Some Definitions

- N^S = Detected pairs with single layer target.
- N^M = Detected pairs with multi-layer target.
- N^B = Background pairs.
- N^C = Coulomb pairs.
- N^{NC} = Non-Coulomb pairs.
- n^A_S = Broken atoms in single-layer.
- n^A_M = Broken atoms in multi-layer.
- N^A = Created atomic pairs.

Some Relations

- The number of background pairs (Coulomb and Non-Coulomb) is the same in the two targets.
- Of course: $N^B = N^C + N^{NC}$
- The number of atomic pairs is also the same in the two targets.
- The number of broken pairs differs and are given by:

$$n_S^A = P^S N^A$$

$$n_M^A = P^M N^A$$

where P^S and P^M are the breakup probabilities of pionium in the single and multi layer targets respectively.

The Main Relation

$$N^B = \frac{P^S N^M - P^M N^S}{P^S - P^M}$$

can be easily proven if we consider:

$$N^S = N^B + n_S^A = N^B + P^S N^A$$

$$N^M = N^B + n_M^A = N^B + P^M N^A$$

The relation can be equivalently expressed as:

$$N^B = N^S - \frac{P^S}{P^S - P^M}(N^S - N^M)$$

or

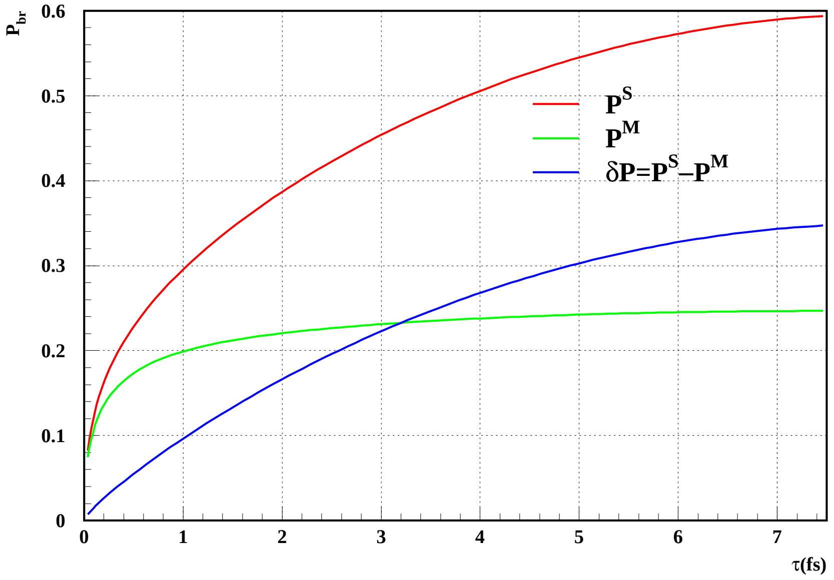
$$N^B = N^M - \frac{P^M}{P^S - P^M}(N^S - N^M)$$

The Main Idea

P^S and P^M depend on the lifetime and we ignore their value. However, we can use some test values P_0^S and P_0^M and compute the errors. As an example we have used:

$$P_0^S = P^S(\tau = 3fs) = 0.454$$

$$P_0^M = P^M(\tau = 3fs) = 0.231$$



The Systematic Error

We want to study wether:

$$N_0^B = \frac{P_0^S N^M - P_0^M N^S}{P_0^S - P_0^M}$$

is a good estimate of N^B.

The systematic error would be:

$$N^B - N_0^B = (N^S - N^M) \times \left[\frac{P_0^S P^M - P_0^M P^S}{(P^S - P^M)(P_0^S - P_0^M)} \right]$$

if we asume $N^{NC} \approx 0$ [a] and consider $N^A = kN^C$ we have ($N^C = N^B$):

$$\frac{N^B - N_0^B}{N^B} = k \frac{P_0^M P_0^S - P^M P^S}{P_0^S - P_0^M}$$

---

[a]Non Coulomb pairs are 2% of the background in the $Q < 2MeV/c$ region.

The Statistical Error

The Statistical Error in the calculation of background with the main Formula is given by:

$$\sigma_{N^B} = \frac{\sqrt{(P^S)^2 N^M + (P^M)^2 N^S}}{P^S - P^M}$$

Notice that $P^M < P^S$, in particular, around $\tau = 3fs$ $P^M \approx P^S/2$. This means that the statistics in the multi-target layer contributes larger to the statistical error. In particular, if we asume $N^{NC} \approx 0$ then:

$$\frac{\sigma_{N^B}}{N^B} = \frac{1}{\sqrt{N^B}} \times$$

$$\frac{\sqrt{(P^S)^2 + (P^M)^2 + kP^S P^M (P^S + P^M)}}{P^S - P^M}$$

Two Cases

We have analyzed two particular cases in the $F < 2$ region [a]:

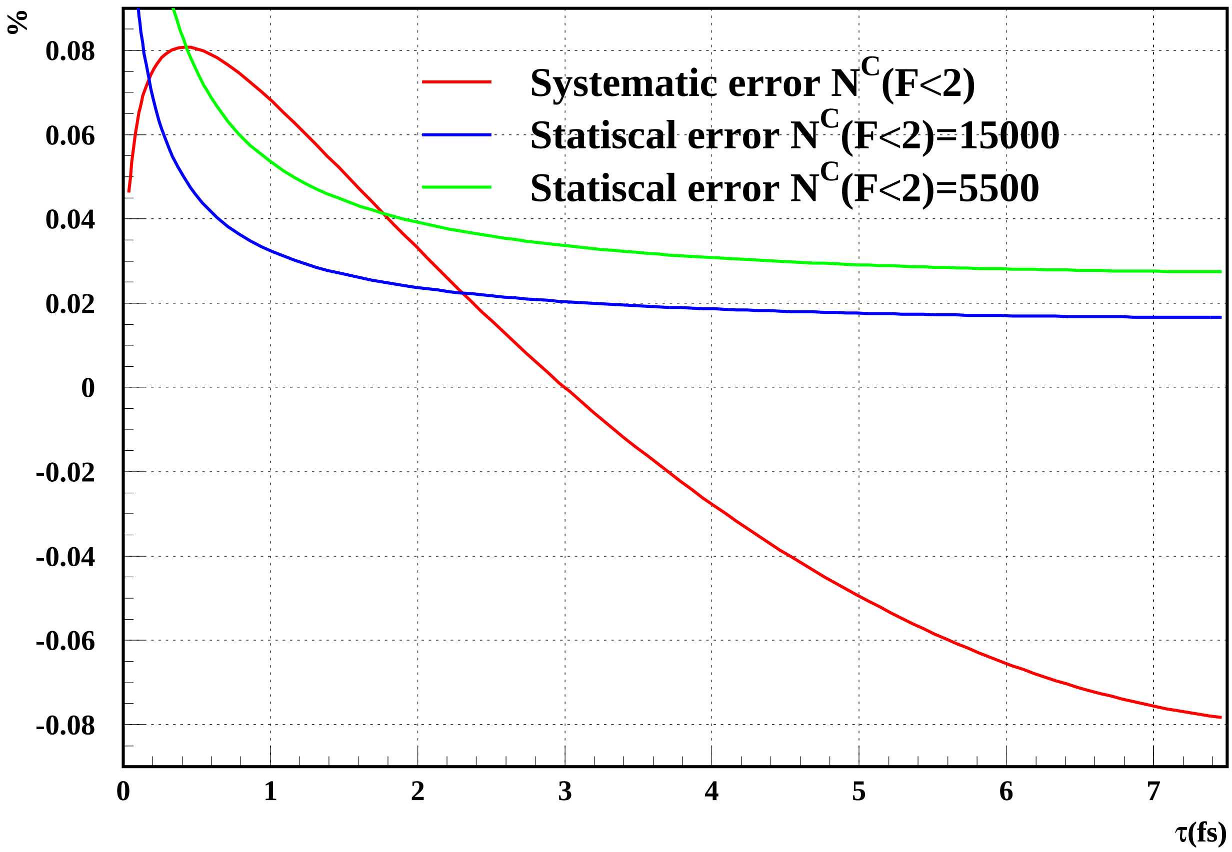
- $N^C = 15000$, acumulated statistic of the single layer target 2001.
- $N^C = 5500$, acumulated statistic of the multi-target layer 2002.

We have used $k = 0.69$ for the k factor.

Errors τ	Stat. $(3fs)$	Sys. $(2.4fs)$	Sys. $(3.6fs)$
$N^C = 15000$	2.0%	1.9%	-1.7%
$N^C = 5500$	3.4%	1.9%	-1.7%

[a] The region with atomic pairs contamination.

Errors as a Function of τ

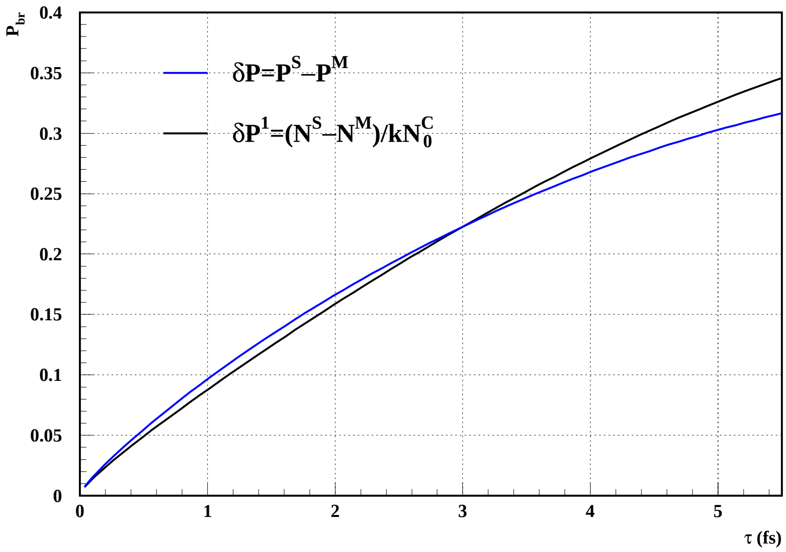


Second approach

We have started a second approach to the Main Formula by analyzing the magnitude:

$$\delta P^1 = \frac{N^S - N^M}{kN_0^C}$$

$$= \frac{(P_0^S - P_0^M)(N^S - N^M)}{k(P_0^S N^M - P_0^M N^S)}$$



Second approach (2)

We have not computed the possible transmision of errors, so, the result should be considered as preliminar.

Errors τ	Sys. $(2.4fs)$	Sys. $(3.6fs)$
2nd app.	-0.36%	0.34%