Experimental Measurement of a K^+K^- Signal at p=2.90 GeV/c in Ni 2001 Data

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Abstract

In this note we make an experimental determination of the ratio $K^+K^-/\pi^+\pi^-$ in Ni 2001 pionium data sample, by measuring the time of flight between upstream detectors and vertical hodoscopes, at low momentum.

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1 $K^+\pi^-$ and $K^-\pi^+$ signals from Time-of-Flight counters only

It is well known that in a V-shaped spectrometer like DIRAC a precision measurement of the difference Δt between the arrival time of positive and negative particles to a given detector (the TOF or Vertical Hodoscopes VH), ensures good discrimination for unequal mass pairs. Such a measurement is insensitive to the time origin, since pair production is assumed to be simultaneous, and therefore it does not require any time measurement upstream the dipole magnet.

In fact, even with 120 ps single-hit precision [1], a good mass discrimination of individual charged particles can only be achieved under the assumption that the mass of the acompanying particle is known. If we assume a leading particle in the pair to be a pion (the negative, for instance), then the squared mass of the other particle (positive) can be measured according to the expression:

$$M_+^2 = p_+^2 \left[\left(\frac{L_-}{L_+} \sqrt{1 + \frac{M_\pi^2}{p_-^2}} - \frac{c\Delta t}{L_+} \right)^2 - 1 \right] \tag{1}$$

where $\Delta t = t_- - t_+$ is the delay in arrival time with respect to the leading particle, L_{+-} and p_{+-} being the respective pathlengths and momenta. Conversely, M_-^2 can be obtained by sign flipping, and figure 1 shows the corresponding spectra for Ni 2001 data. A clear signal is seen for the $K^+\pi^-$ and $p\pi^-$ final states, as well as for $K^-\pi^+$, the latter being significantly suppressed with respect to its charge conjugate. No antiproton ($\bar{p}\,\pi^+$) signal is observed. A low-momentum cut for the test particle 1.4 GeV/c $< p_{+-} <$ 1.5 GeV/c is important. The standard prompt pair cut ($\Delta t < 0.5$ ns) has been removed here in order to see the proton signal, so that a large fraction of the background data are actually accidental pairs. Please note that the $K^\pm\pi^\mp$ signals would be totally suppressed for prompt pairs. It is also worth noting that the standard 5.5 ns time coincidence for upstream tracks (with respect to VH) leaves the $K^\pm\pi^\mp$ signals practically unchanged.

2 Measurement of K^+K^- from TOF counters and upstream detectors

When the positive and negative particles accepted by the spectrometer have equal mass, the previous method does not allow mass discrimination, despite

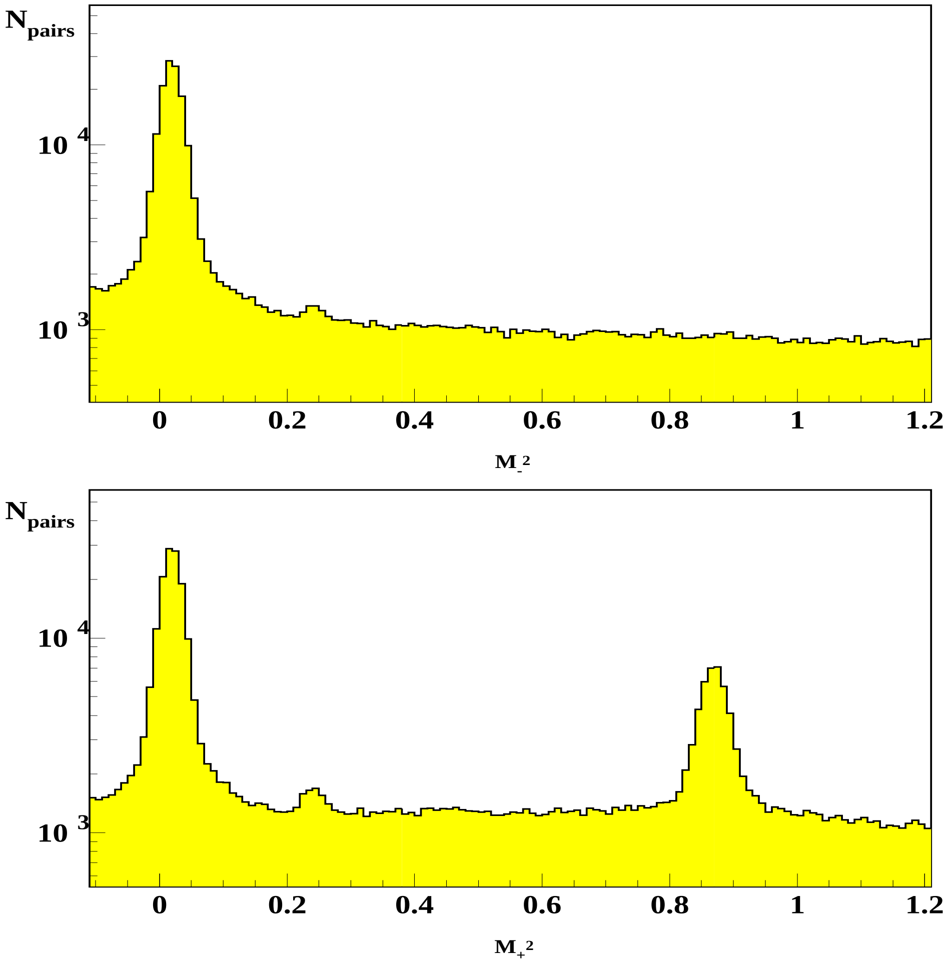

Fig. 1. *Negative (M_-^2) and positive ($M+-^2$) squared mass spectra according to expression (1). Pion mass hyptothesis has been made at the opposite branch. Note no upstream TDCs are used in these measurements.*

the excellent resolution of Time-of-Flight (TOF) hodoscopes. This is because of the narrow Q_L acceptance for normal physics triggers, which makes both particles reach the hodoscopes at practically the same time.

However, upstream tracks contain time information from SFD and Ionization Hodoscope (IH) TDCs. We have up to 6 independent TDC measurements (SFD-X, SFD-Y, IH-X_A, IH-X_B, IH-Y_A, IH-Y_B) for each charged track, so that the time of flight Δt between upstream detectors and vertical hodoscopes can be measured with improved statistical precision. A similar method was used to analyze the $p\pi^-$ background in accidental pairs [2].

For equal-mass pairs, charge ambiguity in the matching procedure does not impeed a clear determination of Δt. This is particularly important for the IH, where the probability that both particles hit the same 6 mm slab is high. In this case, only one TDC will be retained, which is perfectly adequate for the measurement.

The time delay Δt can in general be defined as $\Delta t = t_{VH} - t_{up}$, where the upstream time can be constructed by averaging N measurements among the 6 available detectors $t_{up} = (1/N) \sum_i t_i$.

It is however more convenient to measure squared invariant masses M_i^2 of individual time measurements $\Delta t_i = t_{VH} - t_i$ associated by the upstream tracking to a given charged particle in one arm, with momentum p and path length L:

$$M_i^2 = p^2 (\frac{c^2 {\Delta t_i}^2}{L^2} - 1) \tag{2}$$

The particle M^2 is then obtained by averaging the appropriate N individual measurements $M^2 = (1/N) \sum_i M_i^2$.

All IH detector TDC signals associated to upstream tracks are time-aligned with respect to each other, taking into account propagation time between different layers, as a necessary first step. Despite the excellent detector calibration work available [3] , we have noticed that the IH-X_B detector has a wider TDC distribution and slightly different properties than the others. As a consequence, no attempt has been made to include this detector in subsequent analysis, in order to avoid any doubts.

Searching for a K^+K^- signal implies to suppress a huge background from $\pi^+\pi^-$. The most general way to achieve this goal is to use a certain number of detectors to veto the pion signal and a different detector set to perform the mass measurement. According to this, we have developped two extreme methods. The first (method A) is use the three IH detectors to veto and the two SFD's to perform the measurement. The second (method B) is to perform an unbiased measurement (no pion veto) and concentrate all detector measurements to improve mass resolution at the distribution tail.

In both cases, particle velocity has been reduced by a momentum cut $1.4 < p < 1.5$ GeV/c applied for every charged track.

2.1 Results from method A

Pairs were selected by requiring that $M_L^2 < M_i^2 < M_H^2$ for all IH hits i associated to tracks by upstream tracking (6 measurements), for different choices of the lower cut M_L^2, and the squared mass distributions were analysed using SFD X and Y (4 measurements).

A clear signal is observed at the kaon squared-mass, of about the same size as that observed for the pion. As expected, lowering the cut M_L^2 results in increasing pion background, and the opposite effect is observed by setting higher values of M_L^2, as shown in figure 2. The K^+K^- signal is however not unaffected by the M_L^2 values either, due to the acceptance variations implied by the pion veto, which will be studied below.

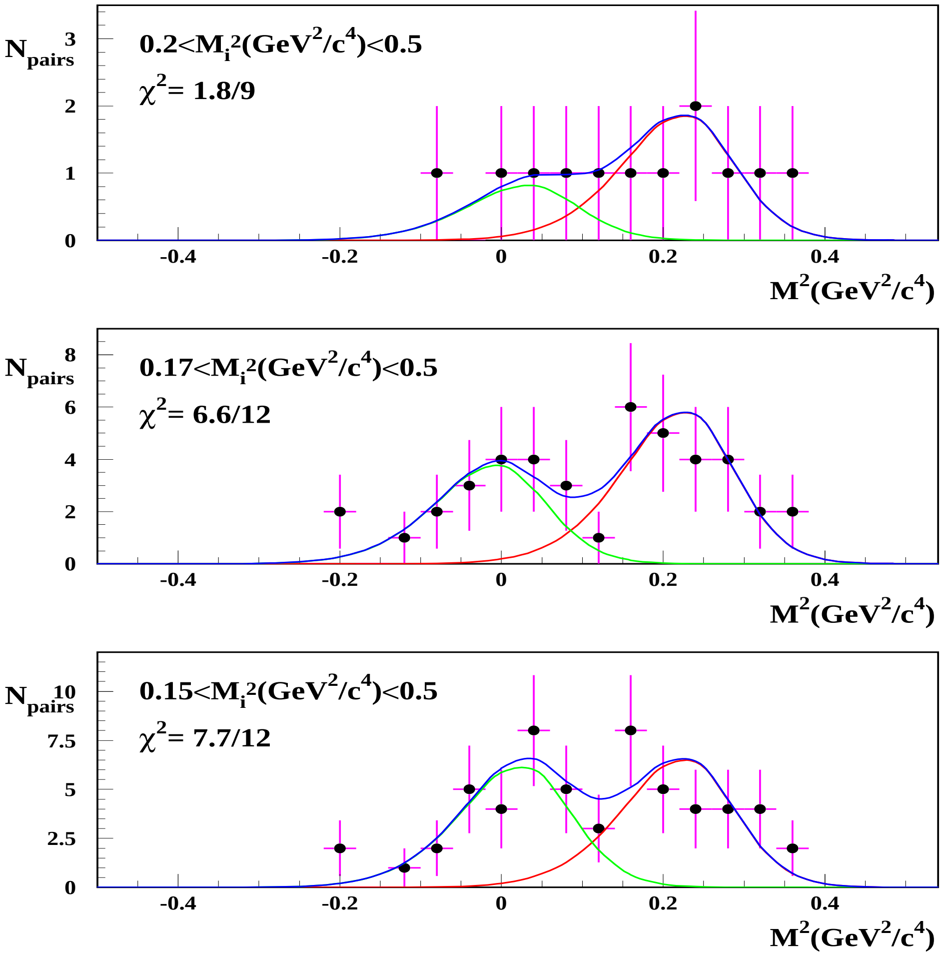

Fig. 2. *Average squared invariant mass of particle pairs measured from TDCs of SFD X-Y according to expression (2). All individual hit measurements M_i^2 associated to any of the two upstream tracks in IH detectors have been restricted to the indicated limits $M_L^2 < M_i^2 < M_H^2$. The results of the maximum likelyhood fit are shown in each case, indicating the progression of the $\pi^+\pi^-$ (green), K^+K^- (red) signals, and of the sum (blue), with decreasing lower limit M_L^2. χ^2 values are indicated only for reference.*

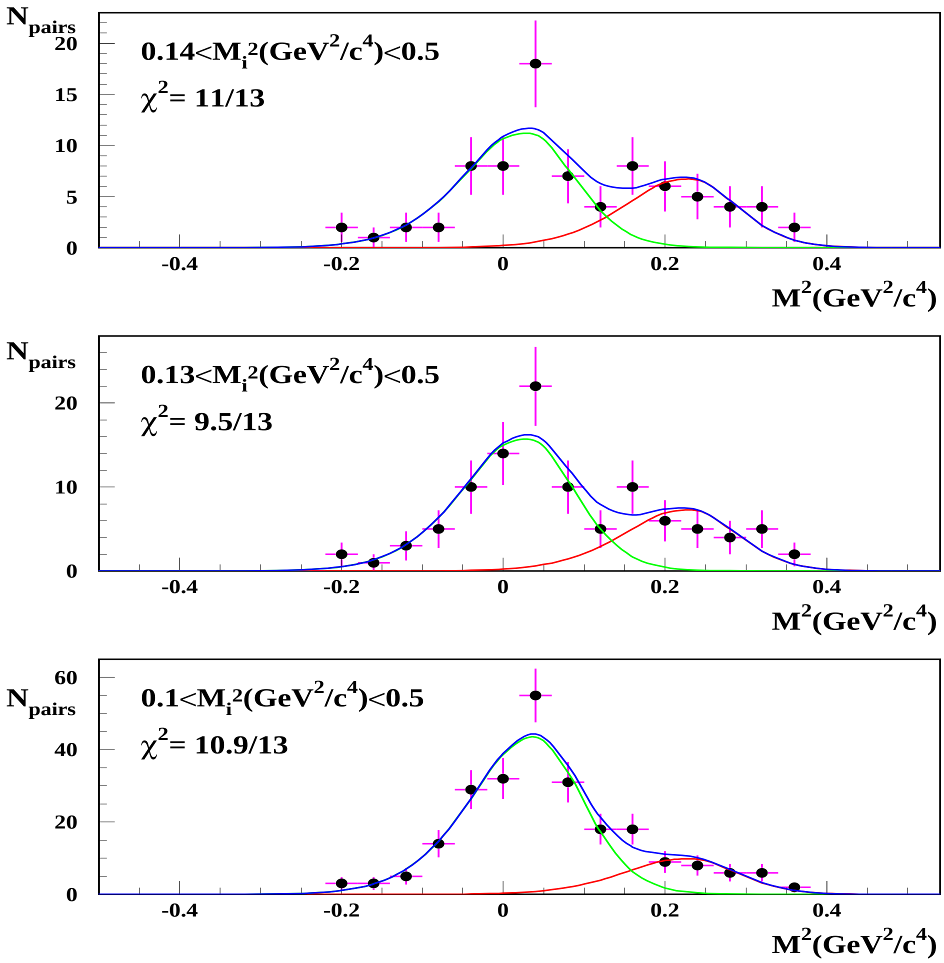

Fig. 3. *Contents are the same as in figure 2, the progression continues from* $M_L^2 = 0.14\ GeV^2/c^4$ *(top) to* $M_L^2 = 0.10\ GeV^2/c^4$ *(bottom)* .

In order to measure this signal we have first determined the pion squared-mass resolution function $R(x)$, which is a continuous curve (normalized to one) peaked at $x = 0$, with $x = M^2 - M_\pi^2$ and M_π^2 the observed pion mass [1]. We determine this curve with high statistical precision by going to lower values of M_L^2, and apply the same resolution function for the kaon, since all instrumental effects are identical in both cases [2].

[1] the analytical expression and parameter values of this function will be reported in the next subsection.

[2] a partial derivative analysis of expression (2) reveals that a resolution factor $\sigma(M_\pi^2)/\sigma(M_K^2) = \sqrt{M_\pi^2 + p^2}/\sqrt{M_K^2 + p^2}$ is expected from the slightly different mass scale. This accounts for a 5% effect only, and its effect is negligible.

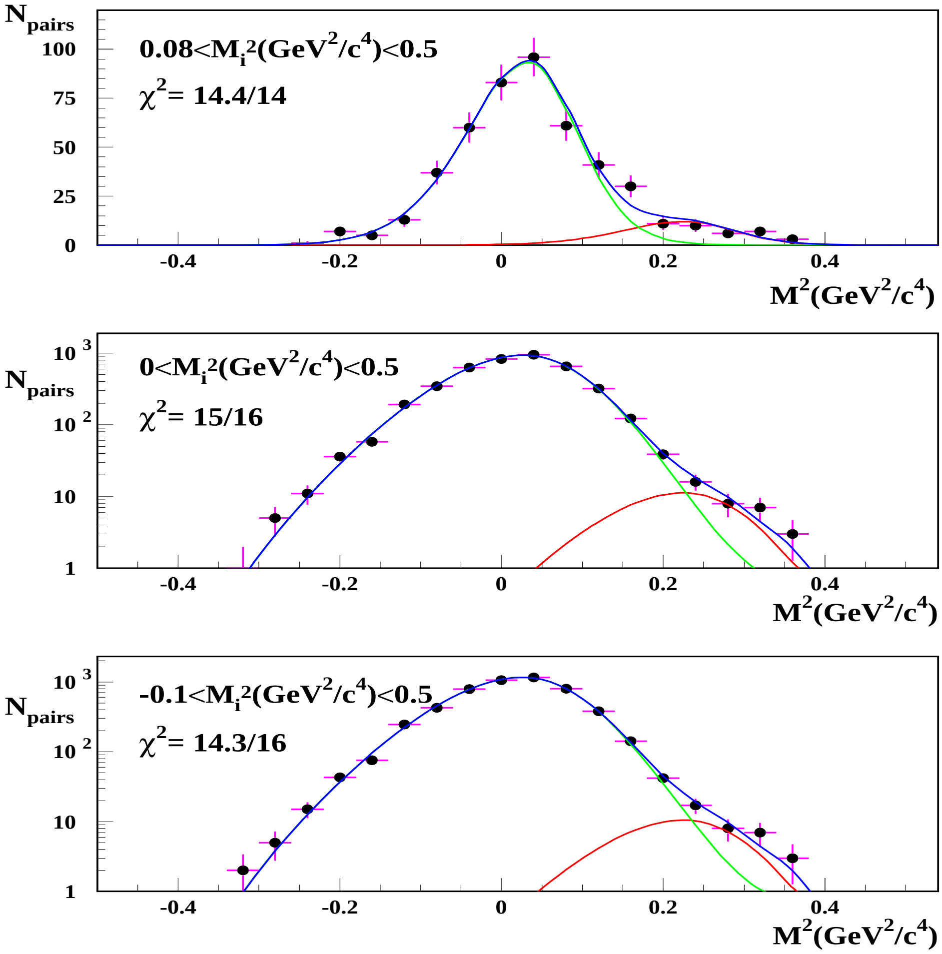

Fig. 4. *Contents are the same as in figure 2, the progression continues from* $M_L^2 = 0.08\ GeV^2/c^4$ *(top) to* $M_L^2 = -0.10\ GeV^2/c^4$ *(bottom)* .

A maximum likelyhood fit is then performed to the entire mass spectra for 8 different choices of M_L^2, having a generic number of events N in the fit. The probability density for an event with mass M_i^2 is given by :

$$L_i = (1-\alpha)R(M_i^2 - M_\pi^2) + \alpha R(M_i^2 - M_K^2) \tag{3}$$

where the M_π^2 value is left free to allow for small biases as function of pion veto. The kaon mass M_K^2 is however a fixed parameter, with its value being calibrated by the observed pion mass M_π^2 without pion veto, after adding the difference $M_K^2 - M_\pi^2$ given by PDG values.

Maximization of $\prod_{i=1}^N L_i$ as function of α , under the constraint $\int_{-C}^{C} R(x)dx = 1$ with $C = 0.4GeV^2/c^4$, provides the number of K^+K^- pairs as $N(K^+K^-) = \alpha N$.

Because of the fact that the kaon resolution function is not entirely contained within the bias interval (M_L^2,M_H^2), the K^+K^- signal efficiency will be a calculable function of the lower limit M_L^2. Note that the relevant resolution function $R(x)$ here involves the measurement with a single generic IH detector [3], and it can be easily determined with pions. Its parametrization is given in table 2 (see next subsection). If we call $\epsilon = \int_{M_L^2}^{\infty} R(x)dx$ then the K^+K^- acceptance probability is given by :

$$A(M_L^2) = \epsilon^{6-3s(1-s)^2-2[3s^2(1-s)]-3s^3} \tag{4}$$

where s is the average probability that both particles intersect the same IH slab. The last three terms in the exponent are related to the probability of single, double and triple same-slab coincidences among the three active counters, respectively. They can easily be determined from real data by using the upstream track extrapolations. The above expression takes into account the fact that rather than having 6 independent detector measurements, there are actually a reduced number of them, due to same-slab intersections. The effective exponent is accurately determined from the real data to be 5.14, which corresponds to $s = 0.30$.

In figure 5 we show that the function $A(M_L^2)$ provides a good description of the K^+K^- signal data found from the 8 maximum likelyhood fits. Errors in α are given by MINOS variation with account taken of the correlation with M_π^2. These are converted into $N(K^+K^-) = \alpha N$ errors, where the observed number of events N is also subject to error. A line for a 0.5% contamination hypothesis is also shown in figure 5 for reference.

The platteau value indicates 61 ± 10 K^+K^- events, where the 1σ error can be determined from any of the M_L^2 choices without significant variation (0.15 GeV^2/c^4 is chosen for reference). It should be noted that this platteau is rather insensitive to small variations of the exponent, compatible with the data. As a matter of fact, a change of $\pm$ 0.2 (contradicting measured values) produces only $\pm$ 1 events in the platteau, still having a good description of the data in figure 5.

It is clear that the observed K^+K^- signal is genuine and it can by no means be explained as a consequence of the applied bias. This point has been checked by applying an opposite bias according to the mirror-symmetric cut $2M_\pi^2 - M_L^2 < M_i^2 < 2M_\pi^2 - M_H^2$ for all active IH detectors i. The result are shown in figures 6 and 7 for 6 different values of M_L^2. In fact, the K^+K^- signal is totally removed, and in addition no significant signal is generated in the bias region, which shows that correlation between IH and SFD measurements

[3] the average is taken between the three active detectors, which differ very little among themselves

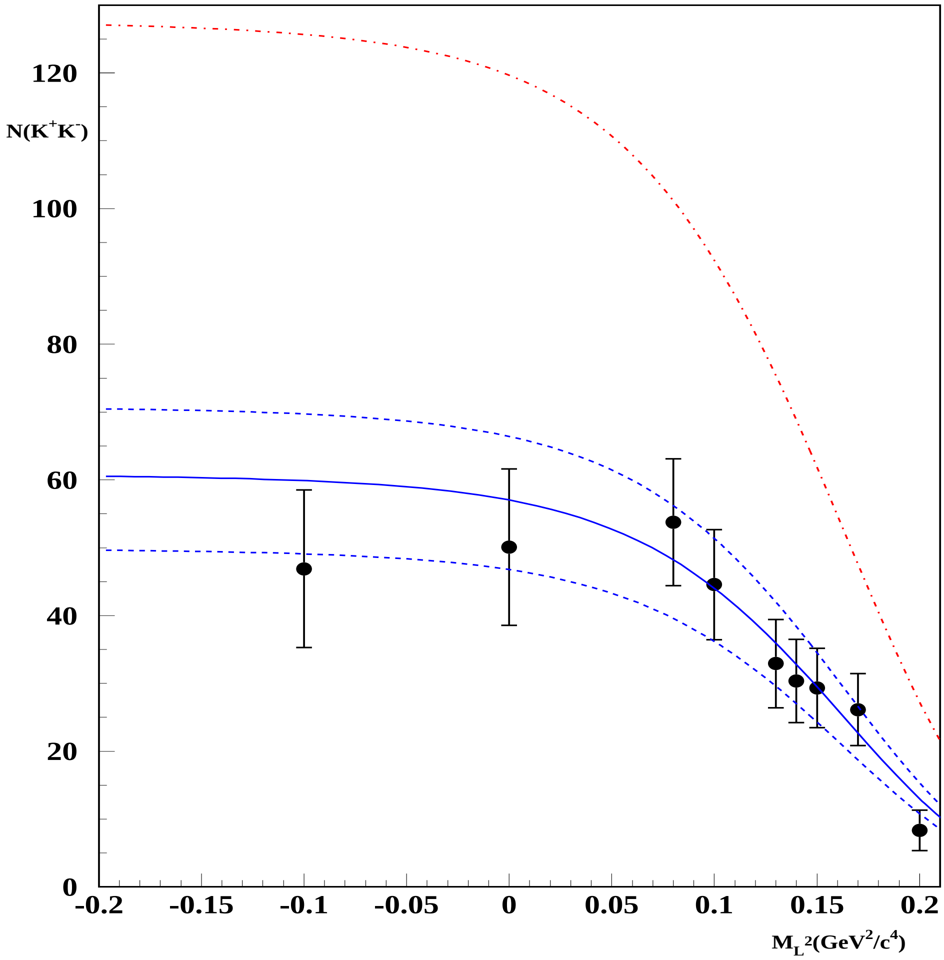

Fig. 5. *Number of K^+K^- pairs determined from the maximum likelyhood fit shown in figures 2 to 4. The continuous line shows the function $A(M_L^2)$ (4) multiplied by $N(K^+K^-)$=61 events.*

is actually quite small. A significant correlation could only be induced by the presence of background tracks crossing all upstream detectors, which is strongly suppressed by the timing requirements ($5.5ns$) of upstream tracking.

When the total number of $\pi^+\pi^-$ pairs is taken into account, the previous result can be converted into a measurement of the ratio $\epsilon_K = N(K^+K^-)/N(\pi^+\pi^-) = 0.238 \pm 0.035$ % .

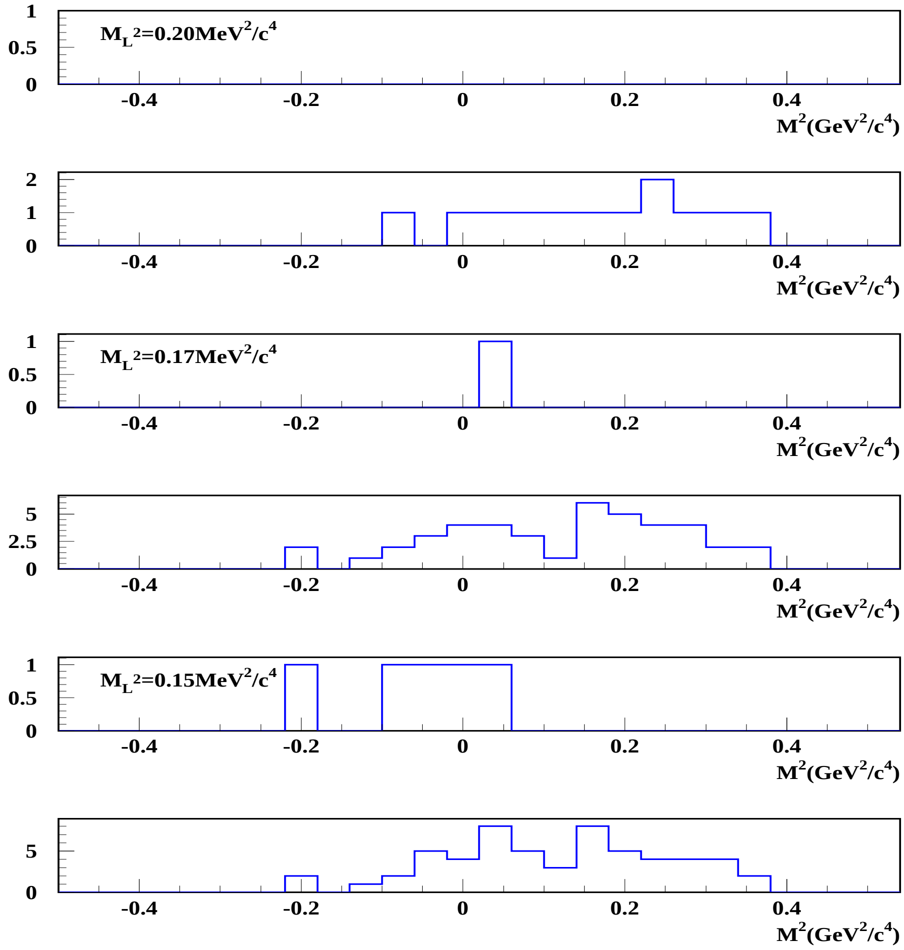

Fig. 6. *Comparison between some of the squared-mass distributions in figures 2 to 4 (bottom) and the ones obtained with the mirror-symmetric cuts* $2M_\pi^2 - M_L^2 < M_i^2 < 2M_\pi^2 - M_H^2$ *(top), with values of* M_L^2 *as indicated, and* $M_H^2 = 0.50\ GeV^2/c^4$ *. Disappearence of* K^+K^- *signal is clearly appreciated in all cases.*

2.2 Results from method B

A second analysis method comes from making a direct fit to the mass spectrum in which all (IH+SFD) detectors are used for the measurement, without any attempt to perform a $\pi^+\pi^-$ veto. The K^+K^- signal is then observed at the distribution tail. A good description of the pion resolution function for the mass measurements M_i^2 is provided by the parametrization:

$$R(x) = Ae^{-(a_1|x|^2 + a_2|x|^3 + a_3|x|^4)} \tag{5}$$

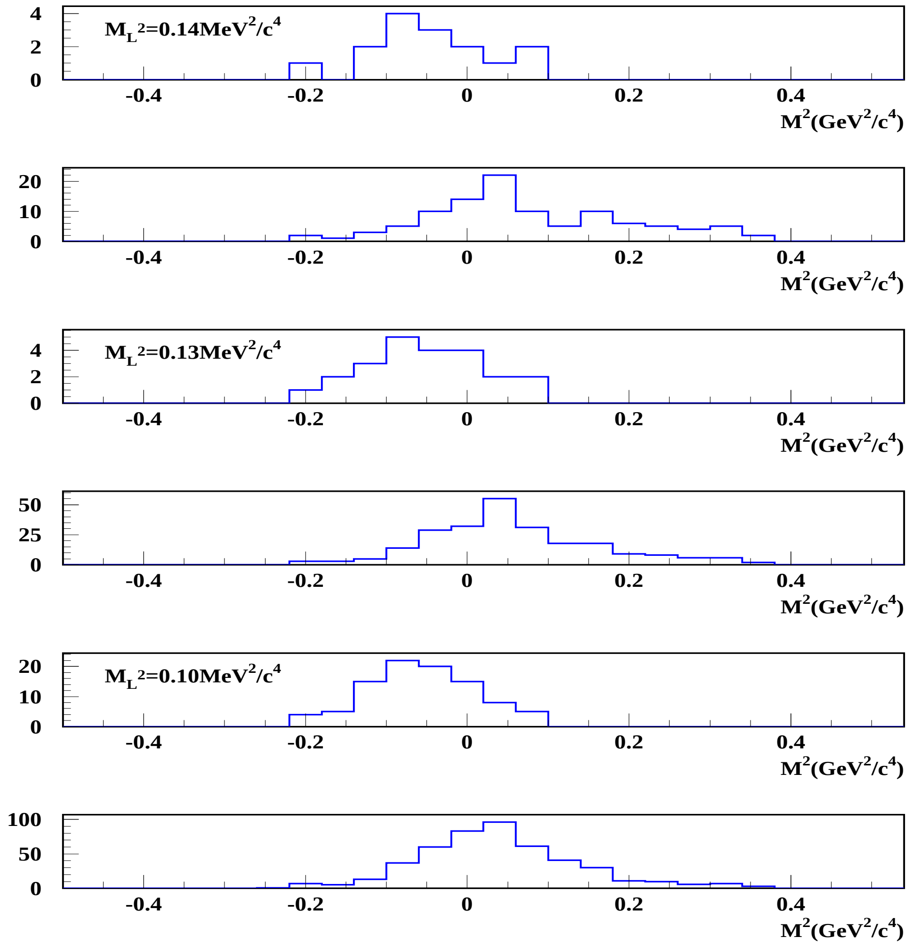

Fig. 7. *Contents the same as figure 6, with higher values of M_L^2.*

for $x \geq 0$ and $R(x) = Ae^{-(b_1|x|^2+b_2|x|^3+b_3|x|^4)}$ for $x \leq 0$, with $x = M_i^2 - M_\pi^2$, where M_π^2 defines the peak value, which is allowed to differ slightly from the true pion squared-mass from PDG, due a systematic error. Since there is no reason to expect left/right symmetry about the peak value, $a_i \neq b_i$ are allowed by the fit. Proper normalization is ensured by the A parameter, according to $1/A = \int_{M_\pi^2-C}^{M_\pi^2+C} R(x)dx$ with C=0.4 GeV^2/c^4 (which is infinity in practical terms).

The K^+K^- signal is parametrized by the same function $R(x)$ used for the pion, with peak value shifted by $M_K^2 - M_\pi^2$ according to PDG values. As in the case described above for method A, a maximum likelyhood fit is made to the hypothesis expressed by equation (3) in the mass interval $I_C = (M_\pi^2 - C, M_\pi^2 + C)$, as shown in figure 8, in order to determine the fraction α, and the corresponding number of K^+K^- pairs.

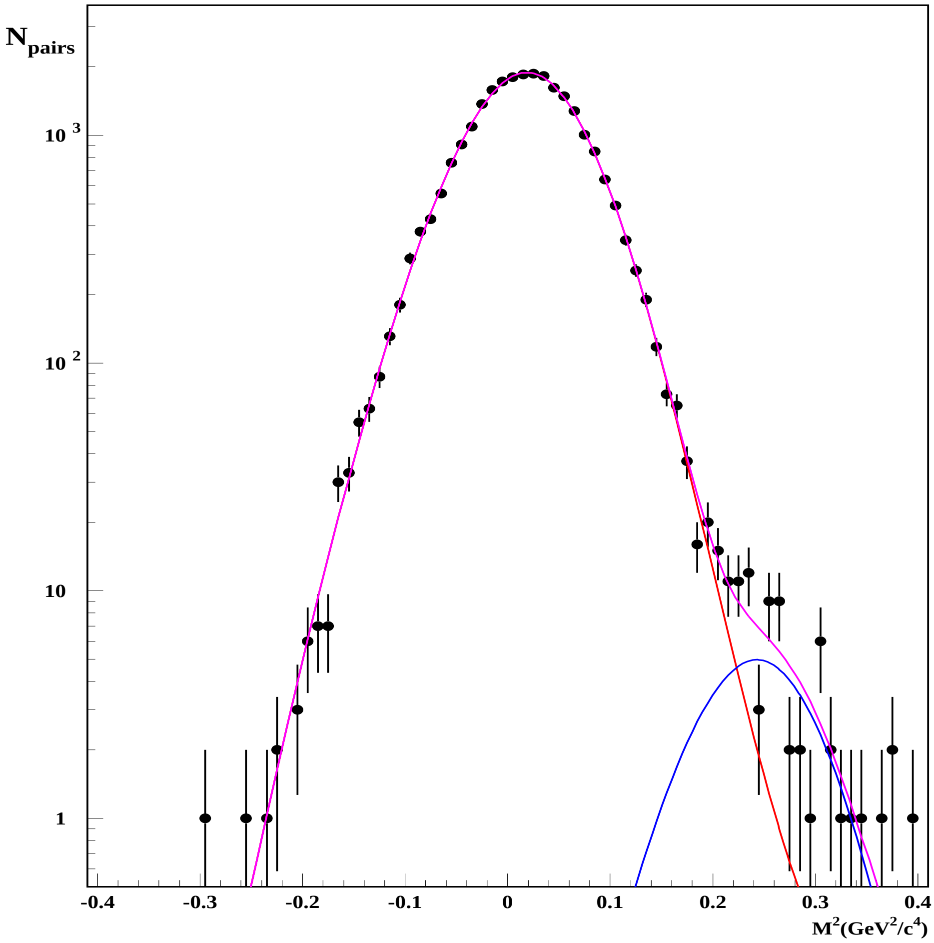

Fig. 8. *Squared invariant mass for particle pairs, measured from the average of all detectors hits asscociated with (positive and negative) upstream tracks in IH and SFD. No attempt has been made to reduce the $\pi^+\pi^-$ background (red), which is described by the parametrization (5). Fit results, indicated in table 1, are shown for the K^+K^- signal (blue) and for the sum (purple).*

The fit has actually been done in two steps: first determine the 7 parameters of the resolution function (a_i, b_i and M_π^2) for $M^2 \leq M_0^2$ ($M_0^2 = 0.135\ GeV^2/c^4$ has been chosen), and then extend the fit to the full region I_C, keeping the $R(x)$ parametrization fixed. Results can be appreciated in figure 8 and in table 1. The fitted number of K^+K^- pairs is 65 ± 10.

For the sake of simplicity, we have refered throughout this note to a generic pion resolution function $R(x)$, due to the fact that all parametrizations of it do have the same functional form. However, the parameter values are different in each case, depending upon which (and how many) detectors are used for the measurement. We give in table 2 the fitted parameters when SFD X and

Table 1

Parameter values for parametrization (5) obtained for mass measurement using all (IH+SFD) upstream detectors (left), and fit results for method B described in the text (right). The variation of the likelihood is also indicated for two hypothesis: 0.5 % K^+K^- signal, and no signal at all.

M_π	0.0188 $\pm$ 0.0014 GeV^2/c^4
a_1	201 $\pm$ 18
a_2	-462 $\pm$ 181
a_3	511 $\pm$ 508
b_1	199 $\pm$ 8
b_2	-137 $\pm$ 46
b_3	-650 $\pm$ 92

α	0.9975 $\pm$ 0.0004
N_k	65 $\pm$ 10
$\epsilon_k(\%)$	0.253 $\pm$ 0.038
χ^2/ndf	69.57 / 55
ΔL(no signal)	38
ΔL(signal 0.5%)	17

Y detectors are used in method A, and also those corresponding to a single (generic) IH detector, as it was used for the acceptance ϵ calculation.

Table 2

Parameter values for the pion squared-mass resolution function according to expresion (5), from minimun-χ^2 fits for measurements using SFD X and Y without pion veto (left), and using a single generic IH detector (right). For the latter, the average between the three active detectors has been taken, as described in the text.

	SFD	IH
$M_\pi(GeV^2/c^4)$	0.0114$\pm$0.0004	0.0378 $\pm$ 0.0007
a_1	103 $\pm$ 8	66.8$\pm$3.4
a_2	-216 $\pm$ 12	105.$\pm$13
a_3	258 $\pm$ 24	47.5$\pm$12.6
b_1	131 $\pm$ 10	43.0$\pm$5.4
b_2	39.7 $\pm$ 33.3	5.79$\pm$29.97
b_3	-713 $\pm$ 29	-85.4$\pm$ 44.5
χ^2/ndf	9.3 / 11	23.3/25

3 Summary

The fact that two radically different methods as those reported A and B give practically the same results for the number of K^+K^- pairs is, in our opinion, a strong indication that systematic effects are under control, and that the statistical errors of the maximum likelihood analysis can be trusted.

Clearly many other intermediate analysis are possible, although they will not improve the result. For example, one can measure invariant masses separately for the positive (M_+^2) and negative (M_-^2) arms, using all available upstream detectors in (2). The scatter plot M_+^2 versus M_-^2 is presented in figure 9, which shows a clear correlated signal for K^+K^-.

Note that this particular method is well suited for equal mass pairs, but it is not adequate to disentangle the $K^+\pi^-$ and $K^-\pi^+$ signals, which are randomized by the charge ambiguity in the low-angle matching, which takes place in conjunction with the IH segmentation effect [4].

Our final result is that the ratio $\epsilon_K = K^+K^-/\pi^+\pi^-$ has been measured at p=2.90 GeV/c, with identical cuts as those used for pionium lifetime analysis [4], and the result is $\epsilon_K = 0.238 \pm 0.035\%$. A value $\epsilon_K > 0.308$ % is excluded at 95 % confidence level.

References

[1] B. Adeva et al., The time-of-flight detector of the DIRAC experiment, Nucl. Instr. Meth. A491 (2002) 41.

[2] DIRAC Note 02-10: Proton to pion ratio in accidental coincidences, S. Trusov.

[3] DIRAC Note 02-09: New ionization hodoscope: design and characteristics, V. Brekhovskikh, M. Jabitski, A. Kuptsov, V. Lapshin, V. Rykalin, L. Tauscher.

[4] DIRAC Note 06-03: Measurement of pionium lifetime, B.Adeva, A.Romero, O.Vazquez Doce.

[4] TOF counters only should be used for this purpose, as demonstrated in section 1.

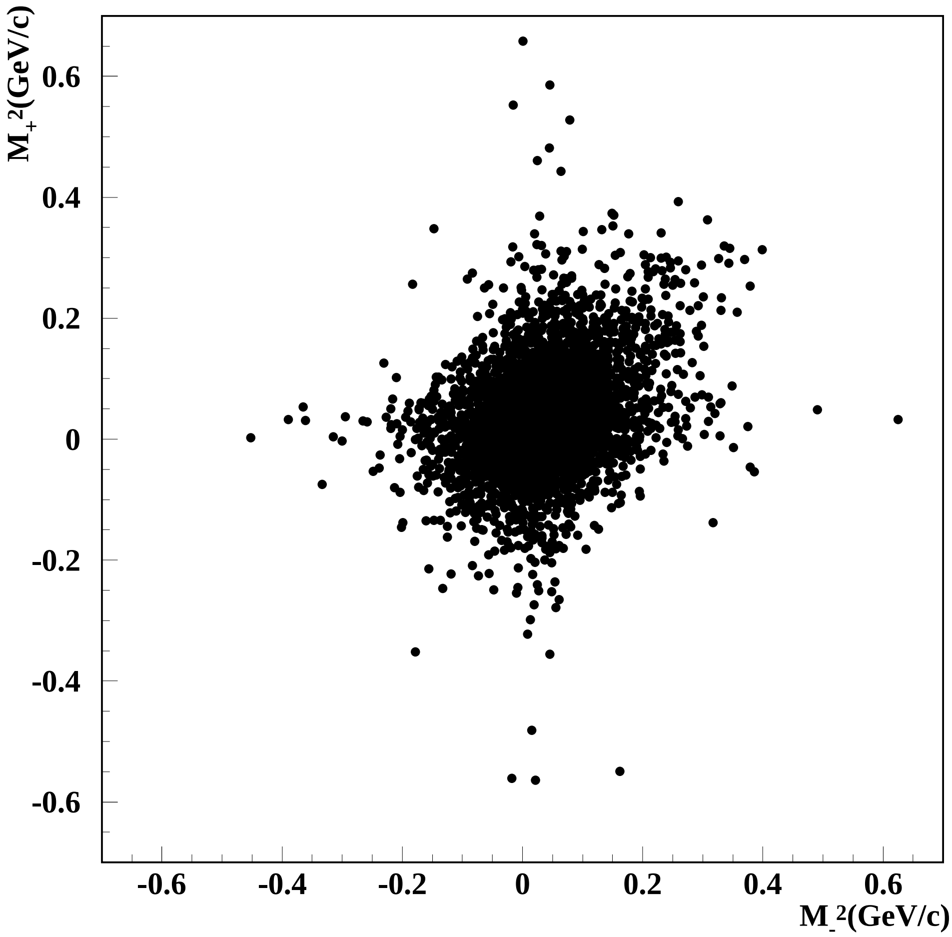

Fig. 9. *Negative (M_-^2) versus positive ($M + -^2$) squared masses of particle pairs, measured from all TDC hits associated to individual upstream tracks. A clear K^+K^- signal is appreciated. Note that a significant correlation is however expected from same-slab hits in IH detector.*