Dirac future :

MEASUREMENT OF ENERGY SPLITTING BETWEEN *ns* AND *np* STATES FOR $A_{2\pi}$ AND $A_{\pi K}$ ATOMS

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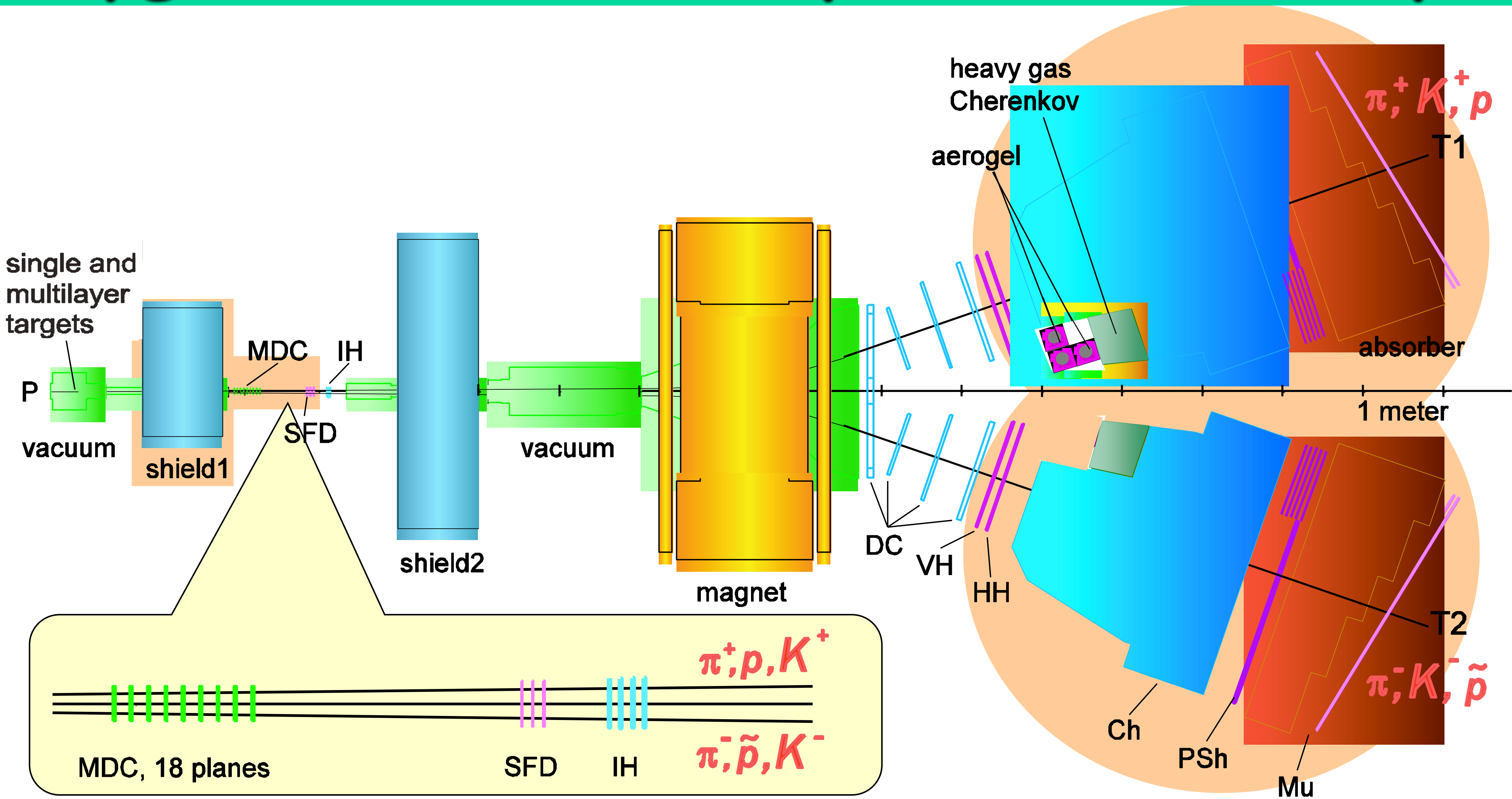
Based on the works :

L.L. Nemenov, **Sov. J. Nucl. Phys. (1985)**

L.L. Nemenov and V.D. Ovsiannikov, **Phys. Lett. (2001)**

Upgraded DIRAC experimental setup



Energy splitting … in theory

Annihilation: $A_{2\pi} \to \pi^0\pi^0$ $1/\tau = W_{ann} \sim (a_0 - a_2)^2$

Energy Splitting between *np* - *ns* states in $A_{2\pi}$ atom

$$\Delta E_n \equiv E_{ns} - E_{np}$$
$$\Delta E_n \approx \Delta E_n^{vac} + \Delta E_n^{s} \qquad \Delta E_n^{s} \sim 2a_0 + a_2$$

For $n = 2$

$$\Delta E_2^{vac} = -0.107 \text{ eV}$$ *from QED calculations*

$$\Delta E_2^{s} \approx -0.45 \text{ eV}$$ *numerical estimated value from ChPT*

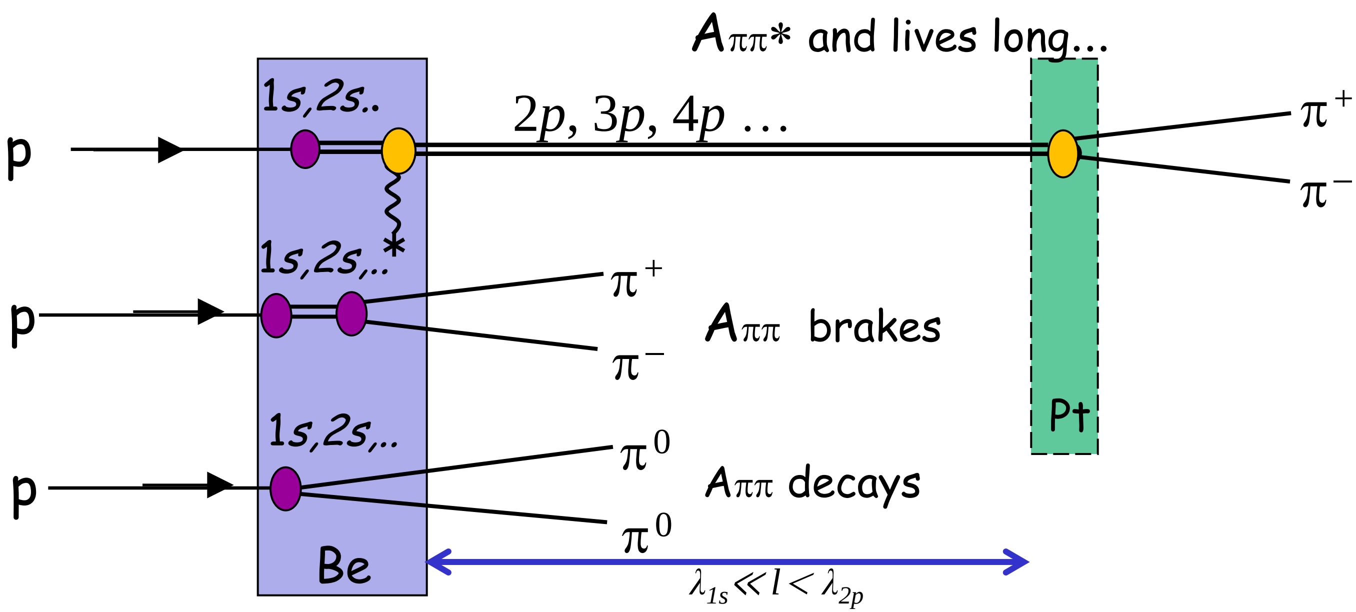
$a_0 = 0.220 \pm 0.005$
$a_2 = -0.0444 \pm 0.0010$

(2001) G. Colangelo, J. Gasser and H. Leutwyler

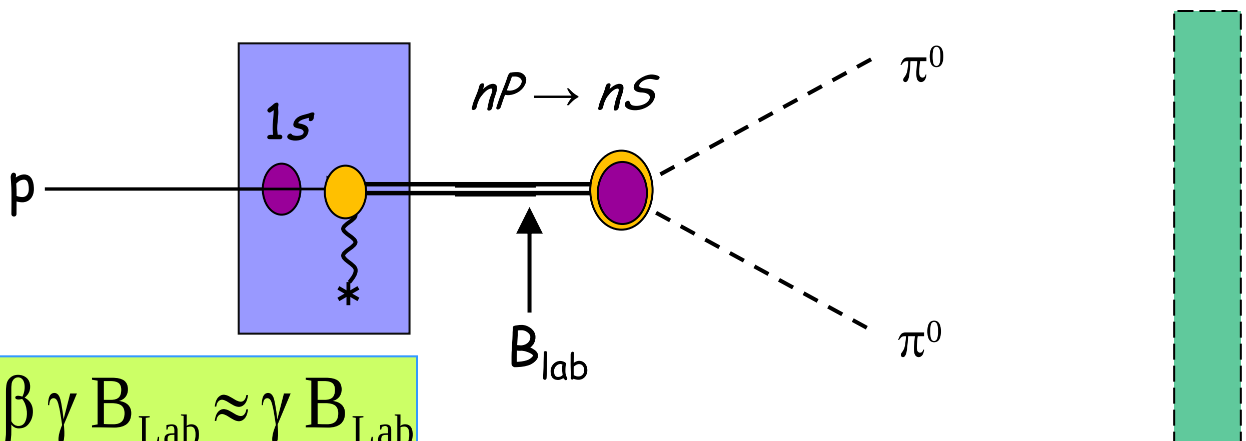
$$\Rightarrow \Delta E_2 \approx -0.56 \text{ eV}$$

Measurement of τ and ΔE allows one to obtain a_0 and a_2 separately

Energy splitting … in practice



Magnetic field → Electric field → mixing $2p_0$-2s,…

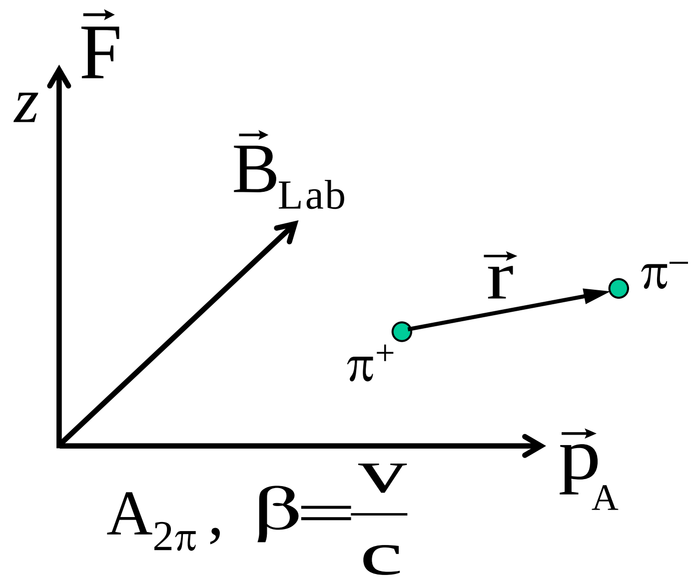


decreased number of atomic pairs

$$F = \beta \gamma B_{Lab} \approx \gamma B_{Lab}$$

External magnetic and electric fields

Atoms in a beam are influenced by external magnetic field
and the relativistic Lorentz factor



$\vec{r} \equiv$ relative distance between π^+ and π^- mesons in $A_{2\pi}$ atom

$\vec{B}_{Lab} \equiv$ laboratory magnetic field

$\vec{F} \equiv$ electric field in the CM system of an $A_{2\pi}$ atom

$$F = \beta \gamma B_{Lab} \approx \gamma B_{Lab}$$

Energy splitting … in formulae

The initial state of the Aππ atom in the 2p state after the target is written as :

$$\Psi(\vec{r},0)=\sum_m \varphi_{2p,m}(\vec{r})\cdot a_m{}^{(0)}$$

At the Hydrogen-like Hamiltonian H0 we add the field interaction V(r) and we study the time evolution of the atom:

$$i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t}=(\hat{H}_0(\vec{r})+\hat{V}(\vec{r}))\Psi(\vec{r},t)$$

$$\hat{V}(\vec{r})=eFz$$

To account for the decay process, the imaginary parts of the Eigen-energy is introduced

$$\Psi(\vec{r},t)=\sum_m a_m(t)e^{-iE_{2p}t/\hbar}\varphi_{2pm}(\vec{r})+a(t)e^{-iE_{2s}t/\hbar}\varphi_{2s}(\vec{r})$$

$$E_{2p}=\mathrm{Re}(E_{2p})-i\frac{\Gamma_{2p}}{2},E_{2s}=\mathrm{Re}(E_{2s})-i\frac{\Gamma_{2s}}{2}$$

The first order correction in the Energy for the new Eigen-states is proportional to :

$$\mathrm{M}\equiv\left\langle\varphi_{2p,0}\middle|\hat{V}\middle|\varphi_{2s}\right\rangle=-\frac{3\mathrm{F}\,\hbar^2}{\mu\,\mathrm{e}}$$

$$\mu=\mathrm{m}_\pi/2$$

The states with m=-1,1 are not touched by the Electric field (perpendicular to the Atom’s velocity)

.. and ..

The wave function assume now the form of a mixture between |2,p,0> and |2,s,0> states that evolves in time

$$\Psi(\vec{r},t) = a_0[f_1(t)\varphi_{2p0}(\vec{r}) + f_2(t)\varphi_{2s}(\vec{r})]\cdot e^{-iE_{2p}t/\hbar}$$

Where f_1 and f_2 depend on ($\Delta E_n = E_{2p} - E_{2s}$) and M

The probability for the $A_{\pi\pi}$ to remain in a state with n=2 :

$$P(t) = \langle \Psi^*(\vec{r},t) \mid \Psi(\vec{r},t) \rangle$$

and the probability to decay .. N(t) = 1-P(t)

$$N(t) = N_0 e^{-t/\tau_{eff}}$$

$$\tau_{eff} = \frac{\tau_{2p}}{1+120|\xi(M,\Delta E_n)|^2}$$

N_0 is extracted from the data already taken in 2010 with Berillim target and Nikel target

The dependence of $A_{2\pi}$ lifetime in 2*p*-states τ_{eff} from a strength of the electric field F

with: $$\tau_{\text{eff}} = \frac{\tau_{2p}}{1+120|\xi|^2}$$

where: $$|\xi|^2 \approx \frac{F^2}{\left(E_{2p} - E_{2s}\right)^2}$$

$B_{\text{Lab}} = 4$ Tesla

$$\begin{cases} \gamma = 20\,, & |\xi| = 0.1 \quad \Rightarrow \quad \tau_{\text{eff}} = \dfrac{\tau_{2p}}{2.2} \\ \gamma = 40\,, & |\xi| = 0.2 \quad \Rightarrow \quad \tau_{\text{eff}} = \dfrac{\tau_{2p}}{6} \end{cases}$$

The End, thank you

Production of the long-lived states

In inclusive processes, $A_{2\pi}$ are produced in *s*-states according to the following distribution ⟹ $\Delta E_n \sim 1/n^3$

Hence: $W_{1s} = 83\%$ $W_{2s} = 10.4\%$ $W_{3s} = 3.1\%$ $W_{n>3s} = 3.5\%$

- excitation in the target, $1s \to 2p, 3p, 4p\ldots$ $2s \to 2p, 3p, 4p\ldots$

For Ni: $1s \to 2p$ – 23%, $3p$ – 4%, $4p$ – 1.5%
$2s \to 2p$ – 32%, $3p$ – 8.6%, $4p$ – 1.8%
$3s \to 3p$ – 38%, $4p$ – 5.2%, $5p$ – 1.1%

Probabilities of *ns* to *np* transitions Without taking into account W_{ns}

- spontaneous, $2p \xrightarrow[\gamma]{} 1s$ $\tau_{2p} = 1.17 \times 10^{-11}$ s

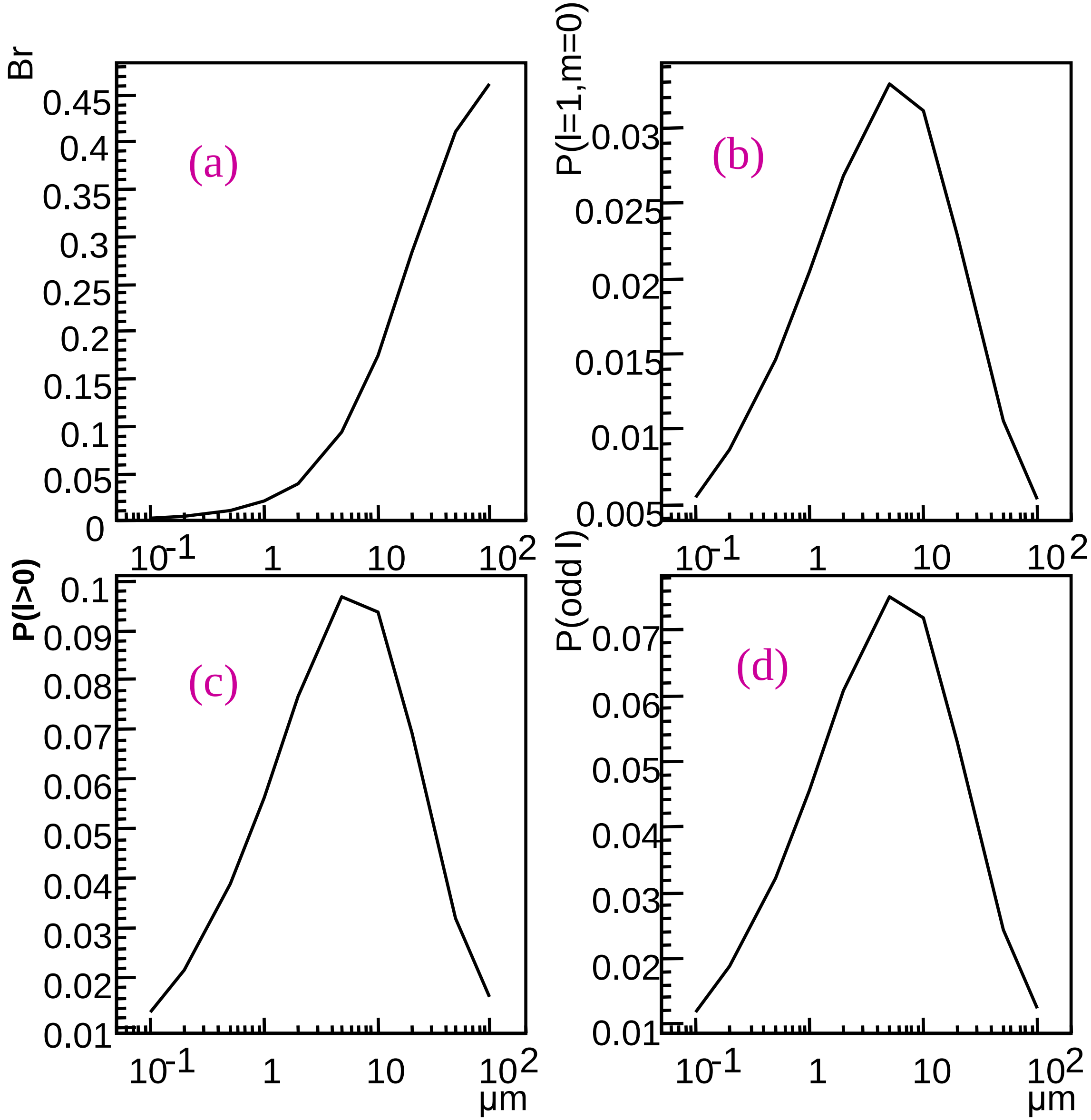
$3p \xrightarrow{\gamma} 1s,\ 2s$

$4p \xrightarrow{\gamma} 1s,\ 2s,\ 3s$

Probabilities of the $A_{2\pi}$ break-up (Br) and yields of the long-lived states for different targets provided the maximum yield of summed population of the long-lived states: $\Sigma(l \geq 1)$

Target Z	Thickness μm	Br	$\Sigma(l \geq 1)$	$2p_0$	$3p_0$	$4p_0$	$\Sigma(l = 1, m = 0)$	$\Sigma(\text{odd } l)$
04	50	2.63%	5.86%	1.05%	0.54%	0.20%	1.93%	4.49%
06	50	5.00%	6.92%	1.46%	0.51%	016%	2.52%	5.24%
13	20	5.28%	7.84%	1.75%	0.57%	0.18%	2.63%	6.05%
28	5	9.42%	9.69%	2.40%	0.58%	0.18%	3.29%	7.52%
78	2	18.8%	10.5%	2.70%	0.54%	0.16%	3.53%	8.10%

Yields of metastable atoms from Nickel target Z = 28 as a function of the target thickness



(a) Probabilities of the $A_{2\pi}$ break-up (Br).

Summed population of the long-lived states:

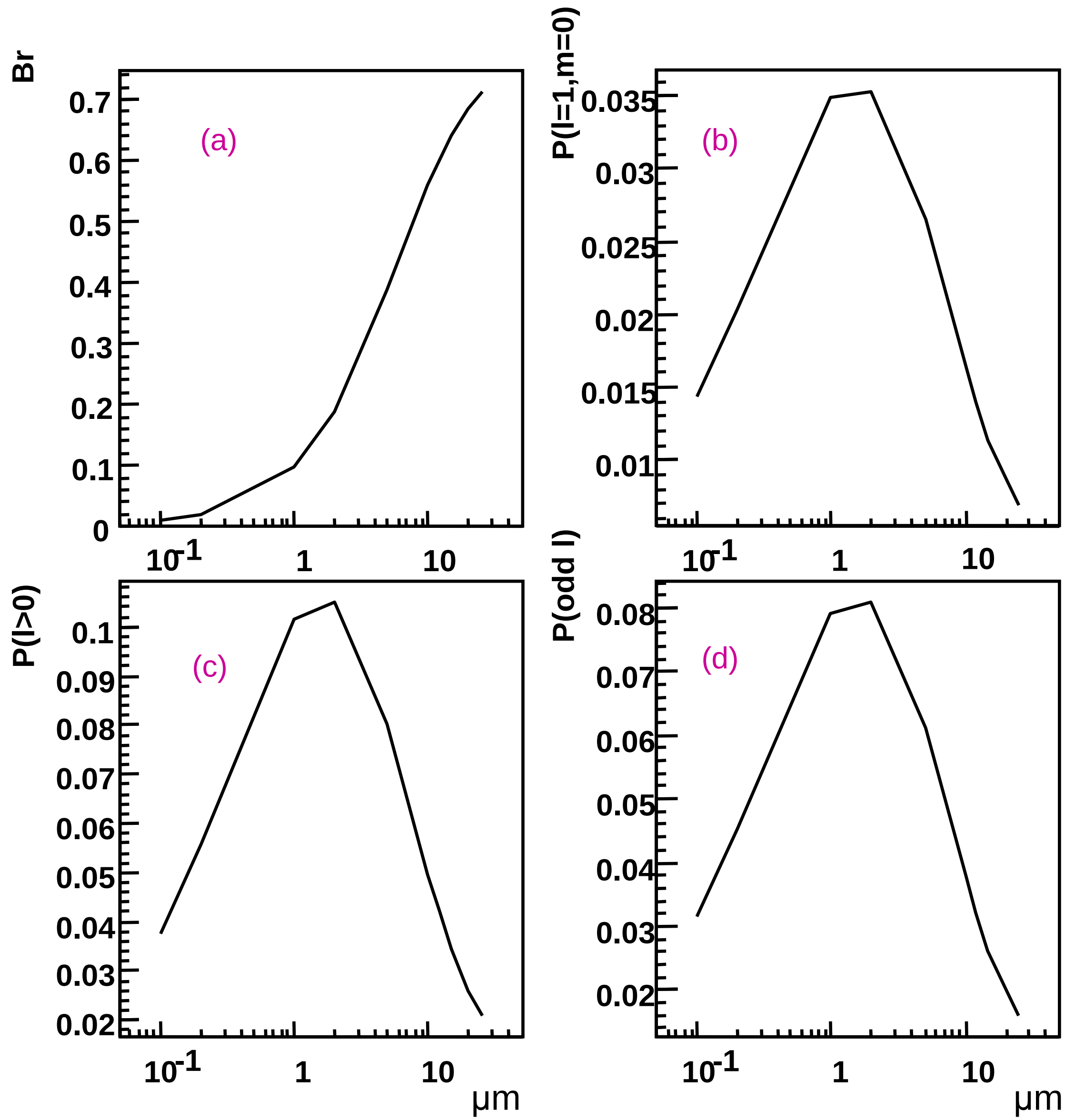
(b) np ($m = 0$) states;

(c) all states with $l > 0$;

(d) states with odd l.

The $A_{2\pi}$ lifetime was assumed to be 3.0×10^{-15} s and the atom momentum 4.5 GeV/c.

Yields of metastable atoms from Platinum target Z = 78 as a function of the target thickness



(a) Probabilities of the $A_{2\pi}$ break-up (Br).

Summed population of the long-lived states:

(b) *np* ($m = 0$) states;

(c) all states with $l > 0$;

(d) states with odd l.

The $A_{2\pi}$ lifetime was assumed to be 3.0×10^{-15} s and the atom momentum 4.5 GeV/c.

DIRAC new set-up