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Reminder of the Edge Effect in Synchrotron Radiation

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Summary

The synchrotron radiation in the LHC will be rather soft and weak, compared to high energy electron machines. Still it is expected to generate non negligible heating and photon-induced gas desorption. A summary of standard formulas and numbers for the LHC have been collected in this note, including a very rough discussion of the spectrum shift expected by the edge effect.

1 Collection of Standard Formulas and values for the LHC at 7 TeV

1.1 Energy Loss and number of Photons

Using the Lorentz factor γ and the velocity β rather than particle energy, the total energy loss in synchrotron radiation per turn for a particle of charge Z (in units of the elementary charge, Z=1 for positrons and protons and Z = 82 for lead ions for example) can be written independently of the particle mass as:

$$U_0 = \frac{(Ze)^2}{3\epsilon_0}\frac{\beta^3\gamma^4}{\rho} \approx 6.0317 \cdot 10^{-9}\ \text{eV m} \quad Z^2\ \frac{\gamma^4}{\rho} \tag{1}$$

At 7 TeV, the Lorentz factor γ is 7460.52 and $\beta = \sqrt{1-\gamma^{-2}} \approx 1 - 9 \cdot 10^{-9}$ for protons and $\gamma = 2962.9$ and $\beta = \sqrt{1-\gamma^{-2}} \approx 1 - 5.7 \cdot 10^{-8}$ for $^{82+}_{208}$Pb ions, such that the approximation $\beta = 1$ can be used.

With the bending radius $\rho = 2784.32$ m of the LHC, the critical energy, frequency and wavelength are (numbers for 7 TeV protons and in brackets for lead ions) [1]:

$$E_c = \frac{3}{2}\,\hbar c\,\frac{\gamma^3}{\rho} = 44.14\ \text{eV} \qquad (2.765\ \text{eV}) \tag{2}$$

$$f_c = \frac{E_c}{2\pi\hbar} = \frac{3c}{4\pi}\frac{\gamma^3}{\rho} = 1.067 \cdot 10^7\ \text{GHz} \qquad (6.686 \cdot 10^5\ \text{GHz}) \tag{3}$$

$$\lambda_c = \frac{c}{f_c} = \frac{4\pi\rho}{3\gamma^3} = 28.087 \text{ nm} \qquad (448.39 \text{ nm}) \tag{4}$$

Half of the synchrotron radiation is radiated below, and the other half above the the critical frequency. The mean photon energy is about 30% of the critical energy:

$$< E_\gamma >= \frac{8}{15\sqrt{3}}\, E_c = \frac{4}{5\sqrt{3}}\, \hbar c\, \frac{\gamma^3}{\rho} \qquad \left(\frac{8}{15\sqrt{3}} \approx 0.30792\right)$$

The number of photons radiated by a particle with charge Z per turn is ($\alpha = e^2/\ 4\pi\epsilon_0\hbar c$ is the fine-structure constant):

$$N_\gamma = \frac{5\pi\ \alpha\ Z^2\ \gamma}{\sqrt{3}} = 493.73 \qquad (1318468.) \tag{5}$$

and the total energy loss by one particle per turn (Eq. 1):

$$U_0 = N_\gamma\ < E_\gamma >= \frac{(Ze)^2}{3\epsilon_0}\frac{\gamma^4}{\rho} \tag{6}$$

or 6.71 keV for protons and 1122.6 keV for ${}^{82+}_{208}$Pb ions. These formulas apply to unscreened, incoherent synchrotron radiation for a circular machine with a constant bending radius.

A summary of synchrotron radiation parameters is given in table 1.

Machine	Particle	Z	p/Z GeV/c	γ	ρ m	E_c eV	n_γ/Z	U_0/Z eV
DaΦne	e	1	0.51	998	97.7	457	66	$9.3{\cdot}10^3$
LEP1	e	1	45.6	89237	3026	$69.5{\cdot}10^3$	5906	$126.4{\cdot}10^6$
LEP2	e	1	94.5	184932	3026	$619{\cdot}10^3$	12239	$2331{\cdot}10^6$
LHC inj.	p	1	450	479.6	2784	$12{\cdot}10^{-3}$	31.7	$114{\cdot}10^{-3}$
LHC	p	1	7000	7460.5	2784	44.1	494	$6.7{\cdot}10^3$
LHC	${}^{82+}_{208}$Pb	82	7000	2962	2784	2.77	16079	$13.7{\cdot}10^3$
SPS	p	1	450	479.6	741.3	$44.1{\cdot}10^{-3}$	31.7	0.430

Table 1: Synchrotron radiation parameters

There is a lot of general, excellent literature on synchrotron radiation or more generally, radiation of relativistic, accelerated charges, discussing more general cases and angular distributions like [2, 3, 4, 5].

1.2 Photon Spectrum

The photon spectrum can be written as

$$\frac{d\,n_\gamma}{dE_\gamma} = \frac{1}{E_c}\ \sqrt{3}\ \alpha\ Z^2\ \gamma\ \ \mathcal{K}_{5/3}(\tfrac{E_\gamma}{E_c}) \tag{7}$$

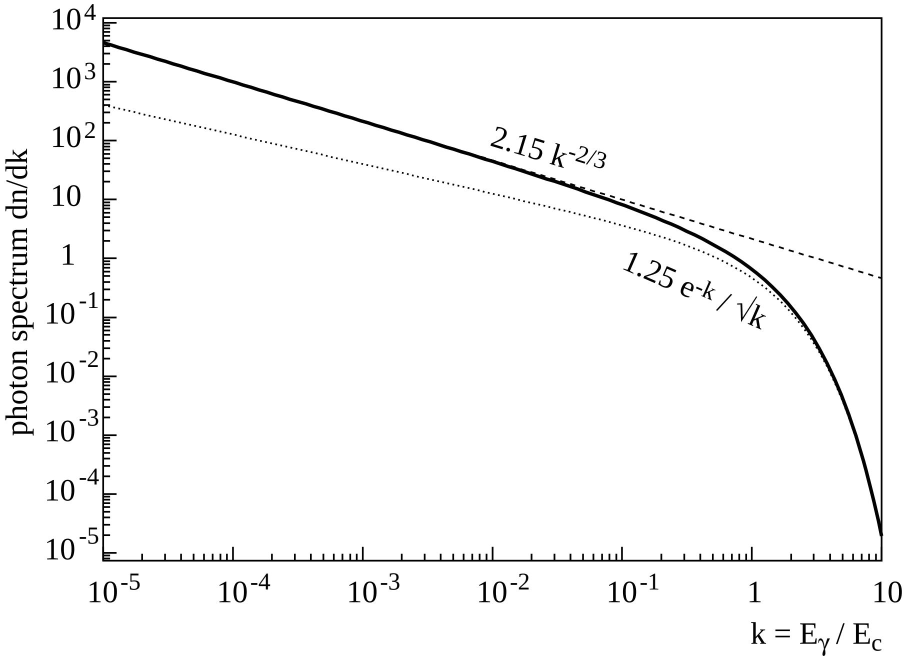

Figure 1: Shape of the synchrotron radiation photon-spectrum $\mathcal{K}_{5/3}(k) = \int_k^\infty K_{5/3}(x)dx$ (solid line). The low energy (dashed line) and high energy (dotted line) approximations are also shown.

where $K_{5/3}$ is a modified Bessel function of the third kind. A universal, dimensionless photon spectrum is obtained, when the photon energy is expressed in units of the critical energy $k = E_\gamma/E_c$:

$$\frac{d\,n_\gamma}{dk} = \sqrt{3}\ \alpha\ Z^2\ \gamma\ \mathcal{K}_{5/3}(k) \quad \text{where} \quad \mathcal{K}_{5/3}(k) = \int_k^\infty K_{5/3}(x)dx \tag{8}$$

For numerical evaluation of the spectrum see [6] and for Monte Carlo generation [7] [1]. The integrated modified Bessel function $\mathcal{K}_{5/3}(k)$ with its approximations for small arguments

$$\mathcal{K}_{5/3}(k) \approx 2^{2/3}\,\Gamma(2/3)\ k^{-2/3} \tag{9}$$

and for large arguments

$$\mathcal{K}_{5/3}(k) \approx \sqrt{\frac{\pi}{2k}}\ e^{-k} \tag{10}$$

are shown in Fig 1. The number of photons radiated per turn can be obtained by integration of Eq. 8:

$$N_\gamma = \int_0^\infty \frac{d\,n_\gamma}{dk}\ dk = \sqrt{3}\ \alpha\ Z^2\ \gamma\ \int_0^\infty K_{5/3}(x)dx = \sqrt{3}\ \alpha\ Z^2\ \gamma\ \cdot \frac{5\pi}{3} = \frac{5\pi\alpha Z^2\gamma}{\sqrt{3}} \tag{11}$$

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[1] available from my home page http://home.cern.ch/∼hbu/Welcome.html

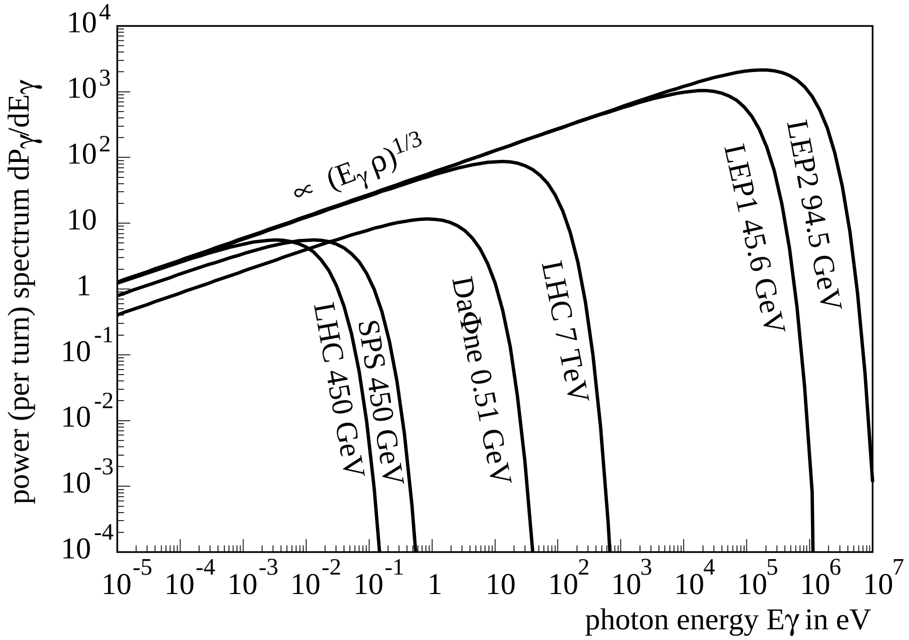

Figure 2: Synchrotron radiation power spectra. Comparison of the LHC and LEP spectra shows, that the power spectrum extends to higher photon energies with the increase in critical energy (proportional to γ^3), but remains the same for low photon energies. The synchrotron spectrum in the LHC at collision energy is already harder than in the low energy electron machine DaΦne.

1.3 Power Spectrum

The power spectrum can be written as

$$\frac{d\,P_\gamma}{dE_\gamma} = E_\gamma\,\frac{d\,n_\gamma}{dE_\gamma} = k\,\frac{d\,n_\gamma}{dk} = \sqrt{3}\,\alpha\,Z^2\,\gamma\,k\,\int_k^\infty K_{5/3}(x)dx \tag{12}$$

For photon energies well below the critical energy we can use Eq. 9 and obtain:

$$\frac{d\,P_\gamma}{dE_\gamma} \approx \sqrt{3}\alpha Z^2\gamma\,k\,2^{2/3}\Gamma(\tfrac{2}{3})\;k^{-2/3} = 2^{2/3}\Gamma(\tfrac{2}{3})\sqrt{3}\alpha Z^2\;\gamma k^{1/3} = 2^{2/3}\Gamma(\tfrac{2}{3})\sqrt{3}\alpha Z^2\,\left(\frac{2E_\gamma\rho}{3\hbar c}\right)^{1/3} \tag{13}$$

which does not depend on γ since

$$\gamma\,k^{1/3} = \gamma\,\left(\frac{E_\gamma}{E_c}\right)^{1/3} = \gamma\,\left(\frac{2E_\gamma\,\rho}{3\hbar c\gamma^3}\right)^{1/3} = \left(\frac{2E_\gamma\rho}{3\hbar c}\right)^{1/3}$$

This can be seen in Fig. 2. For a given bending radius, the power spectrum remains the same below the critical energy. A synchrotron light monitor in the LEP/LHC tunnel receives the same power for visible light ($E_\gamma \approx 3$ eV) as long as the critical energy is sufficient, which is the case for protons at several TeV.

2 Rough estimate of the Edge Effect

The spectra given so far applied to unscreened, incoherent synchrotron radiation for a circular machine with a constant bending radius. Any variation of the magnetic field over a length, that is short compared to the formation length of the synchrotron radiation will shift the spectrum to shorter wavelengths or equivalently higher frequencies or photon energies.

The formation length of synchrotron radiation is:

$$L_0 = \frac{mc}{eB} = \frac{\rho}{\gamma} \tag{14}$$

The ratio L_0/L of formation length to the length of the edge field gives a rough estimate of the wavelength shift to be expected. The edge effect was predicted by Coïsson [8] for the SPS and synchrotron light in the visible spectrum in fact observed [9, 10]. From table 2 we estimate, that the synchrotron spectrum in the SPS (protons 450 GeV) is shifted up by a factor of 15 and very roughly by a factor of 4 in case of the LHC (protons, 7 TeV). With a length of 14.2 m for the LHC dipoles and 0.1 m length of the edge field at both ends it is only $0.2/14.2 = 1.4$% of the synchrotron radiation in the LHC that is affected (for the SPS with 6.26 m long magnets and 0.1 m edge fields it is 3.2%).

Machine	Particle	Z	p/Z GeV/c	γ	ρ m	L_0 m	L_0/L
SPS inj.	p	1	14	14.95	741.3	49.56	496
SPS	p	1	450	479.6	741.3	1.54	15
LHC inj.	p	1	450	479.6	2784	5.81	58
LHC	p	1	7000	7460.5	2784	0.373	3.7
LHC	$^{82+}_{208}$Pb	82	7000	2963	2784	0.94	9.4
LEP1	e	1	45.6	89237	3026	0.0339	0.339

Table 2: Formation length L_0 of Synchrotron radiation and the ratio L_0/L for an edge field of length $L = 0.1$m.

The edge effect can be useful to extend synchrotron radiation monitoring using visible light in the LHC down to LHC injection energy [11]. For the vacuum, there could be consequences for the photon-induced desorption yield.

A more quantitative estimate of the edge effect would best be based on measured field distributions and also consider quadrupole fields. Finally, it may be worth checking if the edge effect has any importance on coherent synchrotron radiation with shielding effects[12, 13].

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