

**nag\_deviates\_chi\_sq (g01fcc)****1. Purpose**

**nag\_deviates\_chi\_sq (g01fcc)** returns the deviate associated with the given lower tail probability of the  $\chi^2$  distribution with real degrees of freedom.

**2. Specification**

```
#include <nag.h>
#include <nagg01.h>
```

```
double nag_deviates_chi_sq(double p, double df, NagError *fail)
```

**3. Description**

The deviate,  $x_p$ , associated with the lower tail probability  $p$  of the  $\chi^2$  distribution with  $\nu$  degrees of freedom is defined as the solution to

$$P(X \leq x_p : \nu) = p = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} \int_0^{x_p} e^{-X/2} X^{\nu/2-1} dX \quad 0 \leq x_p < \infty; \quad \nu > 0.$$

The required  $x_p$  is found by using the relationship between a  $\chi^2$  distribution and a gamma distribution, i.e., a  $\chi^2$  distribution with  $\nu$  degrees of freedom is equal to a gamma distribution with scale parameter 2 and shape parameter  $\nu/2$ .

For very large values of  $\nu$ , greater than  $10^5$ , Wilson and Hilferty's normal approximation to the  $\chi^2$  is used, see Kendall and Stuart (1969).

**4. Parameters****p**

Input: the probability,  $p$ , from the required  $\chi^2$  distribution.  
Constraint:  $0.0 \leq \mathbf{p} < 1.0$ .

**df**

Input: the degrees of freedom,  $\nu$ , of the  $\chi^2$  distribution.  
Constraint: **df** > 0.0.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

**5. Error Indications and Warnings**

On any of the error conditions listed below except **NE\_ALG\_NOT\_CONV** **nag\_deviates\_chi\_sq** returns 0.0.

**NE\_REAL\_ARG\_LT**

On entry, **p** must not be less than 0.0: **p** = *<value>*.

**NE\_REAL\_ARG\_GE**

On entry, **p** must not be greater than or equal to 1.0: **p** = *<value>*.

**NE\_REAL\_ARG\_LE**

On entry, **df** must not be less than or equal to 0.0: **df** = *<value>*.

**NE\_PROBAB\_CLOSE\_TO\_TAIL**

The probability is too close to 0.0 or 1.0.

**NE\_ALG\_NOT\_CONV**

The algorithm has failed to converge in *<value>* iterations.  
The result should be a reasonable approximation.

**NE\_GAM\_NOT\_CONV**

The series used to calculate the gamma probabilities has failed to converge.  
This is an unlikely error exit.

## 6. Further Comments

For higher accuracy the relationship described in Section 3 may be used and a direct call to nag\_deviates\_gamma\_dist (g01ffc) made.

### 6.1. Accuracy

The results should be accurate to 5 significant digits for most parameter values. Some accuracy is lost for  $p$  close to 0.0.

### 6.2. References

Best D J and Roberts D E (1975) The percentage points of the  $\chi^2$  distribution *Appl. Stat.* **24** Algorithm AS91 385–388.

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth.

Kendall M G and Stuart A (1969) *The Advanced Theory of Statistics (Vol 1)* Griffin.

## 7. See Also

nag\_deviates\_gamma\_dist (g01ffc)

## 8. Example

Lower tail probabilities are read for several  $\chi^2$  distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

### 8.1. Program Text

```

/* nag_deviates_chi_sq(g01fcc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
    double df, p, x;
    static NagError fail;

    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vprintf("g01fcc Example Program Results\\n");
    Vprintf("      p      df      x\\n\\n");
    while (scanf("%lf %lf", &p, &df) != EOF)
    {
        x = g01fcc(p, df, &fail);
        if (fail.code==NE_NOERROR)
            Vprintf("%8.3f%8.3f%8.3f\\n", p, df, x);
        else
            Vprintf("%8.3f%8.3f%8.3f\\n Note: %s\\n", p, df, x, fail.message);
    }
    exit(EXIT_SUCCESS);
}

```

### 8.2. Program Data

```

g01fcc Example Program Data
0.0100 20.0
0.4279 7.50
0.8694 45.0

```

**8.3. Program Results**

g01fcc Example Program Results

p	df	x
0.010	20.000	8.260
0.428	7.500	6.200
0.869	45.000	55.759

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