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# Hadronization properties of b quarks compared to light quarks in $e^+e^- \to q\bar{q}$ from 183 to 200 GeV

DELPHI Collaboration

**Abstract**

The DELPHI detector at LEP has collected 54 $\mathrm{pb}^{-1}$ of data at a centre-of-mass energy around 183 GeV during 1997, 158 $\mathrm{pb}^{-1}$ around 189 GeV during 1998, and 187 $\mathrm{pb}^{-1}$ between 192 and 200 GeV during 1999. These data were used to measure the average charged particle multiplicity in $e^+e^- \to b\bar{b}$ events, $\langle n \rangle_{b\bar{b}}$, and the difference $\delta_{bl}$ between $\langle n \rangle_{b\bar{b}}$ and the multiplicity, $\langle n \rangle_{l\bar{l}}$, in generic light quark (u,d,s) events:

$$\begin{aligned} \delta_{bl}(183\,\mathrm{GeV}) &= 4.55 \pm 1.31(stat) \pm 0.73(syst) \\ \delta_{bl}(189\,\mathrm{GeV}) &= 4.43 \pm 0.85(stat) \pm 0.61(syst) \\ \delta_{bl}(200\,\mathrm{GeV}) &= 3.39 \pm 0.89(stat) \pm 1.01(syst)\,. \end{aligned}$$

This result is consistent with QCD predictions, while it is inconsistent with calculations assuming that the multiplicity accompanying the decay of a heavy quark is independent of the mass of the quark itself.

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# 1 Introduction

The study of the properties of the fragmentation of heavy quarks compared to light quarks offers new insights in perturbative QCD. Particularly important is the difference in charged particle multiplicity between light quark and heavy quark initiated events in $e^+e^-$ annihilations.

In a first approximation one could expect that the multiplicity of hadrons produced in addition to the possible decay products of the primary quark-antiquark is a universal function of the available invariant mass; this would give a difference in charged particle multiplicity between light quark and heavy quark initiated events decreasing with the centre-of-mass energy $E_{cm}$ [1]. QCD predicts, somehow counter-intuitively, that this difference is energy independent; this is motivated by mass effects on the gluon radiation (see [2–4] and [5] for a recent review).

The existing experimental tests were not conclusive (see [2] and references therein, [6–9]). At LEP 2 energies, however, the difference between the QCD prediction and the model ignoring mass effects is large, and the experimental measurement can firmly distinguish between the two hypotheses.

# 2 Analysis and Results

A description of the DELPHI detector can be found in [10]; its performance is discussed in [11].

Data corresponding to a luminosity of 54 $\mathrm{pb}^{-1}$ collected by DELPHI at centre-of-mass (c.m.) energies around 183 GeV during 1997, to 158 $\mathrm{pb}^{-1}$ collected around 189 GeV during 1998, and to 187 $\mathrm{pb}^{-1}$ collected between 192 and 200 GeV during 1999, were analysed.

The 1999 data were taken at different energies: 25.8 $\mathrm{pb}^{-1}$ at 192 GeV, 77.4 $\mathrm{pb}^{-1}$ at 196 GeV and 83.8 $\mathrm{pb}^{-1}$ at 200 GeV. Each energy was analyzed separately and the results were then combined as described later and attributed to a c.m. energy of 200 GeV.

A preselection of hadronic events was made, requiring at least 10 charged particles with momentum $p$ above 100 MeV/$c$ and less than 1.5 times the beam energy, with an angle $\theta$ with respect to the beam direction between 20° and 160°, a track length of at least 30 cm, a distance of closest approach to the interaction point less than 4 cm in the plane perpendicular to the beam axis and less than $(4/\sin\theta)$ cm along the beam axis, a relative error on the momentum measurement $\Delta p/p < 1$, and a total transverse energy of the charged particles above $0.2E_{cm}$.

The influence of the detector on the analysis was studied with the full DELPHI simulation program, DELSIM [11]. Events were generated with PYTHIA 5.7 and JETSET 7.4 [12], with parameters tuned to fit LEP1 data from DELPHI [13]. The Parton Shower (PS) model was used. The particles were followed through the detailed geometry of DELPHI giving simulated digitisations in each subdetector. These data were processed with the same reconstruction and analysis programs as the real data.

The hadronic cross-section for $e^+e^-$ interactions above the Z peak is dominated by radiative $\mathrm{q\bar{q}}\gamma$ events; the initial state radiated photons (ISR photons) are generally aligned along the beam direction and not detected. In order to compute the hadronic c.m. energy, the procedure described in [14] was used. In this procedure particles are clustered into jets and the effective centre-of-mass energy of the hadronic system, $\sqrt{s'}$, is computed as being the invariant mass of the system recoiling against an ISR photon, possibly unseen.

Events with reconstructed hadronic c.m. energy ($\sqrt{s'}$) above $0.9E_{cm}$ were used. The selected 1997 (1998, 1999) data sample consisted of 1699 (4583, 4881) hadronic events.

For each year's data, two samples enriched in (1) $b-$ events and in (2) $uds-$ events were selected from the $b$ tagging variable $y$ defined as in Ref. [11]; this variable represents essentially the probability that none of the tracks in the event comes from a vertex separated from the primary one. To select the samples of the type (2), it was required in addition that the narrow jet broadening $B_{min}$ is smaller than 0.065, to reduce the background due to WW and ZZ events. $B_{min}$ is defined as follows. The event is separated into two hemispheres $H_1$ and $H_2$ with respect to the thrust axis, defined by the thrust unit vector $\hat{t}$. Then, calling $\vec{p_k}$ the momentum vector of the $k$-th particle,

$$B_{min} = \min_{i=1,2} \frac{\sum_{k\in H_i} |\vec{p_k} \times \hat{t}|}{2\sum_k |\vec{p_k}|} .$$

The contamination from non-q$\bar{\text{q}}$ events in the samples of type (1) was 7% (8%, 15%), while it was 13% (17%, 20%) in the samples of type (2). After applying the event selection criteria and the cuts to reduce the WW and ZZ background, the purities were approximately 91% (90%, 90%) ($b-$ events) over the total q$\bar{\text{q}}$ in sample (1), and 79% (79%, 79%) ($uds-$ events) over the total q$\bar{\text{q}}$ in sample (2). The fractions of $q$-type quarks in the $(i)$-th sample, $f_q^{(i)}$, were determined from the simulation. The sample (1) consisted of 103 (326, 416) events; the sample (2) of 590 (1450, 1652) events.

The average charge multiplicity was measured in the samples (1) and (2), after subtracting the background bin-by-bin by means of the simulation. It should be noted that the average multiplicity for a given flavour $q$ in each sample is equal to $C_q^{(i)} \times \langle n \rangle_{q\bar{q}}$, with $C_q^{(i)} \neq 1$ in general. The factors $C_q^{(i)}$ account for biases introduced by the application of the $b$ probability and the jet broadening cuts, as well as for detector effects; these factors were computed by means of the simulation.

A third sample (3) was taken into account by considering the measurement of multiplicity described in [15]. This measurement was performed from a sample of 1297 (3444, 3648) hadronic events, with a contamination of 11% (14%, 18%) after applying all the selection criteria; the remaining background mostly comes from the hadronic decay of W and Z pairs. The values $\langle n \rangle^{(3)}$ shown in Table 1 are fully corrected for these backgrounds and for detector effects with their statistical errors; hence the nominal quark flavour ratios appear in the equation (3) below. The systematic errors are reported as the last contribution in Table 3.

The measured mean multiplicities together with the event probability cuts and the factors $f_q^{(i)}$ and $C_q^{(i)}$ are shown in Table 1. For the 1999 data, the values only at $\sqrt{s} = 200$ GeV are tabulated.

In each of the three samples, the average multiplicity $\langle n \rangle$ is a linear combination of the unknowns $\langle n \rangle_{b\bar{b}}$, $\langle n \rangle_{l\bar{l}}$ and $\langle n \rangle_{c\bar{c}}$. One can thus formulate a set of three simultaneous equations to compute these unknowns:

$$\langle n \rangle^{(1)} = f_b^{(1)} C_b^{(1)} \langle n \rangle_{b\bar{b}} + f_{uds}^{(1)} C_{uds}^{(1)} \langle n \rangle_{l\bar{l}} + f_c^{(1)} C_c^{(1)} \langle n \rangle_{c\bar{c}} \ , \quad (1)$$

$$\langle n \rangle^{(2)} = f_b^{(2)} C_b^{(2)} \langle n \rangle_{b\bar{b}} + f_{uds}^{(2)} C_{uds}^{(2)} \langle n \rangle_{l\bar{l}} + f_c^{(2)} C_c^{(2)} \langle n \rangle_{c\bar{c}} \ , \quad (2)$$

$$\langle n \rangle^{(3)} = f_b^{(3)} \langle n \rangle_{b\bar{b}} + f_{uds}^{(3)} \langle n \rangle_{l\bar{l}} + f_c^{(3)} \langle n \rangle_{c\bar{c}} \ . \quad (3)$$

Solving the above equations gave the following mean charge multiplicities at 183 GeV:

$$\langle n \rangle_{b\bar{b}}(183\,\text{GeV}) = 29.79 \pm 1.11 \ ,$$
$$\langle n \rangle_{c\bar{c}}(183\,\text{GeV}) = 29.41 \pm 4.05 \ ,$$

| Data at 183 GeV | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Sample | b-tag prob. | $f_b^{(i)}$ | $C_b^{(i)}$ | $f_{uds}^{(i)}$ | $C_{uds}^{(i)}$ | $f_c^{(i)}$ | $C_c^{(i)}$ | $\langle n\rangle^{(i)}$ |
| (1) | $P_E < 0.00001$ | 0.914 | 0.921 | 0.017 | 1.24 | 0.069 | 0.903 | $27.43 \pm 0.83$ |
| (2) | $0.2 < P_E < 1.0$ | 0.019 | 0.912 | 0.786 | 0.899 | 0.195 | 0.901 | $23.53 \pm 0.33$ |
| (3) | no cut | 0.162 | – | 0.582 | – | 0.256 | – | $27.05 \pm 0.27$ |
| Data at 189 GeV | | | | | | | | |
| Sample | b-tag prob. | $f_b^{(i)}$ | $C_b^{(i)}$ | $f_{uds}^{(i)}$ | $C_{uds}^{(i)}$ | $f_c^{(i)}$ | $C_c^{(i)}$ | $\langle n\rangle^{(i)}$ |
| (1) | $P_E < 0.00001$ | 0.899 | 0.912 | 0.016 | 1.15 | 0.085 | 0.919 | $27.75 \pm 0.48$ |
| (2) | $0.2 < P_E < 1.0$ | 0.016 | 0.896 | 0.789 | 0.893 | 0.195 | 0.913 | $23.93 \pm 0.24$ |
| (3) | no cut | 0.161 | – | 0.580 | – | 0.259 | – | $27.47 \pm 0.18$ |
| Data at 200 GeV | | | | | | | | |
| Sample | b-tag prob. | $f_b^{(i)}$ | $C_b^{(i)}$ | $f_{uds}^{(i)}$ | $C_{uds}^{(i)}$ | $f_c^{(i)}$ | $C_c^{(i)}$ | $\langle n\rangle^{(i)}$ |
| (1) | $P_E < 0.00001$ | 0.880 | 0.928 | 0.026 | 1.11 | 0.094 | 0.881 | $27.31 \pm 0.71$ |
| (2) | $0.2 < P_E < 1.0$ | 0.017 | 0.867 | 0.785 | 0.900 | 0.199 | 0.921 | $23.64 \pm 0.37$ |
| (3) | no cut | 0.159 | – | 0.579 | – | 0.262 | – | $27.52 \pm 0.29$ |

Table 1: Mean multiplicities, $\langle n\rangle$, in three event samples of different flavour content, $f_q$, and correction factors $C_q$. The errors quoted on $\langle n\rangle$ are statistical only. The last dataset contains only the data at 200 GeV from 1999.

$$\begin{aligned} \langle n\rangle_{l\bar{l}}(183\,\mathrm{GeV}) &= 25.25 \pm 1.35\ , \\ \delta_{bl}(183\,\mathrm{GeV}) &= \ 4.55 \pm 1.31\,, \end{aligned}$$

with correlation coefficient of $-0.45$ between $\langle n\rangle_{b\bar{b}}$ and $\langle n\rangle_{l\bar{l}}$, and at 189 GeV:

$$\begin{aligned} \langle n\rangle_{b\bar{b}}(189\,\mathrm{GeV}) &= 30.53 \pm 0.70\ , \\ \langle n\rangle_{c\bar{c}}(189\,\mathrm{GeV}) &= 28.63 \pm 2.81\ , \\ \langle n\rangle_{l\bar{l}}(189\,\mathrm{GeV}) &= 26.10 \pm 0.97\ , \\ \delta_{bl}(189\,\mathrm{GeV}) &= \ 4.43 \pm 0.85\,, \end{aligned}$$

with correlation coefficient of $-0.52$ between $\langle n\rangle_{b\bar{b}}$ and $\langle n\rangle_{l\bar{l}}$.

From the 1999 data, the results obtained for each energy are tabulated in Table 2. The values were scaled to 200 GeV using JETSET and then a weighted average was calculated using the inverse of the square of the statistical error as weight. One obtains

$$\begin{aligned} \langle n\rangle_{b\bar{b}}(200\,\mathrm{GeV}) &= 29.38 \pm 0.65\ , \\ \langle n\rangle_{c\bar{c}}(200\,\mathrm{GeV}) &= 29.89 \pm 2.92\ , \\ \langle n\rangle_{l\bar{l}}(200\,\mathrm{GeV}) &= 25.99 \pm 1.03\ , \\ \delta_{bl}(200\,\mathrm{GeV}) &= \ 3.39 \pm 0.89\,, \end{aligned}$$

with average correlation coefficient of $-0.52$ between $\langle n\rangle_{b\bar{b}}$ and $\langle n\rangle_{l\bar{l}}$. The difference between the average of the values rescaled to 200 GeV and the average of the values without the scaling was added in quadrature to the final systematic error. This difference is anyway small (0.16 units for $\langle n\rangle_{b\bar{b}}$ and less than 0.01 units for $\delta_{bl}$).

The relatively large uncertainty of the measured mean multiplicities for charm stems from the inability of the $P_E$ variable to extract a $c$-enriched sample of events.

It should be noted that the transition between particle and detector level measurements in equations (1) and (2) is done via multiplicative factors $C$ applied to the mean value

| $E_{cm}$ | $\langle n \rangle_{b\bar{b}}$ | $\langle n \rangle_{c\bar{c}}$ | $\langle n \rangle_{l\bar{l}}$ | $\delta_{bl}$ |
|---|---|---|---|---|
| 192 GeV | $27.57 \pm 1.56$ | $30.63 \pm 7.70$ | $25.54 \pm 2.75$ | $2.03 \pm 2.36$ |
| 196 GeV | $29.58 \pm 0.97$ | $26.75 \pm 4.45$ | $27.12 \pm 1.58$ | $2.46 \pm 1.37$ |
| 200 GeV | $29.55 \pm 1.06$ | $32.42 \pm 4.43$ | $24.75 \pm 1.54$ | $4.79 \pm 1.34$ |

Table 2: Multiplicities measured for each energy during 1999.

of the distributions. The validity of this procedure requires that the simulation used to compute the $C$ values reproduces the real data well; the $chi^2$/DF at centre of mass energies of 183, 189 and 200 GeV are respectively 0.81, 1.17 and 0.67 for the sample (1) and 0.93, 1.44 and 1.36 for the sample (2).

The analysis was repeated with different cuts applied to the $b$-tag probability, $P_E$, and the results for the $\delta_{bl}$ were found to be quite stable (see Figure 1). A systematic error was evaluated as half of the difference between the greatest and the smallest multiplicity values obtained from varying the cut on $P_E$ from $0.5 \times 10^{-5}$ to $1.5 \times 10^{-5}$.

The uncertainty due to the event selection in sample (2) was investigated by repeating the analysis after variation of the narrow jet broadening cut, from 0.05 to 0.08. Half of the differences between the greatest and the smallest multiplicities were added in quadrature to the systematic error previously calculated. The propagated systematic error in the total multiplicity in equation (3) from [15] was also added in quadrature to the systematic error. Uncertainties arising from the modelling of short-lived particles in the simulation were considered. The main physics sources of these uncertainties come from the assumed lifetime of B-hadrons ($\tau_B = 1.564 \pm 0.014$ ps) [16], and the $D^+$, $D^0$ lifetimes and production rates [16]. Also a variation in the modelling of the $b$ fragmentation was investigated, by allowing the average fractional energy of a $B$ hadron to vary by 1.5%. The same relative uncertainty was assumed as in [6].

The effect of a variation of 1% in the fraction $R_b$ of $b\bar{b}$ events and of 3% in the fraction $R_c$ of $c\bar{c}$ events was found to be negligible. Since the multiplicity difference, $\delta_{bl}$, was found to be independent of energy within errors, the effect of the modelling of the initial state radiation is also expected to be negligible.

The contributions to the systematic error are summarized in Table 3.

| | 183 GeV | | 189 GeV | | 200 GeV | |
|---|---|---|---|---|---|---|
| Source | $\langle n \rangle_{b\bar{b}}$ | $\delta_{bl}$ | $\langle n \rangle_{b\bar{b}}$ | $\delta_{bl}$ | $\langle n \rangle_{b\bar{b}}$ | $\delta_{bl}$ |
| $b$-tag probability cut | 0.14 | 0.10 | 0.16 | 0.11 | 0.25 | 0.17 |
| Narrow jet broadening cut | 0.06 | 0.32 | 0.05 | 0.18 | 0.15 | 0.63 |
| Modelling in the simulation | 0.10 | 0.33 | 0.10 | 0.32 | 0.09 | 0.23 |
| $E_{cm}$ rescaling | – | – | – | – | 0.16 | 0.00 |
| Systematic error on $\langle n \rangle^{(3)}$ | 0.21 | 0.56 | 0.27 | 0.47 | 0.36 | 0.74 |
| TOTAL | 0.28 | 0.73 | 0.34 | 0.61 | 0.50 | 1.01 |

Table 3: Contributions to the systematic errors on $\langle n \rangle_{b\bar{b}}$ and $\delta_{bl}$.

The final mean values of the event multiplicity in b events are $\langle\ n\ \rangle_{b\bar{b}}(183\,\mathrm{GeV}) = 29.79 \pm 1.11(stat) \pm 0.28(syst)$, $\langle\ n\ \rangle_{b\bar{b}}(189\,\mathrm{GeV}) = 30.53 \pm 0.70(stat) \pm 0.34(syst)$, and $\langle\ n\ \rangle_{b\bar{b}}(200\,\mathrm{GeV}) = 29.38 \pm 0.65(stat) \pm 0.50(syst)$. The multiplicity difference between

$b\bar{b}$ and light quark-antiquark events measured at the different energies is:

$$\delta_{bl}(183\,\mathrm{GeV}) = 4.55 \pm 1.31(stat) \pm 0.73(syst)\ , \quad (4)$$
$$\delta_{bl}(189\,\mathrm{GeV}) = 4.43 \pm 0.85(stat) \pm 0.61(syst)\ , \quad (5)$$
$$\delta_{bl}(200\,\mathrm{GeV}) = 3.39 \pm 0.89(stat) \pm 1.01(syst)\ . \quad (6)$$

These values include the products of $\mathrm{K}^0_S$ and $\Lambda$ decays. The uncertainties on the modelling of the detector largely cancel out in the difference.

Our results on $\delta_{bl}$ are plotted in Figure 2 and compared with previous results in the literature.

# 3 Comparison with Models and QCD Predictions

**Flavour-Independent Fragmentation —** In a model in which the hadronization is independent of the mass of the quarks, one can assume that the non-leading multiplicity in an event, i.e., the light quark multiplicity which accompanies the decay products of the primary hadrons, is governed by the effective energy available to the fragmentation system following the production of the primary hadrons [1]. One can thus write:

$$\delta_{bl}(E_{cm}) = 2\langle n_B^{(decay)}\rangle + \int_0^1 dx_B f_{E_{cm}}(x_B) \int_0^1 dx_{\bar{B}} f_{E_{cm}}(x_{\bar{B}})\ n_{l\bar{l}}\left(\left(1 - \frac{x_B + x_{\bar{B}}}{2}\right) E_{cm}\right) - n_{l\bar{l}}(E_{cm})\,, \quad (7)$$

where $\langle n_B^{(decay)}\rangle$ is the average number of charged particles coming from the decay of a $B$ hadron, $x_B$ ($x_{\bar{B}}$) is the fraction of the beam energy taken by the $B$ ($\bar{B}$) hadron, and $f_{E_{cm}}(x_B)$ is the $b$ fragmentation function.

We assumed $2\langle n_B^{(decay)}\rangle = 11.0 \pm 0.2$ [2], consistent with the average $\langle n_B^{(decay)}\rangle = 5.7 \pm 0.3$ measured at LEP [17]. For $f_{E_{cm}}(x_B)$, we assumed a Peterson function with hardness parameter $\epsilon_p = 0.0047^{+0.0010}_{-0.0008}$ [16], evolving with energy as in [12] to take into account the effects of scaling violations. The value of $n_{l\bar{l}}(E)$ was computed from the fit to a perturbative QCD formula [18] including the resummation of leading (LLA) and next-to-leading (NLLA) corrections, which reproduces well the measured charged multiplicities [15], with appropriate corrections to remove the effect of heavy quarks [19] and leading particles.

The prediction of the model in which the hadronization is independent of the quark mass is plotted in Figure 2. The reason for the drop with collision energy is that the heavy quark system carries away a large fraction of the available energy, approximately (i.e., neglecting scaling violations) linear with $\sqrt{s}$, while the multiplicity growth with $\sqrt{s}$ is less than linear. There are several variations of this model in the literature, leading to slightly different predictions (see [17] and references therein). The result from substituting in Eq. (7) $n_{l\bar{l}}\left(\left(1 - \frac{x_B + x_{\bar{B}}}{2}\right) E_{cm}\right)$ with $n_{l\bar{l}}\left(E_{cm}\sqrt{(1-x_B)(1-x_{\bar{B}})}\right)$ as in [7], or approximating the Peterson fragmentation function with a Dirac delta function at $\langle x_B\rangle$, are within the errors. Also by using for $n_{l\bar{l}}$ the expression in [7] one stays within the band in Figure 2. The prediction as plotted in Figure 2 agrees with the one calculated in [5].

**QCD Calculation —** The large mass of the $b$ quark, in comparison to the scale of the strong interaction, $\Lambda \simeq 0.2$ GeV, results in a natural cut off for the emission of gluon bremsstrahlung. Furthermore, where the c.m. energy greatly exceeds the scale of the $b$ quark mass, the inclusive spectrum of heavy quark production is expected to be well described by perturbative QCD in the Modified Leading Logarithmic Approximation (MLLA, [20]).

The value of $\delta_{bl}$ has been calculated in perturbative QCD[2,3]:

$$\delta_{bl} = 2\langle n_B^{(decay)} \rangle - \langle n_{l\bar{l}} \rangle(\sqrt{s} = e^{1/2} m_b) + O(\alpha_s(m_b))\langle n_{l\bar{l}} \rangle(\sqrt{s} = m_b)\,. \qquad (8)$$

The reason for the appearance of the $e^{1/2}$ factor in the above expression is discussed in detail in [3]. The calculation of the actual value of $\delta_{bl}$ in [2] on the basis of the first two terms in (8) gives a value of $5.5 \pm 0.8$. A different calculation of $\delta_{bl}$ gives 3.68 [3]. These two calculations assume $m_b = 5$ GeV/$c^2$ and $m_b = 4.8$ GeV/$c^2$ respectively, and different parametrizations for the function $\langle n_{l\bar{l}} \rangle(\sqrt{s})$. The dependence of the perturbative part in Eq. (8) on $m_b$ is such that moving the $m_b$ value from 5 GeV/$c^2$ to 4 GeV/$c^2$ induces a change of +0.6 units of multiplicity.

The difference of the results in [2] and in [3] demonstrates the importance of the contribution proportional to $\alpha_s(m_b)$. A less restrictive condition is the calculation of upper limits: an upper limit $\delta_{bl} < 4.1$ is given in [3], based on the maximization of the nonperturbative term; $\delta_{bl} < 4$ is obtained from phenomenological arguments in Ref. [4].

Although the presence of the last term in the equation limits the accuracy in the calculation of $\delta_{bl}$, QCD tells that $\delta_{bl}$ is fairly independent of $E_{cm}$. In this article the average of the experimental values of $\delta_{bl}$ up to $m_Z$ included, $\langle \delta_{bl} \rangle = 2.96 \pm 0.20$ (dominated by the LEP 1 data), is taken as the high energy prediction from QCD. The accuracy of the measurement at the Z is thus used to constrain the theoretical prediction.

Our measurement of $\delta_{bl}$, as seen in Figure 2, is consistent with the prediction of energy independence based on perturbative QCD, and more than three standard deviations larger than predicted by the naive model presented in the beginning of this section.

# 4 Conclusions

The difference $\delta_{bl}$ between the average charged particle multiplicity $\langle n \rangle_{b\bar{b}}$ in $e^+e^- \to b\bar{b}$ events and the multiplicity in generic light quark $l = u, d, s$ events has been measured at centre-of-mass energies of 183, 189 and 200 GeV:

$$\begin{aligned}
\delta_{bl}(183\,\mathrm{GeV}) &= 4.55 \pm 1.31(stat) \pm 0.73(syst) \\
\delta_{bl}(189\,\mathrm{GeV}) &= 4.43 \pm 0.85(stat) \pm 0.61(syst) \\
\delta_{bl}(200\,\mathrm{GeV}) &= 3.39 \pm 0.89(stat) \pm 1.01(syst)\,.
\end{aligned}$$

This difference is in agreement with QCD predictions, while it is inconsistent with calculations assuming that the multiplicity accompanying the decay of a heavy quark is independent of the mass of the quark itself.

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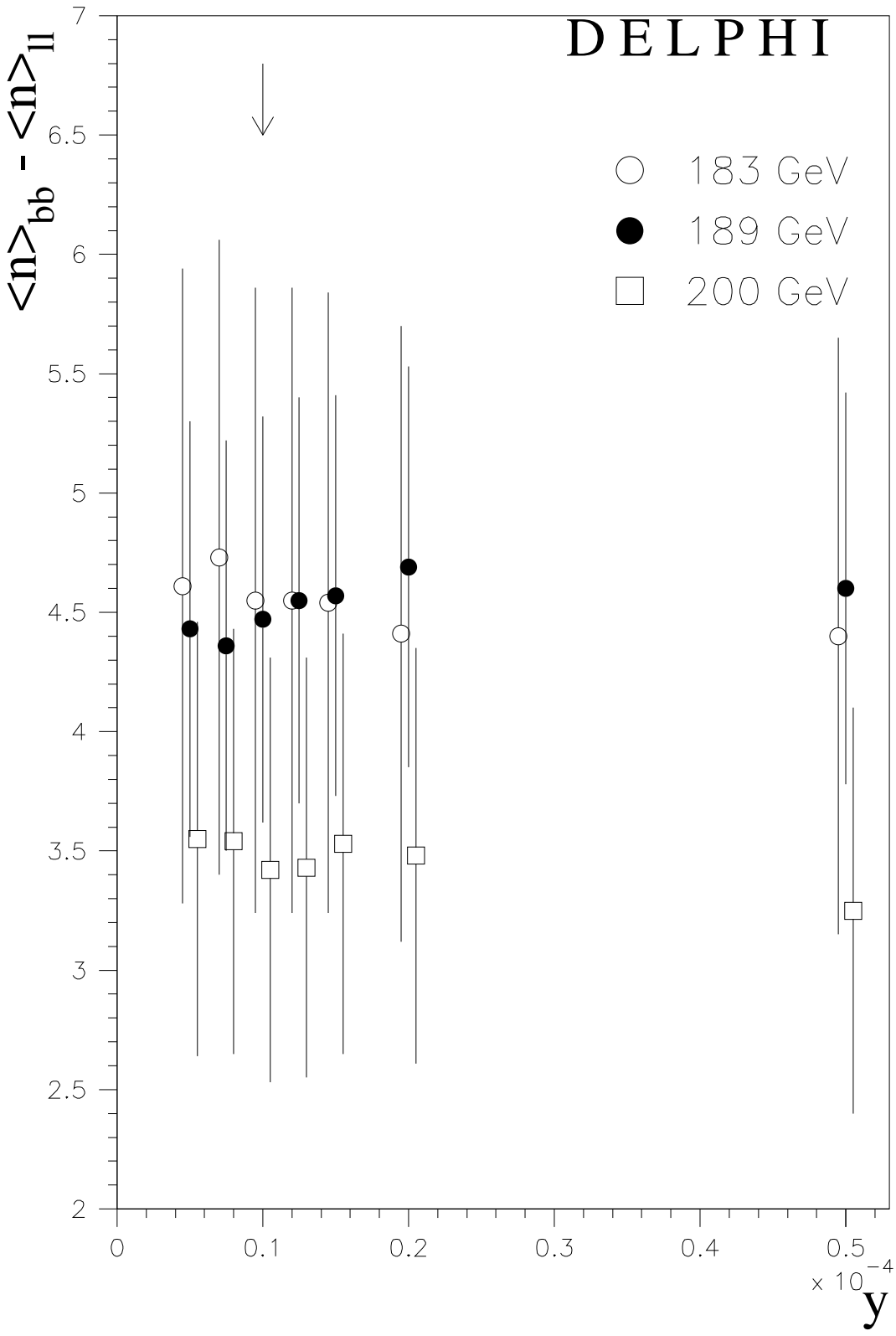


Figure 1: Stability of $\delta_{bl} = \langle n \rangle_{b\bar{b}} - \langle n \rangle_{l\bar{l}}$ with respect to variations of the cut on the b-tagging variable, $y$. Notice that the errors in the plot are correlated (see text). The arrow indicates the value used in the analyses.

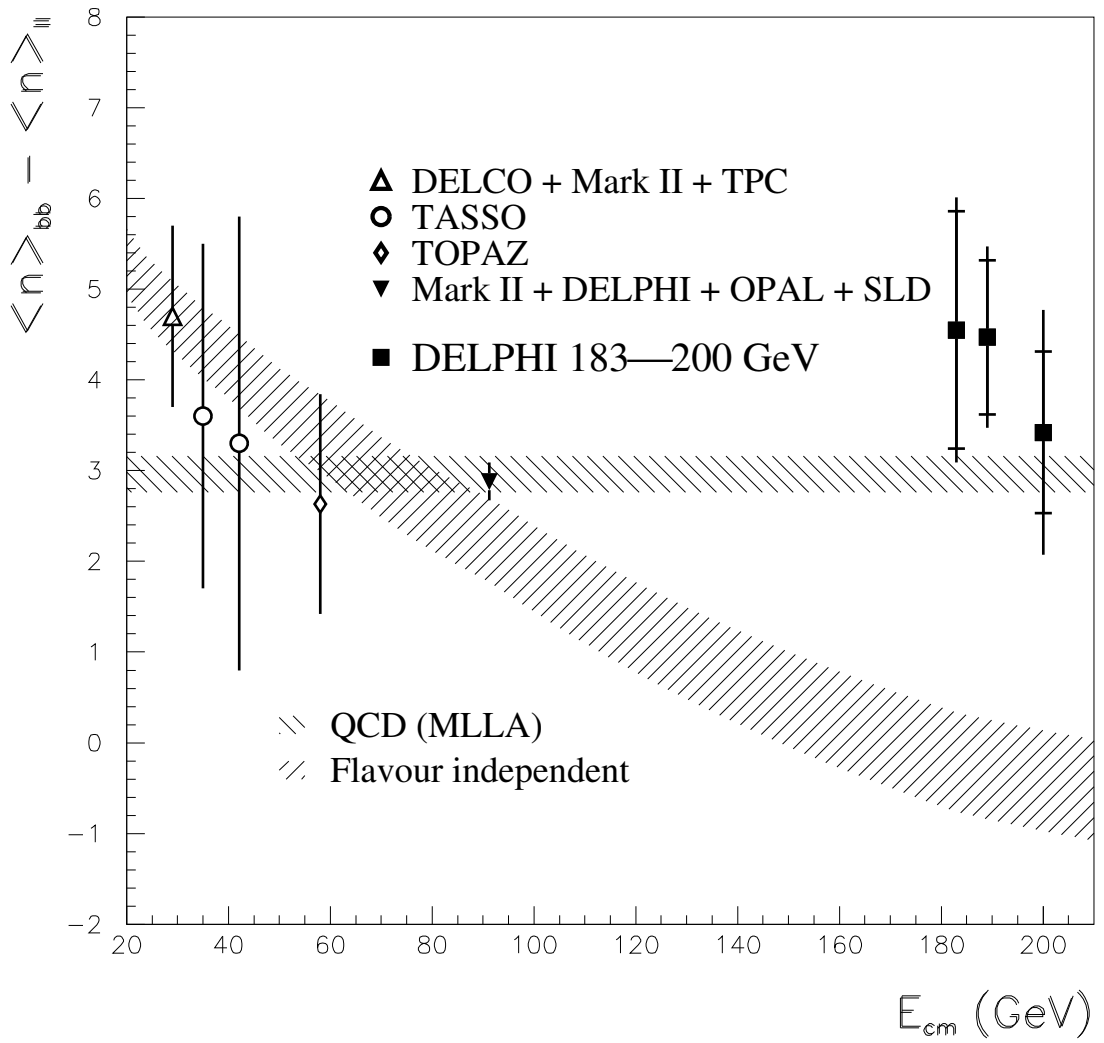


Figure 2: The present measurement of $\delta_{bl}$ compared to previous measurements as a function of the centre-of-mass energy, to the QCD prediction (taken as the average of the values up to the Z included, see the text), and to the expectation from flavour-independent fragmentation. The inner error bars represent the statistical error; the full bars show the sum in quadrature of the statistical and systematic errors.