1 Introduction

Improving the coordinate resolution of wire chambers is impossible without comprehending basic mechanisms for their intrinsic accuracy limitation. Among them there are fluctuations of energy losses for ionization in the sensitive volume, diffusion of ionization clusters during their drift to the signal wire, gas amplification fluctuations in the course of electron avalanche formation near the wire [1, 2]. The present paper deals with a phenomenon which can sometimes limit the coordinate resolution too.

Up to now signal wires of wire chambers have been thought to be stationary, if in electrostatic equilibrium. But we have faced vibration of signal wires under radiation in drift tubes. The wires started oscillating when a radioactive source was brought near the tube and stopped when it was removed. The oscillation amplitude depended both on radiation intensity and on voltage applied to the tube. No changes were observed in performance of the detector.

One already encountered vibration of signal wires in construction of large proportional chambers, when the free length of signal wires exceeded the electrostatic stability limit [3]. This type of vibration is accompanied by considerable noise, relatively high current in the high-voltage supply circuit [4, 5] and is indifferent to presence of an external radiation source.

The phenomenon that we observed took place in a drift tube where the electrostatic stability condition for the signal wire was known to be fulfilled. The calculations showed that variation in the force of electrostatic interaction between the signal wire and the cathode in recording of ionizing particles could not result in noticeable oscillation either. Furthermore, in this case the oscillation amplitude should not depend on the irradiation spot coordinate along the wire while we observed the dependence.

Underlying the phenomenon is repulsion of a positively charged signal wire and a cloud of ions that remained after neutralization of the electron part of the avalanches formed on the wire. Yet, the amount of force due to repulsion of the wire and the avalanche charge from a single particle is about 10^{-9}–10^{-8} N and not enough to excite noticeable vibration of a wire stretched with a force of 0.5–5 N. So the signal wire vibration is most like to arise from repulsive force fluctuations caused by recording of many particles. To prove that fluctuations of insignificantly small forces can be observed is Brown motion of an oil drop caused by fluctuations of the surrounding liquid pressure, though the pressure itself results from impact of tiny liquid molecules with the drop surface.

2 Physical model of phenomenon

Let us consider a cylindrical tube of radius R and length L, inside which a wire of radius r and linear density ρ is stretched with a force T (see Fig.1). Let $y(x, t)$ be the projection of wire shape on the chosen direction. Then, in the approxi-

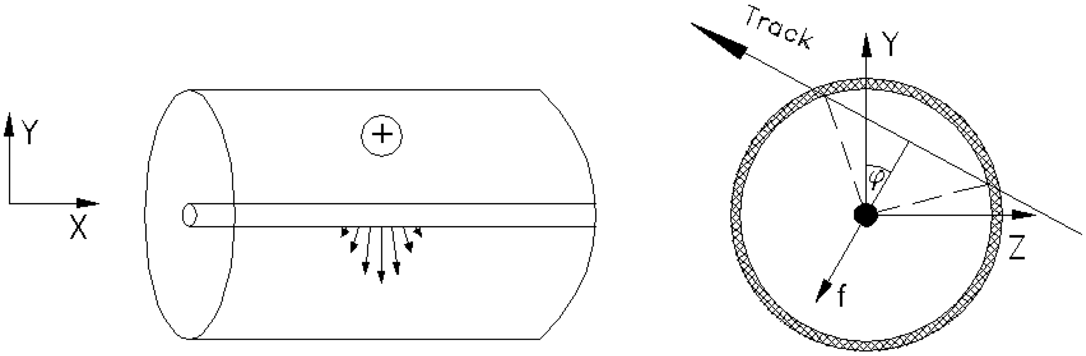

Fig. 1: Schematic view of the drift tube

mation of small deviations ($(dy/dx)^2 \ll 1$) wire oscillations under the action of perturbing forces will be described by the equation

$$\rho\frac{\partial^2 y}{\partial t^2} + \gamma_0\frac{\partial y}{\partial x} - T\frac{\partial^2 y}{\partial x^2} - \kappa y = -g\rho + f(x,t),$$

where γ_0 is the coefficient of the gas friction force ($F_{fr} = \gamma_0\frac{dy}{dt}$) acting on a unit length of the wire, $\kappa = \frac{2\pi\epsilon_0 V^2}{(R\ln\frac{R}{r})^2}$ is the coefficient of the electrostatic force ($F_{el} = \kappa y$), V is the voltage at the tube, ϵ_0 is the dielectric permittivity of free space, $f(x,\, t)$ is the linear density of the external perturbing force, which is a random force generated by particles that chaotic pass through the detector. The force from electrons acting on the wire can be ignored.

Linear density of the forces from a single ion acting on the wire is the largest in the region $|x| \sim R$ which is substantially smaller than the spatial period of the first wire oscillation harmonics $R \ll L$. Therefore the real distribution of forces on the wire can be replaced by their resultant applied at one point:

$$f_j = F_j\delta(x), \quad \text{где} \quad F_j = \int f_j dx = \frac{q_j V}{y_j ln(\frac{R}{r})},$$

where y_j is the distance between the wire and the ion of charge q_j.

Another approximation of the model is based on the fact that the maximum drift time of ions ($t_{max} < 1$ ms) in a detector of normal size is much smaller than the period of first harmonics of intrinsic wire oscillations (> 20 ms). Therefore one can regard the action of the repulsive force as prompt impact with the wire:

$$F_j = P_j\delta(t), \quad P_j = \int F_j dt = \frac{q_j}{\mu}\int v dt \approx \frac{q_j R}{\mu}, \tag{1}$$

where μ is the mobility of positive ions.

Then the resultant of all ions in the ith event can be written down as

$$f_i(x,t) = \sqrt{K}\frac{Q_i R}{\mu}\delta(t-t_i)\delta(x-x_i)\cos\phi_i,$$

where Q_i is the total charge in the event, ϕ_i is the angle of axis y with a perpendicular to the particle trajectory in the detector (see Fig.1), t_i, x_i are the time and coordinate of the event respectively, K is the coefficient of proportionality between the "effective" and the measured average square charges

$$K = \overline{[\sum_j q_j \cos(\phi_j - \overline{\phi})]^2} \Big/ \overline{[\sum_j q_j]^2}$$

Its value depends on how the cloud of positive ions surrounds the wire. It can run from 0, if ions are uniformly arranged around the wire, to 1, if ions are concentrated in a narrow cone on one side of the wire. The arrangement of ions around the wire depends on gas amplification, wire diameter, gas mixture composition [6]. Ions entirely surround the wire only when the gas amplification coefficient is large and the wire is thin. In other cases an avalanche is asymmetrical about the wire. One can estimate the coefficient K assuming that positive ions come back along drift trajectories of primary ionization electrons. Averaging over all possible particle trajectories one must perform calculations with the average square charge because it is this charge that appear in the final expressions for the oscillation amplitude.

$$K \approx \int_0^1 \left[\int_0^{\sqrt{1-y^2}} \frac{y dx}{\sqrt{x^2+y^2}} \right]^2 dy \Big/ \int_0^1 \left[\int_0^{\sqrt{1-y^2}} dx \right]^2 dy = G - \frac{1}{2} \approx 0.416$$

where

$$G = \sum_{k=0}^{+\infty} \frac{(-1)^k}{(2k+1)^2} \approx 0.916$$

is the Catalan constant.

The final form of the expression for linear density of the perturbing force is

$$f(x,t) = \sqrt{K}(R/\mu) \sum_i Q_i \cos\phi_i \delta(t-t_i)\delta(x-x_i).$$

If the wire is irradiated symmetrically, $\overline{\cos\phi} = 0$ and thus the average force is $\overline{f(x,t)} = 0$. However the autocorrelation function of the perturbing force is different from zero:

$$\overline{\delta f(x_1,t_1)\delta f(x_2,t_2)} = K K_1 \frac{\overline{Q^2} R^2}{\mu^2} j(x_1)\delta(x_2-x_1)\delta(t_2-t_1) \tag{2}$$

where $j(x)$ is the flux per unit length, the coefficient $K_1 = \overline{\cos^2\phi}$ depends on the angular distribution of recorded particles and takes on values 1/2 for a distribution uniform in angle and 1 for particles moving mainly at right angles to the observation plane.

We are interested in deviation of the wire from its mean position $U = \delta y = y - \overline{y}$. Subtracting the equation for the stationary wire form $\overline{y(x)}$ from the equation for $y(x,t)$, we get

$$\rho\frac{\partial^2 U}{\partial t^2} + \gamma_0\frac{\partial U}{\partial t} - T\frac{\partial^2 U}{\partial x^2} - \kappa U = \delta f(x,t)\ ; \quad U(0,t) = U(L,t) = 0$$

The Fourier solution of this problem is

$$U(x,t) = \frac{2}{\rho L}\sum_{n=1}^{+\infty} X_n(x)\int_0^L X_n(x_1)[T_n(t) * \delta f(x_1,t)]dx_1,$$

where $X_n(x) = \sin\left(\frac{\pi n x}{L}\right)$, $T_n(t) = \frac{\theta(t)}{w_n}e^{-\gamma t}\sin(w_n t)$, $w_n^2 = \omega_{0n}^2 - \gamma^2 = \frac{T}{\rho}(\frac{\pi n}{L})^2 - \frac{\kappa}{\rho} - \gamma^2$, $\gamma = \gamma_0/(2\rho)$ is the logarithmic attenuation decrement. Then the length-average square of the deviation is

$$\frac{1}{L}\int_0^L U^2(x,t)dx = \frac{2}{\rho^2 L^2}\sum_{n=1}^{+\infty}\int_0^L\int_0^L X_n(x_1)X_n(x_2)[T_n(t)*\delta f(x_1,t)][T_n(t)*\delta f(x_2,t)]dx_1dx_2$$

Knowing that f(x, t) is a stationary random process we get the time-average square of the deviation

$$\frac{1}{L}\int_0^L \overline{U^2(x,t)}dx = \frac{1}{\rho^2 L^2\gamma}\sum_{n=1}^{+\infty}\frac{1}{\omega_{0n}^2}\int_0^L\int_0^L X_n(x_1)X_n(x_2)\times$$

$$\times\int_0^{+\infty} e^{-\gamma\tau}\left[\cos(w_n\tau) + \frac{\gamma}{w_n}\sin(w_n\tau)\right]\overline{\delta f(x_1,t)\delta f(x_2,t+\tau)}d\tau dx_1dx_2$$

Substituting (2) in this solution, we get

$$\frac{1}{L}\int_0^L \overline{U^2(x,t)}dx = \frac{KK_1}{2\rho^2 L^2\gamma}\frac{\overline{Q^2}R^2}{\mu^2}\sum_{n=1}^{+\infty}\frac{1}{\omega_{0n}^2}\int_0^L j(x)X_n^2(x)dx$$

Let us calculate

$$Z = \sum_{n=1}^{+\infty}\frac{X_n^2(x)}{\omega_{0n}^2} = \frac{\rho L^2}{T\pi^2}\sum_{k=1}^{+\infty}\frac{\sin^2(kt)}{k^2 - a^2} = \frac{\rho L^2}{T\pi^2}\frac{\pi}{4a}\left[\cot a\pi(\cos 2at - 1) + \sin 2at\right],$$

where $a^2 = (\kappa L^2)/(\pi^2 T)$, $t = \pi x/L$. Ensuring electrostatic stability of the wire ($a^2 \ll 1$), when the electric field effect on frequencies of harmonics is insignificant, we get

$$\int_0^L j(x)Z(x)dx \approx \frac{\nu\rho L^2}{12T}K_2,$$

where ν is the total flux through the detector. In the case of the radiation flux distributed uniformly over all tube length, $K_2 \approx 1+\frac{\pi^2 a^2}{15}$. If the particle flux is concentrated at a distance s from the tube center, then $K_2 \approx H(s)\left[1+\frac{\pi^2 a^2}{18}H(s)\right]$, where $H(s)=\frac{3}{2}[1-(\frac{2s}{L})^2]$.

The final expression for the root-mean-square (rms) deviation of the wire is

$$\langle U\rangle = \frac{\langle Q\rangle R}{\mu}\sqrt{\frac{KK_1K_2\ \nu}{24\ T\rho\gamma}} \tag{3}$$

Thus the average amplitude of wire vibration due to irradiation mainly depends on gas amplification and radiation flux through the detector. Besides, the scale of oscillations is determined by parameters of the detector and the gas mixture used. If the maximum ion drift time is smaller than the first harmonic period, the electric field configuration does not affect the size of vibration. In this case the oscillation amplitude only depends on the total drift distance of positive ions, and the formula obtained applies to detectors with any field configuration. The oscillation amplitude is not directly related to the wire length or diameter and depends only on the wire tension and density.

Fig.2 shows results of calculating the average amplitude of signal wire vibration in drift tubes of radius 1 cm against the flux for different average charges in the event. Upper limits of corridors for a fixed average charge correspond to tension of 50 g, lower ones to that of 500 g. Calculations were carried out with positive ion mobility of 1.7×10^{-4} $m^2 s^{-1}V^{-1}$, which corresponds, as an example, to mobility of CO_2 or isobutane ions in Ar. Product of the linear density of the wire by the logarithmic attenuation decrement was taken to be 3.5×10^{-5} s^{-1} kg/m.

Calculations show that wire vibration can be a decisive factor for the coordinate resolution of detectors operating in a limited streamer mode at fluxes $\geq 10^5$ particles/s per wire. In the proportional mode vibration makes an insignificant contribution to the coordinate resolution, and only in some cases (higher pressure, low mobility of ions in the gas, thin wire) it can be as large as 25–30 μm.

3 Measurement technique

Model prediction were verified with an apparatus schematically shown in Fig.3. A cylindrical stainless steel tube 2.7 m long and 3 cm in outer diameter was used.

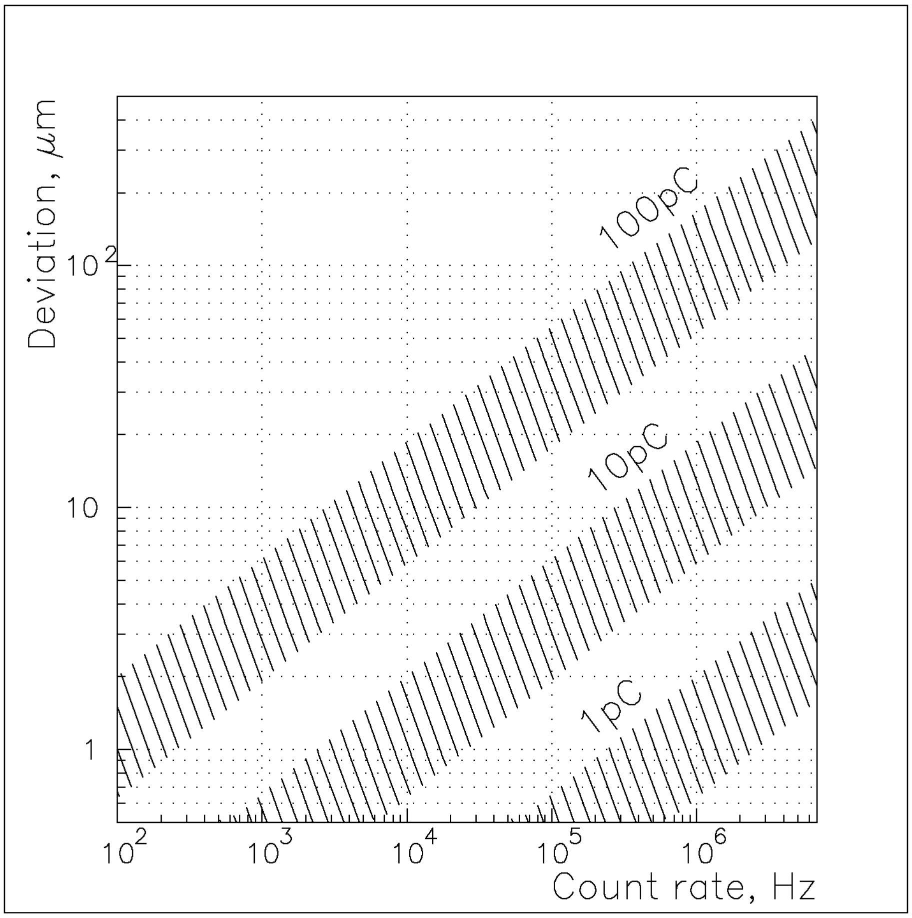

Fig. 2: The calculated average vibration amplitude versus the count rate

In the middle of the tube there were two transparent windows, one to illuminate the wire, the other to observe its deviation with a microscope. Visual observation of the wire allowed the average oscillation amplitude to be estimated in the range from 5 μm to half a diameter of the wire. Oscillations of larger amplitude made the wire invisible. Measurements were carried out with a 70 μm tungsten wire stretched at 100 and 300 g and with a beryllium bronze wire 80 μm with tension of 100, 230 and 250 g. The tube was irradiated by a ^{106}Ru β-source not far from the tube center. A gas mixture $Ar + 15\%CO_2 + 2.5\%iC_4H_{10}$ was used.

The charge in an event was measured with an ADC "LeCroy 2249A" that was triggered directly by the same signal from the tube. Two preamplifiers with current amplification 70 and 25 were used. The triggering threshold of the discriminator was 3 μA, which corresponded to the minimum recorded charge of about 0.5 pC in an event. The ADC gate was 500 ns.

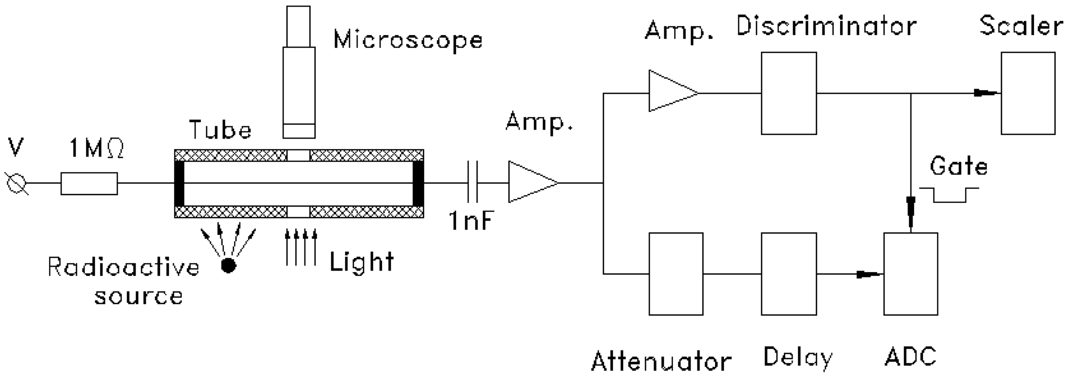

Fig. 3: Schematic view of the experimental setup

4 Experimental results

While experimentally verifying model predictions, we investigated the effect of the main parameters from (3) on the oscillation amplitude. There were only two uncontrolled parameters, the logarithmic attenuation decrement and coefficient K that depends on the extend of surrounding of the wire by positive ions.

Fig.4 shows the average oscillation amplitude in the center of the tube against

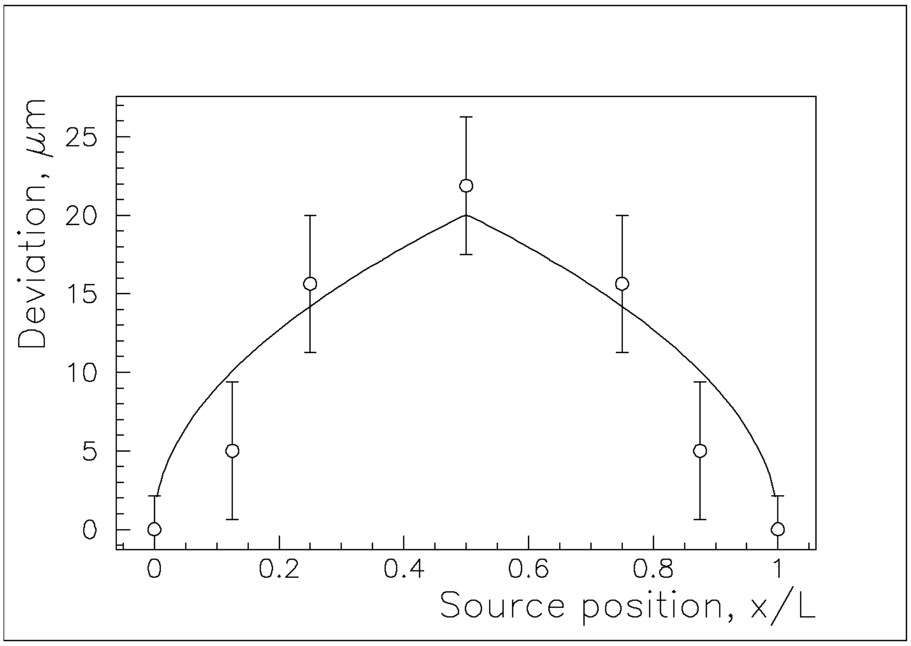

Fig. 4: Vibration amplitude as a function of the radioactive source position

the position coordinate of the radioactive source. The data were obtained at 3 atm gas pressure and at 4.5 kV on the tube. The amplitude is seen to depend on the location of detector irradiation. The solid curve is the result of fitting by a relation of the type $C\sqrt{1-|2s/L|}$ predicted by the model. The measured dependence of the average oscillation amplitude on the flux through the detector is shown in Fig.5. The fitting by a relation of the type $C_1\nu^{C_2}$ yields an exponent of 0.74 ± 0.25.

The results do not contradict the model predictions. And yet, large errors in oscillation amplitude measurements do not allow exact conclusions. On the other hand, varying the flux or voltage at the tube one can reproduce an earlier observed amplitude with an accuracy higher than the measurement accuracy. Therefore we used the following procedure to verify the model more thoroughly. At a fixed flux through the detector the voltage at the tube was adjusted so that the average oscillation amplitude was 10 μm, and an amplitude spectrum of signals was measured. The rms charge in the event was found by analyzing the spectrum. Measurements were carried out at different radiation fluxes and gas mixture pressures.

The voltage applied to the tube usually corresponded to the mode with both proportional and limited streamer discharges. When deducing the rms charge from the measured spectra, we corrected charges of proportional signals for an

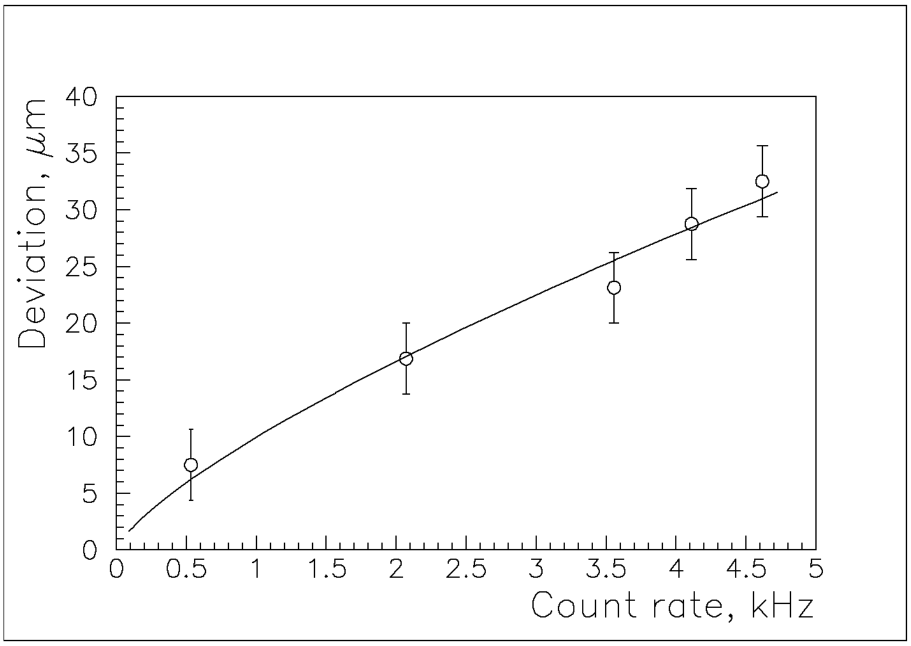

Fig. 5: Vibration amplitude as a function of the detector count rate

inadequate measurement time

$$\frac{Q}{Q_{tot}} = \frac{ln(1+t/t_0)}{2ln(R/r)}, \quad t_0 = r^2 ln(R/r)/2\mu V_0$$

In Fig.6 there is the rms charge in the event as a function of the radiation intensity for an 80-μm beryllium bronze wire stretched at 250 g. The data were fitted by the expected relation $Q = C/\sqrt{\nu}$. Within the experimental errors the data are well fitted by this relation. The coefficient C derived from fitting is given in Fig.7 as a function of the gas mixture pressure. The coefficient C is directly proportional to ion mobility which is inversely proportional to pressure. Therefore the values of C were fitted by the relation 1/P. One can see that the vibration magnitude is inverse proportional to the gas mixture pressure. The product of the logarithmic oscillation attenuation coefficient by the linear density of the wire was found from the fitting.

If one assumes that attenuation is mainly determined by the gas resistance force, then the major parameter determining the product $\gamma\rho$ is dynamic gas viscosity. We measured the attenuation coefficient of wires in the air in the

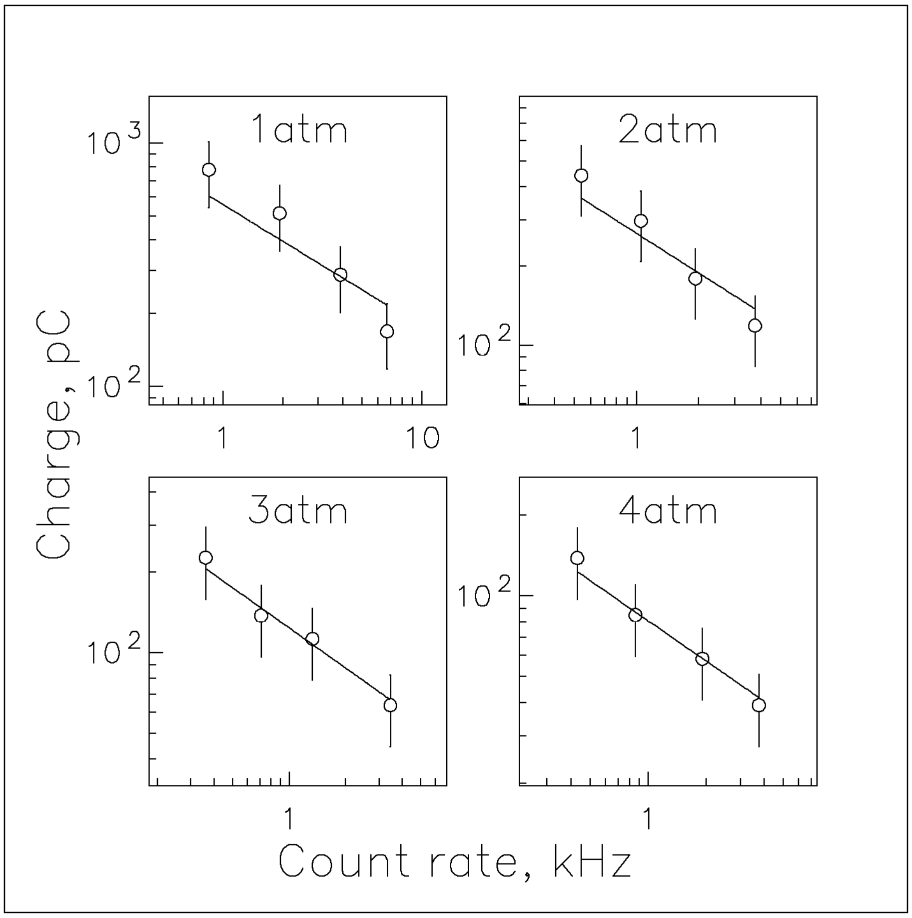

Fig. 6: Average charge in an event versus the tube count rate at different gas pressures

amplitude range from 5 mm to 0.5 mm. Wires were of the same length as in the tube. The results were independent of the wire material and tension within the measurement errors. The dependence of $\gamma\rho$ on the oscillation amplitude in the air is shown in Fig.8 together with the value of $\gamma\rho$ deduced from fitting of the experimental data on wire vibration. Extrapolation to the region of amplitudes around 20 μm yields $\gamma\rho \sim 3 \times 10^{-5}\frac{kg}{s\cdot m}$ for the air, which coincides with the measured value in magnitude. It should be mentioned that dynamic viscosity of Ar is about 20% greater than that of the air, therefore $\gamma\rho$ for the gas mixture used must be somewhat larger than for the air.

The results of similar measurements for a beryllium bronze wire stretched at 100 g and with a tungsten wire with different tensions show that the wire material and diameter do not affect the oscillation amplitude. Within the experimental errors the values of $\gamma\rho$ obtained are in a good agreement for all cases.

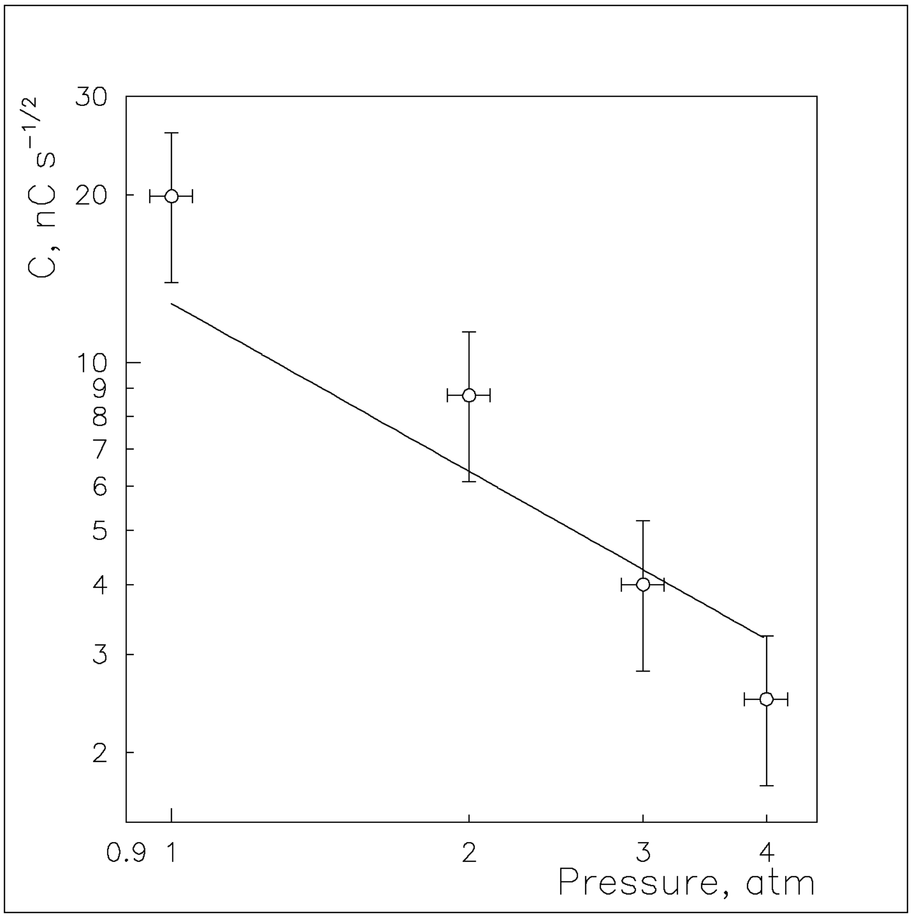

Fig. 7: The product of the charge and the square root of the count rate measured at fixed vibration amplitude as a function of the gas pressure

5 Conclusion

Vibration of signal wires in wire detectors under irradiation is described and explained. The vibration arises from fluctuations of repulsive forces acting between the wire with a positive potential and clusters of ions produced in gas amplification of primary ionization of recorded particles. A formula is obtained which makes it possible to calculate the scale of the phenomenon for different wire detectors. The main features predicted by the proposed model are verified by measurements.

The formula allows in principle more accurate predictions to be made provided that two model parameters will be estimated or found experimentally. These are a logarithmic coefficient of wire oscillation attenuation and a coefficient related to arrangement of positive ions around the wire. If the logarithmic attenuation

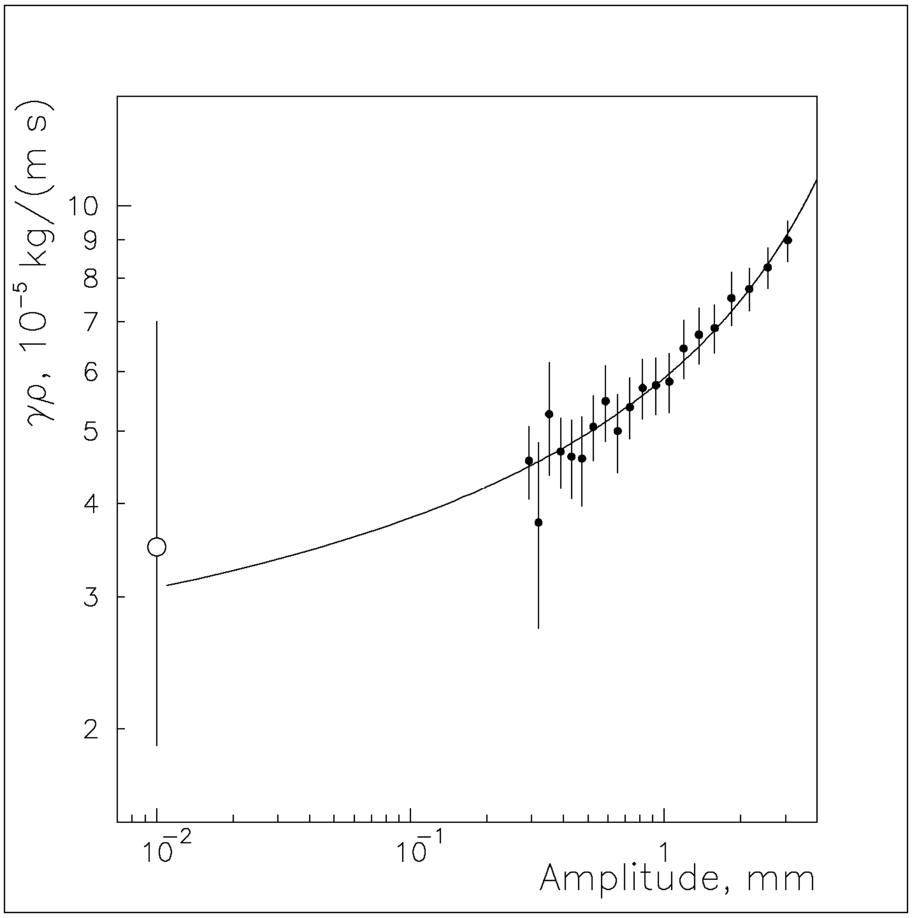

Fig. 8: The $\gamma\rho$ value versus vibration amplitude in the air. Empty circle shows the value obtained from the data of the wire vibration in the tube

coefficient is constant for a given gas mixture, the arrangement of ions around the wire depends on many parameters and varies with the gas amplification coefficient.

Estimations show that in some cases vibration of signal wires can be a decisive factor limiting the space resolution of wire detectors, especially in the limited streamer discharge mode. In the proportional mode the oscillation amplitude can be as large as 20–30 μm only in some cases and at large count rates. The fact that wire vibration has not been observed up to now appears to be due to its small amplitude in the experiments, where was obtained the best coordinate resolution.

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