Chapter 1

Drift velocity

1.1 Motion in vacuum

In vacuum, a charge q with mass m subject to an electric field E and a magnetic field B obeys a second order equation of motion, driven by a Lorentz force:

$$m\dot{v} = q\left(E + v \times B\right) \tag{1.1}$$

Typically, this results in spiraling motion.

1.2 The Langevin equation

Electrons in a gas do not follow such orbits when subject to uniform E and B fields, at least not on a scale much larger than the mean free path which is a few microns in detector gases at standard temperature and pressure. Rather, they move along straight lines. This is a consequence of the frequent encounters with gas molecules which continuously randomise, even reverse, the electron velocity.

It can be shown that the steady-state drift velocity v_{D} is determined by the first order equation:

$$v_{\mathrm{D}} = \frac{q\tau}{m}\left(E + v_{\mathrm{D}} \times B\right) \tag{1.2}$$

In this expression, τ stands for the transient time needed to reach the steady-state drift velocity. The equation becomes intuitively clear when one substitutes $\dot{v} = v/\tau$ in Eq. 1.1. The factor $\mu = q\tau/m$ is called the mobility, and it depends on the electric and magnetic fields. Note that contrary to the commonly used convention, μ in these formulae is negative for negatively charged particles, such as electrons.

A solution of Eq. 1.2 is given by the Langevin equation:

$$v_{\mathrm{D}} = \frac{\mu}{1 + \mu^2 B^2}\left(E + \mu E \times B + \mu^2 B (E \cdot B)\right) \tag{1.3}$$

1.3 Magboltz

Magboltz computes the drift velocity using Monte-Carlo techniques, tracing electrons at the microscopic level through numerous collisions with gas molecules. The program does not use the Langevin equation. As can be seen from Fig. 1.1, there can be sizeable differences between the Magboltz and Langevin velocity components.

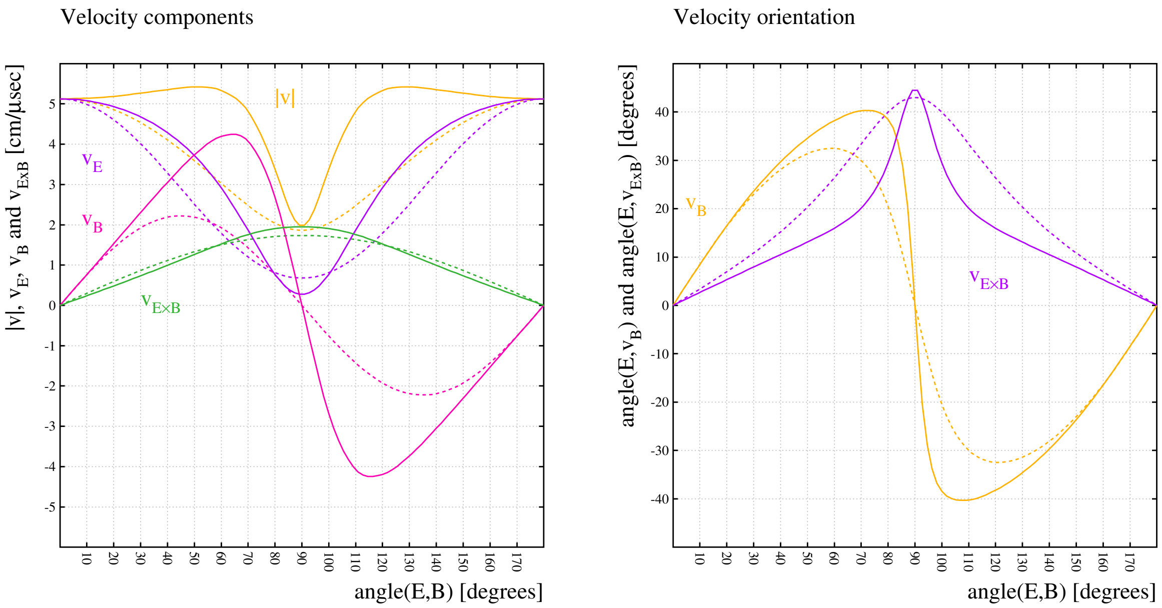

Figure 1.1: For an electric field of $E = 200\ \mathrm{V/cm}$ and a magnetic field of $B = 1\ \mathrm{T}$, oriented with respect to each other at angles ranging over $[0, \pi]$, the graphs compare the drift velocity components according to the Langevin equation (dashed) and according to Magboltz (solid) for a mixture of $90\ \%\ \mathrm{Ar} + 10\ \%\ \mathrm{CH_4}$. The mobility used for the Langevin equation has been chosen so as to make the curves match for parallel E and B fields. One notes good agreement at small angles, but not for large angles.

Magboltz uses a right-handed coordinate system in which the fields are oriented as follows:

- E: in the negative z direction;
- B: in the x-z plane, at an angle θ to E, such that B points in the negative x direction if $\theta = \pi/2$.

Basis vectors for the Magboltz coordinate frame can therefore be obtained from E and B as follows (the normalisation is not shown):

- e_x^M: $-B_{\mathrm{T}} = E \times (E \times B) = -B(E \cdot E) + E(E \cdot B)$,
- e_y^M: $E \times B$,
- e_z^M: $-E$.

In case E and B are parallel, then e_x^M and e_y^M are chosen to be any 2 axes perpendicular to E such the coordinate system is right-handed. Without loss of generality, Magboltz assumes that $0 < \theta < \pi$. Magboltz, as part of its output, shows the electron velocities along these 3 axes.

The velocities along these axes are shown when the transport properties are printed or plotted (using the `GAS-PRINT` and `GAS-PLOT` options in the gas section). The components can be retrieved using procedures:

- v_x^M: `VELOCITY_BTRANSVERSE`
- v_y^M: `VELOCITY_EXB`
- v_z^M: `VELOCITY_E`

1.4 Sign and symmetries of the velocity components

Although Magboltz is believed to be more precise than the Langevin equation, it is easier to determine the signs and symmetry properties from the Langevin equation.

Projecting the Langevin drift velocity vector Eq. 1.3 on the Magboltz axes (assuming $0 < \theta < \pi$), one finds the expressions used to make Fig. 1.1, and which are not explicitely used in Garfield:

$$v = \frac{-\mu E}{1+\mu^2 B^2} \begin{pmatrix} \mu^2 B^2 \cos\theta \sin\theta \\ -\mu B \sin\theta \\ 1 + \mu^2 B^2 \cos^2\theta \end{pmatrix} \tag{1.4}$$

Bearing in mind that $\mu < 0$ for electrons, one sees from Eq. 1.4 that all velocity components are expected to be positive in the Magboltz frame, except for v_x^M which is negative when $\pi/2 < \theta < \pi$.

One also notes from this equation that v_y^M and v_z^M are invariant under $\theta \leftrightarrow \pi - \theta$ while v_x^M changes sign. This behaviour can be seen in Fig. 1.1. The interface between Garfield and Magboltz (not Magboltz itself) uses this symmetry to limit the angles between E and B to the range $[0, \pi/2]$, which therefore does not constitute a loss of generality.

Furthermore, one sees that the velocity is anti-symmetric ($v_D \leftrightarrow -v_D$) under $q \leftrightarrow -q$ only when $E \times B = 0$, i.e. when the fields are parallel or anti-parallel.

1.5 Use in Garfield

The Garfield interface uses a right-handed coordinate system which is a cyclic variation of the one used in Magboltz, except that 2 of the 3 basis vectors have the opposite orientation:

- e_x^G: E,
- e_y^G: $B_\mathrm{T} = (E \times B) \times E = B(E \cdot E) - E(E \cdot B)$,
- e_z^G: $E \times B$.

When using this system, the velocity components have to be undergo the same cyclic permutation as the basis vectors and the signs of 2 velocity components have to be changed:

- $v_x^G = -v_z^M$
- $v_y^G = -v_x^M$
- $v_z^G = +v_y^M$

Transport tables computed by Magboltz are the preferred source of drift velocity vectors in Garfield. The Langevin equation is used whenever the transverse components of the drift velocity vector are not available. This may happen when incomplete measured drift velocities have been entered, or when the transverse components computed by Magboltz are explicitely deleted by the user. Garfield always uses the Langevin equation for ions.