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Dimesoatoms and Correlation Functions in Two-Body Inclusive Reactions

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The yields of the $\pi^+\pi^-$- and $K^\pm\pi^\mp$-atoms in proton-nuclear interactions are supposed to increase with increasing energy [1]. In a recent DIRAC Note [2] the yields of the $\pi^+\pi^-$-, $K^+\pi^-$- and $K^-\pi^+$-atoms in pNi-collisions 450 GeV/c were estimated and compared with the results of the running DIRAC experiment on the CERN PS, where the production rates and properties of these atoms are investigated in pNi-interactions at 24 GeV/c. As it is shown in [2], a possible continuation of the DIRAC experiment, with the same setup, on the CERN SPS in the proton beam of 450 GeV/c and emission angle 4° might allow to increase the yields of the $\pi^+\pi^-$-, $K^+\pi^-$- $K^-\pi^+$-atoms by a factor of 17, 27 and 35 respectively, besides an additional factor of more than 4 due to higher beam time during the same supercycle on SPS than on PS and besides some other advantages (see [2]).

The inclusive yield of dimesoatoms as a function of inclusive production rates of π^+- and π^--mesons (or $K^\pm$- and $\pi^\mp$-mesons) is defined in [2] as

$$\frac{\mathrm{d}^2N_A}{\mathrm{d}p_A\mathrm{d}\Omega} = 1.202\cdot 8\pi^2(\mu\alpha)^3\frac{E_A}{M_A}\frac{p_A^2}{p_1^2p_2^2}\frac{\mathrm{d}^2N_1}{\mathrm{d}p_1\mathrm{d}\Omega}\frac{\mathrm{d}^2N_2}{\mathrm{d}p_2\mathrm{d}\Omega}R(p_1,p_2,s), \tag{1}$$

where M_A is the atom mass, E_A and p_A - its energy and momentum, p_1 and p_2 are the momenta of the decay particles in the lab. system, $\mu = \frac{m_1m_2}{m_1+m_2}$ is the reduced mass of the atom, α the fine structure constant, Ω the solid angle, s the total c.m. energy squared and $R(p_1,p_2,s)$ the correlation function depending on strong interactions only and expressed like:

$$R(p_1,p_2,s) = (\frac{\mathrm{d}^2N}{\mathrm{d}p_1\mathrm{d}p_2})/(\frac{\mathrm{d}N}{\mathrm{d}p_1}\frac{\mathrm{d}N}{\mathrm{d}p_2}). \tag{2}$$

The yields of the $K^\pm$- and $\pi^\pm$-mesons and correlation function $R(p_1,p_2,s)$ in [2] were obtained by Monte Carlo simulation with the generator FRITIOF 6.0 both for the present DIRAC setup at CERN PS (at 24 GeV/c and emission angle $\theta = 5.7°$) and for the same setup at CERN SPS (at 450 GeV/c and $\theta = 0°, 2°, 4°$ and 5.7°). As claimed in [2], the

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simulated yields of mesons agreed with those measured in p(Be,Al,Cu,Pb)-interactions at 24 GeV/c (for secondary lab. momenta > 4 GeV/c) and in pBe-interactions at 450 GeV/c (for secondary lab. momenta > 7 GeV/c) with accuracy from 20% to 50%. The correlation function R at $p_{\rm p} = 450$ GeV/c and $\theta = 4^\circ$ was found higher than at $p_{\rm p} = 24$ GeV/c and $\theta = 5.7^\circ$ by about 30% at the smallest lab. momenta of the $\pi^+\pi^-$ or $\rm K^\pm\pi^\mp$ pairs and by a factor of ≈ 2.5 higher at the largest pair lab. momenta due to significant fall of R with increasing pair momentum at $p_{\rm p} = 24$ GeV/c and R practically independent on pair momentum at $p_{\rm p} = 450$ GeV/c.

The correlation functions between particles in two-particle inclusive pp reactions

$$\rm p + p \to c_1 + c_2 + X \tag{3}$$

and p-nuclear reactions at different energies were studied in many experiments (see, for example, review [3]). These reactions are described by 5 independent variables, besides the total c.m. energy $\sqrt{s}$, if one ignores polarization effects. In studies of correlations one usually considers such variables as y_1, y_2, p_{t1}, p_{t2} and ϕ_{12}, where y is the rapidity

$$y = \frac{1}{2}\ln\frac{E + p_L}{E - p_L} \tag{4}$$

(E and p_L are the energy and longitudinal momentum), p_t the transverse momentum and ϕ_{12} the relative azimuthal angle. The rapidity is often approximated by the pseudorapidity

$$\eta = -ln\ tg\theta/2, \tag{5}$$

which can be determined in experiments in which only angles θ, and not momenta, are measured. The quantities (4) and (5) are close, i. e. $y \approx \eta$, when $m^2/p_t^2 << 1$. If one integrates the inclusive cross section over p_t at fixed θ, the values of y and η differ by less than 10%, over nearly the full rapidity range [3].

With the rapidity densities for one and two particles as

$$\rho(y) = \frac{1}{\sigma_{in}}\int\frac{{\rm d}^2\sigma}{{\rm d}p_t^2{\rm d}y}{\rm d}p_t^2, \tag{6}$$

$$\rho(y_1, y_2) = \frac{1}{\sigma_{in}}\int\frac{{\rm d}^4\sigma}{{\rm d}p_{t1}^2{\rm d}y_1{\rm d}p_{t2}^2{\rm d}y_2}{\rm d}p_{t1}^2{\rm d}p_{t2}^2, \tag{7}$$

where σ_{in} is the total inelastic cross section, the correlation function in the rapidity space can be written as

$$C(y_1, y_2) = \rho(y_1, y_2) - \rho(y_1)\rho(y_2) \tag{8}$$

or as a normalized quantity[2]

$$R(y_1, y_2) = \frac{C(y_1, y_2)}{\rho(y_1)\rho(y_2)} = \frac{\rho(y_1, y_2)}{\rho(y_1)\rho(y_2)} - 1, \tag{9}$$

which is less sensitive to experimental errors than $C(y_1, y_2)$. If particles in reaction are produced independently, then $\rho(y_1, y_2) = \rho(y_1)\rho(y_2)$ and, consequently, $R(y_1, y_2) = 0$. If

[2]Notice that definitions (2) and (9) differ by 1.

otherwise, they are correlated and it is convenient to separate correlations in two types. They are the short range and the long range correlations respectively, where the range refers to the relevant rapidity interval. Observing a particle may give information about what to be expected near by in rapidity space. This is a short range effect. Resonance (as well as dimesoatom) formation and decay is an obvious short range effect. Observing a particle may also give information about the reaction process as a whole. For instance observing a large momentum proton is likely to signal an event with a rather small associated multiplicity. This is a typical long range effect. Short range and long range effects are clearly both present and happen together.

What are results of the experiments in which a behaviour of the correlation function $R(y_1, y_2)$ have been studied in details? Fig. 1 taken from reference [4] shows the dependence of $R(\eta_1, \eta_2)$ on η_2 at $\eta_1 = \eta_2$ and $\eta_1 = -\eta_2$ for all charged particles measured on CERN ISR at $\sqrt{s} = 23$ and 63 GeV. Function $R(\eta_1, \eta_2)$ at $\eta_1 = \eta_2$ is almost constant at these energies in

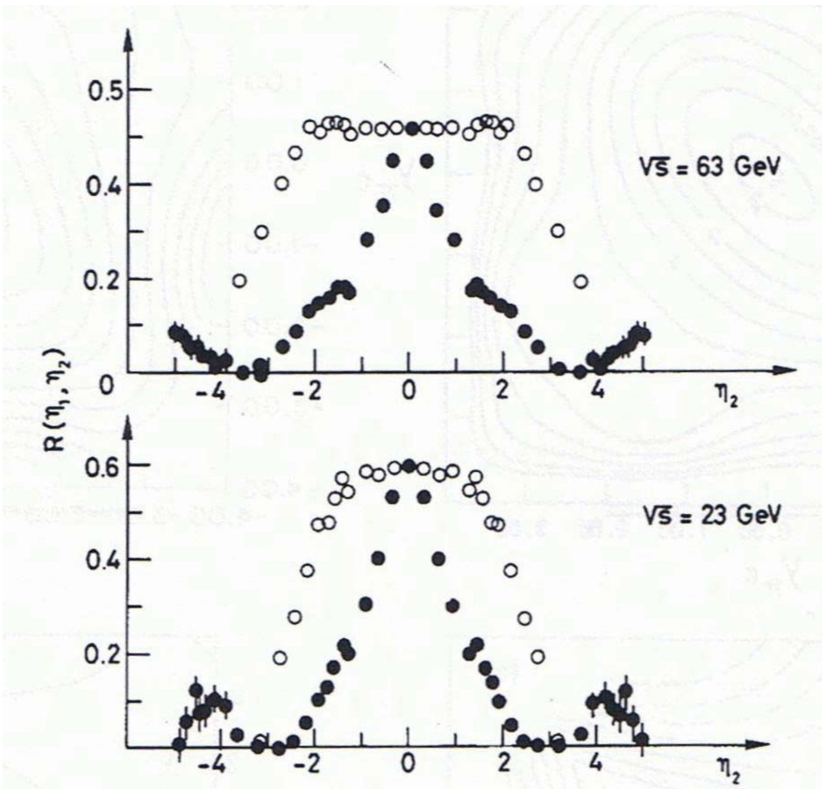

Figure 1: Correlation function $R(\eta_1, \eta_2)$ at $\sqrt{s} = 63$ GeV and 23 GeV [4]: open circles for $\eta_1 = \eta_2$, black circles for $\eta_1 = -\eta_2$.

the pseudorapidity interval $-2 < \eta_2 < 2$ (with broader plateau at higher energy) and shows a rapid decrease as one moves along the diagonal $\eta_1 = -\eta_2$. This a clear evidence of the short range correlations: existence of the particle with a certain rapidity favours existence of another particle with a similar rapidity. Fig. 2 illustrates the dependence of $R(\eta_1, \eta_2)$ on η_2 at $\eta_1 = 0$ for different energies for the same ISR data and the data at $\sqrt{s} = 14$ GeV from

FNAL [5]. The top value of the function, $R(0,0)$, is constant over the full energy range,

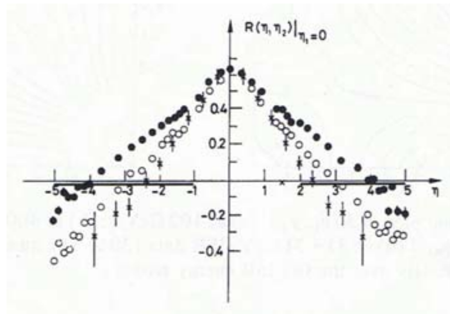

Figure 2: Correlation function $R(\eta_1, \eta_2)$ for $\eta_1 = 0$ as a function of η_2 at $\sqrt{s} = 63$ GeV, 23 GeV [4] and 14 GeV [5] (black, open circles and crosses, respectively).

while the width increases rapidly with increasing energy. The same shows fig. 3, where the ISR data are compared with the Serpukhov results at $p_{\mathrm{p}} = 70$ GeV/c taken from review [6]. For η_1 values different from zero, the top value of correlation function $R(\eta_1, \eta_2)$ as a function

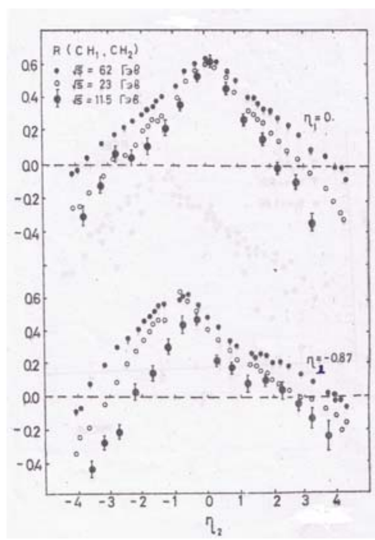

Figure 3: Correlation function $R(\eta_1, \eta_2)$ for $\eta_1 = 0$ as a function of η_2 at $\sqrt{s} = 63$ GeV, 23 GeV [4] and 11.5 GeV [6] (small black and open circles and large black circles, respectively).

of η_2 at $\sqrt{s} = 63$ GeV is positioned at the chosen values of rapidity η_1 (fig. 4). Moreover the peak value of $R(\eta_1, \eta_2)$ is almost the same in a rather broad η_1 range at $\sqrt{s} = 63$ GeV, while it decreases with decreasing η_1 at $\sqrt{s} = 23$ GeV. Finally fig. 5, taken from [3], shows energy dependence of the correlation functions $R^{+-}(y_1, y_2)$ and $R^{--}(y_1, y_2)$ at $y_1 = y_2 = 0$ for the particles with the opposite and the same charge, respectively. As could be expected $R^{+-}(0,0) > R^{--}(0,0)$ and these values are fairly energy independent and do not dependent on the nature of the incoming projectile.

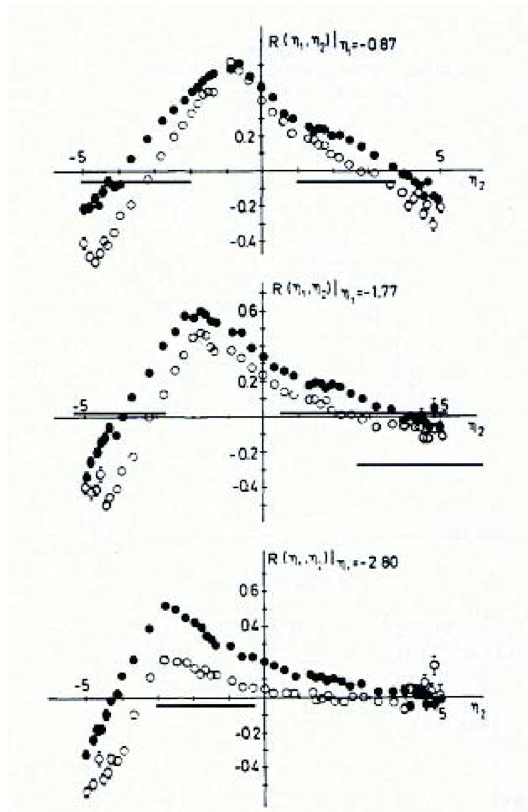

Figure 4: Correlation function $R(\eta_1, \eta_2)$ for the fixed different values of η_1 as a function of η_2 at $\sqrt{s} =$ 63 GeV and 23 GeV [4] (black and open circles, respectively).

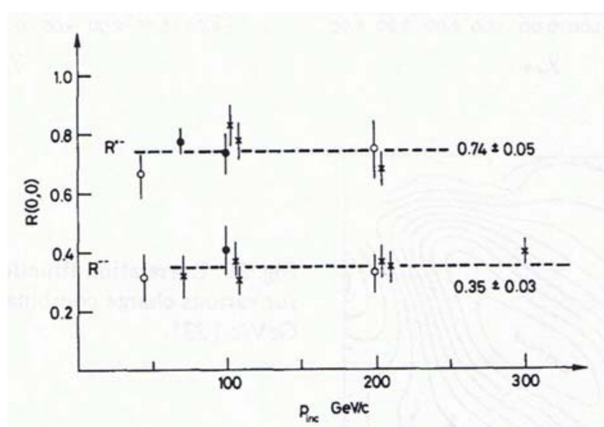

Figure 5: $R(0,0)$ as a function of incident momentum in pp-, π^-p- and π^+p- interactions (crosses, open and black circles, respectively). Figure is taken from [3].

Now one may notice that the invariant mass M of two particles with the masses m_1 and m_2 is related to the difference of their rapidities as

$$M^2 = m_1^2 + m_2^2 + m_{t_1} m_{t_2} [\exp(\Delta y) + \exp(-\Delta y)] - 2 p_{t_1} p_{t_2} \cos(\phi_1 - \phi_2), \qquad (10)$$

where p_t is the transverse momentum, $m_t = \sqrt{m^2 + p_t^2}$ the transverse mass and ϕ the azimuthal angle. For $M = m_1 + m_2$ one has $\phi_1 = \phi_2$ and (for any value of p_t) $\Delta y = 0$. With increasing $M > m_1 + m_2$ the values of Δy increase also. Clearly for two particles with the invariant mass M only slightly higher than $m_1 + m_2$ (as for dimesoatoms) the difference of their rapidities is close to zero. Therefore from the data presented above it follows that the correlation function $R^{+-}(y_1, y_2)$ for such particles might be expected to be energy independent or almost energy independent for the values of the c.m. rapidities close to zero.

The c.m. rapidity spectra have a plateau around $|y| = 0$, with the width increasing with increasing energy. On the other hand, the lab. momentum distributions fall rapidly with increasing momentum. Therefore a study of correlations in the lab. momentum space is difficult and, as far as I know, has been never attempted. Obviously, the large rapidities correspond to large momenta. Therefore the correlation function R in the lab. momentum space have to decrease much faster with increasing momentum at smaller energies than at the same momentum interval at higher energies. This is what is observed in [2] both at the two-dimensional plots of $R(p_1, p_2)$ in figs. 1-3 from [2] and at the dependence of R on the total momentum of two particles in figs. 4-6 from [2] (where R rapidly decreases with increasing lab. momentum of the particle pairs for the data at $p_\mathrm{p} = 24$ GeV/c but only slightly decreases in the same momentum interval for the data at 450 GeV/c). Unfortunately, from this behaviour it is rather difficult to draw any conclusion about behaviour of the correlation function for the particle pairs with small invariant masses close to the dimesoatom masses. Relatively small contribution of the particle pairs with small masses may possibly explain why R values are close to zero or negative (for R defined as in (9)) at 24 GeV/c for even small pair momenta. Besides, these R values have been determined in the quite narrow interval 10^{-3} sr of the solid angle. On the other hand, this possibly needs clarification bearing in mind not a perfect agreement of the experimental and generated momentum spectra (achieved with the accuracy of $\approx 20 - 50\%$ only). In this respect one may also notice that significant increase of the dimesoatom yields at 450 GeV/c in comparison with the present DIRAC setup at 24 GeV/c claimed in [2] has been obtained at best with the same accuracy. In any case for the precise determination of the dimesoatom yields it would be very useful to measure the $\pi^\pm$ and $\mathrm{K}^\pm$ inclusive spectra with a good precision on the DIRAC spectrometer and to achieve agreement of the generated spectra with the measured ones. It would be also useful to analyse the behaviour of the correlation functions for the Monte Carlo generated events at 24 and 450 GeV/c in the c.m. rapidity space and also to study their dependence on the invariant mass of particle pairs.

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