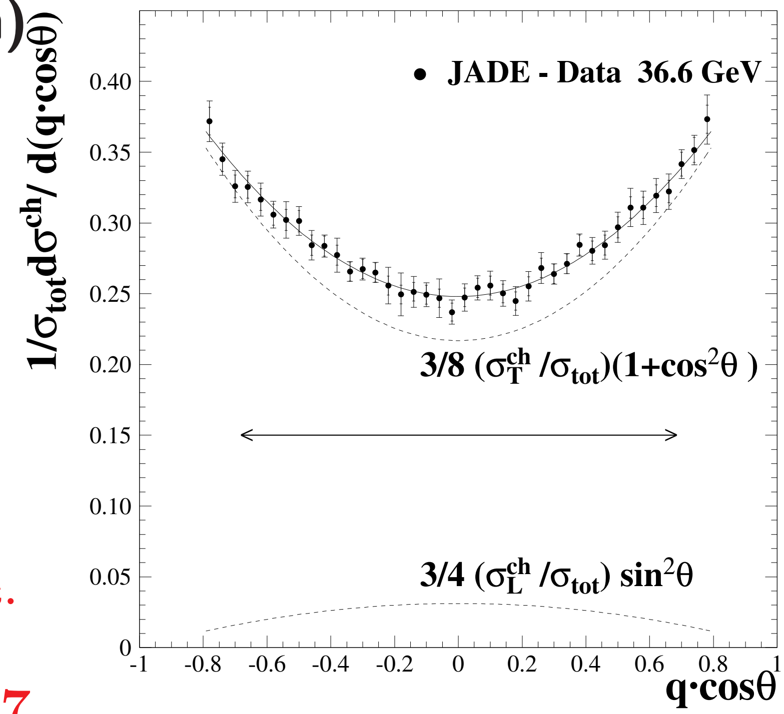
- $\frac{\sigma_L}{\sigma_{tot}} = \frac{\alpha_s}{\pi} + \frac{\alpha_s^2}{\pi^2}(13.583 - 1.028 N_f)$
  (Rijken & van Neerven)
- One can measure $F_L = \frac{1}{\sigma_{tot}}\frac{\mathrm{d}\sigma_L}{\mathrm{d}x_p}$, which is closely related to the gluon fragmentation function.

JADE/MPI (München)

(MPI-PhE/2001-11)

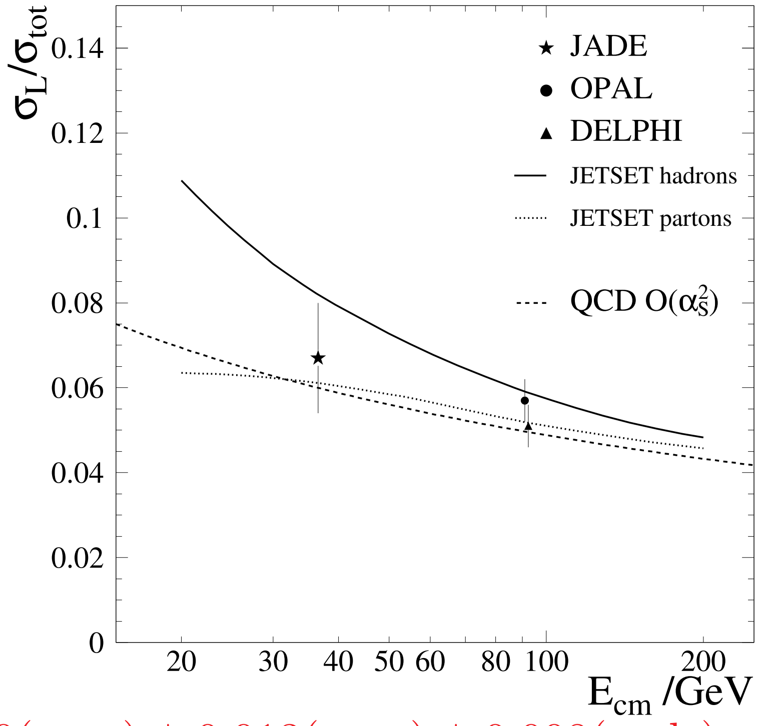
$\sqrt{s} = 35 - 44$ GeV

(1979-1986)



$$\frac{\sigma_L}{\sigma_{tot}} \pm \Delta_{stat.} \pm \Delta_{syst.}$$

$$0.067 \pm 0.011 \pm 0.007$$



Using NLO formula (Rijken & van Neerven):

$\alpha_s(36.6 \text{ GeV}) = 0.150 \pm 0.020(\text{stat.}) \pm 0.013(\text{syst.}) \pm 0.008(\text{scale})$

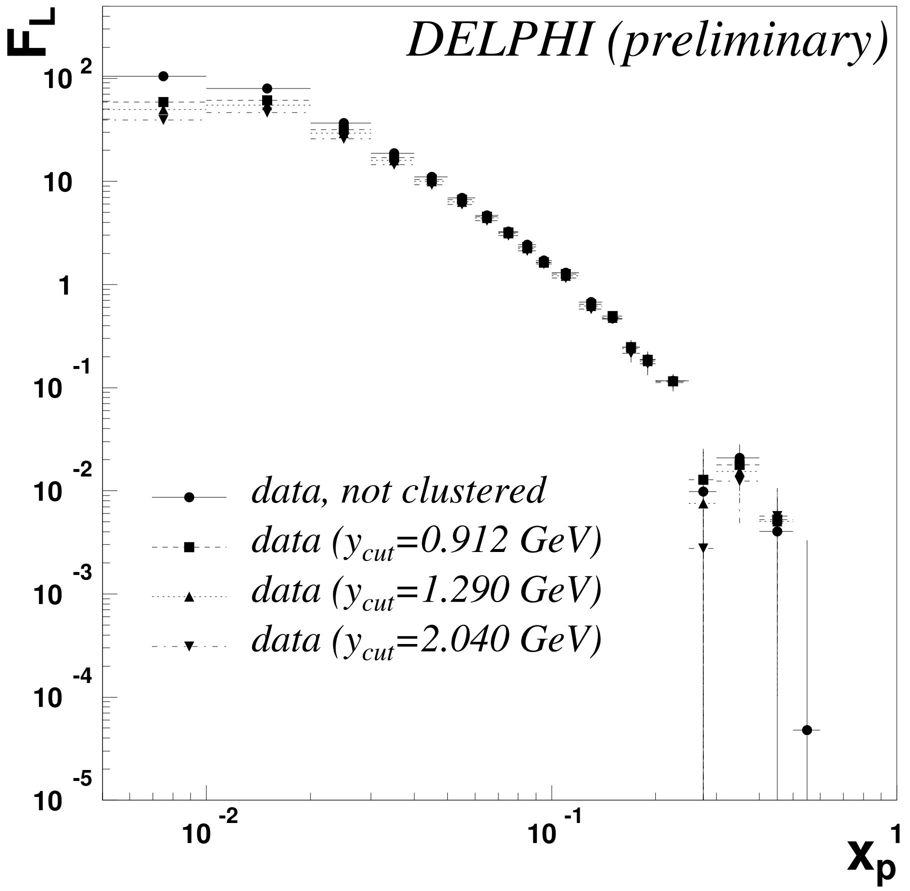
$\rightarrow$ evolved to M_Z: $\alpha_s(M_Z) = 0.127 \pm^{0.017}_{0.018}$

Measurement of F_T, F_L and of α_s from σ_L/σ_{tot}

Prediction σ_L/σ_{tot} assumes hadrons are parallel to parent partons.

But there is angular smearing in the fragmentation.

(DELPHI 2001-058): use θ_{jet} to reduce angular smearing in the fragmentation.



y_{cut}	σ_L/σ_{tot}	α_s
-	$0.0531 \pm 0.0006 \pm 0.0048$	$0.1249 \pm 0.0011 \pm 0.0090$
0.912 GeV	$0.0470 \pm 0.0006 \pm 0.0056$	$0.1132 \pm 0.0012 \pm 0.0109$
1.290 GeV	$0.0445 \pm 0.0006 \pm 0.0060$	$0.1083 \pm 0.0012 \pm 0.0119$
2.040 GeV	$0.0409 \pm 0.0006 \pm 0.0062$	$0.1010 \pm 0.0012 \pm 0.0126$

where $y_{ij}(\text{Durham}) = \sqrt{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}$

$$\alpha_s(M_Z) = 0.1083 \pm 0.0012(\text{stat.}) \pm 0.0119(\text{syst.})$$

Summary

4-jet rates

- DELPHI: α_s from R_4 using NLO + optimal scale setting. Results agrees with world average and earlier results using NLO + NLL for R_4.

simultaneous measurement of α_s, C_A and C_F

- OPAL: Test of QCD group structure by measuring α_s, C_A and C_F in a single fit from R_4, D_2 and 4-jet angles. Agreement with QCD expectation. Light gluino is disfavoured.

charged multiplicities in 3-jet events

- DELPHI: measured $\mathrm{dN}_{\mathrm{q\bar{q}g}}/\mathrm{d}\theta_1$ and tests new predictions for this quantity. Predictions work well and are used to extract C_A/C_F, which agrees with QCD expectation.

helicity components of fragmentation functions

- MPI/JADE: $\sigma_L/\sigma_{\mathrm{tot}} = 0.067 \pm 0.011 \pm 0.007$. Extracted value of $\alpha_s(36.6\ \mathrm{GeV})$ agrees with world average.
- DELPHI: measurement F_L, F_T and $\sigma_L/\sigma_{\mathrm{tot}}(M_Z)$ using jet angles to reduce hadronisation effects. The extracted value of $\alpha_s(M_Z)$ agrees with the world average.