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Background Estimation with Single and Multi Layer Targets

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1 Introduction

The new multi target installation widenes the possibilities to obtain physics results in the Dirac experiment. In this note we aim to study a particular way to calculate the time correlated background getting rid of the contamination from atomic pairs with relative momentum $Q < 2MeV/c$. In order to do this we will need to make a hypothesis on the lifetime value. We will analyze which is the systematic error that this choice produces and how it compares to the estimated statistical error.

2 Some Definitions and Some Relations

Let us start by defining some basic magnitudes for Dirac's experimental procedure:

- N^S = Detected prompt [1] pairs sample with the single layer target.
- N^M = Detected prompt pairs sample with the multi-layer target.
- N^B = Background pairs [2].
- N^C = Coulomb pairs.
- N^{NC} = Non-Coulomb pairs.
- n^A_S = Broken atoms in the single-layer target.
- n^A_M = Broken atoms in the multi-layer target.

[1] A prompt event is an event with time difference $-0.5ns < \Delta t < 0.5ns$ between the two particles of the pair.

[2] In other notes or presentations also called free pairs (N_{free}).

- N^A = Created atoms.

These magnitudes are not all independent. In particular the following relations hold:

- The number of background pairs (Coulomb and Non-Coulomb) is the same in the two targets.
- $N^B = N^C + N^{NC}$.
- The number of created atoms is the same in the two targets.
- The number of broken atoms differs and is given by:

$$n_S^A = P^S N^A, \tag{1}$$

  for the single-layer target and:

$$n_M^A = P^M N^A, \tag{2}$$

  for the multi-layer target, where P^S and P^M are the breakup probabilities of pionium in the single and multi layer targets, respectively.

Finally, the number of detected prompt events with the single and the multi layer targets are defined as:

$$\begin{aligned} N^S &= N^B + n_S^A, && (3)\\ N^M &= N^B + n_M^A, && (4) \end{aligned}$$

which, making use of (1) and (2), can be transformed into:

$$\begin{aligned} N^S &= N^B + P^S N^A, && (5)\\ N^M &= N^B + P^M N^A, && (6) \end{aligned}$$

that is a system of linear equations relating the number of atoms and background pairs with the direct measurement of single and multi layer target prompt events [3].

[3]This relation, expressed for integrated samples of prompt events, can be also applied, due to its linearity, to distributions:

$$\begin{aligned} \frac{dN^S}{dQ} &= \frac{dN^B}{dQ} + P^S \frac{dN^A}{dQ}, && (7)\\ \frac{dN^M}{dQ} &= \frac{dN^B}{dQ} + P^M \frac{dN^A}{dQ}. && (8) \end{aligned}$$

3 The Background Estimation

The equations (5) (6) can be inverted to give:

$$N^B = \frac{P^S N^M - P^M N^S}{P^S - P^M} \tag{9}$$

$$N^A = \frac{N^S - N^M}{P^S - P^M} \tag{10}$$

in particular we are interested in (9) which allows us to obtain N^B as a function of the direct measurements of N^S and N^M [4].

We know that P^S and P^M depend on the lifetime, as we can see in Figure 1, but we ignore their true value. However, we can take some test values P_0^S and P_0^M and compute the error to the real value. As an example we have used:

$$P_0^S = P^S(\tau = 3 \cdot 10^{-15} s) = 0.454,$$

$$P_0^M = P^M(\tau = 3 \cdot 10^{-15} s) = 0.231,$$

and hence we want to study whether

$$N_0^B = \frac{P_0^S N^M - P_0^M N^S}{P_0^S - P_0^M} \tag{11}$$

is a good estimate of N^B.

4 The Systematic Error

The systematic error we would made in estimating the N^B value with (11) is given by:

$$N^B - N_0^B = (N^S - N^M)\left[\frac{P_0^S P^M - P_0^M P^S}{(P^S - P^M)(P_0^S - P_0^M)}\right]. \tag{12}$$

We can perform a quantitative calculation of this systematic error if we assume $N^{NC} \approx 0$ [5]. Taking into account that $N^A = kN^C$ we have ($N^C = N^B$):

$$\frac{N^B - N_0^B}{N^B} = k\frac{P_0^M P_0^S - P^M P^S}{P_0^S - P_0^M}$$

which can be calculated for any value of τ. The result of this calculation is shown in Figure 2 where we can see that, if the absolute ratio of the real lifetime to the test value is less than 20% the systematic error is less than 2%.

[4] The relation can be equivalently expressed as:

$$N^B = N^S - \frac{P^S}{P^S - P^M}(N^S - N^M),$$

or

$$N^B = N^M - \frac{P^M}{P^S - P^M}(N^S - N^M).$$

[5] Non Coulomb pairs are 2% of the background in the $Q < 2MeV/c$ region.

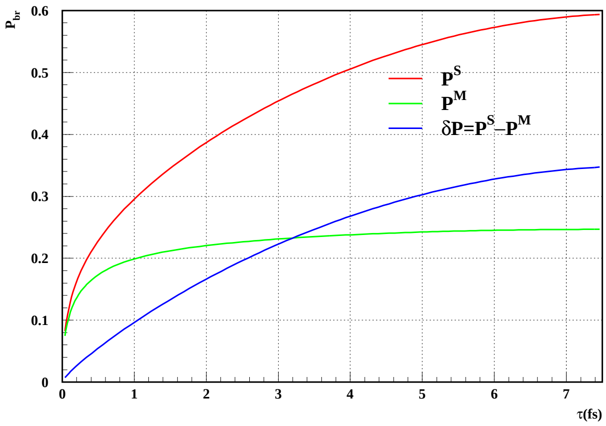

Figure 1: *Breakup probability dependence on lifetime for the single layer target, the multi layer target.*

5 The Statistical Error

The statistical error in the calculation of background with equation (9) is given by:

$$\sigma_{N^B} = \frac{\sqrt{(P^S)^2(\sigma^{N^M})^2 + (P^M)^2(\sigma^{N^S})^2}}{P^S - P^M} \tag{13}$$

where σ^{N^S} and σ^{N^M} are the standard deviations of N^S and N^M. Notice that $P^M < P^S$, in particular, around $\tau = 3 \cdot 10^{-15}s$ $P^M \approx P^S/2$. This means that the statistics in the multi-layer target contributes larger to the statistical error.

If we consider $\sigma^{N^S} = \sqrt{N^S}$ and $\sigma^{N^M} = \sqrt{N^M}$ equation (13) becomes:

$$\sigma_{N^B} = \frac{\sqrt{(P^S)^2 N^M + (P^M)^2 N^S}}{P^S - P^M} \tag{14}$$

which in particular, if we assume $N^{NC} \approx 0$ gives us:

$$\frac{\sigma_{N^B}}{N^B} = \frac{1}{\sqrt{N^B}} \frac{\sqrt{(P^S)^2 + (P^M)^2 + kP^S P^M (P^S + P^M)}}{P^S - P^M}. \tag{15}$$

6 Two Cases

We have analyzed two particular cases in the $F < 2$ region [6]:

[6] The region with atomic pairs contamination.

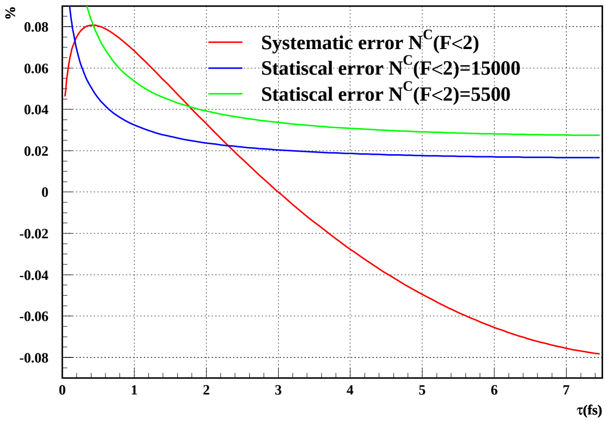

Figure 2: *Computation of the systematic and statistical errors in the background determination.*

- $N^C = 15000$, accumulated statistic of the single layer target 2001.
- $N^C = 5500$, accumulated statistic of the multi-target layer 2002.

We have used $k = 0.69$ for the k factor. The results can be seen in Table 1 and in Figure 2.

Errors τ	Stat. $(3 \cdot 10^{-15} s)$	Sys. $(2.4 \cdot 10^{-15} s)$	Sys. $(3.6 \cdot 10^{-15} s)$
$N^C = 15000$	2.0%	1.9%	−1.7%
$N^C = 5500$	3.4%	1.9%	−1.7%

Table 1: *Statistical and systematic errors in the estimation of the background N^C of these two samples by assuming a lifetime of $3 \cdot 10^{-15} s$.*