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An improved πK atom lifetime measurement

V. Yazkov
Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Russia

M. Zhabitsky
Joint Institute for Nuclear Research, Dubna, Russia

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Abstract

This note describes details of analysis of data samples collected by DIRAC experiment on a Pt target in 2007 and Ni targets in 2008–2010 in order to estimate the lifetime of πK atoms. Experimental results consist of eight distinct data samples: both charge combinations (π^+K^- and $K^+\pi^-$ atoms) obtained in different experimental conditions corresponding to each year of data taking. Estimations of systematic errors are presented. Taking into account both statistical and systematic uncertainties, the lifetime of πK atoms is estimated by the maximum likelihood method. The above sample comprises the total statistics, available for the analysis, thus the improvement over the previous estimation [1,3] of the πK atom lifetime is achieved.

1 Basic relations

The lifetime of $A_{\pi K}$ in the ground state is related to $a_0^- = \frac{1}{3}\left(a_0^{1/2} - a_0^{3/2}\right)$ scattering length [4]:

$$\frac{1}{\tau} = \Gamma_{\pi K} \approx \Gamma(A_{\pi K} \to \pi^0\overline{K^0}) = \Gamma(A_{K\pi} \to \pi^0 K^0) = 8\alpha^3\mu_+^2 p^*(a_0^-)^2(1+\delta_K), \tag{1}$$

$$\delta_K = (4.0 \pm 2.2) \times 10^{-2}. \tag{2}$$

By using $a_0^- m_{\pi^+} = 0.090 \pm 0.005$ [5], theory estimates the πK atom lifetime

$$\tau_{1S}^{\text{th}} = (3.5 \pm 0.4) \times 10^{-15} \text{ s}. \tag{3}$$

There is a one-to-one relation between the probability for πK atoms to break-up P_{br} in a thin foil and the lifetime τ of πK atoms. In a foil of thickness s the probability of break-up is a function of an exotic atom lifetime and its momentum p in the laboratory frame $P_{\text{br}} = P_{\text{br}}(\tau, p)$. This relation has been calculated [6] as a solution of a system of kinetic equations. Cross sections in Born approximation have been used [7]. Thicknesses of all target foils, used in DIRAC, have been thoroughly measured [8]. Distributions $P_{\text{br}}(\tau, p)$ have been integrated over the experimentally measured spectra dN/dp of Coulomb-correlated πK pairs with low relative momenta. The corresponding $P_{\text{br}}(\tau)$ relations for Pt and Ni targets are shown in Fig. 1.

2 Experimental data

Experimental values P_{br} from $(|Q_L|, Q_T)$-, Q- and $|Q_L|$-analyses of statistics, collected on Pt and Ni targets, are presented in Tabs. 1–6. Performed analyses follow procedures described in articles [1,2]. Here we cite only results of fits of experimental distributions. An excessive number of digits is preserved to avoid round-off errors.

As statistics for each period of data taking is very limited, the Feldman–Cousins approach (Appendix A) has been adopted to obtain the best estimate $\hat{\tau}$ of the lifetime and the corresponding confidence interval (1σ coverage).

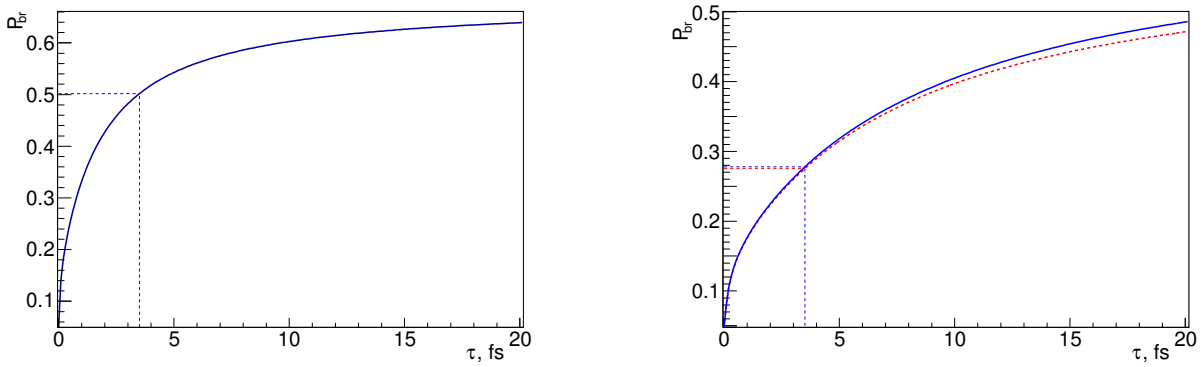

Figure 1: The probability of break-up as a function of πK atom lifetime in the ground state in the DIRAC experimental conditions. Left figure is for the 25.7 μm thick Pt target. Right plots are for Ni targets of thicknesses 98 μm (red dashed line) and 108 μm (solid blue line). The theoretical prediction (3) is projected on axes.

Table 1: Experimental $P_{\rm br}$ from $(|Q_L|, Q_T)$-analyses ($Q_T < 4$ MeV/c). Only statistical uncertainties are cited. The best estimate of the lifetime $\hat{\tau}$ and the corresponding confidence interval (1σ) are based on the Feldman–Cousins approach (see Appendix A).

Atom	Year	s, μm	$P_{\rm br}$	$\hat{\tau}$, fs	FC(1σ), fs
$A_{\pi K}$	2007	Pt, 25.7	0.2728 ± 0.5613		
$A_{\pi K}$	2008	Ni, 98	0.4226 ± 0.3751	12.57	$(0.06, \infty)$
$A_{\pi K}$	2009	Ni, 108	0.3346 ± 0.2381	5.53	$(0.16, 62.73)$
$A_{\pi K}$	2010	Ni, 108	0.2096 ± 0.1992	1.58	$(0.01, 9.85)$
$A_{K\pi}$	2007	Pt, 25.7	1.4384 ± 0.5851		
$A_{K\pi}$	2008	Ni, 98	0.4369 ± 0.2237	14.23	$(1.76, \infty)$
$A_{K\pi}$	2009	Ni, 108	0.1591 ± 0.1458	0.73	$(0.02, 4.47)$
$A_{K\pi}$	2010	Ni, 108	0.3419 ± 0.1599	5.93	$(1.07, 23.13)$
$A_{K\pi}\&A_{\pi K}$	2007	Pt, 25.7	0.8314 ± 0.4051	∞	$(1.98, \infty)$

For data obtained on the platinum target both πK pairs combinations (π^+K^- and π^-K^+) have been combined to provide the statistically meaningful best estimate $\hat{\tau}$ and the confidence interval.

From analysis of results presented in Tabs. 1–6 one can formulate following conclusions. The two-dimensional $(|Q_L|, Q_T)$-analysis and the one-dimensional Q-analysis show good sensitivity towards the lifetime measurement, while the $|Q_L|$-analysis provides inferior results. Here we cite results from the $|Q_L|$-analysis for completeness, they will not be used in the overall analysis. Two type of cuts have been applied in the relative momentum Q-space in the plane perpendicular to the atom's velocity: either cylindrical ($Q_T < 4$ MeV/c) or rectangular ($|Q_X|, |Q_Y| < 4$ MeV/c) one. Both cuts provide similar sensitivity and provide consistent results.

Sources of systematic errors in $P_{\rm br}$ are shown in Tabs. 7–8. While for Ni targets systematic uncertainty is small, for the Pt target it is of the same order of magnitude as the statistical one.

Table 2: Experimental $P_{\rm br}$ from Q-analyses ($Q_T < 4\,{\rm MeV}/c$). Abbreviations as in Tab. 1.

Atom	Year	s, μm	$P_{\rm br}$	$\hat{\tau}$, fs	FC(1σ), fs
$A_{\pi K}$	2007	Pt, 25.7	1.1931 ± 1.2691		
$A_{\pi K}$	2008	Ni, 98	0.5294 ± 0.3862	∞	$(0.53, \infty)$
$A_{\pi K}$	2009	Ni, 108	0.2917 ± 0.2011	3.84	$(0.13, 20.63)$
$A_{\pi K}$	2010	Ni, 108	0.3259 ± 0.2203	5.11	$(0.20, 39.24)$
$A_{K\pi}$	2007	Pt, 25.7	1.0945 ± 0.5239		
$A_{K\pi}$	2008	Ni, 98	0.3205 ± 0.2019	5.26	$(0.29, 38.61)$
$A_{K\pi}$	2009	Ni, 108	0.2298 ± 0.1593	2.12	$(0.09, 8.82)$
$A_{K\pi}$	2010	Ni, 108	0.4086 ± 0.1734	10.07	$(2.21, \infty)$
$A_{K\pi}\&A_{\pi K}$	2007	Pt, 25.7	1.1089 ± 0.4843	∞	$(14.51, \infty)$

Table 3: Experimental $P_{\rm br}$ from $|Q_L|$-analyses ($Q_T < 4\,{\rm MeV}/c$). Abbreviations as in Tab. 1.

Atom	Year	s, μm	$P_{\rm br}$	$\hat{\tau}$, fs	FC(1σ), fs
$A_{\pi K}$	2007	Pt, 25.7	0.8753 ± 1.5913		
$A_{\pi K}$	2008	Ni, 98	0.5037 ± 0.6185	29.62	$(0.00, \infty)$
$A_{\pi K}$	2009	Ni, 108	0.4924 ± 0.3669	20.77	$(0.33, \infty)$
$A_{\pi K}$	2010	Ni, 108	-0.1265 ± 0.1979	0.00	$(0.00, 0.09)$
$A_{K\pi}$	2007	Pt, 25.7	1.4155 ± 0.9531		
$A_{K\pi}$	2008	Ni, 98	0.4131 ± 0.3394	11.49	$(0.09, \infty)$
$A_{K\pi}$	2009	Ni, 108	0.0406 ± 0.2151	0.00	$(0.00, 2.82)$
$A_{K\pi}$	2010	Ni, 108	0.1510 ± 0.2030	0.60	$(0.00, 6.54)$
$A_{K\pi}\&A_{\pi K}$	2007	Pt, 25.7	1.2729 ± 0.8177	∞	$(2.44, \infty)$

Table 4: Experimental $P_{\rm br}$ from ($|Q_L|$, Q_T)-analyses ($|Q_X|, |Q_Y| < 4\,{\rm MeV}/c$). Abbreviations as in Tab. 1.

Atom	Year	s, μm	$P_{\rm br}$	$\hat{\tau}$, fs	FC(1σ), fs
$A_{\pi K}$	2007	Pt, 25.7	0.9901 ± 0.8588		
$A_{\pi K}$	2008	Ni, 98	0.4429 ± 0.3757	15.15	$(0.08, \infty)$
$A_{\pi K}$	2009	Ni, 108	0.3502 ± 0.2309	6.27	$(0.29, \infty)$
$A_{\pi K}$	2010	Ni, 108	0.1293 ± 0.1714	0.38	$(0.00, 4.10)$
$A_{K\pi}$	2007	Pt, 25.7	3.3810 ± 1.2488		
$A_{K\pi}$	2008	Ni, 98	0.4374 ± 0.2124	14.30	$(2.04, \infty)$
$A_{K\pi}$	2009	Ni, 108	0.1461 ± 0.1372	0.56	$(0.00, 3.68)$
$A_{K\pi}$	2010	Ni, 108	0.3046 ± 0.1430	4.35	$(0.75, 13.89)$
$A_{K\pi}\&A_{\pi K}$	2007	Pt, 25.7	1.7578 ± 0.7076	∞	$(1.00, \infty)$

Table 5: Experimental $P_{\rm br}$ from Q-analyses ($|Q_X|, |Q_Y| < 4\,\mathrm{MeV}/c$). Abbreviations as in Tab. 1.

Atom	Year	s, μm	$P_{\rm br}$	$\hat{\tau}$, fs	FC(1σ), fs
$A_{\pi K}$	2007	Pt, 25.7	2.8177 ± 2.7831		
$A_{\pi K}$	2008	Ni, 98	0.5279 ± 0.3746	42.58	$(0.66, \infty)$
$A_{\pi K}$	2009	Ni, 108	0.3357 ± 0.2053	5.58	$(0.37, 36.43)$
$A_{\pi K}$	2010	Ni, 108	0.2842 ± 0.2029	3.56	$(0.10, 19.46)$
$A_{K\pi}$	2007	Pt, 25.7	1.4241 ± 0.5735		
$A_{K\pi}$	2008	Ni, 98	0.3357 ± 0.1965	5.99	$(0.48, 45.62)$
$A_{K\pi}$	2009	Ni, 108	0.2339 ± 0.1548	2.22	$(0.10, 8.80)$
$A_{K\pi}$	2010	Ni, 108	0.3719 ± 0.1561	7.53	$(1.75, 31.27)$
$A_{K\pi}\&A_{\pi K}$	2007	Pt, 25.7	1.4809 ± 0.5617	∞	∞

Table 6: Experimental $P_{\rm br}$ from $|Q_L|$-analyses ($|Q_X|, |Q_Y| < 4\,\mathrm{MeV}/c$). Abbreviations as in Tab. 1.

Atom	Year	s, μm	$P_{\rm br}$	$\hat{\tau}$, fs	FC(1σ), fs
$A_{\pi K}$	2007	Pt, 25.7	1.3066 ± 2.1868		
$A_{\pi K}$	2008	Ni, 98	0.6911 ± 0.7339	∞	$(0.00, \infty)$
$A_{\pi K}$	2009	Ni, 108	0.7869 ± 0.4624	∞	$(5.03, \infty)$
$A_{\pi K}$	2010	Ni, 108	-0.2863 ± 0.1820	0.00	?
$A_{K\pi}$	2007	Pt, 25.7	2.3988 ± 1.4466		
$A_{K\pi}$	2008	Ni, 98	0.4977 ± 0.3735	27.22	$(0.34, \infty)$
$A_{K\pi}$	2009	Ni, 108	0.0374 ± 0.2262	0.00	$(0.00, 3.05)$
$A_{K\pi}$	2010	Ni, 108	0.1969 ± 0.2220	1.35	$(0.00, 10.94)$
$A_{K\pi}\&A_{\pi K}$	2007	Pt, 25.7	2.0663 ± 1.2065	∞	∞

Table 7: Sources of systematic errors in $P_{\rm br}$ on Ni targets

	Type of analysis	Q	$\lvert Q_L\rvert$	$(\lvert Q_L\rvert, Q_T)$
$\sigma^{\rm syst}_{\Lambda}$	Uncertainty in Λ width correction	0.0006	0.0013	0.0006
$\sigma^{\rm syst}_{MS}$	Uncertainty of multiple scattering in a Ni target	0.0051	0.0006	0.0036
$\sigma^{\rm syst}_{SFD}$	Accuracy of SFD simulation	0.0002	0.0001	0.0003
$\sigma^{\rm syst}_{FS}$	Correction of the Coulomb correlation function on finite size production region	0.0001	0.0000	0.0000
$\sigma^{\rm syst}_{s}$	Uncertainty in πK pair lab. momentum spectrum	0.0052	0.0031	0.0050
$\sigma^{\rm syst}_{bg}$	Uncertainty in the laboratory momentum spectrum of background pairs	0.0011	0.0010	0.0011
$\sigma^{\rm syst}_{cs}$	Uncertainty in the $P_{\rm br}(\tau)$ relation	0.005	0.005	0.005
	Total	0.0089	0.0062	0.0080

Table 8: Sources of systematic errors in $P_{\rm br}$ on the Pt target

	Type of analysis	Q	$\lvert Q_L\rvert$	$(\lvert Q_L\rvert, Q_T)$
$\sigma^{\rm syst}_{\Lambda}$	Uncertainty in Λ width correction	0.011	0.099	0.0732
$\sigma^{\rm syst}_{MS}$	Uncertainty of multiple scattering in the Pt target	0.0087	0.0086	0.0141
$\sigma^{\rm syst}_{SFD}$	Accuracy of SFD simulation	0.	0.	0.
$\sigma^{\rm syst}_{FS}$	Correction of the Coulomb correlation function on finite size production region	0.0001	0.0002	0.0002
$\sigma^{\rm syst}_{s}$	Uncertainty in πK pair lab. momentum spectrum	0.089	0.27	0.25
$\sigma^{\rm syst}_{bg}$	Uncertainty in the laboratory momentum spectrum of background pairs	0.22	0.068	0.21
$\sigma^{\rm syst}_{cs}$	Uncertainty in the $P_{\rm br}(\tau)$ relation	0.005	0.005	0.005
	Total	0.24	0.29	0.34

3 Lifetime estimation based on all data periods

Estimation of πK atom lifetime in the ground state has been performed by the maximum likelihood method according to [9]:

$$L(\tau) = \exp\left(-U^T G^{-1} U/2\right), \tag{4}$$

where $U_i = m_i - P_{{\rm br},i}(\tau)$ is a vector of differences between measured values of break-up m_i and corresponding theoretical functions $P_{{\rm br},i}(\tau)$ for a data sample i. G is the error matrix on U, which includes both statistical σ_i and systematic uncertainties:

$$G_{ij} = \delta_{ij}\left(\sigma_i^2 + (\sigma^{\rm syst}_{i,{\rm global}})^2\right) + (1-\delta_{ij})\delta_{i,{\rm Ni}}(\sigma^{\rm syst}_{i,{\rm global}})^2 + (\sigma^{\rm syst}_{cs})^2, \tag{5}$$

$$(\sigma^{\rm syst}_{i,{\rm global}})^2 = (\sigma^{\rm syst}_{\Lambda})^2 + (\sigma^{\rm syst}_{ms})^2 + (\sigma^{\rm syst}_{SFD})^2 + (\sigma^{\rm syst}_{fs})^2 + (\sigma^{\rm syst}_{s})^2 + (\sigma^{\rm syst}_{bg})^2. \tag{6}$$

Only the term $\sigma^{\rm syst}_{cs}$, corresponding to uncertainty in the $P_{\rm br}(\tau)$ relation, is considered as correlated between Ni and Pt targets (this is a conservative approach, which overestimates this error). All other systematic uncertainties are uncorrelated between data samples, collected on Ni and Pt targets. At the same time systematic uncertainties for all data samples from Ni targets are correlated.

Likelihood functions for $(\lvert Q_L\rvert, Q_T)$- and (Q)-analyses are shown in Fig. 2. Summary of results for both type of analysis and different cuts in Q-space is presented in Tab. 9. Usage of Pt data samples doesn't significantly improve the final result. As the magnitude of the systematic error on Pt is about the statistical uncertainty, taking into account systematic errors changes the relative weights of Pt and Ni data samples, thus shifts the best estimate $\hat{\tau}_{\rm tot}$ with respect to $\hat{\tau}_{\rm stat}$.

3.1 Lifetime from combined $(\lvert Q_L\rvert, Q_T)$-analyses

From $(\lvert Q_L\rvert, Q_T)$-analyses with $Q_T < 4\,{\rm MeV}/c$ if one combines both charge combinations (π^+K^- and π^-K^+) and uses all statistics collected in 2007–2010, the following best estimates for πK atoms lifetime are obtained

$$\hat{\tau}_{\rm stat} = 3.96^{+3.49}_{-2.12}\big|_{\rm stat}\ {\rm fs}, \tag{7}$$

$$\hat{\tau}_{\rm tot} = 3.79^{+3.48}_{-2.12}\big|_{\rm tot}\ {\rm fs}. \tag{8}$$

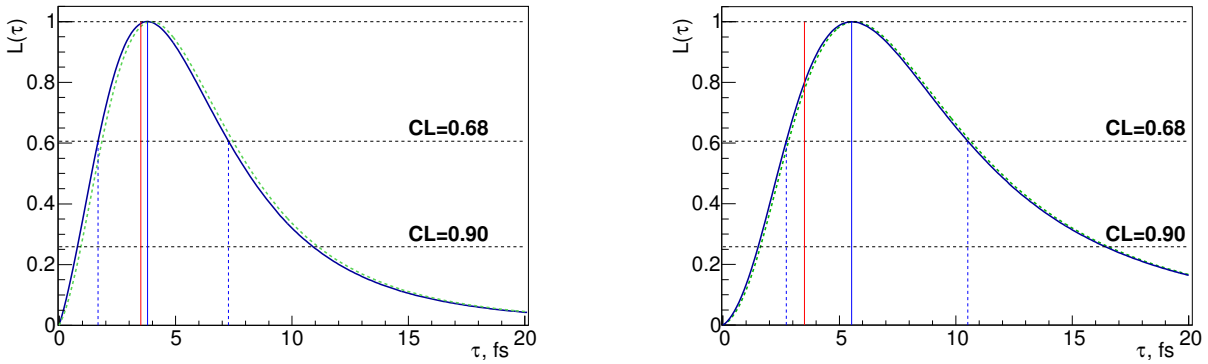

Figure 2: Likelihood functions $L(\tau)$ for $(|Q_L|, Q_T)$ (left) and (Q) (right) analyses with $Q_T < 4\text{MeV}/c$. The likelihood function, which takes into account both statistical and systematic errors, is shown by a dashed green line, only statistical uncertainties — by a solid blue line. Vertical blue lines show the best estimate $\hat{\tau}_{\text{tot}}$ and the corresponding confidence interval. The vertical red line indicates the theoretical prediction (3).

Table 9: Estimations of πK atom lifetime

Analysis	Cuts	Period	Stat. only, fs	Total, fs
$(\|Q_L\|, Q_T)$	$Q_T < 4\,\text{MeV}/c$	Pt 2007 & Ni 2008–2010	$3.96^{+3.49}_{-2.12}$	$3.79^{+3.48}_{-2.12}$
$(\|Q_L\|, Q_T)$	$Q_T < 4\,\text{MeV}/c$	Ni 2008–2010	$3.52^{+3.40}_{-2.10}$	$3.52^{+3.42}_{-2.11}$
$(\|Q_L\|, Q_T)$	$\|Q_x\|, \|Q_y\| < 4\,\text{MeV}/c$	Pt 2007 & Ni 2008–2010	$3.16^{+2.67}_{-1.73}$	$2.89^{+2.63}_{-1.70}$
$(\|Q_L\|, Q_T)$	$\|Q_x\|, \|Q_y\| < 4\,\text{MeV}/c$	Ni 2008–2010	$2.66^{+2.56}_{-1.66}$	$2.66^{+2.58}_{-1.66}$
Q	$Q_T < 4\,\text{MeV}/c$	Pt 2007 & Ni 2008–2010	$5.64^{+4.99}_{-2.82}$	$5.53^{+4.98}_{-2.81}$
Q	$Q_T < 4\,\text{MeV}/c$	Ni 2008–2010	$5.07^{+4.73}_{-2.74}$	$5.07^{+4.77}_{-2.75}$
Q	$\|Q_x\|, \|Q_y\| < 4\,\text{MeV}/c$	Pt 2007 & Ni 2008–2010	$5.62^{+4.65}_{-2.71}$	$5.60^{+4.68}_{-2.72}$
Q	$\|Q_x\|, \|Q_y\| < 4\,\text{MeV}/c$	Ni 2008–2010	$4.98^{+4.37}_{-2.60}$	$4.98^{+4.41}_{-2.62}$

The measured πK atom lifetime (Eq. 8) corresponds through the relation (1) (see Fig. 3) to the following value of the πK scattering length a_0^-:

$$\left|a_0^-\right| m_{\pi^+} = 0.087^{+0.044}_{-0.024}\big|_{\text{tot}}\,. \tag{9}$$

3.2 Lifetime from combined Q-analyses

As a main result from combined Q-analyses we use the same cut $Q_T < 4$ MeV/c as in 2-dimensional analysis (3.1). The following best estimates for πK atoms lifetime are obtained, based on all statistics collected in 2007–2010 for both charge combinations (π^+K^- and π^-K^+):

$$\hat{\tau}_{\text{stat}} = 5.64^{+4.99}_{-2.82}\big|_{\text{stat}}\ \text{fs}, \tag{10}$$

$$\hat{\tau}_{\text{tot}} = 5.53^{+4.98}_{-2.81}\big|_{\text{tot}}\ \text{fs}. \tag{11}$$

Thus the πK scattering length a_0^- from Q-analyses is

$$\left|a_0^-\right| m_{\pi^+} = 0.072^{+0.031}_{-0.020}\big|_{\text{tot}}\,. \tag{12}$$

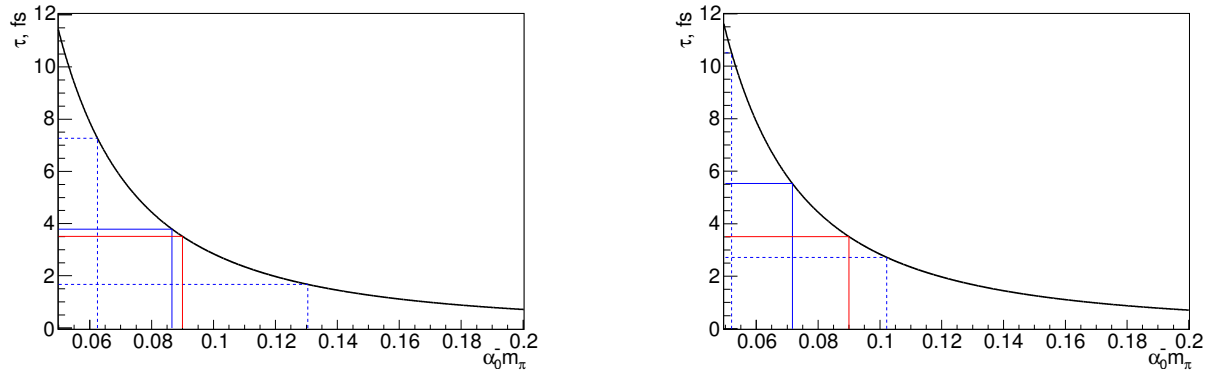

Figure 3: Dependence of $A_{\pi K}$ lifetime in the ground state τ_{1S} on a_0^-. Experimental results (blue lines) are compared to the theoretical prediction (red lines). $(|Q_L|, Q_T)$-analysis (left) and (Q)-analysis (right).

A Feldman–Cousins approach to estimate confidence intervals for πK atom lifetime from measurements by DIRAC

DIRAC experiment performed a series of lifetime measurements of double exotic πK-atoms. These atoms are rarely produced in proton-nuclei collisions at 24 GeV/c. Corresponding experimental statistics are very limited, thus special attention should be paid to calculate either double-sided or one-sided interval estimates with proper coverage. We apply a unified approach by G. Feldman and R. Cousins [10] to construct confidence intervals on measurements of πK-atom's lifetime by DIRAC.

Analysis of a πK data sample results in a *measurement* m of the break-up probability p. The probability density function is assumed to be a normal distribution with a known standard deviation σ:

$$f(m|p,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(m-p)^2}{2\sigma^2}\right). \tag{13}$$

Value of m is unbound in the analysis. In fact a *true value* of break-up probability p_{true} is limited: $p_{\text{true}} \in (0; p_{\text{as}})$, where p_{as} is an asymptotic break-up probability for non-decaying atoms. Thus we enforce a *best estimate* of break-up probability p_{best} to be within the allowed range

$$p_{\text{best}} = \min(\max(0, m), p_{\text{as}}). \tag{14}$$

An *acceptance interval* $(m_{\min}, m_{\max})$ is selected to provide a *confidence level*

$$CL = \int_{m_{\min}}^{m_{\max}} f(m|p)dm = \frac{1}{2}\left(\text{erf}\left\{\frac{m_{\max}-p}{\sigma\sqrt{2}}\right\} - \text{erf}\left\{\frac{m_{\min}-p}{\sigma\sqrt{2}}\right\}\right). \tag{15}$$

Then a *confidence interval* $(p_{\text{low}}, p_{\text{up}})$ is a union of all values of p, for which the acceptance interval $(m_{\min}, m_{\max})$ includes a particular measurement m_0. This construction has an ambiguity in the way to select an acceptance interval for a particular p, thus an ambiguity in the definition of a confidence interval. We choose Feldman-Cousins approach [10] to fix a confidence interval $(p_{\text{low}}, p_{\text{up}})$ in a unique way.

The probability density function (Eq. 13) at the most-probable estimate (Eq. 14) reads

$$f(m|p_{\text{best}}) = \begin{cases} \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(-\dfrac{m^2}{2\sigma^2}\right) & \text{if } m < 0, \quad (16a) \\ \dfrac{1}{\sigma\sqrt{2\pi}} & \text{if } m \in [0, p_{\text{as}}], \quad (16b) \\ \dfrac{1}{\sigma\sqrt{2\pi}} \exp\left(-\dfrac{p_{\text{as}} - m^2}{2\sigma^2}\right) & \text{if } m > p_{\text{as}}. \quad (16c) \end{cases}$$

We construct a ratio

$$R(m|p) = \frac{f(m|p)}{f(m|p_{\text{best}})} = \begin{cases} \exp\left(-\dfrac{p^2 - 2mp}{2\sigma^2}\right) & \text{if } m < 0, \quad (17a) \\ \exp\left(-\dfrac{(m-p)^2}{2\sigma^2}\right) & \text{if } m \in [0, p_{\text{as}}], \quad (17b) \\ \exp\left(-\dfrac{(p_{\text{as}} - p)(2m - p - p_{\text{as}})}{2\sigma^2}\right) & \text{if } m > p_{\text{as}}. \quad (17c) \end{cases}$$

$$\max_m R(m|p) = \begin{cases} \exp\left(-\dfrac{p^2}{2\sigma^2}\right) & \text{at } m \to 0 \text{ if } m < 0, \quad (18a) \\ 1 & \text{at } m = p \text{ if } m \in [0, p_{\text{as}}], \quad (18b) \\ \exp\left(-\dfrac{(p - p_{\text{as}})^2}{2\sigma^2}\right) & \text{at } m \to p_{\text{as}} \text{ if } m > p_{\text{as}}. \quad (18c) \end{cases}$$

The ratio R determines the order for a measurement m to enter into an interval $(m_{\text{min}}, m_{\text{max}})$: measurements with higher R are preferred when a confidence interval is expanded around $m = p_{\text{best}}$ to fulfill a selected confidence level (Eq. 15).

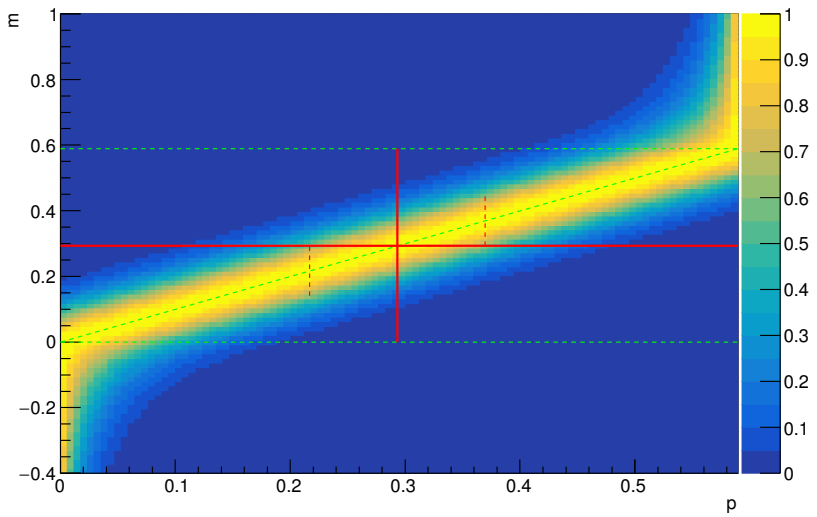

Figure 4: Ratio $R(m|p)$ with sufficiently small σ. Dotted green lines are for $m = 0$, $m = p_{\text{as}}$ and $m = p$. Measurement m_0 and corresponding p_{best} are shown by solid red lines. Dotted red lines correspond to optimal integration paths in Eq. (19) and (20).

For a particular measurement m_0 construction of a confidence interval is reduced to a solution of two system of equations with respect to an interval $(p_{\rm low}, p_{\rm up})$:

$$\begin{cases} CL = \int\limits_{m_{\rm min}}^{m_0} f(m|p_{\rm low})dm, \\ R(m_{\rm min}|p_{\rm low}) = R(m_0|p_{\rm low}); \end{cases} \tag{19}$$

$$\begin{cases} CL = \int\limits_{m_0}^{m_{\rm max}} f(m|p_{\rm up})dm, \\ R(m_{\rm max}|p_{\rm up}) = R(m_0|p_{\rm up}). \end{cases} \tag{20}$$

As long as integration limits $m_{\rm min}$ and $m_{\rm max}$ are within the interval $[0, p_{\rm as}]$, Feldman-Cousins approach is equivalent to a symmetric confidence interval costruction with

$$(1 - CL)/2 = \int\limits_{-\infty}^{m_{\rm min}} f(m|p)dm = \int\limits_{m_{\rm max}}^{\infty} f(m|p)dm. \tag{21}$$

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