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Three target method for the lifetime measurement ($2Z$-method)

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The extrapolation method of the $A_{2\pi}$ lifetime measurement which suggested in the DIRAC proposal has few week points:

- To separate "Coulomb" and "non-Coulomb" pairs we simulate they in very rough approach. For "Coulomb" pairs we assume the Coulomb enhancement (weight) such as for the point-like $\pi^+\pi^-$ source. Still there is no formulas accounting dimensions and shape of the $\pi^+\pi^-$ source. In any case this formulas will include some model assumption about the $\pi^+\pi^-$ source.
- The fitting procedure require that the efficiency of the true and accidental coincidence should be equal with a high precision. And many other requires.

Let us consider another method which do not include these points. The main idea based on using of three different targets and formulas of this method are the same as for the subtraction method described in Appendix C of the DIRAC proposal.

Consider the total number of $\pi^+\pi^-$ pairs with the relative momentum $q < 2$ MeV/c obtained for the same number of prime proton-nucleus interactions (or the same flux of secondary particles) in measurements with three targets consisted of different materials, a different number of lays and having equal thicknesses in radiation length units: N_1 for the Platinum target (the highest available Z), N_2 for target with an intermediate Z (Ti, Ni or Mo) and N_3 for the Platinum target consisting of a few lays with the gap of 1 mm. These pair numbers can be represented as

$$\begin{aligned} N_1 &= n_{A1} + N_{f1} \\ N_2 &= n_{A2} + N_{f2} \\ N_3 &= n_{A3} + N_{f3}. \end{aligned} \tag{1}$$

Here n_{A1}, n_{A2} and n_{A3} are the numbers of the atomic pairs and N_{f1}, N_{f2} and N_{f3} are the numbers of free pairs.

For simplicity sake assume that fractions of the Coulomb and non-Coulomb free pairs are the same for different target materials. In this case a shape of the q-distributions for the free $\pi^+\pi^-$ pairs from the different targets should be identical. Possible corrections to the method arising because of an inaccuracy of this assumption are discussed below.

Consider the numbers of free pairs N_{f1}, N_{f2} and N_{f3}. Pairs of $\pi^+\pi^-$-mesons are produced mainly in the interaction of an incident proton with a single nucleon of a target nucleus. So N_{f1}, N_{f2} and N_{f3} should be the same, taking into account also the same multiple scattering in these targets.

$$N_{f1} = N_{f2} = N_{f3}\,. \tag{2}$$

In practice the distributions of free pairs will be normalized to each other in the range of the relative momentum $q > 3$ MeV/c and taking into account the identical shape of the q-distributions we get the above equality for the range of $q < 2$ MeV/c.

The numbers of the atomic pairs n_{A1}, n_{A2} and n_{A3} are determined by the probabilities of $A_{2\pi}$ breakup which are different for the targets (see Figure 1) and by the number of the produced atoms N_A which are proportional to the number of the Coulomb pairs and thus, under the above assumption, are the same for all targets.

$$n_{Ai} = P_{\mathrm{br}\,i} N_A \tag{3}$$

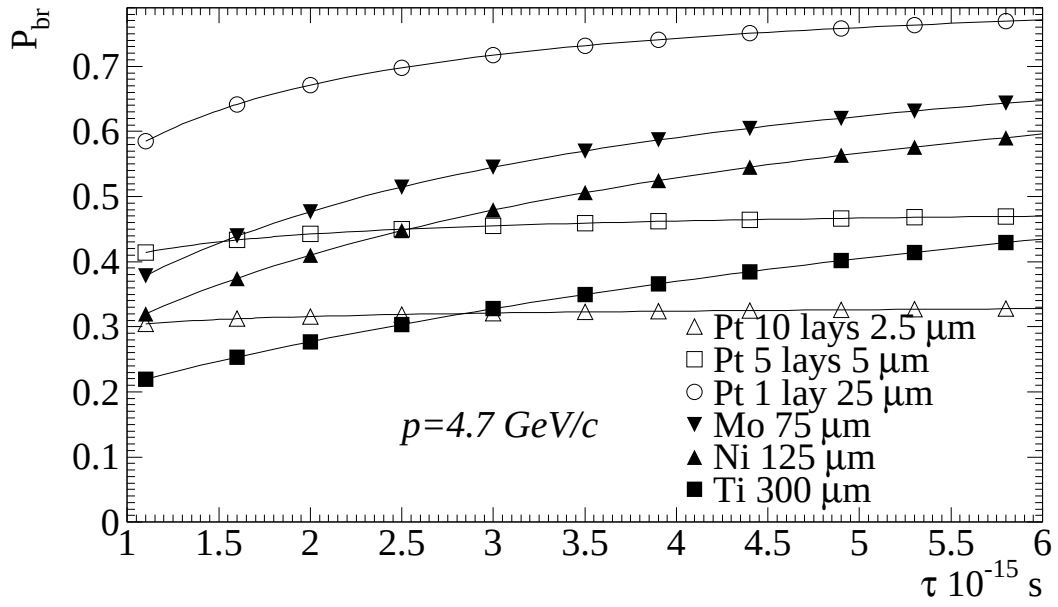

Figure 1: Probability of the $A_{2\pi}$ breakup as a function of the atom lifetime for different materials of targets having equal thicknesses in radiation length units and different lay numbers in the Platinum target with 1 mm gaps between lays.

In order to have the same efficiency of a pair registration, which could depend on the flux of secondary particles, the prime beam intensity in the measurement with other target should be decreased in comparison with the Platinum target one to compensate a higher efficiency of other targets for proton-nucleus interactions.

Let us consider the value

$$x \;=\; \frac{N_2 - N_3}{N_1 - N_3}. \tag{4}$$

Taking into account (1) and (2) we obtain

$$x \;=\; \frac{n_{A2} - n_{A3}}{n_{A1} - n_{A3}}\,. \tag{5}$$

The x value depends on the atomic pair numbers only and can with account of (3) be expressed via the calculated probabilities of $A_{2\pi}$ breakup for the correspondent targets:

$$x \;=\; \frac{P_{\mathrm{br}\,2} - P_{\mathrm{br}\,3}}{P_{\mathrm{br}\,1} - P_{\mathrm{br}\,3}}\,. \tag{6}$$

The calculations have shown that x is a simple function of the $A_{2\pi}$ lifetime ($x = x(\tau)$) for fixed pair momentum (see Figure 2). Therefore the measurement of x value allows one to determine the dimesoatom lifetime.

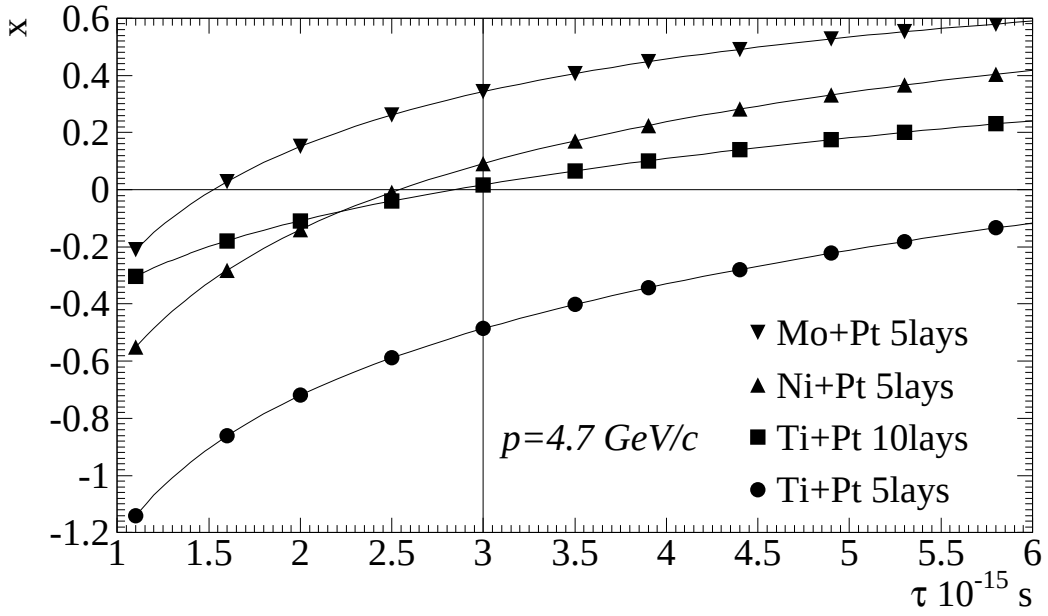

Figure 2: Values of x as a function of the atom lifetime for different materials of the second targets and different lay numbers in the third target. The straight lines are drawn for $x = 0$ and the expected lifetime of $3.0 \cdot 10^{-15}$ s.

The expression for the standard deviation for x is

$$\sigma_x \;=\; \frac{1}{N_1 - N_3}\,\sqrt{\sigma_{N_2}^2 + x^2\sigma_{N_1}^2 + (1-x)^2\sigma_{N_3}^2}\;. \tag{7}$$

The correspondent error in the lifetime is

$$\sigma_\tau \;=\; \sigma_x/(dx/d\tau)\;. \tag{8}$$

Using the procedure of the atom production simulation described in the proposal one can estimate the number of atoms to be produced in the three targets to obtain the $A_{2\pi}$ lifetime within required accuracy. In Figure 3 the number of $A_{2\pi}$, to be produced in all three targets to obtain the $A_{2\pi}$ lifetime within 10%, is shown as a function of the atom lifetime for the different materials of the second target. The ratio of accidental to real coincidences used in the simulation was equal to 2.0.

Note that the minimums for the Nickel's and Molybdenum's curves in Fig.3 correspond to the points $x = 0$ in Fig.2. It is due to the structure of Eq.(7).

Some illustrative and final numbers are given in Table 1. Calculations were performed for the $A_{2\pi}$ lifetime $3.0 \cdot 10^{-15}$ s and momentum 4.7 GeV/c. The total thicknesses of the Platinum targets is 25 μm and the probabilities of atoms breakup in these targets are 0.717, 0.455 and 0.321 for 1, 5 and 10 lays, respectively. For the different materials of the second target with nucleus charge Z and the different lay numbers in the third target n_l there are

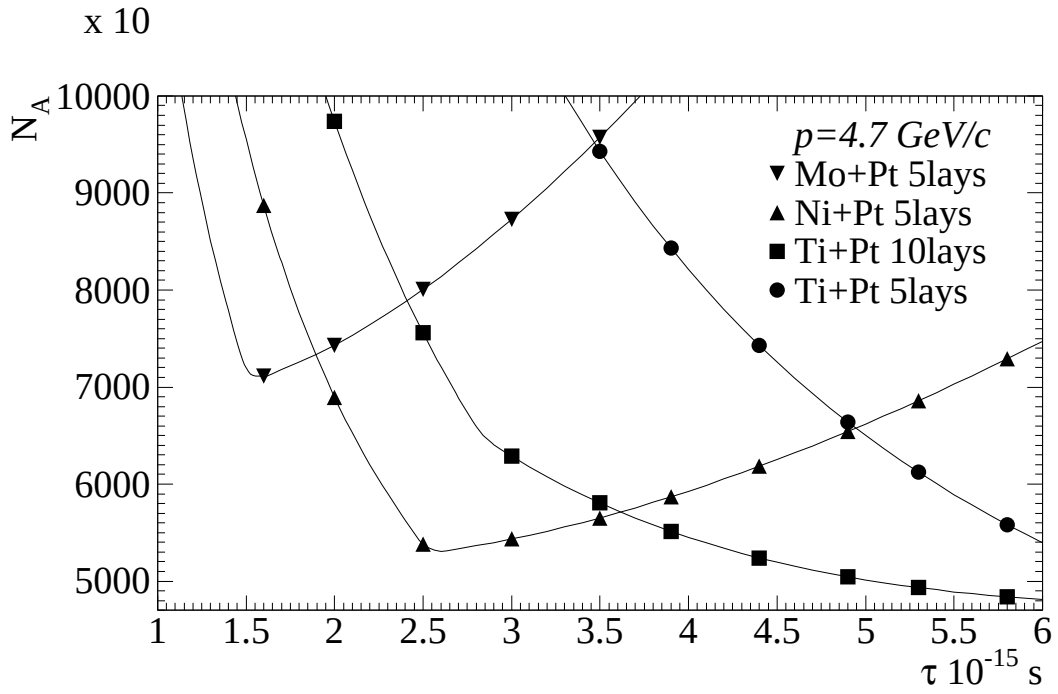

Figure 3: The number of $A_{2\pi}$ to be produced in the three target method to obtain the $A_{2\pi}$ lifetime within 10% as a function of atom lifetime for the different materials of the second target and different lay numbers in the third target.

presented the probabilities of $A_{2\pi}$ breakup $P_{\text{br}\,2}$; the value of x; the relative accuracy δx in measurement of x and the number of produced atoms in all three targets N_A, required to obtain the lifetime within 10%. The ratio $w_1 : w_2 : w_3$ between the numbers of produced $A_{2\pi}$ in each target provides the smallest value of N_A.

Table 1:

	Z	$P_{\text{br}\,2}$	n_l	x	δx	$w_1 : w_2 : w_3$	N_A
Ti	22	0.3279	10	0.0172	$3.1 \cdot 10^{-2}$	0.010 : 0.500 : 0.490	$6.3 \cdot 10^4$
Ti	22	0.3279	5	-0.4862	$5.5 \cdot 10^{-2}$	0.175 : 0.325 : 0.500	$1.1 \cdot 10^5$
Ni	28	0.4792	5	0.0913	$5.3 \cdot 10^{-2}$	0.049 : 0.500 : 0.451	$5.4 \cdot 10^4$
Mo	42	0.5450	5	0.3428	$4.3 \cdot 10^{-2}$	0.179 : 0.500 : 0.321	$8.7 \cdot 10^4$

From Fig.3 one can conclude that Ni is the most suitable material for the second target for the expected lifetime about $3.0 \cdot 10^{-15}$ s. In this case the third target consists of 5 lays of 5 μm Platinum foil with 1 mm gap, which could easy be produced. The total required number of the produced atoms in the three targets with this method is almost the same as for the singe Ni target with the extrapolation method.

Foils from different materials for the three target method should have the value of the multiple scattering as close as possible in order to exclude big corrections based on the simulation of the multiple scattering.

Possible correction to the method

If with the extrapolation method we will find that the assumption about the equality of the fractions of the Coulomb and non-Coulomb free pairs for different target materials is not valid the following corrections to the $2Z$-method should be applied.

1. If the shape of the q-distribution for Pt and Ni targets will be significantly different in the range $q > 3$ MeV/c than the distributions should be normalized on the Coulomb free pairs only that leads to the equal number of these pairs for $q < 2$ MeV/c. The difference in the non-Coulomb pairs number for Pt and Ni targets in the range $q < 2$ MeV/c ΔN_f^{nC} can be calculated based on the results of the extrapolation method. The value of ΔN_f^{nC} could not exceed a few percent of N_2. Then the modified x' value is written as:

$$x' = \frac{N_2 - N_3 + \Delta N_f^{nC}}{N_1 - N_3}. \tag{9}$$

2. As the number of the produced atoms is proportional to the Coulomb pairs only, the calculated function $x(\tau)$ should be modified also:

$$x' = \frac{rP_{\text{br}\,2} - P_{\text{br}\,3}}{P_{\text{br}\,1} - P_{\text{br}\,3}}\,. \tag{10}$$

Here r is the ratio between the Coulomb pair fractions for Ni and Pt targets obtained with the extrapolation method.

Due to uncertainties in this corrections the required number of produced atoms will increase. But it will be of the same order as for the extrapolation method taking into account that we are going to measure the lifetime with this method for the different target materials. Thus, adding of the 5 lays Platinum target to the set of our targets will open the new possibility to obtain the lifetime in the simple and reliable way.