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# Influence of the $\omega$ and $\eta'$ Decays on Determining the Lifetime of $\pi^+\pi^-$ Atoms in the DIRAC Experiment

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## Abstract

It is shown that for pp interactions at 24 GeV/$c$ the fractions of the $\pi^{\pm}_{\omega}\pi^{\mp}$ and $\pi^{\pm}_{\eta'}\pi^{\mp}$ pairs with one pion from the $\omega$ and $\eta'$ decays at small values of the relative momentum $Q \leq 4$ MeV/$c$ of pions in the c. m. system of $\pi^+\pi^-$ pair are $f_{\omega} = 0.167 \pm 0.022$ and $f_{\eta'} = 0.0126 \pm 0.0042$. With these values of $f_{\omega}$ and $f_{\eta'}$, the uncertainty in the pionium breakup probability and the corresponding overestimation of the pionium life-time in the DIRAC experiment at CERN are $(3.8 \pm 1.0)\%$ and $(9.9 \pm 2.6)\%$.

## 1. Introduction

The goal of the DIRAC experiment [1] at CERN is to measure the lifetime of the $\pi^+\pi^-$ atom with a purpose of determining the difference $|a_0 - a_2|$ of the $s$-wave $\pi\pi$-scattering lengths for isospins $I = 0$ and 2. The theory predictions for the difference $|a_0 - a_2|$ depend on the structure of the QCD vacuum. The precise measurement of the $\pi\pi$ scattering length allows therefore to test chiral symmetry breaking of QCD. For this it is necessary to measure the lifetime of the $\pi^+\pi^-$ atoms in the ground state with the precision of better than 3%.

The detection of the pionic atoms formed in interaction of a 24 GeV/$c$ proton beam with a thin Ni target in this experiment is based on their breakup, after interacting with target atoms, into $\pi^+\pi^-$ pair with low relative momentum $Q \leq 4$ MeV/$c$ of pions in the atomic pair c. m. system [2]. The creation of atomic pairs is accompanied by a large background which should be accounted for in the measured $Q$ distribution in order to observe the excess due to the atomic signal. The background is formed by the accidental $\pi^+\pi^-$ pairs originating from different primary pNi interactions and uncorrelated in time, by the $\pi^+\pi^-$ pairs originating from short-lived sources and undergoing Coulomb interactions in the final state and by the $\pi^+\pi^-$ pairs with at least one pion coming from decay of relatively long-lived particles, without Coulomb final state interaction, such as $\eta$ and most of the $\eta'$ mesons decaying electromagnetically or $K^0_S$ and $\Lambda$ decaying weakly far away from the interaction point.

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The $Q$ distribution of $\pi^+\pi^-$ pairs with pions produced either directly or from decays of the short-lived resonances, such as $\rho$, K$^*$(892), $\Delta$(1232), etc., can be described by point-like Coulomb approximation applying the so called Gamow-Sommerfeld factor. However, this approximation must be corrected if the distance between produced pions is larger than $\sim$ 10 fm. Such pion pairs, with the respective path length of about 30 fm in the pair c.m. system, are formed when at least one pion originates from the $\omega$ decay. Smaller contribution give also pion pairs with one pion from $\eta'$ decay. The corresponding final-size correction, representing one of the main sources of systematic error in the pionium lifetime measurement in the DIRAC experiment [3], is basically determined by the fractions $f_\omega$ and $f_{\eta'}$ of such pion pairs. The $f_\omega$ represents the fraction of $\pi^\pm_\omega\pi^\mp$ pairs with one pion from the $\omega$ decay and another from other sources (except for pions from $\eta$ or $\eta'$ decays or from the decay of the same $\omega$) to the total number of $\pi^+\pi^-$ pairs (except such pairs where only one of the pions comes from $\eta$ decay). The $f_{\eta'}$ represents the fraction of $\pi^\pm_{\eta'}\pi^\mp$ pairs with one pion from the $\eta'$ decay and another from other sources (except for pions from $\eta$ decay or from the decay of the same $\eta'$) to the total number of $\pi^+\pi^-$ pairs (except such pairs where only one of the pions comes from $\eta$ decay).

As shown in [4], the values of these fractions $f_\omega$ and $f_{\eta'}$ are directly related to the uncertainty in the pionium breakup probability, $\Delta P_{br}/P_{br}$, where $P_{br} = N_A^{br}/N_A$ is the ratio of the number $N_A^{br}$ of breakup atoms to the number $N_A$ of atoms produced in the target. Using the previous estimates of $f_\omega = 0.19$ and $f_{\eta'} = 0.01$ for the low-$Q$ $\pi^+\pi^-$ pairs obtained from the Monte Carlo simulation of pNi collisions at 24 GeV/$c$ [5] performed with the UrQMD generator [6] and arbitrary assumed 30% uncertainties in these values, the overestimation of the breakup probability was found equal to $\Delta P_{br}/P_{br} = (4.6 \pm 2.4)\%$. The corresponding overestimation of the pionium life-time is then $\Delta\tau/\tau = (12 \pm 6)\%$ [4].

In this note, the fractions $f_\omega$ and $f_{\eta'}$ as functions of $Q$ are determined for pp interactions at 24 GeV/$c$ by means of Monte Carlo simulation performed with the Fritiof generator [7] with parameters tuned to reproduce the experimental inclusive cross-sections of meson and baryon resonances in pp interactions at 24 GeV/$c$. In order to check the reliability of the simulation, the value of $f_\omega$ integrated over all values of $Q$ was also obtained in the topological approach [8] based on the experimental results on topological cross-sections (with the fixed number of charged particles in the final states) and $\omega$ production rate.

In section 2, the total inclusive cross-sections of the meson and baryon resonances in pp interactions at 24 GeV/$c$ are presented. They have been either directly measured in pp interactions at 24 GeV/$c$ or estimated by us from other experimental results in hadronic reactions at different energies or at LEP. In section 3, the fraction $f_\omega$, integrated over all $Q$ values, is determined in the topological approach. In section 4, the fractions $f_\omega$ and $f_{\eta'}$ are determined by means of Monte Carlo simulation with the Fritiof generator. The Fritiof predictions for the resonance production rates are compared with the experimental results and the Fritiof parameters are tuned to reproduce the experimental results. A comparison with the results of the previous simulation [5] for pNi collisions with the UrQMD generator is presented. The value of $f_\omega$, integrated over all values of $Q$, is determined. The influence of the $\eta$ and $\eta'$ decays on the value of the $f_\omega$ and $f_{\eta'}$ at the smallest $Q$ values is analyzed. The values of $f_\omega$ and $f_{\eta'}$ at the smallest $Q$ are used for determination of the uncertainty in the pionium breakup probability, $\Delta P_{br}/P_{br}$ and resulting overestimation of the pionium life-time $\Delta\tau/\tau$. The conclusions are summarized in section 5.

## 2. Resonance production rates in pp interactions

The total inclusive $\rho^0$ cross-sections measured in pp interactions at 24 GeV/$c$ [9] and at nearby beam momentum of 32 GeV/$c$ [10] amount to

$$\sigma(\rho^0)_{24} = 3.49 \pm 0.42 \text{ mb} \quad \text{and} \quad \sigma(\rho^0)_{32} = 4.04 \pm 0.48 \text{ mb}. \tag{1}$$

The production of the $\rho^+$ and $\rho^-$ in pp interactions at 24 GeV/$c$ has been only studied in the final states with a single $\pi^0$, i.e. in quasi-inclusive reactions pp→ $\rho^\pm$ + *charged particles*, applying the energy and momentum conservation in one constraint (1C) fit [9]. Assuming that the ratios $\rho^+/\rho^0$ and $\rho^-/\rho^0$ of the total inclusive cross-sections are the same as in these quasi-inclusive reactions one obtained:

$$\sigma(\rho^+)/\sigma(\rho^0) = 0.92 \pm 0.25 \quad \text{and} \quad \sigma(\rho^-)/\sigma(\rho^0) = 0.86 \pm 0.23.$$

The $\rho^+/\rho^0$ and $\rho^-/\rho^0$ ratios in the Feynman-$x$ interval $x \geq 0.1$ extracted from the results of the 400 GeV/$c$ pp experiment [11] are equal to $1.16 \pm 0.13$ and $0.82 \pm 0.10$, respectively. The $\rho^+/\rho^0$ ratio has been found equal to $1.15 \pm 0.14$ for $x \geq 0.3$ in $\pi^+$p and $0.94 \pm 0.28$ for $x \geq 0.2$ in K$^+$p interactions at 250 GeV/$c$ [12, 13]. The weighted averages of these ratios are

$$\sigma(\rho^+)/\sigma(\rho^0) = 1.108 \pm 0.085 \quad \text{and} \quad \sigma(\rho^-)/\sigma(\rho^0) = 0.826 \pm 0.092.$$

Then the corresponding $\rho^+$ and $\rho^-$ cross-sections are

$$\sigma(\rho^+) = 3.87 \pm 0.47 \text{ mb} \quad \text{and} \quad \sigma(\rho^-) = 2.88 \pm 0.47 \text{ mb}. \tag{2}$$

There are no experimental data on inclusive $\omega$ production in pp collisions at this energy range. However with $\sigma(\rho^0)$ from (1) and the value of the ratio $\omega/\rho^0 = 0.92 \pm 0.05$ found in [8] by averaging the results of many experiments, one obtains:

$$\sigma(\omega) = 3.21 \pm 0.42 \text{ mb}. \tag{3}$$

The corresponding value at 32 GeV/$c$ is $3.72 \pm 0.49$ mb.

The $\rho^0$ total cross-section at 32 GeV/$c$ in (1) is found from the fit of the overall $\pi^+\pi^-$ mass spectrum. However the sum of the $\rho^0$ topological cross-sections also measured in [10] is slightly different: $4.31 \pm 0.46$ mb. With this value, instead of (1), one obtains $\sigma(\omega) = 3.97 \pm 0.47$ at 32 GeV/$c$. Another estimate of $\sigma(\omega)$ at 24 GeV/$c$ can be found from a study [9] of quasi-inclusive $\omega$ production in the final states pp→ pp$k(\pi^+\pi^-)\pi^0$ (for $k \geq 1$) with a single $\pi^0$, i.e. in reaction pp→ $\omega$ + *charged particles*, applying the energy and momentum conservation in one constraint (1C) fit. Determining the $\rho^0$ cross section in analogous quasi-inclusive reaction pp→ $\rho^0$ + *charged particles* one obtained the ratio $\omega/\rho^0 = 1.07 \pm 0.20$. Assuming that the same ratio can be applied, instead of $0.92 \pm 0.05$, for fully inclusive reactions, one gets $\sigma(\omega) = 3.73 \pm 0.83$ mb. All these values are consistent within errors.

The production cross-section of the tensor $f_2(1270)$ meson measured in pp interactions at 32 GeV/$c$ is $0.88 \pm 0.22$ mb [10]. Its estimate at 24 GeV/$c$ is obtained by multiplying this value by the ratio of the total inelastic cross-sections $\sigma_{inel} = 30.6 \pm 0.25$ [14] and $30.89 \pm 0.39$ mb [15] at 24 and 32 GeV/$c$ respectively. This gives:

$$\sigma(f_2(1270)) = 0.87 \pm 0.22 \text{ mb}. \tag{4}$$

One can also notice that the $f_2(1270)/\rho^0$ ratios in the 32 and 400 GeV/$c$ pp interactions [10, 11] are practically the same: $0.22 \pm 0.06$ and $0.24 \pm 0.03$ and agree within errors with $f_2(1270)/\rho^0 = 0.180 \pm 0.035$ at LEP [16, 17].

The inclusive $\mathrm{K}^{*+}(892)$ and $\mathrm{K}^{*-}(892)$ cross-sections in pp interactions at 24 GeV/$c$ [18] amount to

$$\sigma(\mathrm{K}^{*+}(892)) = 0.66 \pm 0.06 \text{ mb} \quad \text{and} \quad \sigma(\mathrm{K}^{*-}(892)) = 0.19 \pm 0.04 \text{ mb}. \tag{5}$$

The $\mathrm{K}^{*-}(892)$ cross-section is significantly smaller than that of the $\mathrm{K}^{*+}(892)$, since the former can only be produced from the sea quarks, while the fragmentation of the initial proton $u$ quarks into the $\mathrm{K}^{*+}(892)$ plays very important role at this energy. This is supported by a large contribution of the associated $\mathrm{K}^{*+}(892)$ and $\Lambda$ production with $\sigma(\mathrm{K}^{*+}(892)\Lambda) = 0.29 \pm 0.06$ mb [18]. Such excess of the $\mathrm{K}^{*+}(892)$ over $\mathrm{K}^{*-}(892)$ production cross-section due to proton fragmentation processes persists even at much higher energies. Thus for the 400 GeV/$c$ pp interactions $\sigma(\mathrm{K}^{*+}(892)) = 4.33 \pm 0.53$ mb and $\sigma(\mathrm{K}^{*-}(892)) = 2.87 \pm 0.39$ mb [11].

The $\mathrm{K}^{*0}(892)$ and $\overline{\mathrm{K}}^{*0}(892)$ cross-sections were not measured in pp interactions at 24 GeV/$c$. However, since the $\overline{\mathrm{K}}^{*0}(892)$ can be only produced from the sea quarks, as well as $\mathrm{K}^{*-}(892)$, we assumed that $\sigma(\overline{\mathrm{K}}^{*0}(892)) = \sigma(\mathrm{K}^{*-}(892))$. This assumption is supported by results of the 400 GeV/$c$ pp experiment [11] where $\sigma(\overline{\mathrm{K}}^{*0}(892)) = 2.96 \pm 0.54$ mb and $\sigma(\mathrm{K}^{*-}(892)) = 2.87 \pm 0.39$ mb. The $\mathrm{K}^{*0}(892)$ cross-section is expected to be smaller than the one for the $\mathrm{K}^{*+}(892)$, since contribution of the proton $d$-quark fragmentation into $\mathrm{K}^{*0}(892)$ is expected to be smaller than its $u$-quark fragmentation into $\mathrm{K}^{*+}(892)$. Therefore the $\mathrm{K}^{*0}(892)$ cross-section was taken equal to the $\mathrm{K}^{*+}(892)$ cross-section in (5) multiplied by the ratio $\mathrm{K}^{*0}(892)/\mathrm{K}^{*+}(892) = 0.67$ taken from predictions of Fritiof with default parameters[3]. Thus one obtains:

$$\sigma(\mathrm{K}^{*0}(892)) = 0.441 \pm 0.040 \text{ mb} \quad \text{and} \quad \sigma(\overline{\mathrm{K}}^{*0}(892)) = 0.19 \pm 0.04 \text{ mb}. \tag{6}$$

The relative production cross-sections of the strange tensor mesons can be estimated from the results of the $\mathrm{K}^+\mathrm{p}$ experiment at 32 GeV/$c$: $\mathrm{K}^{*+}(1430)/\mathrm{K}^{*+}(892) = 0.247 \pm 0.042$ and $\mathrm{K}^{*0}(1430)/\mathrm{K}^{*0}(892) = 0.225 \pm 0.080$ [19], well consistent with an estimate of this ratio from $f_2(1270)/\rho^0 = 0.250 \pm 0.069$ with $f_2(1270)$ and $\rho^0$ rates taken from (1) and (4). Together with (5) and (6) this gives:

$$\sigma(K_2^{*+}(1430)) = 0.163 \pm 0.031 \text{ mb}, \qquad \sigma(K_2^{*-}(1430)) = 0.047 \pm 0.013 \text{ mb}, \tag{7}$$

$$\sigma(K_2^{*0}(1430)) = 0.099 \pm 0.036 \text{ mb}, \qquad \sigma(\overline{K_2^{*0}}(1430)) = 0.047 \pm 0.013 \text{ mb}. \tag{8}$$

The total inclusive $\phi$ cross-section in pp interactions at 24 GeV/$c$ equals [20]

$$\sigma(\phi) = 0.158 \pm 0.35 \text{ mb}. \tag{9}$$

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[3] In the 400 GeV/$c$ pp experiment [11], this ratio is higher: $\sigma(\mathrm{K}^{*0}(892))/\sigma(\mathrm{K}^{*+}(892)) = 0.91 \pm 0.19$. This can be explained by significantly higher contribution of the centrally produced $\mathrm{K}^{*}(892)$ at 400 GeV/$c$. Indeed, if one defines the fragmentation contributions at 400 GeV/$c$ as $\sigma(\mathrm{K}^{*+}(892))_{frag} = \sigma(\mathrm{K}^{*+}(892)) - \sigma(\mathrm{K}^{*-}(892))$ and $\sigma(\mathrm{K}^{*0}(892))_{frag} = \sigma(\mathrm{K}^{*0}(892)) - \sigma(\overline{\mathrm{K}}^{*0}(892))$ then $\sigma(\mathrm{K}^{*0}(892))_{frag}/\sigma(\mathrm{K}^{*+}(892))_{frag} = 0.66 \pm 0.60$, practically coinciding with the Fritiof prediction for 24 GeV/$c$.

Its reliability is supported by close values of the $\phi/\rho^0$ ratios amounting to $0.045 \pm 0.011$ in pp interactions at 24 GeV/$c$ and $0.049 \pm 0.005$ at 400 GeV/$c$ [11].

The total inclusive $\Sigma^{*+}(1385)$ and $\Sigma^{*-}(1385)$ cross-sections measured in pp interactions at 24 GeV/$c$ [21] are:

$$\sigma(\Sigma^{*+}(1385)) = 0.276 \pm 0.026 \text{ mb} \quad \text{and} \quad \sigma(\Sigma^{*-}(1385)) = 0.119 \pm 0.018 \text{ mb}. \tag{10}$$

They can be compared with $\sigma(\Sigma^{*+}(1385)) = 0.119 \pm 0.014$ mb and $\sigma(\Sigma^{*-}(1385)) = 0.049 \pm 0.011$ mb measured in the 32 GeV/$c$ $\mathrm{K^+p}$ experiment [22]. The last ones are smaller than in (10) by factors of $2.32 \pm 0.35$ and $2.42 \pm 0.66$, which agree within errors, as could be expected, with the ratio $(\sigma_{inel})_{\mathrm{pp}}/(\sigma_{inel})_{\mathrm{K^+p}} = 2.00 \pm 0.02$ of the corresponding total inelastic cross-sections.

This agreement allows also to estimate the inclusive $\Delta^{++}(1232)$ cross-section in pp collisions at 24 GeV/$c$ from the value of the $\Delta^{++}/\Sigma^{*+}$ ratio taken from the experiments at nearby energies. In the 32 GeV/$c$ $\mathrm{K^+p}$ experiment $\sigma(\Delta^{++}) = 2.25 \pm 0.30$ mb [22] and $\Delta^{++}/\Sigma^{*+} = 18.9 \pm 3.4$. In the 16 GeV/$c$ $\pi^+$p experiment $\sigma(\Delta^{++}) = 4.67 \pm 0.11$ mb [23][4]. Together with $\sigma(\Sigma^{*+}) = 0.193 \pm 0.037$ mb from the 15 GeV/$c$ $\pi^+$p experiment [25] this gives $\Delta^{++}/\Sigma^{*+} = 24.2 \pm 4.7$. The weighted average of these two $\Delta^{++}/\Sigma^{*+}$ ratios is $20.7 \pm 2.7$. Then, under reasonable assumption that this ratio is the same for pp interactions at 24 GeV/$c$, and using (10), one obtains:

$$\sigma(\Delta^{++}(1232)) = 5.72 \pm 0.92 \text{ mb}. \tag{11}$$

The $\Delta^{+}(1232)$ and $\Delta^{0}(1232)$ total inclusive cross-sections can be estimated from (11) and the ratios

$$\sigma(\Delta^+)_{x \geq 0.2}/\sigma(\Delta^{++})_{x \geq 0.2} = 0.572 \pm 0.097, \quad \sigma(\Delta^0)_{x \geq 0}/\sigma(\Delta^{++})_{x \geq 0} = 0.645 \pm 0.037,$$

taken from the results of the 400 GeV/$c$ pp experiment [11]. This gives:

$$\sigma(\Delta^+) = 3.27 \pm 0.76 \text{ mb} \quad \text{and} \quad \sigma(\Delta^0) = 3.69 \pm 0.63 \text{ mb}. \tag{12}$$

There are no data on inclusive $\eta$, $\eta'$, $f_0(980)$, $a_0(980)$ production in pp collisions at nearby beam momenta. However the relative production for the most of particles is practically independent of beam momenta, as have been shown here earlier for $f_2(1270)/\rho^0$ and $\phi/\rho^0$ ratios. Moreover, good agreement of particle yields, apart from their overall normalization, has been demonstrated for the data collected in $e^+e^-$ collisions at LEP and in pp interactions (see, for example, [26, 27] and refs. therein).

Using the averaged values of the $\eta$ and $\rho^0$ rates [28] obtained in the LEP experiments [16, 17, 29, 30] one finds $\eta/\rho^0 = 0.815 \pm 0.092$. The same ratio from the results of pp experiment at 400 GeV/$c$ equals $0.778 \pm 0.060$ [11]. They are indeed very similar and their weighted average is $0.789 \pm 0.050$. Assuming that the value of this ratio is the same for pp

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[4] The $\Delta^{++}(1232)$ production rate is higher in $\pi^+$p than in $\mathrm{K^+p}$ interactions not only due to higher total inelastic cross-section. Not negligible part of the $\Delta^{++}(1232)$ is produced in the beam fragmentation processes. The $u$ quark of the incident $\pi^+$ is significantly more energetic than $u$ quark of the incident $\mathrm{K}^+$, since more heavier strange $\bar{s}$ quark in $\mathrm{K}^+$ carries a larger fraction of the incident kaon momentum [24].

collisions at 24 GeV/$c$ and using the corresponding value (1) of the $\rho^0$ cross section, one finds :

$$\sigma(\eta) = 2.75 \pm 0.38 \text{ mb}. \tag{13}$$

Similarly, with the ratio $\eta'/\rho^0 = 0.137 \pm 0.042$ from the averaged LEP data [28] and the value (1) of the $\rho^0$ cross-section at 24 GeV/$c$ one obtains:

$$\sigma(\eta') = 0.48 \pm 0.16 \text{ mb}. \tag{14}$$

There are no data on the $\eta'$ inclusive production in pp interactions. Rough estimate of the relative $\eta'$ and $\eta$ production rates in quasi-inclusive reactions in $\pi^+ p$ experiment at 16 GeV/$c$ yielded $\eta'/\eta \sim 5 - 10\%$ [31]. Our estimate of this ratio, $\eta'/\eta = 0.17 \pm 0.05$, is higher but does not contradict it within large errors.

The $f_0(980)$ production rate was measured in the LEP [17, 30] and in the 400 GeV/$c$ pp [11] experiments. The corresponding $f_0(980)/\rho^0$ ratios $0.119 \pm 0.013$ and $0.059 \pm 0.021$ differ by 2.4 s.d. Their weighted average is $0.086 \pm 0.011$. With this value of the $f_0(980)/\rho^0$, an estimate of the $f_0(980)$ cross-section in pp interactions at 24 GeV/$c$ is

$$\sigma(f_0(980)) = 0.300 \pm 0.053 \text{ mb}. \tag{15}$$

The ratio $(a_0^+(980) + a_0^-(980))/f_0(980) = 1.84 \pm 0.75$ at LEP [28] is close to the expected value of 2. Therefore it has been assumed that for pp interactions at 24 GeV/$c$:

$$\sigma(a_0^+(980) + a_0^-(980)) = 2\sigma(f_0(980)) = 0.60 \pm 0.10 \text{ mb}. \tag{16}$$

Finally, production rates of two $1^{++}$ and $1^{+-}$ states, $f_1(1285)$ and $f_1(1420)$, were measured at LEP [32]. The corresponding rates for pp interactions at 24 GeV/$c$ are estimated assuming the same $f_1(1285)/\rho^0$ and $f_1(1420)/\rho^0$ ratios as in LEP. This gives:

$$\sigma(f_1(1285)) = 0.46 \pm 0.12 \text{ mb} \quad \text{and} \quad \sigma(f_1(1420)) = 0.158 \pm 0.066 \text{ mb}. \tag{17}$$

The values of the total inclusive cross-sections (1-17) presented above are also collected in Table 1, together with the values of the average rates per inelastic collision $\langle res \rangle = \sigma(res)/\sigma_{inel}$, with $\sigma_{inel} = 30.6 \pm 0.25$ mb [14].

## 3. Topological approach

We are interested in the $f_\omega$ and $f_{\eta'}$ values at the low relative momentum $Q$ of pions in the $\pi^+\pi^-$ c. m. system, where the $\pi^+\pi^-$ atoms are produced. Still the determination of these fractions for all $Q$ values based on the experimental results is also of interest since it allows to test reliability of the results obtained from simulation in the next section. Two different approaches have been suggested in [8] for determination of the fraction $f_\omega$ for all $Q$-values in the DIRAC experiment. The first approach, based on counting the resonances produced in an association with the $\omega$, suffers from the fact that production rates of many resonances are not known. Besides, there is practically no experimental information on an associated production of several resonances. The second approach has been based on the use of topological cross-sections, $\sigma_n$, for the final states with $n$ charged particles. In this approach, the fraction $f_\omega$ is defined as

$$f_\omega = \frac{\Sigma_n \sigma_n(\omega) R_n}{\Sigma_n \sigma_n S_n}, \tag{18}$$

Table 1: The total inclusive cross-sections of the resonances and their averaged rates per inelastic pp collision, $\langle res \rangle = \sigma(res)/\sigma_{inel}$, either measured at 24 GeV/$c$ or extracted from other experimental data (see the text), together with predictions of Fritiof with default and tuned parameters for the averaged rates at 24 GeV/$c$.

| Resonance | Experiment | | Fritiof $\langle res \rangle$ | |
|---|---|---|---|---|
| | $\sigma(res)$, mb | $\langle res \rangle$ | default | tuned |
| $\rho^0$ | $3.49 \pm 0.42$ | $0.114 \pm 0.014$ | 0.196 | 0.116 |
| $\rho^+$ | $3.87 \pm 0.47$ | $0.126 \pm 0.015$ | 0.198 | 0.121 |
| $\rho^-$ | $2.88 \pm 0.47$ | $0.0943 \pm 0.015$ | 0.158 | 0.0960 |
| $\omega$ | $3.21 \pm 0.42$ | $0.105 \pm 0.014$ | 0.174 | 0.105 |
| $\phi$ | $0.158 \pm 0.35$ | $0.0052 \pm 0.0011$ | 0.00282 | 0.0013 |
| $\mathrm{K}^{*+}(892)$ | $0.66 \pm 0.06$ | $0.0216 \pm 0.0020$ | 0.0401 | 0.0214 |
| $\mathrm{K}^{*-}(892)$ | $0.19 \pm 0.04$ | $0.0062 \pm 0.0013$ | 0.0326 | 0.0060 |
| $\mathrm{K}^{*0}(892)$ | $0.441 \pm 0.040$ | $0.0144 \pm 0.0013$ | 0.0268 | 0.0184 |
| $\overline{\mathrm{K}}^{*0}(892)$ | $0.19 \pm 0.04$ | $0.0062 \pm 0.0013$ | 0.0291 | 0.0058 |
| $\eta$ | $2.75 \pm 0.38$ | $0.0900 \pm 0.012$ | 0.159 | 0.0901 |
| $\eta'$ | $0.48 \pm 0.16$ | $0.0156 \pm 0.0051$ | 0.0666 | 0.0157 |
| $f_2(1270)$ | $0.87 \pm 0.22$ | $0.0285 \pm 0.0071$ | —- | 0.0291 |
| $\mathrm{K}^{*+}(1430)$ | $0.163 \pm 0.031$ | $0.0053 \pm 0.0010$ | —- | 0.0051 |
| $\mathrm{K}^{*-}(1430)$ | $0.047 \pm 0.013$ | $0.0015 \pm 0.0004$ | —- | 0.0017 |
| $\mathrm{K}^{*0}(1430)$ | $0.099 \pm 0.036$ | $0.0032 \pm 0.0012$ | —- | 0.0028 |
| $\overline{\mathrm{K}}^{*0}(1430)$ | $0.047 \pm 0.013$ | $0.0015 \pm 0.0004$ | —- | 0.0019 |
| $f_0(980)$ | $0.300 \pm 0.053$ | $0.0098 \pm 0.0017$ | —— | 0.0095 |
| $a_0^+(980)+a_0^-(980)$ | $0.60 \pm 0.10$ | $0.0196 \pm 0.0034$ | —— | 0.0258 |
| $f_1(1285)$ | $0.46 \pm 0.12$ | $0.0152 \pm 0.0038$ | —— | 0.0155 |
| $f_1(1420)$ | $0.158 \pm 0.066$ | $0.0052 \pm 0.0022$ | —— | 0.00018 |
| $\Delta^{++}(1232)$ | $5.72 \pm 0.92$ | $0.187 \pm 0.030$ | 0.206 | 0.185 |
| $\Delta^+(1232)$ | $3.27 \pm 0.76$ | $0.107 \pm 0.025$ | 0.156 | 0.107 |
| $\Delta^0(1232)$ | $3.69 \pm 0.63$ | $0.121 \pm 0.021$ | 0.0704 | 0.125 |
| $\Sigma^{*+}(1385)$ | $0.276 \pm 0.026$ | $0.00902 \pm 0.00085$ | 0.00127 | 0.00931 |
| $\Sigma^{*-}(1385)$ | $0.119 \pm 0.018$ | $0.00389 \pm 0.00059$ | 0.00014 | 0.00454 |

where $\sigma_n(\omega)$ is the $\omega$ topological cross-sections multiplied by branching fraction $Br(\omega \rightarrow \pi^+\pi^- X)$, $R_n$ is the number of $\pi^{\pm}_{\omega}\pi^{\mp}$ combinations in the final state with $n$ charged particles and $S_n$ is the corresponding number of all possible $\pi^+\pi^-$ combinations[5] including $\pi^+\pi^-$ pairs from the same $\omega$.

The topological cross-sections, $\sigma_n$, measured in pp interactions at 24 GeV/$c$ [14] and 32 GeV/$c$ [15] are presented in Table 2, together with the $\rho^0$ topological cross-sections, $\sigma_n(\rho^0)$, measured at 32 GeV/$c$ [10]. The topological cross-sections, $\sigma_n(\omega)$, at 24 GeV/$c$ also given in Table 2 are defined as $\sigma_n(\omega) = \sigma(\omega)(\sigma_n(\rho^0)/\sigma(\rho^0))$ assuming $[\sigma_n(\rho^0)/\sigma(\rho^0)]_{24} = [\sigma_n(\rho^0)/\sigma(\rho^0)]_{32}$.

Table 2: Topological cross-sections, $\sigma_n$, measured in pp interactions at 32 GeV/$c$ [15] and 24 GeV/$c$ [14], $\rho^0$ topological cross-sections, $\sigma_n(\rho^0)$, measured at 32 GeV/$c$ [10] and $\omega$ topological cross-sections at 24 GeV/$c$, $\sigma_n(\omega)$, as defined in the text (all cross-sections are in mb)

|  | 32 GeV/$c$ |  | 24 GeV/$c$ |  |
|---|---|---|---|---|
| n | $\sigma_n$ | $\sigma_n(\rho^0)$ | $\sigma_n$ | $\sigma_n(\omega)$ |
| $2_{inel}$ | $7.47 \pm 0.28$ | - | $8.70 \pm 0.25$ | - |
| 4 | $11.32 \pm 0.21$ | $1.29 \pm 0.17$ | $12.55 \pm 0.10$ | $0.87 \pm 0.19$ |
| 6 | $7.74 \pm 0.15$ | $1.46 \pm 0.27$ | $6.71 \pm 0.07$ | $0.99 \pm 0.25$ |
| 8 | $3.32 \pm 0.08$ | $1.33 \pm 0.26$ | $2.17 \pm 0.04$ | $0.90 \pm 0.23$ |
| 10 | $0.92 \pm 0.04$ | $0.23 \pm 0.21$ | $0.404 \pm 0.016$ | $0.16 \pm 0.14$ |
| 12 | $0.171 \pm 0.016$ | - | $0.041 \pm 0.005$ | - |
| 14 | $0.030 \pm 0.007$ | - | $0.005 \pm 0.002$ | - |
| 16 | $0.003 \pm 0.002$ | - | $0.002 \pm 0.001$ | - |
| $\Sigma$ | $30.89 \pm 0.39$ | $4.31 \pm 0.46$ | $30.60 \pm 0.25$ | $2.92 \pm 0.41$ |

Knowing the topological $\sigma_n$ and $\sigma_n(\omega)$ cross-sections one can calculate the value of $f_\omega$ from (18) bearing in mind that there are three different final-state topologies: pp→pp+$X^0$, pp→pn+$X^+$ and pp→nn+$X^{++}$. Their relative weights $g_1$, $g_2$ and $g_3$ for each value of $n$ were determined from simulation with the Fritiof generator. A contribution of the final states with the strange baryons was neglected not only because it was small but also because the values of $f_\omega$ calculated separately for each of the pp→pp+$X^0$, pp→pn+$X^+$ and pp→nn+$X^{++}$ topologies were found rather similar and one could expect the same situation for the topologies with strange baryons. Thus one can write:

$$f_\omega = \frac{\sigma_4(\omega)(g_2+2g_3)+\sigma_6(\omega)(2g_1+3g_2+4g_3)+\sigma_8(\omega)(4g_1+5g_2+6g_3)+\sigma_{10}(\omega)(6g_1+7g_2+8g_3)}{\sigma_4(g_1+2g_2+3g_3)+\sigma_6(4g_1+6g_2+8g_3)+\sigma_8(9g_1+12g_2+15g_3)+\sigma_{10}(16g_1+20g_2+24g_3).} \quad (19)$$

This resulted in $f_\omega = 0.095 \pm 0.018$.

However eq. (19) is obtained under assumption that there is only one $\omega$ in the final state. The double $\omega$ production does not affect the number of $\pi^+\pi^-$ combinations in the denominator of (19), but increases the number of $\pi^{\pm}_{\omega}\pi^{\mp}$ combinations in the numerator. For events with pp, pn and nn in the final states, the numbers of additional

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[5] In [8], due to some misunderstanding, the denominator in (18) was calculated not for all possible $\pi^+\pi^-$ combinations, but only for the $\pi^+\pi^-$ pairs produced in association with the $\omega$. This resulted in incorrect high value of $f_\omega$.

$\pi^{\pm}_{\omega}\pi^{\mp}$ combinations are, respectively, 0, 1, 2 for 6-prong events, 2, 3, 4 for 8-prongs and 4, 5, 6 for 10-prongs. The double $\omega$ production cross-section was estimated from relation[6] $\sigma(\omega\omega) = \sigma^2(\omega)/\sigma_{inel}$. The corresponding topological cross-sections were found assuming $\sigma_n(\omega\omega)/\sigma(\omega\omega) = \sigma_n(\rho^0)/\sigma(\rho^0)$. After accounting for $Br(\omega \to \pi^+\pi^- X)$, this gives $f_\omega = 0.099 \pm 0.018$.

According to the definition of $f_\omega$ in introduction, the contributions of the $\pi^{\pm}_{\omega}\pi^{\mp}_{\eta}$ and $\pi^{\pm}_{\omega}\pi^{\mp}_{\eta'}$ pairs have to be removed from the numerator of (19)[7]. For this it is necessary to know the cross-sections of associated $\omega\eta$ and $\omega\eta'$ production. They have been estimated assuming again that

$$\sigma(\omega\eta) = \sigma(\omega)\sigma(\eta)/\sigma_{inel} \quad \text{and} \quad \sigma(\omega\eta') = \sigma(\omega)\sigma(\eta')/\sigma_{inel},$$

$\sigma_n(\omega\eta)/\sigma(\omega\eta) = \sigma_n(\rho^0)/\sigma(\rho^0)$ and $\sigma_n(\omega\eta')/\sigma(\omega\eta') = \sigma_n(\rho^0)/\sigma(\rho^0)$. Besides, a contribution of $\pi^{\pm}_{\eta}\pi^{\mp}$ pairs, with one pion from $\eta$ decay, must be removed from the denominator of (19). Then, after accounting for the $\omega$, $\eta$ and $\eta'$ branching fractions, one finally arrives to the following value of $f_\omega$ integrated over all $Q$ values in the topological approach:

$$f_\omega(all\ Q) = 0.100 \pm 0.018\ (exp)\ \pm 0.008\ (syst). \tag{20}$$

The first error in (20) is essentially determined by uncertainty in the $\omega$ production rates. The dominant sources of systematic uncertainties in (20) arise from the following.

In defining the ratio $\sigma_n(\rho^0)/\sigma(\rho^0)$ at 32 GeV/$c$, we obviously took the total $\rho^0$ cross-section equal to the sum of the topological cross-sections $\sigma(\rho^0) = 4.31 \pm 0.46$ mb. However, as already mentioned earlier, the total $\rho^0$ cross-section obtained in [10] from the fit of overall $\pi^+\pi^-$ mass spectrum is $4.04 \pm 0.48$ mb. A relative uncertainty in these two values of $\sigma(\rho^0)$ gives an uncertainty of $\delta f_\omega = \pm 0.0062$. A half of the difference between the $f_\omega$ values obtained after and before accounting for the double $\omega$ production gives $\delta f_\omega = -0.0019$. The error $\delta f_\omega = -0.0005$ arising from possible biases in estimates of the $\eta$ and $\eta'$ contributions is taken as a half of the difference between the $f_\omega$ values obtained after and before accounting for these contributions. The $\rho^0$ cross-section for $n \geq 12$ at 32 GeV/$c$ was found in [10] to be negligible. Indeed the cross-section $0.22 \pm 0.19$ mb for $n \geq 10$ was found compatible within error with $\sigma_{10}(\rho^0) = 0.23 \pm 0.22$ mb. Still, since combinatorics of $\pi^+\pi^-$ pairs increases with rise of $n$, we recalculated the value of $f_\omega$ assuming, rather arbitrary, $\sigma_{12}(\rho^0) = 0.05 \pm 0.10$ mb. The corresponding shift in the $f_\omega$ value gives $\delta f_\omega = +0.0022$. Finally, a rough estimate of systematics due to contamination of charged pions in the final states by charged kaons gives $\delta f_\omega = \pm 0.0050$. The total systematic error in (20) is obtained by combining all these values in quadrature.

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[6] As shown in [33], an assumption that the associated production cross-section of two particles 1 and 2 can be expressed as $\sigma_{12} = \sigma_1\sigma_2/\sigma_{inel}$ works well for independent production of particles 1 and 2. The double $\omega$ or associated $\omega$ and $\eta$ production can hardly be independent. In this case this equation represents an upper estimate of the associated production rate.

[7] Notice, that pions from $\eta$ decays in the cascade decays of $\eta' \to (\eta \to \pi^+\pi^-)X$, $\omega \to (\eta \to \pi^+\pi^-)\gamma$, $\phi \to (\eta \to \pi^+\pi^-)\gamma$ and $a_0^{\pm}(980) \to (\eta \to \pi^+\pi^-)\pi^{\pm}$ were considered as originating from $\eta$ decays, while pions from $\eta' \to (\rho^0 \to \pi^+\pi^-)\gamma$, $\eta' \to (\omega \to \pi^+\pi^-)\gamma$ and $\eta' \to \pi^+\pi^-\eta$ were attributed to the $\eta'$ decays.

## 4. Monte Carlo simulation

In previous section, the fraction $f_\omega$ was determined for all values of the relative momentum $Q$ of pions in the $\pi^+\pi^-$ rest system. The $Q$-dependences of $f_\omega$ and $f_{\eta'}$ and their values at small $Q$ can be found only by means of Monte Carlo simulation. Such simulation for pp collisions at 24 GeV/$c$ was performed with the Fritiof generator (version 7.02) [7] applying the JETSET fragmentation.

Clearly one may rely on results of such simulation provided the Fritiof predictions for the particle production rates are consistent with the experimental results. In addition, the $f_\omega$ value integrated over all $Q$ values have to be similar in simulation and topological approach.

The averaged production rates per inelastic collision predicted by Fritiof with default values of parameters are compared with the experimental rates in Table 1. The Fritiof predictions are not consistent with the data. In particular, they are larger than the experimental rates by a factor of 1.7–1.8 for the $\rho^0$, $\omega$, $K^{*+}(892)$ and $\eta$ and by a factor of 4.3 for the $\eta'$. On the other hand, one may notice that the $\omega/\rho^0$, $K^{*+}(892)/\rho^0$ and $\eta/\rho^0$ ratios in Fritiof are well consistent with the experimental values of these ratios, while the $\eta'/\rho^0$ ratio is higher[8] by a factor of 2.5.

Therefore, in the next step the model parameters were tuned in order to describe the data. The resonance rates obtained after this tuning are also presented in Table 1. As one can see, the results of tuning were quite successful. The large disagreement between the experimental and tuned rates is seen for the $\phi$ and $f_1(1420)$ only. However these states are produced with small rates and besides their decay channels into pions are also small.

The value of $f_\omega$ integrated over all $Q$ values obtained in simulation with tuned Fritiof,

$$f_\omega(all\ Q) = 0.104 \pm 0.14, \tag{21}$$

is in excellent agreement with the corresponding value (20) found in the topological approach. We stress that this agreement with the value found in the topological approach, based on experimental results only, is of importance, giving confidence to the subsequent investigation of $Q$ dependence of the fractions $f_\omega$ and $f_{\eta'}$ by means of simulation with tuned Fritiof. Another important result of this agreement is that the errors of $f_\omega$ and $f_{\eta'}$ obtained from simulation are essentially determined only by the errors of the $\omega$ and $\eta'$ rates.

The $Q$ dependence of $f_\omega$ obtained in the previous simulation [5] performed with the UrQMD generator [6] for pNi collisions at 24 GeV/$c$ in approximate DIRAC conditions is presented in Fig. 1a. It shows that the fraction $f_\omega$ increases with decreasing $Q$ and reaches the value $f_\omega \approx 0.18 - 0.19$ at the smallest $Q$ values. Our results for pp collisions at 24 GeV/$c$ obtained with the Fritiof generator with tuned parameters and without accounting for the DIRAC acceptance are presented in Fig. 1b. The $Q$ dependence of $f_\omega$ in Fig. 1b for $Q \geq 35$ MeV/$c$ is different from the one in Fig. 1a, since acceptance of the detector, close to 100% at the smallest $Q$ values, deteriorates with increasing $Q$. However the $f_\omega$ values for $Q \leq 35$ MeV/$c$ are the same within errors in Figs 1a and 1b. Thus results of simulation

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[8] The failure of the original JETSET Monte Carlo to reproduce the $\eta'$ rate is well known (see [16, 29, 30], for example). It might be related to the assumed similar strange quark content for the $\eta$ and $\eta'$ and neglect the effect of their mass difference on their relative production rate. For this reason, the suppression of the $\eta'$ relative to the $\eta$ is a free parameter in JETSET 7.4 with the default value of 0.4.

for pp collisions for the smallest $Q$ values are very similar to those for pNi collisions. This is of importance, allowing to use the results of simulation for pp collisions for determining the $f_\omega$ value for $Q \leq 4$ MeV/$c$ in the DIRAC experiment.

As one can see from Fig. 1b, the fraction $f_\omega$ is practically constant in the $Q \leq 80$ MeV/$c$ interval where it is equal to

$$f_\omega = 0.167 \pm 0.022. \tag{22}$$

The $Q$ dependence of the fraction $f_{\eta'}$ is shown in Fig. 1c. The fraction $f_{\eta'}$ is practically constant in the $Q \leq 200$ MeV/$c$ interval where it is equal to

$$f_{\eta'} = 0.0126 \pm 0.0042. \tag{23}$$

The errors in (22) and (23) are determined by uncertainties in the $\omega$ and $\eta'$ production rates (3) and (14).

For completeness we also show in Fig. 2a and 2b the behaviour of the numerators of $f_{\eta'}$ and $f_\omega$ and in Fig. 2c the behaviour of their denominator as a function of $Q$. Numerators of $f_{\eta'}$ and $f_\omega$ are smooth functions of $Q$. The denominator exhibits structures at $Q \approx 0.28$ GeV/$c$ and $Q \approx 0.74$ GeV/$c$.

A large broad peak at $Q \approx 0.74$ GeV/$c$ is obviously related to the $\rho^0$ decay. A small narrow peak at $\approx 0.73$ GeV/$c$ is due to $\omega \to \pi^+\pi^-$ decay.

The peak at $Q \approx 0.28$ GeV/$c$ in Fig. 2c is mainly due to the $\pi^+\pi^-$ pairs with both pions from the decay of the same $\eta$ or $\eta'$. This is illustrated in Figs. 2 d-f. Fig. 2d shows a contribution to the denominator of the $\pi^+\pi^-$ pairs from $\eta \to \pi^+\pi^-\pi^0$ decay (for either direct $\eta$ or resulting from $\eta' \to (\eta \to \pi^+\pi^-\pi^0) + X$ decay), all concentrated at $Q \leq 0.3$ GeV/$c$, and smaller contribution of $\pi^+\pi^-$ pairs from $\eta \to \pi^+\pi^-\gamma$ and $\eta' \to (\eta \to \pi^+\pi^-\gamma) + X$ decays in a broader $Q \leq 0.47$ GeV/$c$ region. As one can see $\approx 15\%$ of $\pi^+\pi^-$ pairs contributing to the denominator of fractions $f_\omega$ and $f_{\eta'}$ at the smallest $Q$ values come from these $\eta$ decays. Fig. 2e shows a contribution to the denominator of the $\pi^+\pi^-$ pairs from $\eta' \to \pi^+\pi^- + neutrals$ and $\eta' \to \pi^+\pi^-(\eta \to \pi^+\pi^-)$ (not counting $\pi^+\pi^-$ from $\eta$ decay). Finally, Fig. 2f shows a contribution of $\pi^\pm_{\eta'}\pi^\mp$ pairs with the second pion not from $\eta$ or $\eta'$ decays. Thus we see that the fraction of $\pi^\pm_{\eta_i}\pi^\mp_{\eta_j}$, $\pi^\pm_{\eta'_i}\pi^\mp_{\eta'_j}$ (for i=j) and $\pi^\pm_{\eta'}\pi^\mp_{(not\ \eta,\eta')}$ to the total number of $\pi^+\pi^-$ pairs, contributing to the denominator of $f_\omega$ and $f_{\eta'}$ at the smallest $Q$ values, amounts to $\approx$20%.

With the values of $f_\omega$ and $f_{\eta'}$ given in (22) and (23), the corresponding uncertainty in the pionium breakup probability from [4] is

$$\Delta P_{br}/P_{br} = (3.8 \pm 1.0)\%.$$

Then the corresponding overestimation of the pionium life-time is

$$\Delta\tau/\tau = (9.9 \pm 2.6)\%.$$

## 5. Conclusions

One source of background in the DIRAC experiment, which should be accounted for in the measured $Q$ distribution of the $\pi^+\pi^-$ pairs in order to observe the excess due to the

$\pi^+\pi^-$ atomic signal, arises from the $\pi^\pm_\omega\pi^\mp$ pairs with one pion from the $\omega \to \pi^+\pi^- X$ decay and another from other sources (apart from the pion from the same $\omega$ and pions from $\eta$ and $\eta'$ decays). The precise determination of the corresponding fraction $f_\omega$ of such pairs to the total number of $\pi^+\pi^-$ pairs (except pairs where one pion comes from $\eta$ decay), representing one of the most important sources of systematic error in the pionium lifetime measurement, is therefore very important.

In this note, the fraction $f_\omega$ was first determined for all values of the relative momentum $Q$ of pions in the $\pi^+\pi^-$ rest frame in the topological approach using experimental results on the charged particle multiplicity distribution and $\omega$ production rate in pp interactions at 24 GeV/$c$. The fraction $f_\omega$ is found to be: $0.100 \pm 0.018$ $(exp) \pm 0.008$ $(syst)$.

The $Q$ dependence of the fractions $f_\omega$ and $f_{\eta'}$ and their values for $Q \leq 4$ MeV/$c$, where the most of $\pi^+\pi^-$ atoms are detected in the experiment, have been determined from simulation with the Fritiof generator tuned to the experimental resonance production rates.

No difference in the $f_\omega$ values at small $Q$ is found between the earlier analysis [4, 5] based on the use of the UrQMD generator for pNi collisions at 24 GeV/$c$ and our results for pp collisions obtained with the Fritiof generator tuned to the experimental data. This proves that the results of simulation for pp collisions with the Fritiof generator can be used for determining the fractions $f_\omega$ and $f_{\eta'}$ in the DIRAC experiment.

From the results of such simulation it is found that the value of $f_\omega$ for all $Q$, $f_\omega = 0.104 \pm 0.014$, is practically the same as the one found in the topological approach. This excellent agreement gives confidence to the results of simulation.

The fractions $f_\omega$ and $f_{\eta'}$ as a function of Q are practically constant for $Q \leq 80$ MeV/$c$ and, respectively, $Q \leq 200$ MeV/$c$ and amount to

$$f_\omega = 0.167 \pm 0.022 \quad \text{and} \quad f_{\eta'} = 0.0126 \pm 0.0042,$$

with the errors determined by uncertainties in the $\omega$ and $\eta'$ production rates from the experimental data.

It is also shown that $\pi^+\pi^-$ pairs from the $\eta$ and $\eta'$ decays are concentrated in the region of small $Q$, so that the fraction of $\pi^\pm_{\eta_i}\pi^\mp_{\eta_j}$, $\pi^\pm_{\eta'_i}\pi^\mp_{\eta'_j}$ (for i=j) and $\pi^\pm_{\eta'}\pi^\mp_{(not\ \eta,\eta')}$ to the total number of $\pi^+\pi^-$ pairs, contributing to the denominator of $f_\omega$ and $f_{\eta'}$, at the smallest $Q$ values, amounts to ≈20%.

With the found values of $f_\omega$ and $f_{\eta'}$, the corresponding uncertainty in the pionium breakup probability is

$$\Delta P_{br}/P_{br} = (3.8 \pm 1.0)\%.$$

Then the corresponding overestimation of the pionium life-time is

$$\Delta\tau/\tau = (9.9 \pm 2.6)\%,$$

against the previous estimate $\Delta\tau/\tau = (12 \pm 6)\%$ [4].

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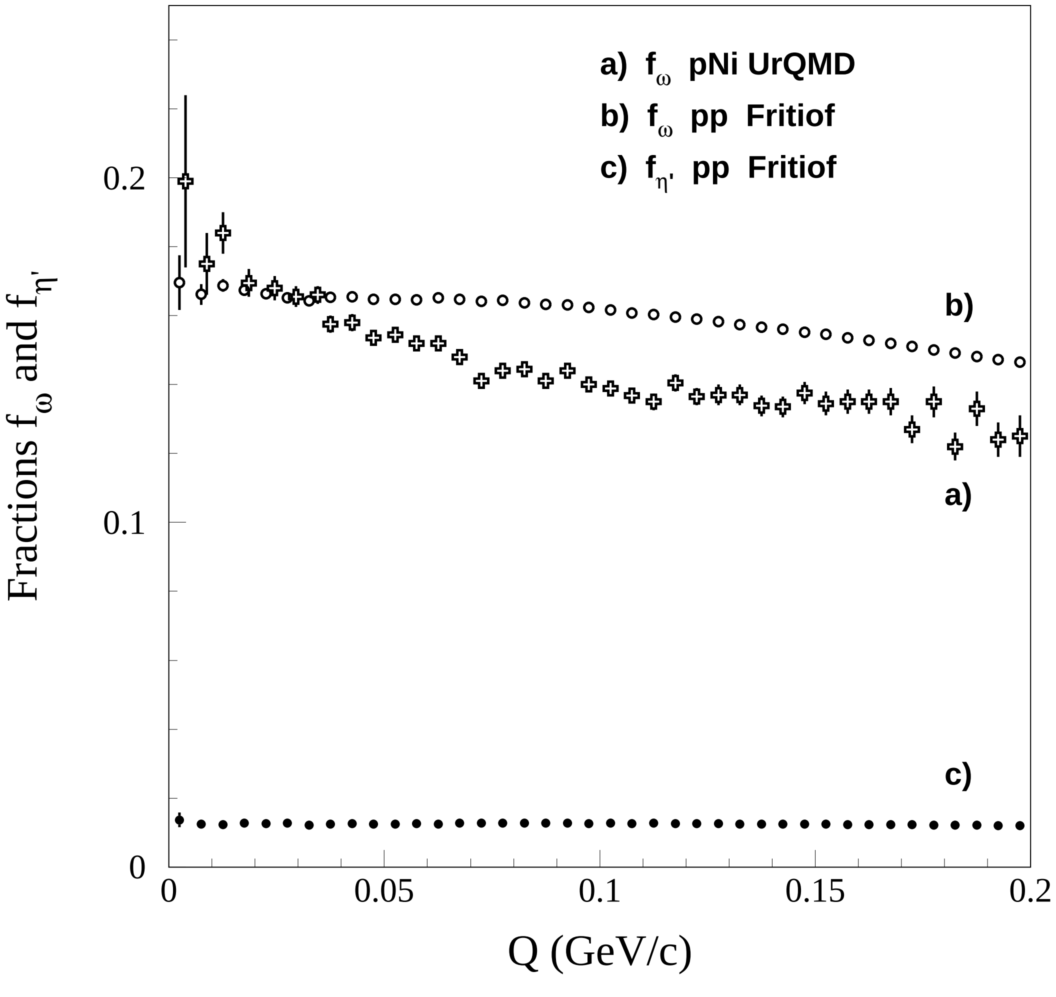

Figure 1: $Q$ dependence of fractions $f_\omega$ and $f_{\eta'}$ as found from Monte Carlo simulations: a) $f_\omega$ for pNi collisions at 24 GeV/$c$ in approximate DIRAC conditions obtained with the UrQMD generator [5] (crosses, slightly shifted to the right in some first intervals); b) $f_\omega$ for pp collisions at 24 GeV/$c$ obtained with the Fritiof generator, without accounting for detector acceptance (open dots); c) as in b) but for the fraction $f_{\eta'}$ (closed dots).

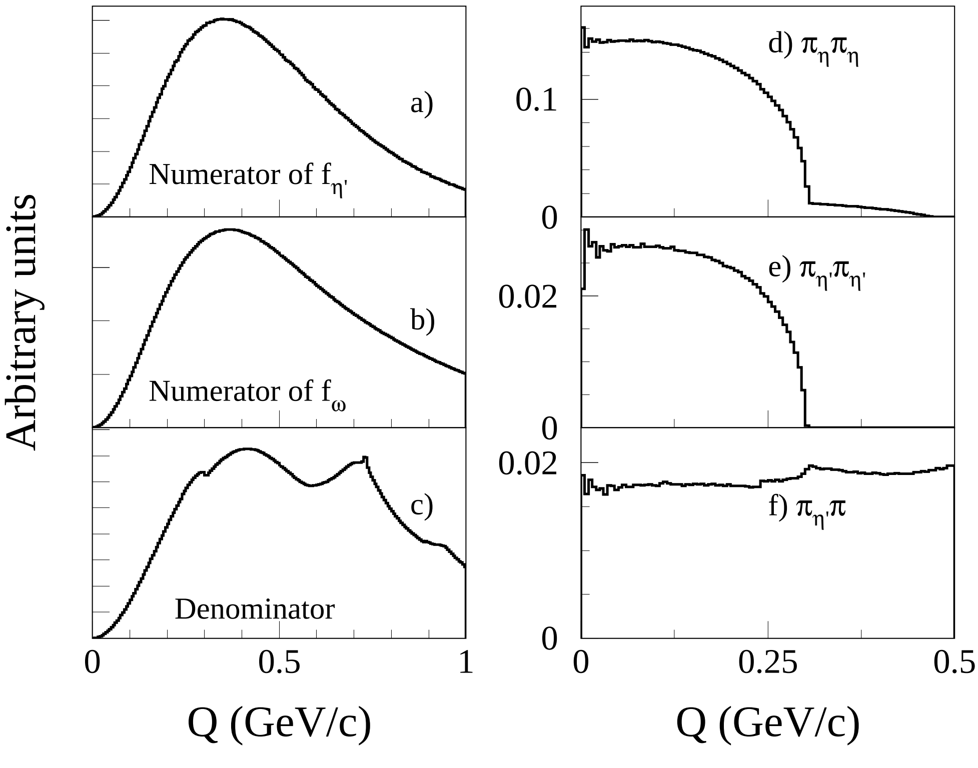

Figure 2: $Q$ dependence of a) numerator of the fraction $f_{\eta'}$; b) numerator of the fraction $f_\omega$; c) denominator of the fractions $f_{\eta'}$ and $f_\omega$; d) contribution of $\pi^\pm_{\eta_i}\pi^\mp_{\eta_j}$ (i=j) pairs from $\eta$ decay to the denominator of the fractions $f_{\eta'}$ and $f_\omega$; e) contribution of $\pi^\pm_{\eta'_i}\pi^\mp_{\eta'_j}$ (i=j) pairs from $\eta'$ decay to the denominator of the fractions $f_{\eta'}$ and $f_\omega$; f) contribution of $\pi^\pm_{\eta'_i}\pi^\mp$ with the second pion not from $\eta'$ decay to the denominator of the fractions $f_{\eta'}$ and $f_\omega$.