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What is known about the branching ratio $\Gamma_{A_{2\pi}\to\gamma\gamma}/\Gamma_{tot}$?

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The DIRAC experiment is measuring the total (1s) lifetime of pionium ($A_{2\pi}$). In order to extract from this measurement precise pion-pion scattering length data, we need to know the hadronic fraction of the pionium decay width, or - in other words - the non-hadronic admixture at some precision.

The aim of this note is to investigate the status of theoretical and experimental information about the main non-hadronic decay channel $A_{2\pi} \to \gamma\gamma$.

1 Introduction

The electromagnetic decay of pionium into $\gamma\gamma$ corresponds to the well-known para-positronium decay $A^{para}_{e^+e^-} \to \gamma\gamma$. The width of this decay is given in lowest order (LO) by

$$\Gamma^{LO}_{A_{e^+e^-}\to\gamma\gamma} = \frac{1}{2}\alpha^5 m_e \tag{1}$$

with $\alpha \simeq \frac{1}{137}$ and m_e the electron mass.

Accordingly, the decay $A_{2\pi} \to \gamma\gamma$ with structureless pions was studied by Uretsky

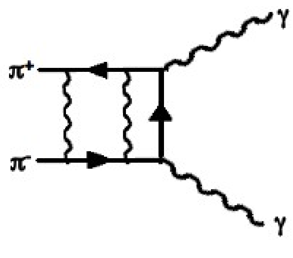

Figure 1: $A_{2\pi} \to \gamma\gamma$ with structureless pions

and Palfrey [1] in 1961, yielding the following expression (up to a factor of 16) for the decay width:

$$\Gamma^0_{A_{2\pi}\to\gamma\gamma} = \frac{1}{4}\alpha^5 M_\pi, \tag{2}$$

where M_π is the charged pion mass.

In the following we notice shortly the pion polarizability correction to the "pointlike" formula (2) with the aim to present for the branching ratio $R_\gamma = \Gamma_{A_{2\pi}\to\gamma\gamma} \big/ \Gamma_{tot}$ theoretical and experimental results with best known errors.

2 Partial decay width $\Gamma_{A_{2\pi}\to\gamma\gamma}$

Calculating Compton scattering on pions, $\gamma + \pi^+ \to \gamma + \pi^+$, one has to take into account polarizability effects due to the non-pointlike electromagnetic structure of the charged pions. Hammer and Ng [2] determined the partial decay width $\Gamma_{A_{2\pi}\to\gamma\gamma}$ as follows:

$$\Gamma_{A_{2\pi}\to\gamma\gamma} = \frac{2\pi\alpha^2}{M_\pi^2}|\Psi(0)|^2 \cdot [1 + \frac{M_\pi^3}{\alpha}(\alpha_\pi - \beta_\pi)]^2 = \frac{1}{4}\alpha^5 M_\pi \cdot [1+P]^2 \,. \tag{3}$$

Here, the Coulomb wave function is given by $|\Psi(0)| = [\frac{1}{\pi}(\frac{\alpha M_\pi}{2})^3]^{\frac{1}{2}}$ and $P = \frac{M_\pi^3}{\alpha}(\alpha_\pi - \beta_\pi)$ is the correction due to the electric (α_π) and magnetic (β_π) pion polarizabilities.

With formula (3) there is a tool to compare numerical values for $\Gamma_{A_{2\pi}\to\gamma\gamma}$ obtained from different sources:
On the theoretical side, we consider a recent recalculation for the reaction $\gamma\gamma \to \pi^+\pi^-$ in the framework of chiral perturbation theory (ChPT) by Gasser, Ivanov and Sainio [3]. With updated values for the so-called low-energy constants at order p^4 (two-loop), these authors found for the dipole polarizabilities $(\alpha_\pi - \beta_\pi) = (5.7 \pm 1.0) \cdot 10^{-4}\ fm^3$. By inserting this result in (3), we get the following theoretical partial decay width:

$$\Gamma^{th}_{A_{2\pi}\to\gamma\gamma} = (764 \pm 7)\mu eV \quad [0.9\% \ \ accuracy]. \tag{4}$$

On the experimental side, pion polarizabilities can be extracted by measuring Compton scattering off pions. One typical experiment, done at Serpukhov [4] in 1983, investigated Primakoff radiative pion scattering on a heavy nucleus ($\pi^- Z \to \gamma\pi^- Z$). The result of a conservative analysis [4] is $(\alpha_\pi - \beta_\pi) = (15.6 \pm 7.8) \cdot 10^{-4}\ fm^3$, yielding

$$\Gamma^{exp1}_{A_{2\pi}\to\gamma\gamma} = (835 \pm 59)\mu eV \quad [7\% \ \ accuracy]. \tag{5}$$

Recently, a similar experiment used the radiative pion photoproduction process ($\gamma p \to \gamma\pi^+ n$) at MAMI [5] in Mainz and derived $(\alpha_\pi - \beta_\pi) = (11.6 \pm 3.4) \cdot 10^{-4}\ fm^3$. With these polarizabilities we get

$$\Gamma^{exp2}_{A_{2\pi}\to\gamma\gamma} = (807 \pm 25)\mu eV \quad [3\% \ \ accuracy]. \tag{6}$$

3 Partial decay width $\Gamma_{A_{2\pi}\to\pi^0\pi^0}$

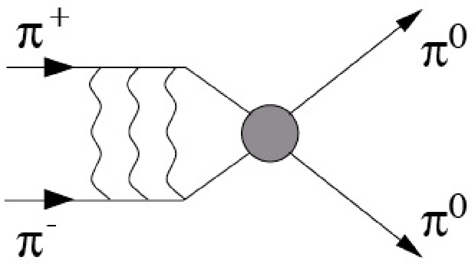

Figure 2: The dominant decay channel of *pionium* $A_{2\pi} \to \pi^0\pi^0$

In low energy QCD the decay rate $A_{2\pi}(ground\ state) \to \pi^0\pi^0$ [6] (Fig. 2) is given by

$$\Gamma_{A_{2\pi}\to\pi^0\pi^0} = \frac{2}{9}\,\alpha^3\,p\,|a_0 - a_2|^2\,(1+\delta_\Gamma). \tag{7}$$

In formula (7) α is the fine structure constant, p the π^0 momentum in the pionium system, and a_0 and a_2 the S-wave $\pi\pi$ scattering lengths (units of inverse charged pion mass) for isopin $I = 0$ and 2, respectively. The small term $\delta_\Gamma = (5.8 \pm 1.2)\cdot 10^{-2}$ [6] accounts for corrections of order α as well as for those due to the quark mass difference $m_u \neq m_d$.

In the framework of ChPT the scattering length difference $|a_0 - a_2|$ has been calculated at the 2% level: $a_0 - a_2 = 0.265 \pm 0.004$ [7]. Inserting this value in (7) one gets the following theoretical prediction for $\Gamma_{A_{2\pi}\to\pi^0\pi^0}$:

$$\Gamma^{th}_{A_{2\pi}\to\pi^0\pi^0} = (228 \pm 7)meV \quad [3\%\ \ accuracy]. \tag{8}$$

Experimentally, the CERN NA48 experiment presented at KAON07 [8, 9] a result for $|a_0 - a_2| = 0.261 \pm 0.015$ by studying the decay $K^+ \to \pi^+\pi^0\pi^0$. With this measured (non-DIRAC) scattering length difference an experimental partial decay width $\Gamma_{A_{2\pi}\to\pi^0\pi^0}$ can be deduced:

$$\Gamma^{exp}_{A_{2\pi}\to\pi^0\pi^0} = (221 \pm 25)meV \quad [12\%\ \ accuracy]. \tag{9}$$

4 Branching ratio $R_\gamma \equiv \Gamma_{A_{2\pi}\to\gamma\gamma}\big/\Gamma_{tot}$

To estimate the contribution of $\Gamma_{A_{2\pi}\to\gamma\gamma}$ to the total width, we first calculate the ratio $\Gamma_{A_{2\pi}\to\gamma\gamma}\big/\Gamma_{A_{2\pi}\to\pi^0\pi^0}$. As will be seen, this ratio of partial widths approximates very well the branching ratio R_γ.

Using the theoretical values as given in eq. (4) and (8), one finds

$$R^{th}_\gamma \approx \Gamma^{th}_{A_{2\pi}\to\gamma\gamma}\big/\Gamma^{th}_{A_{2\pi}\to\pi^0\pi^0} = (3.35 \pm 0.10)\cdot 10^{-3} \quad [3.1\%\ \ accuracy]. \tag{10}$$

This result (10) from theory is to be compared with two further R_γ values evaluated with some input from experiments:

- By inserting eq. (5) and (9) in R_γ, one gets

$$R_\gamma^{exp1} \approx \Gamma^{exp1}_{A_{2\pi}\to\gamma\gamma} \Big/ \Gamma^{exp}_{A_{2\pi}\to\pi^0\pi^0} = (3.78 \pm 0.51) \cdot 10^{-3} \quad [13.5\% \;\; accuracy]. \qquad (11)$$

- Similarly with eq. (6) and (9) in R_γ, leads to

$$R_\gamma^{exp2} \approx \Gamma^{exp2}_{A_{2\pi}\to\gamma\gamma} \Big/ \Gamma^{exp}_{A_{2\pi}\to\pi^0\pi^0} = (3.65 \pm 0.43) \cdot 10^{-3} \quad [11.9\% \;\; accuracy]. \qquad (12)$$

5 Conclusion

We are confident, that the $A_{2\pi} \to \gamma\gamma$ contribution to the total decay rate is small. This means, the total width Γ_{tot} as measured by DIRAC corresponds within the uncertainties to $\Gamma_{A_{2\pi}\to\pi^0\pi^0}$.

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