nag_regsn_mult_linear_upd_model (g02ddc)

1. Purpose

nag_regsn_mult_linear_upd_model (g02ddc) calculates the regression parameters for a general linear regression model. It is intended to be called after nag_regsn_mult_linear_addrem_obs (g02dcc), nag_regsn_mult_linear_add_var (g02dec) or nag_regsn_mult_linear_delete_var (g02dfc).

2. Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_regsn_mult_linear_upd_model(Integer n, Integer ip, double q[],
     Integer tdq, double *rss, double *df, double b[], double se[],
     double cov[], Boolean *svd, Integer *rank, double p[], double tol,
     NagError *fail)
```

3. Description

A general linear regression model fitted by nag_regsn_mult_linear (g02dac) may be adjusted by adding or deleting an observation using nag_regsn_mult_linear_addrem_obs (g02dcc), adding a new independent variable using nag_regsn_mult_linear_add_var (g02dec) or deleting an existing independent variable using nag_regsn_mult_linear_delete_var (g02dfc). These functions compute the vector c and the upper triangular matrix R. nag_regsn_mult_linear_upd_model takes these basic results and computes the regression coefficients, $\hat{\beta}$, their standard errors and their variance-covariance matrix.

If R is of full rank, then $\hat{\beta}$ is the solution to:

$$R\hat{\beta} = c_1,$$

where c_1 is the first p elements of c.

If R is not of full rank a solution is obtained by means of a singular value decomposition (SVD) of R,

$$R = Q_* \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} P^T$$

where D is a k by k diagonal matrix with non-zero diagonal elements, k being the rank of R, and Q_* and P are p by p orthogonal matrices. This gives the solution

$$\hat{\beta} = P_1 D^{-1} Q_{*_1}^T c_1$$

P_1 being the first k columns of P, i.e., $P = (P_1 P_0)$ and Q_{*_1} being the first k columns of Q_*.

Details of the SVD, are made available, in the form of the matrix P^*:

$$P^* = \begin{pmatrix} D^{-1} P_1^T \\ P_0^T \end{pmatrix}$$

This will be only one of the possible solutions. Other estimates may be obtained by applying constraints to the parameters. These solutions can be obtained by calling nag_regsn_mult_linear_tran_model (g02dkc) after calling nag_regsn_mult_linear_upd_model. Only certain linear combinations of the parameters will have unique estimates, these are known as estimable functions. These can be estimated using nag_regsn_mult_linear_est_func (g02dnc).

The residual sum of squares required to calculate the standard errors and the variance-covariance matrix can either be input or can be calculated if additional information on c for the whole sample is provided.

4. Parameters

n

Input: number of observations.
Constraint: $\mathbf{n} \geq 1$.

ip

Input: the number of terms in the regression model, p.
Constraint: $\mathbf{ip} \geq 1$.

q[n][tdq]

Input: **q** must be the array **q** as output by nag_regsn_mult_linear_addrem_obs (g02dcc), nag_regsn_mult_linear_add_var (g02dec) or nag_regsn_mult_linear_delete_var (g02dfc). If on entry $\mathbf{rss} \leq 0.0$ then all **n** elements of c are needed. This is provided by functions nag_regsn_mult_linear_add_var (g02dec) or nag_regsn_mult_linear_delete_var (g02dfc).

tdq

Input: **tdq** the last dimension of the array **q** as declared in the function from which nag_regsn_mult_linear_upd_model is called.
Constraint: $\mathbf{tdq} \geq \mathbf{ip}+1$.

rss

Input: either the residual sum of squares or a value less than or equal to 0.0 to indicate that the residual sum of squares is to be calculated by the function.
Output: if $\mathbf{rss} \leq 0.0$ on entry, then on exit **rss** will contain the residual sum of squares as calculated by nag_regsn_mult_linear_upd_model.

If **rss** was positive on entry, then it will be unchanged.

df

Output: the degrees of freedom associated with the residual sum of squares.

b[ip]

Output: the estimates of the p parameters, $\hat{\beta}$.

se[ip]

Output: the standard errors of the p parameters given in **b**.

cov[ip∗(ip+1)/2]

Output: the upper triangular part of the variance-covariance matrix of the p parameter estimates given in **b**. They are stored packed by column, i.e., the covariance between the parameter estimate given in $\mathbf{b}[i]$ and the parameter estimate given in $\mathbf{b}[j]$, $j \geq i$, is stored in $\mathbf{cov}[j(j+1)/2+i]$, for $i = 0, 1, \ldots, \mathbf{ip} - 1$ and $j = i, i+1, \ldots \mathbf{ip} - 1$.

svd

Output: if a singular value decomposition has been performed, then **svd** = **TRUE**, otherwise **svd** = **FALSE**.

rank

Output: the rank of the independent variables.
If **svd** = **FALSE**, then **rank** = **ip**.
If **svd** = **TRUE**, then **rank** is an estimate of the rank of the independent variables.
rank is calculated as the number of singular values greater than **tol** × (largest singular value).
It is possible for the singular value decomposition to be carried out but **rank** to be returned as **ip**.

p[ip∗ip+2∗ip]

Output: **p** contains details of the singular value decomposition if used.
If **svd** = **FALSE**, **p** is not referenced.
If **svd** = **TRUE**, the first **ip** elements of **p** will not be referenced, the next **ip** values contain the singular values. The following **ip∗ip** values contain the matrix P^* stored by rows.

tol

Input: the value of **tol** is used to decide if the independent variables are of full rank and, if not, what is the rank of the independent variables. The smaller the value of **tol** the stricter the criterion for selecting the singular value decomposition. If **tol** = 0.0, then the singular

value decomposition will never be used, this may cause run time errors or inaccuracies if the independent variables are not of full rank.
Suggested value: **tol** = 0.000001.
Constraint: **tol** ≥ 0.0.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_INT_ARG_LT
On entry, **n** must not be less than 1: **n** = ⟨*value*⟩.
On entry, **ip** must not be less than 1: **ip** = ⟨*value*⟩.

NE_2_INT_ARG_LT
On entry **tdq** = ⟨*value*⟩ while **ip** + 1 = ⟨*value*⟩. These parameters must satisfy **tdq** ≥ **ip** + 1.
On entry, **n** = ⟨*value*⟩ while **ip** = ⟨*value*⟩. These parameters must satisfy **n** ≥ **ip**.

NE_DOF_LE_ZERO
The degrees of freedom for error are less than or equal to 0. In this case the estimates, $\hat{\beta}$, are returned but not the standard errors or covariances.

NE_SVD_NOT_CONV
The singular value decomposition has failed to converge.
See nag_real_svd (f02wec). This is an unlikely error exit.

NE_REAL_ARG_LT
On entry, **tol** must not be less than 0.0: **tol** = ⟨*value*⟩.

NE_ALLOC_FAIL
Memory allocation failed.

6. Further Comments

6.1. Accuracy

The accuracy of the results will depend on the accuracy of the input R matrix, which may lose accuracy if a large number of observations or variables have been dropped.

6.2. References

Golub G H and Van Loan C F (1983) *Matrix Computations* Johns Hopkins University Press, Baltimore.
Hammarling S (1985) The Singular Value Decomposition in Multivariate Statistics *ACM Signum Newsletter* **20** (3) 2–25.
Searle S R (1971) *Linear Models* Wiley.

7. See Also

nag_real_svd (f02wec)
nag_regsn_mult_linear (g02dac)
nag_regsn_mult_linear_addrem_obs (g02dcc)
nag_regsn_mult_linear_add_var (g02dec)
nag_regsn_mult_linear_delete_var (g02dfc)
nag_regsn_mult_linear_tran_model (g02dkc)
nag_regsn_mult_linear_est_func (g02dnc)

8. Example

A data set consisting of 12 observations and four independent variables is input and a regression model fitted by calls to nag_regsn_mult_linear_add_var (g02dec). The parameters are then calculated by nag_regsn_mult_linear_upd_model and the results printed.

8.1. Program Text

```
/* nag_regsn_mult_linear_upd_model(g02ddc) Example Program
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg02.h>

#define NMAX 12
#define MMAX 5
#define TDX MMAX
#define TDQ MMAX+1

main()
{
  double   rss, tol;
  Integer  i, ip, rank, j, m, n;
  double df;
  Boolean  svd;
  char     weight;
  double   b[MMAX], cov[MMAX*(MMAX+1)/2], p[MMAX*(MMAX+2)],
  q[NMAX][MMAX+1], se[MMAX], wt[NMAX], x[NMAX][MMAX], xe[NMAX];
  double   *wtptr;
  static   NagError fail;

  Vprintf("g02ddc Example Program Results\n");
  /*  Skip heading in data file */
  Vscanf("%*[^\n]");
  Vscanf("%ld %ld %c", &n, &m, &weight);
  if (weight=='w')
    wtptr = wt;
  else
    wtptr = (double *)0;

  if (n<=NMAX && m<MMAX)
    {
      if (wtptr)
        {
          for (i=0; i<n; i++)
            {
              for (j=0; j<m; j++)
                Vscanf("%lf", &x[i][j]);
              Vscanf("%lf%lf", &q[i][0], &wt[i]);
            }
        }
      else
        {
          for (i=0; i<n; i++)
            {
              for (j=0; j<m; j++)
                Vscanf("%lf", &x[i][j]);
              Vscanf("%lf", &q[i][0]);
            }
        }
      /*    Set tolerance */
      tol = 0.000001e0;
      ip = 0;
      for (j=0; j<m; ++j)
        {
          /*
           *        Fit model using g02dec
           */
          for (i=0; i<n; i++)
            xe[i] = x[i][j];
          g02dec(n, ip, (double *)q, (Integer)(TDQ), p, wtptr, xe, &rss,
```

```
                  tol, &fail);
           if (fail.code==NE_NOERROR)
             ip += 1;
           else if (fail.code==NE_NVAR_NOT_IND)
             Vprintf(" * New variable not added * \n");
           else
             {
               Vprintf("%s\n", fail.message);
               exit(EXIT_FAILURE);
             }
         }
       rss = 0.0;
       g02ddc(n, ip, (double *)q, (Integer)(TDQ), &rss, &df, b, se, cov, &svd,
              &rank, p, tol, NAGERR_DEFAULT);

       Vprintf("\n");
       if (svd)
         Vprintf("Model not of full rank\n\n");
       Vprintf("Residual sum of squares = %12.4e\n", rss);
       Vprintf("Degrees of freedom = %3.1f\n\n", df);
       Vprintf("Variable    Parameter estimate   Standard error\n\n");
       for (j=0; j<ip; j++)
         Vprintf("%6ld%20.4e%20.4e\n", j+1, b[j], se[j]);
       Vprintf("\n");
     }
   else
     {
       Vfprintf(stderr, "One or both of m and n are out of range:\
 m = %-3ld while  n = %-3ld\n", m, n);
       exit(EXIT_FAILURE);
     }
   exit(EXIT_SUCCESS);
 }
```

8.2. Program Data

```
g02ddc Example Program Data
 12 4 u
1.0 0.0 0.0 0.0 33.63
0.0 0.0 0.0 1.0 39.62
0.0 1.0 0.0 0.0 38.18
0.0 0.0 1.0 0.0 41.46
0.0 0.0 0.0 1.0 38.02
0.0 1.0 0.0 0.0 35.83
0.0 0.0 0.0 1.0 35.99
1.0 0.0 0.0 0.0 36.58
0.0 0.0 1.0 0.0 42.92
1.0 0.0 0.0 0.0 37.80
0.0 0.0 1.0 0.0 40.43
0.0 1.0 0.0 0.0 37.89
```

8.3. Program Results

```
g02ddc Example Program Results

Residual sum of squares =   2.2227e+01
Degrees of freedom = 8.0

Variable    Parameter estimate   Standard error

     1          3.6003e+01          9.6235e-01
     2          3.7300e+01          9.6235e-01
     3          4.1603e+01          9.6235e-01
     4          3.7877e+01          9.6235e-01
```