

# BEAM DYNAMICS SIMULATION IN $e^-$ RINGS IN SRFF REGIME

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## Abstract

Obtaining very short bunches is an issue for colliders and Coherent Synchrotron Radiation (CSR) sources. In strong RF focusing regime (SRFF) bunch length is not constant along the ring, but a bunch length modulation (BLM) occurs; thus allowing to obtain short bunches at a given position in the ring. Until now the bunch length modulation has been studied only in the limit of zero current; in this paper we present a simulation code suitable to study the effects of CSR and of the vacuum chamber wakefields on the single bunch longitudinal dynamics in a regime of strong RF focusing. The code has been applied to lattices that can be realized in DAΦNE for a possible experiment on bunch length modulation.

## INTRODUCTION

Short bunches are important for both  $e^+/e^-$  colliders and synchrotron light sources, the first to reach high luminosities, the second to produce CSR in a controlled way. SRFF allows to obtain short bunches thanks to the property that in this regime the bunch length changes along the ring. Length modulation in lattices with high and low momentum compaction have been widely studied at zero current without considering the microwave and microbunching instability that could strongly limit to the maximum stored current [3]. A simulation code has been written to study the effect of the CSR and vacuum chamber wakefields.

## STRONG RF FOCUSING AND APPLICATIONS IN DAΦNE

In a ring with one RF cavity placed at  $s_{RF}$  the voltage gradient can be defined as:

$$U = \frac{2\pi V_{RF}}{E_0 \lambda_{RF}} \quad (1)$$

and the longitudinal drift function is:

$$R_1(s) = \int_s^{s_{RF}} ds' \frac{\eta(s')}{\rho(s')} \quad (2)$$

where  $V_{RF}$  is the peak voltage,  $\lambda_{RF}$  the RF wavelength,  $E_0$  the nominal energy,  $\eta(s)$  the dispersion and  $\rho(s)$  the bending radius. When the voltage gradient is high and the drift function is large, the analysis of the longitudinal one turn matrix [1] shows that the natural energy spread is constant along the ring but the natural bunch length changes. There are two different ways to obtain a regime of bunch length modulation depending on the monotonicity of the

drift function. If the drift function is monotonic the momentum compaction  $\alpha_c$  and synchrotron tune  $Q_s$  are high and the minimum bunch length occurs in the zone of the ring opposite to the cavity [1]. On the contrary, when  $R_1(s)$  is non-monotonic, with large derivatives with opposite signs in two different zones of the ring, the lattice has low  $\alpha_c$  and  $Q_s$  and the minimum bunch length position occurs nearer to the cavity the lower is the momentum compaction [2].

In DAΦNE [4] there is the possibility to tune the dispersion on a wide range to reach the necessary  $R_1(s)$  variation; the  $U$  parameter can become large enough by installing, for example, a Tesla type RF cavity at 1.3 GHz with a maximum voltage of 10 MV [5] in one of the two interaction regions. Three different structures are considered: structure A in which  $R_1(s)$  is monotonic and  $\alpha_c = 0.073$ ; structure B corresponding to a non-monotonic regime with  $\alpha_c = 0.02$ ; structure C also non-monotonic but with a much lower value of  $\alpha_c$  (0.004).

## SIMULATION CODE

The C simulation code SPIDER (Simulation Program for Impedances Distributed in Electron Rings) has been written in order to study the effects of the CSR and of the vacuum chamber wakefields on the longitudinal single bunch dynamics in bunch lengthening modulation regime. In this program a bunch, described by  $N$  macroparticles, runs in the machine that is divided into an arbitrary number of longitudinal drift spaces and RF cavities. The code calculates the longitudinal phase space coordinates of each macroparticle at the end of each section (drift space or cavity) turn by turn. The used coordinates are the energy deviation  $\epsilon \equiv E - E_0$  and the displacement from the synchronous particle  $z$ .

If the section represents one RF cavity the single bunch dynamics equations are

$$\begin{cases} z(s_2) = z(s_1) \\ \epsilon(s_2) = \epsilon(s_1) + V_{cav}(z(s_2)) + V_{RF} \cos(\Phi - 2\pi c \frac{z(s_1)}{\lambda_{RF}}) \end{cases}$$

where  $s_1$  and  $s_2$  are the beginning and the end of the section, respectively,  $\Phi$  is the synchronous phase and  $V_{cav}(z(s_2))$  is the energy loss due to the cavity wakefields calculated with the distribution at the end of the section. If the section is a longitudinal drift space the equations are:

$$\begin{cases} z(s_2) = z(s_1) - \int_{s_1}^{s_2} ds' \frac{\eta(s')}{\rho(s')} \epsilon(s_1) / E_0 \\ \epsilon(s_2) = \epsilon(s_1) + V_{pipe}(z(s_2)) + V_{CSR}(z(s_2)) + V_{ISR} \end{cases}$$

where  $V_{CSR}$  and  $V_{pipe}$  are the energy losses due to the CSR

and vacuum chamber wakefields and  $V_{ISR}$  is the contribution of the incoherent synchrotron radiation given by

$$-U_0 - D\epsilon(s_1) + G\sigma_E\sqrt{2D} \quad (3)$$

where  $U_0 = 1.4 \cdot 10^{-32} E_0^4 I_2(s_2; s_1)$  is the energy radiated by the synchronous particle in the section,  $D = 1.4 \cdot 10^{-32} E_0^3 [2I_2(s_2; s_1) + I_4(s_2; s_1)]$  is the damping factor of the section,  $G$  is a gaussian random number with zero mean and unitary rms,  $\sigma_E = 1.2 \cdot 10^{-12} E_0^2 \sqrt{\frac{I_3(s_2; s_1)}{2I_2(s_2; s_1) + I_4(s_2; s_1)}}$  is the energy spread of the section without BLM, and  $I_2(s_2; s_1)$ ,  $I_3(s_2; s_1)$  and  $I_4(s_2; s_1)$  are the usual synchrotron radiation integrals calculated in the section.

## WAKEFIELDS CONTRIBUTION

CSR and vacuum chamber wakefield contributions are calculated by the convolution of the bunch longitudinal distribution and the wake functions of the considered sections.

### *Vacuum chamber wake*

The DAΦNE wake function has been calculated from the wake potential of a 2.5 mm gaussian bunch obtained by numerical codes [6]; since in the case of SRFF the bunch length is comparable with 2.5 mm, the wake function of each section has been approximated by an RLC equivalent model whose parameters have been found by fitting the wake potential. For the Tesla SC cavity, the analytic approximation of the wake function per unit length given in [7] has been used

$$w(z) = 38.1(1.165 \cdot e^{-\sqrt{z/3.65}} - 0.165) \left[ \frac{\text{V}}{\text{pC m}} \right] \quad (4)$$

where  $z$  is expressed in mm.

### *CSR wakefields*

Since the bunch length is of the order of few mm, CSR effects could be relevant. In the code different models of CSR wake function are implemented, that can be chosen depending on the considered magnet. For the dipole contribution the code includes:

- a steady state CSR wakefield model [8];
- a model considering entrance and exit transients [9];
- a model considering the pipe shielding modeled by the contribution of two parallel plates [10].

For wigglers the code includes the steady state wakefield given in [11]. All these models need the bunch distribution derivative that is obtained by a Savitzky-Golay filter [12].

## SIMULATIONS FOR THE DAΦNE EXPERIMENT ON BLM

The code has been applied to study wakefields effect in the SRFF regime of the lattices A, B and C that can be

realized in DAΦNE and with a voltage in the SC cavity equal to 3 and 9 MV. The ring has been divided in five sections: one SC cavity and four drift spaces 25 meters long. In these cases the longitudinal distributions of the bunch have been studied at the end of each section. The number of macroparticles used has been  $1.5 \cdot 10^5$ . At zero current the code reproduces the theoretical length modulation. For different bunch currents it has been studied the effect of

- beam pipe wakefield only;
- CSR wakefield only;
- beam pipe and CSR wakefields;
- beam pipe, CSR and SC cavity wakefields.

## RESULTS OF THE SIMULATIONS

For each case the instability threshold, defined by the current at which the energy spread begins to increase, has been obtained. Above the threshold the strength of the instability, up to a current of 25 mA, has been studied. The instability threshold is higher in monotonic regime (structure A) but the instability grows faster than in the non-monotonic regimes (structures B, C) and by comparing the structures B and C we may conclude that this consideration is the more relevant the lower the momentum compaction. These properties are due to the fact that, in the non-monotonic regime at a given RF voltage, the modulation factor  $F_m \equiv \sigma_{LMAX}/\sigma_{LMIN}$  is higher than in the monotonic structure. Therefore, at a given minimum bunch length, the average length along the ring is higher. Furthermore it has been verified that in the monotonic regime the modulation factor is independent on the current, while in the structure B and, above all, in the structure C,  $F_m$  increases immediately after the threshold: thanks to this, bunch lengthening and instability increase more slowly for the minimum bunch length. CSR effects are not dominant but, especially in the non-monotonic regimes, their contribution is important for the actual lengthening and the shape of the longitudinal distributions. The effect of the SC cavity wake is negligible even in cases in which the minimum bunch length is in the cavity itself. Furthermore, in order to evidence the effect of the bunch lengthening modulation, results have been compared with the ones obtained with a lattice with a very low momentum compaction (0.004) but without bunch length modulation: in the following figures this case is labelled as NO SRFF. In Figs. 1, 2 and 3 we show respectively the minimum bunch length, the length modulation factor and the energy spread as a function of current in the different cases. In Fig. 4 we report the ratio  $N^2/\sigma_{LMIN}$  where  $N$  represents the number of particles in the bunch and  $\sigma_{LMIN}$  is the minimum bunch length. This quantity gives informations about luminosity in the case of short bunch at interaction point, in the hypothesis of a vertical betatron function (in the interaction point) equal to the bunch length. In this figure the full lines represent

the trend of the function  $N^2/\sigma_{LMIN}$  if we neglect wakefields effects. As a comparison a point showing the peak luminosity of DAΦNE (in the same units) is shown [13]. In each figure error bars correspond to the strength of the instability.

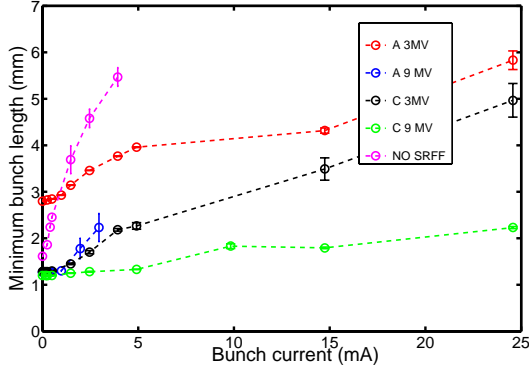


Figure 1: Minimum bunch length as a function of current

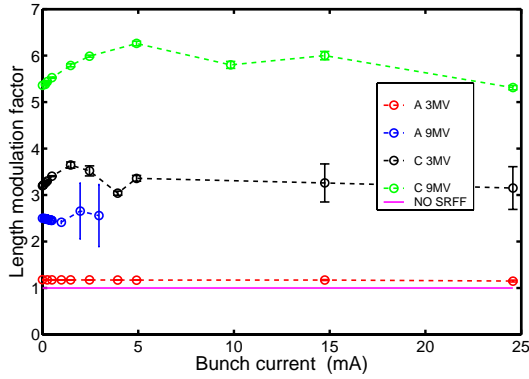


Figure 2: Modulation factor as a function of current

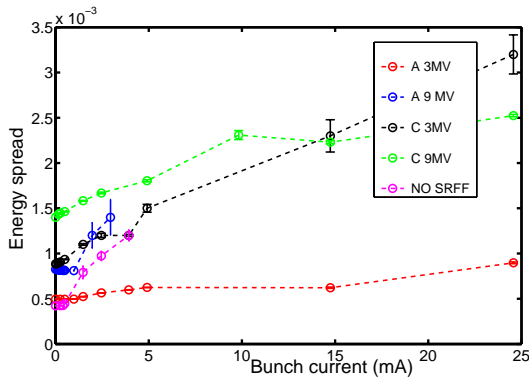


Figure 3: Energy spread as a function of current

## CONCLUSIONS

The principle of bunch length modulation along a storage ring was studied under the effect of wakefields due to

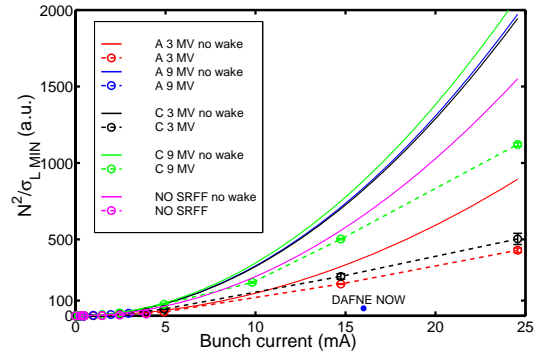


Figure 4: Ratio of the second power of the stored current over the minimum bunch length

the CSR and the interaction of the beam with the vacuum chamber by the dedicated program SPIDER. Simulations have been performed for different kinds of SRFF lattices realizable in DAΦNE to study the bunch lengthening and the instability threshold as a function of the number of particles stored in the bunch. This study showed that also in presence of wakefields the bunch length modulation given by the SRFF is maintained. In the present DAΦNE rings, bunches with lengths in the range of few mm could be obtained with bunch currents above 10 mA, while in a quasi isochronous regime and no SRFF only currents below the mA can be steadily stored.

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