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Remarks about the Pionium Lifetime in the femtosecond region

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1 Introduction

The experiment DIRAC, PS212 at CERN, aims to extract precise pion-pion scattering length data by means of measuring the pionium ($\pi^+\pi^-$ atom or $A_{2\pi}$) lifetime. Hence, the relation between lifetime and scattering lengths constitutes a key element of the experiment. Many references (eg. [1], [2], [3]) about this topic can be found in literature, and recently a new publication [4] appeared, claiming a much smaller (but wrong [5]) pionium lifetime!

It is the aim of this study to collect important material available, to make comparisons with new experimental results from other related hadronic atoms (pionic hydrogen) and to draw conclusions concerning limitations and accuracy of relations and quantities.

2 Decay width and level shift of hadronic atoms

The energy shift (ε) and width (Γ) of the 1s level in hadronic atoms ($A_{h_1h_2}$) can be set in relation to the real and imaginary part of the complex s-wave h_1h_2 scattering length $a_{h_1h_2}$. In first order approximation the total complex hadronic level shift is given by

$$\varepsilon^* = \varepsilon + i\left(-\frac{\Gamma}{2}\right) = 2\alpha^2\mu\,(\alpha\mu\ a_{h_1h_2}) \tag{1}$$

with μ the reduced mass and α the fine structure constant. This expression is called Deser-Trueman formula [1], [3].

In the case of $A_{2\pi}$ the level shift amounts to

$$\varepsilon^* = 27.1 a_{\pi^+\pi^-} eV m_\pi, \tag{2}$$

where the scattering length is measured in units of inverse charged pion mass. Inserting $\Re(a_{\pi^+\pi^-}) = \frac{1}{3}(2a_0 + a_2)$ and $\Im(a_{\pi^+\pi^-}) = -\frac{2}{9}q_0\,|a_0 - a_2|^2$ [3] in equation (1), the following expressions are found:

$$\varepsilon_{1s}(A_{2\pi}) = -\frac{4}{3}\frac{(2a_0 + a_2)}{r_B}E_{1s}, \tag{3}$$

$$\Gamma_{1s}(A_{2\pi}) = \frac{16}{9}q_0 r_B\frac{|a_0 - a_2|^2}{r_B^2}E_{1s}, \tag{4}$$

where a_0 and a_2 are the isoscalar and isotensor scattering lengths, respectively, q_0 is the c.m. π^0 momentum and r_B the Bohr radius.

For pionic hydrogen ($A_{\pi^- p}$) same kind of formulas like equations (3, 4) are valid as shown in the Table.

Since the nineties the two-pion atom, pionium, is accessible to experiments, and a first crude measurement of its lifetime has been published [6]. In order to relate precise experimental lifetimes of hadronic atoms to this estimated [6] and also to predicted pionium lifetimes [7], the very recently measured (preliminary) $A_{\pi^- p}$ decay width [8] (see Table) together with the corresponding formulas of the type (4) are used.

Assuming reasonable values for the hadronic scattering lengths (from theory and experiment, see [9]), one realizes that the width (and shift) should be quite smaller for $A_{2\pi}$ compared to $A_{\pi^- p}$, due to the much larger $A_{2\pi}$ Bohr radius. As shown in the Table, a $A_{2\pi}$ width of about 0.2eV is obtained, five times smaller than that one for $A_{\pi^- p}$.

The decay width of hadronic atoms can also be expressed through the Coulomb ground-state wave function at the origin (see Table). To get an idea about the quality of this relation, the square of the $A_{\pi^- p}$ wave function around the origin is derived by inserting the experimental width $\Gamma_{1s}(A_{\pi^- p})$ ([8] and Table) and corresponding Karlsruhe scattering lengths [10]. Within the error of about 6% the wave function value found agrees perfectly with the pure Coulomb one!

3 Conclusion

From the above considerations *a pionium lifetime of several femtoseconds* ($\sim 3fs$) *is expected.*

What remains to be done and *is a project* [11] for the near future, is a precision study of the lifetime dependence on strong interaction parameters (eg. isospin violation).

References

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