SM and MSSM Higgs Theory

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1. SM Higgs Theory

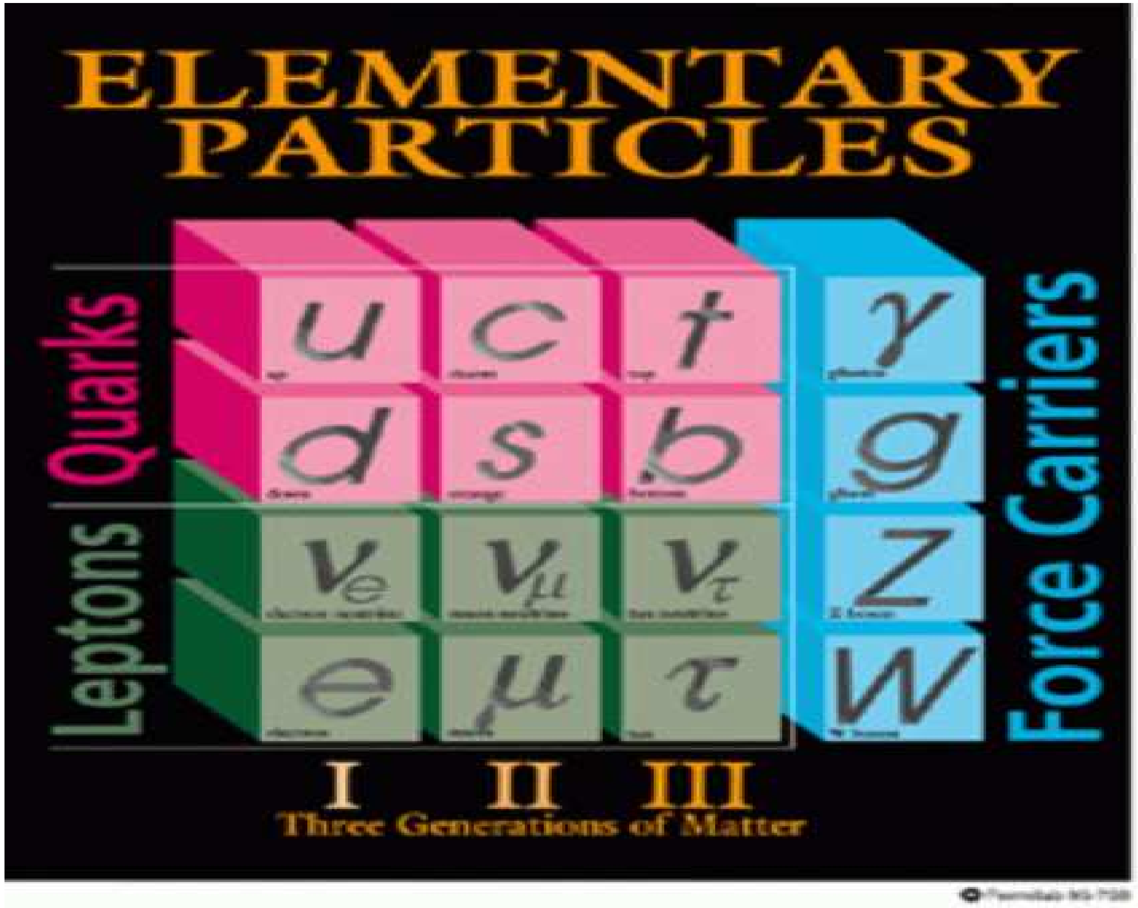
2. Motivation for SUSY

3. MSSM Higgs Theory

4. Conclusions

1. SM Higgs Theory

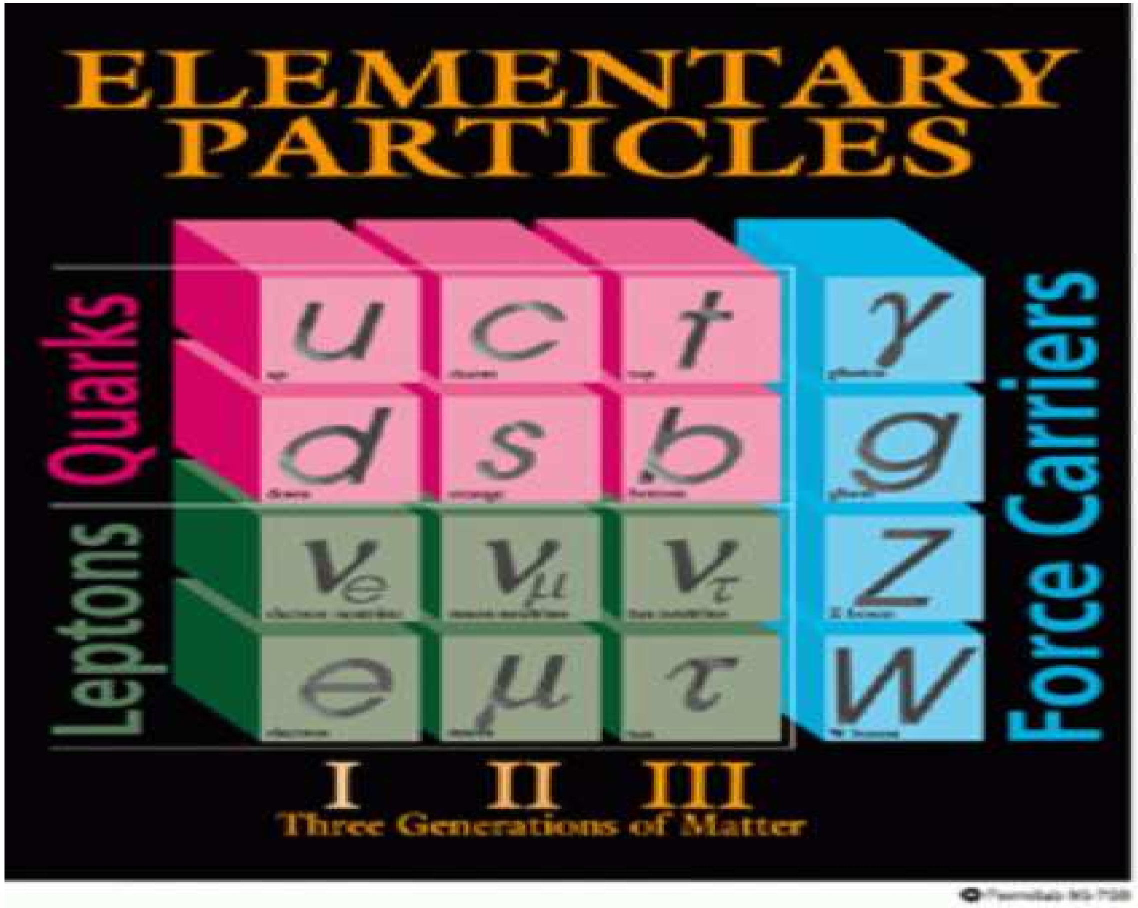
Current status of knowledge: the Standard Model (SM)



$\Rightarrow$ all particles experimentally seen

1. SM Higgs Theory

Current status of knowledge: the Standard Model (SM)



⇒ all particles experimentally seen

⇒ but one particle is missing . . .

Problem:

Gauge fields Z, W^+, W^- are massive

explicite mass terms in the Lagrangian $\Leftrightarrow$ breaking of gauge invariance

Solution: Higgs mechanism

scalar field postulated, mass terms from coupling to Higgs field

Higgs sector in the Standard Model:

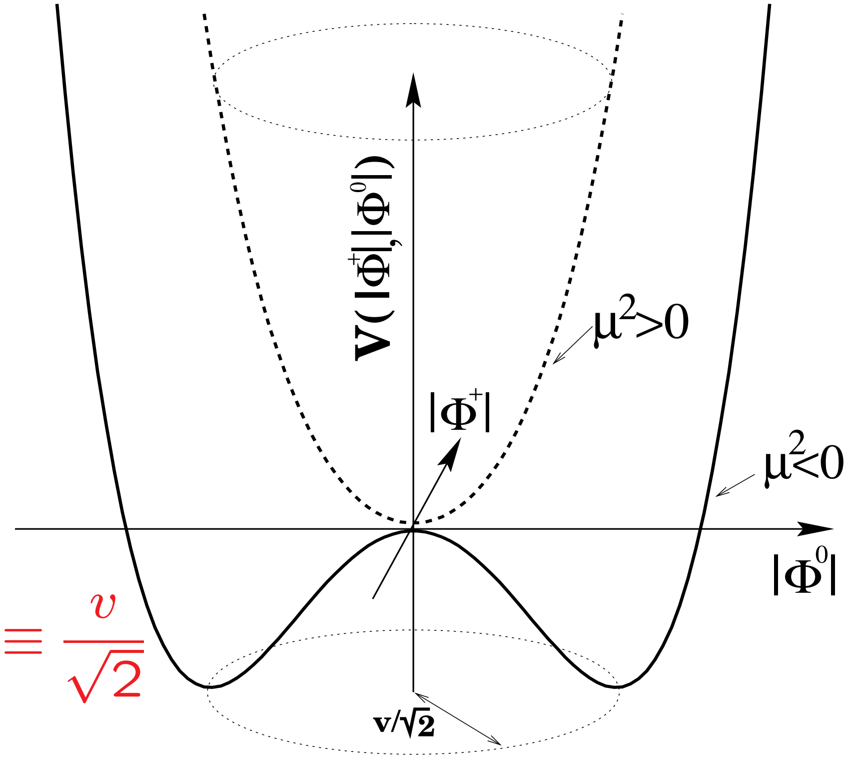
Scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger\Phi\right| + \lambda \left|\Phi^\dagger\Phi\right|^2, \quad \lambda > 0$$

$\mu^2 < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle\Phi_0\rangle| = \sqrt{\frac{-\mu^2}{2\,\lambda}} \equiv \frac{v}{\sqrt{2}}$



$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad \text{(unitary gauge)}$$

H: elementary scalar field, Higgs boson

Lagrange density:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = \quad & (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ & - g_d \bar{Q}_L \Phi d_R \; - \; g_u \bar{Q}_L \Phi_c u_R \\ & - V(\Phi) \end{aligned}$$

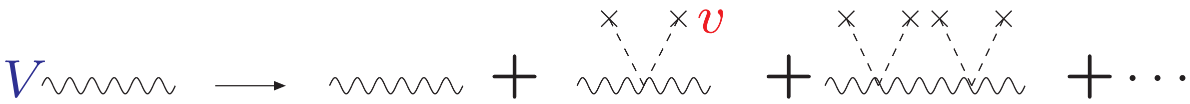
with

$$\begin{aligned} iD_\mu &= i\partial_\mu - g_2 \vec{I}\vec{W}_\mu - g_1 Y B_\mu \\ \Phi_c &= i\sigma_2 \Phi^\dagger \qquad\qquad Q_L \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; \Phi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}, \; \Phi_c \sim \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}$$

Gauge invariant coupling to gauge fields

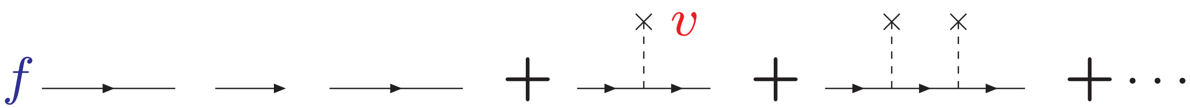
$\Rightarrow$ mass terms for gauge bosons and fermions

1.) $VV\Phi\Phi$ coupling:



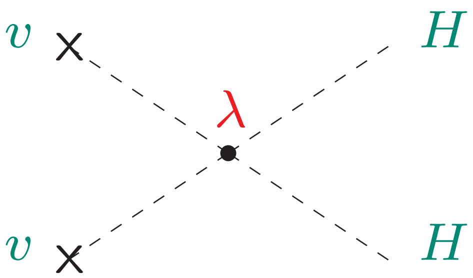
$$\frac{1}{q^2} \to \frac{1}{q^2} + \sum_j \frac{1}{q^2}\left[\left(\frac{gv}{\sqrt{2}}\right)^2 \frac{1}{q^2}\right]^j = \frac{1}{q^2 - M^2} \; : \; M^2 = g^2\frac{v^2}{2} \qquad \Rightarrow M \propto g$$

2.) fermion mass terms: Yukawa couplings:



$$\frac{1}{\not{q}} \to \frac{1}{\not{q}} + \sum_j \frac{1}{\not{q}}\left[\frac{g_f v}{\sqrt{2}} \frac{1}{\not{q}}\right]^j = \frac{1}{\not{q} - m_f} \; : \; m_f = g_f\frac{v}{\sqrt{2}} \qquad \Rightarrow m_f \propto g_f$$

3.) mass of the Higgs boson: self coupling



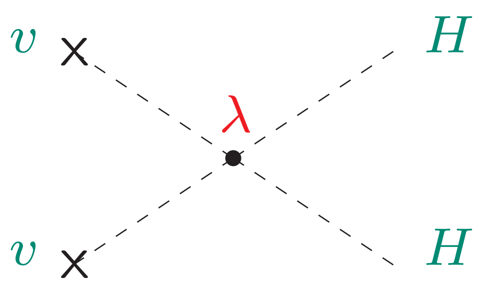
. . .

$$\lambda = M_H^2/v$$

$$M_H = v\sqrt{\lambda}$$ free parameter

$\rightarrow$ last unknown parameter of the SM

3.) mass of the Higgs boson: self coupling



. . .

$$\lambda = M_H^2/v$$

$M_H = v\sqrt{\lambda}$ free parameter

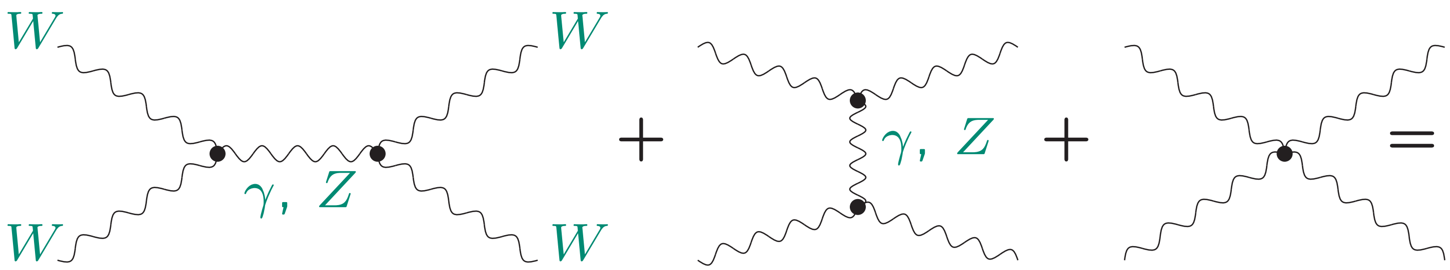
$\rightarrow$ last unknown parameter of the SM

$\Rightarrow$ establish Higgs mechanism $\equiv$ find the Higgs $\oplus$ measure its couplings

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \to W_L W_L$

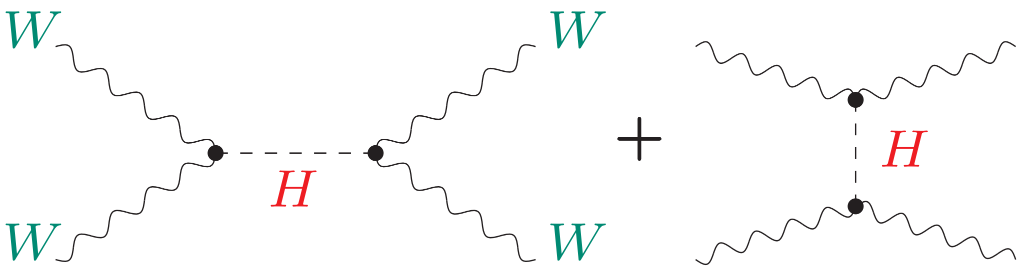
$$\mathcal{M}_V = \text{[diagrams]} = -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1) \quad \text{for } E \to \infty$$



$\Rightarrow$ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$$\mathcal{M}_S = \text{[diagrams]} = g_{WWH}^2 \frac{E^2}{M_W^4} + \mathcal{O}(1) \quad \text{for } E \to \infty$$



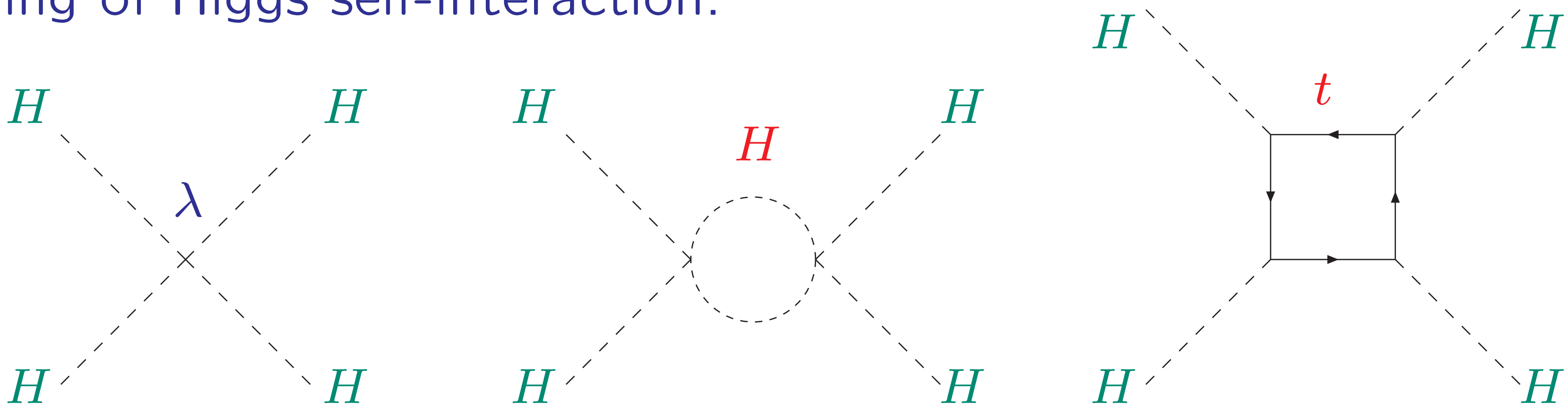
$$\mathcal{M}_{\text{tot}} = \mathcal{M}_V + \mathcal{M}_S = \frac{E^2}{M_W^4}\left(g_{WWH}^2 - g^2 M_W^2\right) + \dots$$

$\Rightarrow$ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g\, M_W$$

What else do we know about the Higgs boson?

Running of Higgs self-interaction:



Renormalization group equation:

$$\frac{d\lambda}{dt} = \frac{3}{8\pi^2}\left[\lambda^2 + \lambda g_t^2 - g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right] , \quad t = \log\left(\frac{Q^2}{v^2}\right)$$

Two conditions:

1.) avoid Landau pole (for large $\lambda \sim M_H^2$)

2.) avoid vacuum instability (for small/negative λ)

1.) avoid Landau pole (for large $\lambda \sim M_H^2$)

$$\frac{d\,\lambda}{d\,t} = \frac{3}{8\,\pi^2}\left[\lambda^2\right]$$

$$\Rightarrow \qquad \lambda(Q^2) = \frac{\lambda(v^2)}{1 - \frac{3\lambda(v^2)}{8\,\pi^2}\log\left(\frac{Q^2}{v^2}\right)}$$

$$\lambda(\Lambda) < \infty \;\Rightarrow\; M_H^2 \le \frac{8\,\pi^2\,v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}$$: upper bound on M_H

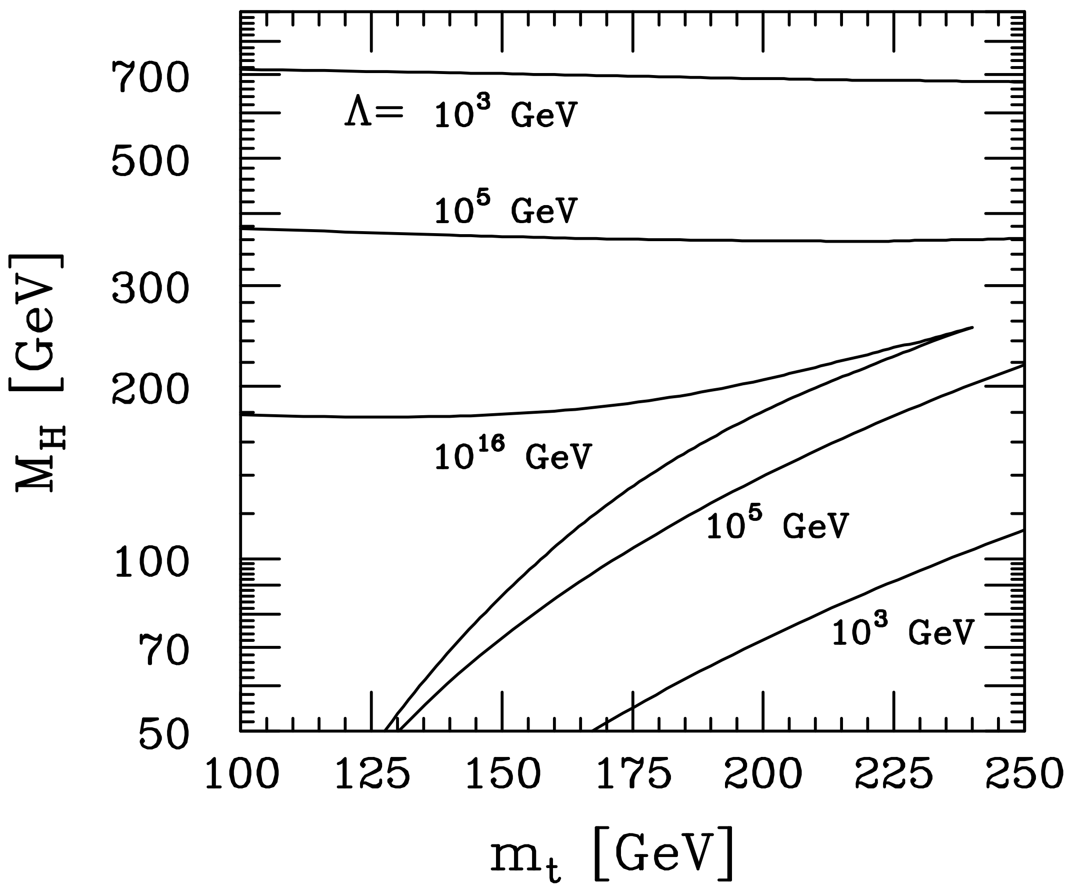
2.) avoid vacuum instability (for small/negative λ): $V(v) < V(0) \Rightarrow \lambda(\Lambda) > 0$

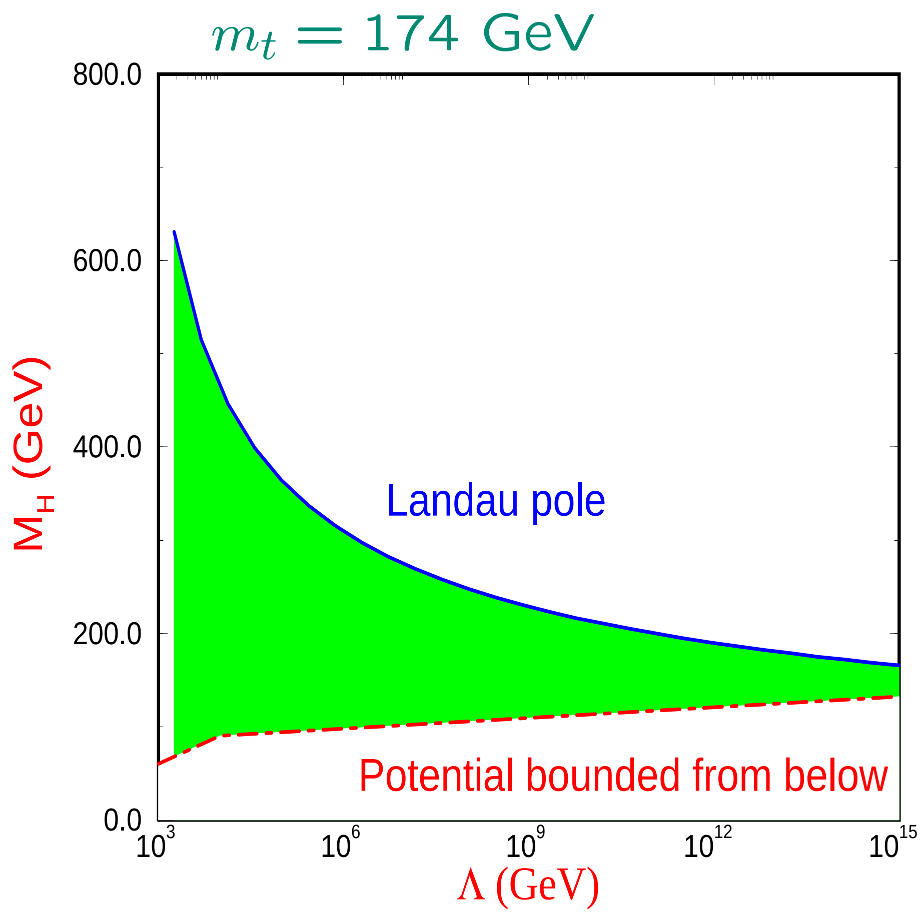
$$\frac{d\,\lambda}{d\,t} = \frac{3}{8\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

$$\Rightarrow \qquad \lambda(Q^2) = \lambda(v^2)\frac{3}{8\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\left(\frac{Q^2}{v^2}\right)$$

$$\lambda(\Lambda) > 0 \;\Rightarrow\; M_H^2 > \frac{v^2}{4\,\pi^2}\left[-g_t^4 + \frac{1}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]\log\left(\frac{\Lambda^2}{v^2}\right)$$: lower bound

Both limits combined:





Λ: scale up to which the SM is valid

$\Lambda = M_{\text{GUT}} \Rightarrow 130\text{ GeV} \lesssim M_H \lesssim 180\text{ GeV}$

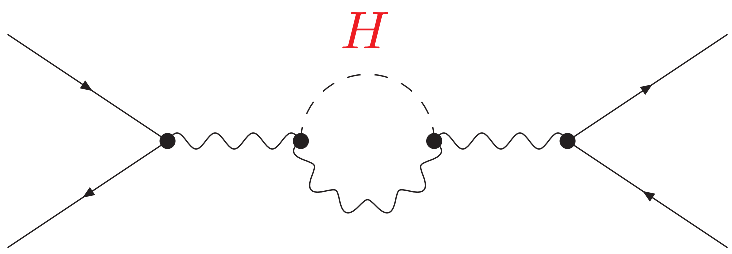
Electroweak Precision Observables (EWPO):

Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$M_W, \sin^2\theta_{\mathrm{eff}}, a_\mu$	$\leftrightarrow$	SM, MSSM , . . .

$\Downarrow$

Test of theory at quantum level: Sensitivity to loop corrections, e.g. H



$\Downarrow$

SM: limits on M_H

Very high accuracy of measurements and theoretical predictions needed

Example: prediction of M_W

Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}(1 + \Delta r)$$

$\Updownarrow$

loop corrections

Evaluate Δr from μ decay $\Rightarrow M_W$

One-loop result for M_W in the SM:

[*A. Sirlin '80*] , [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \quad \Delta\alpha \quad - \quad \frac{c_W^2}{s_W^2}\Delta\rho \quad + \quad \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2 \qquad \log(M_H/M_W)$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Comparison of SM prediction of M_W with direct measurements:

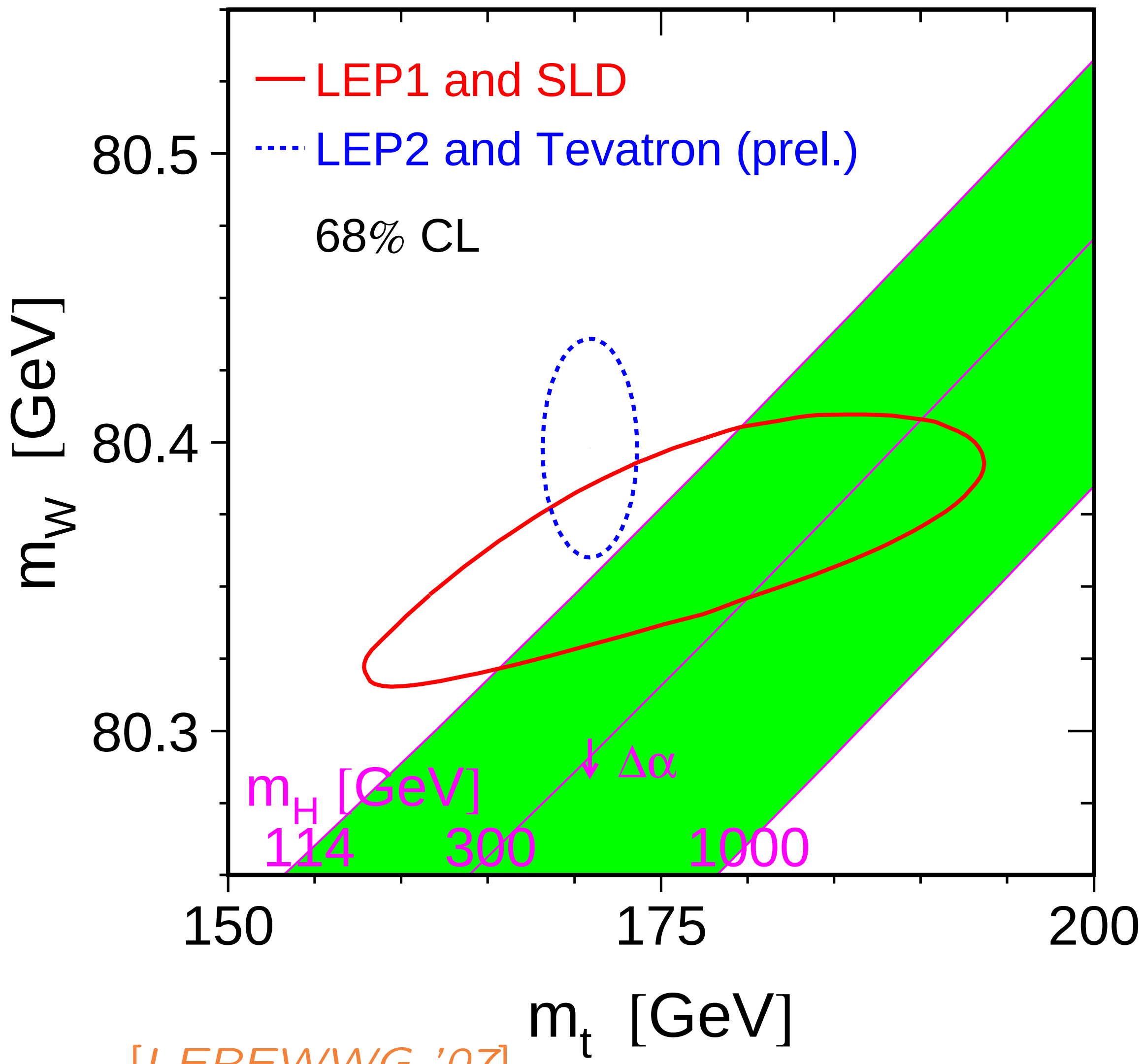
$$\Delta r = -\frac{11 g_2^2\, s_W^2}{96\, \pi^2\, c_W^2} \log\left(\frac{M_H}{M_W}\right)$$

general for EWPO:

$$\Delta \sim g_2^2 \left[\log\left(\frac{M_H}{M_W}\right) + g_2^2 \frac{M_H^2}{M_W^2}\right]$$

leading term: $\log(M_H)$

first term $\sim M_H^2$ with g_2^4



[LEPEWWG '07]

⇒ light Higgs boson preferred

Results for M_H from other EWPO:

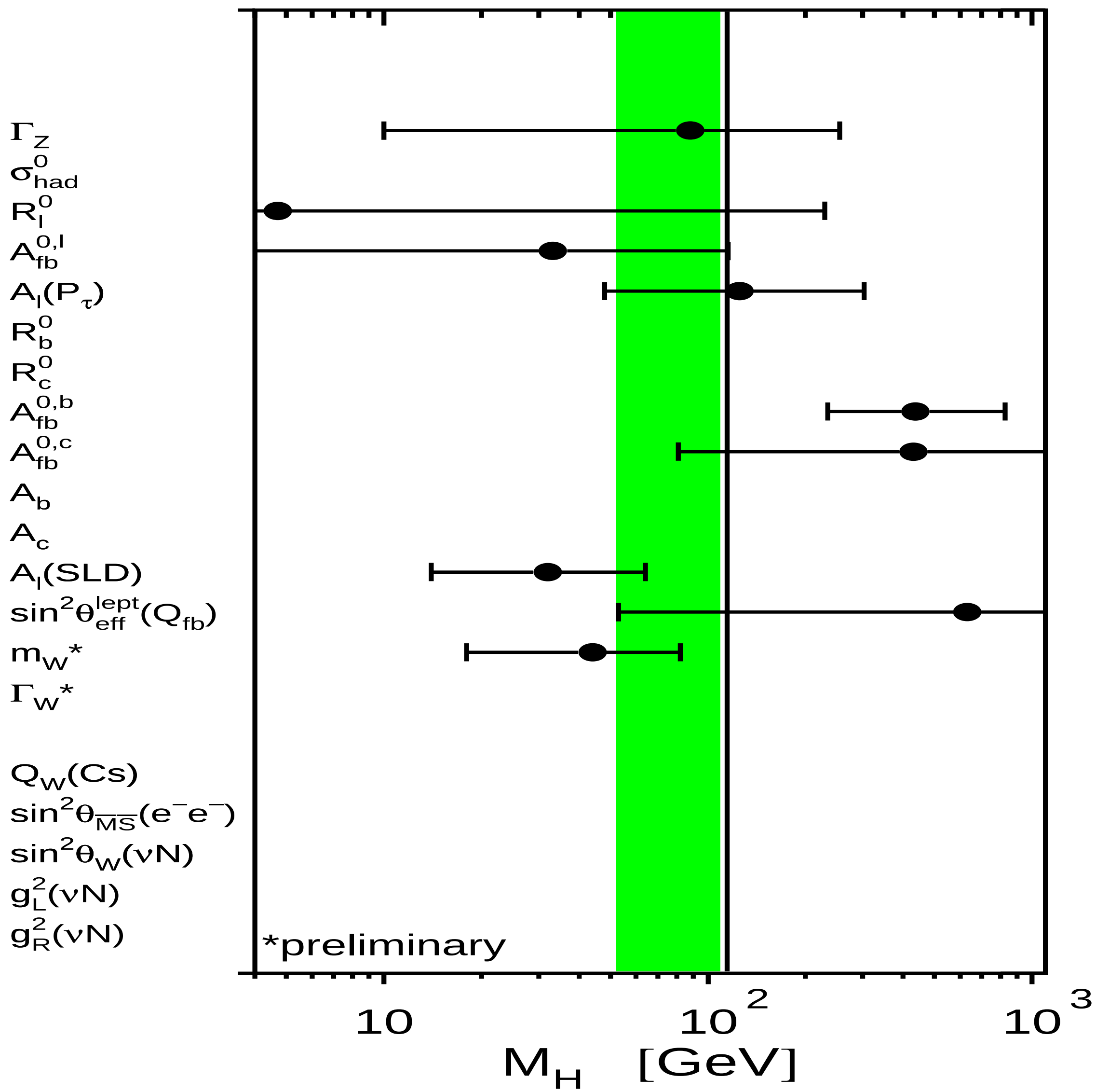
light Higgs preferred by:

M_W, $A_l^{\rm LR}$ (SLD)

heavier Higgs preferred by:

$A_b^{\rm FB}$ (LEP)

$\Rightarrow$ keeps SM alive



$\Rightarrow$ light Higgs boson preferred

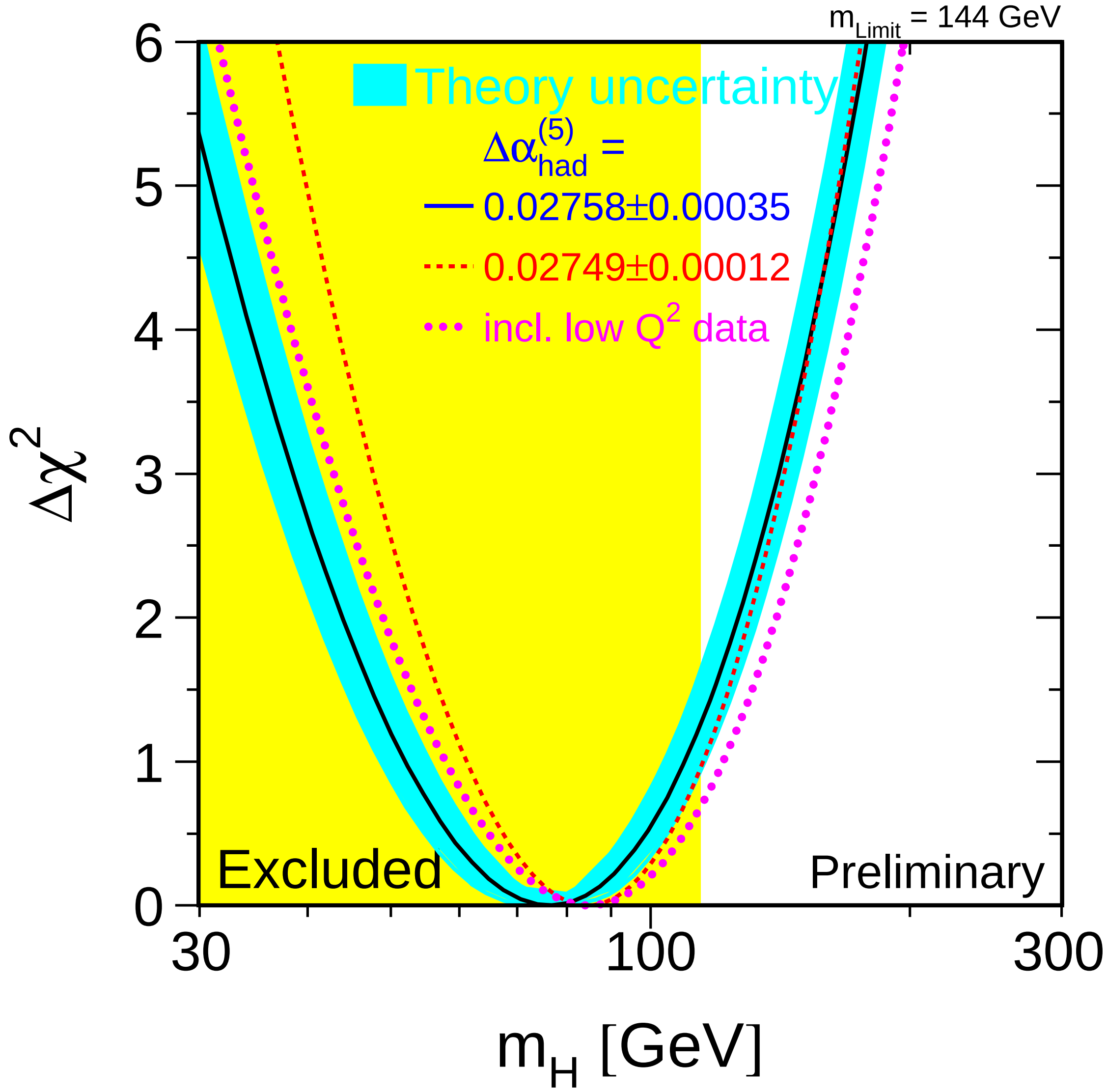
[*LEPEWWG '07*]

Global fit to all SM data:

[*LEPEWWG '07*]

$\Rightarrow M_H = 76^{+33}_{-24}$ GeV

$M_H < 144$ GeV, 95% C.L.



Assumption for the fit:
SM incl. Higgs boson

$\Rightarrow$ no confirmation of
Higgs mechanism

$\Rightarrow$ Higgs boson seems to be light, $M_H \lesssim 150$ GeV

Properties of the SM Higgs boson

1.) Decay to fermions:

coupling:

$$g_{f\bar{f}H} = \left[\sqrt{2}\,G_\mu\right]^{1/2} m_f$$

decay width:

$$\Gamma(H \to f\bar{f}) = N_c \frac{G_\mu\, M_H}{4\sqrt{2}\,\pi} m_f^2(M_H^2) \left(1 - 4\frac{m_f^2}{M_H^2}\right)^{3/2}$$

with N_c = number of colors

Bulk of QCD corrections for decays to quarks are mapped into

$$m_q^2(\text{pole}) \to m_q^2(M_H^2)$$

Dominant decay process: $H \to b\bar{b}$

2.) Decay to heavy gauge bosons ($V = W, Z$):

coupling:

$$g_{VVH} = 2 \left[\sqrt{2}\, G_\mu\right]^{1/2} M_V^2$$

on-shell decay width ($M_H > 2M_V$):

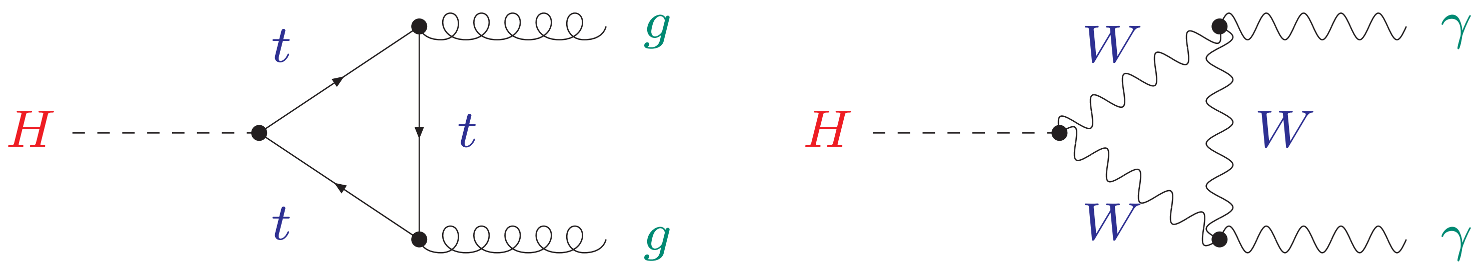
$$\Gamma(H \to VV) = \delta_V \frac{G_\mu M_H^3}{16\sqrt{2}\,\pi} \left(1 - 4\frac{M_V^2}{M_H^2} + 12\frac{M_V^4}{M_H^4}\right) \left(1 - 4\frac{M_V^2}{M_H^2}\right)^{1/2}$$

with $\delta_{W,Z} = 2, 1$

off-shell decay width ($M_H < 2M_V$):

$$\Gamma(H \to VV^*) = \delta'_V \frac{3G_\mu^2 M_H}{16\,\pi^3} M_V^4 \times \text{Integral}$$

3.) Decay to massless gauge bosons (gg, $\gamma\gamma$):



$$\Gamma(H \to gg) = \frac{G_\mu \, \alpha_s^2(M_H^2) \, M_H^3}{36 \sqrt{2} \, \pi^3} \left[1 + C \frac{\alpha_s(\mu)}{\pi} \right]$$

via the top quark loop with

$$C = \frac{215}{12} - \frac{23}{6} \log\left(\frac{\mu^2}{M_H^2}\right) + \mathcal{O}(\alpha_s)$$

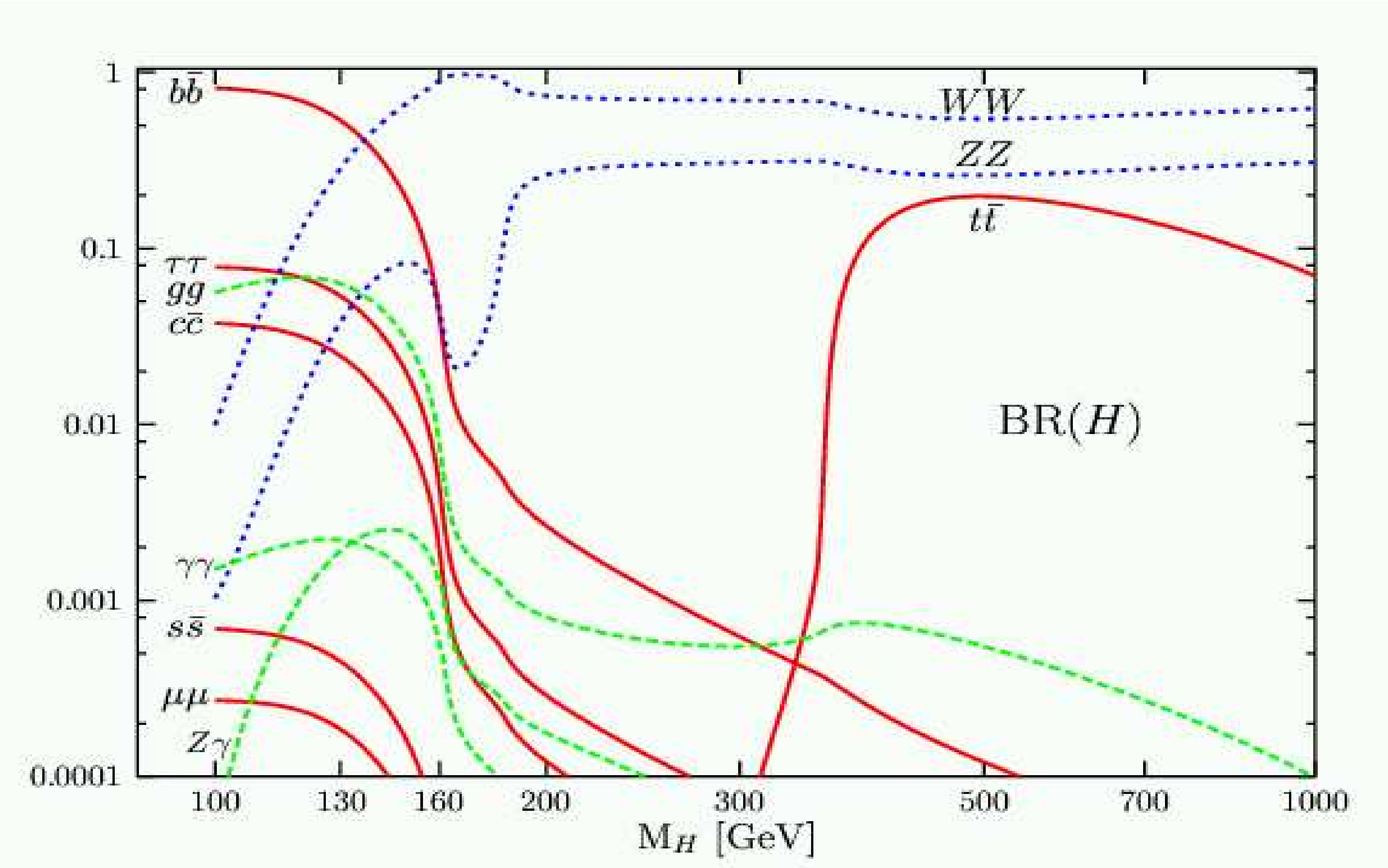
⇒ huge QCD corrections

$$\Gamma(H \to \gamma\gamma) = \frac{G_\mu \, \alpha^2 \, M_H^3}{128 \sqrt{2} \, \pi^3} \left| \frac{4}{3} e_t^2 - 7 \right|^2$$

via the top quark and W boson loop

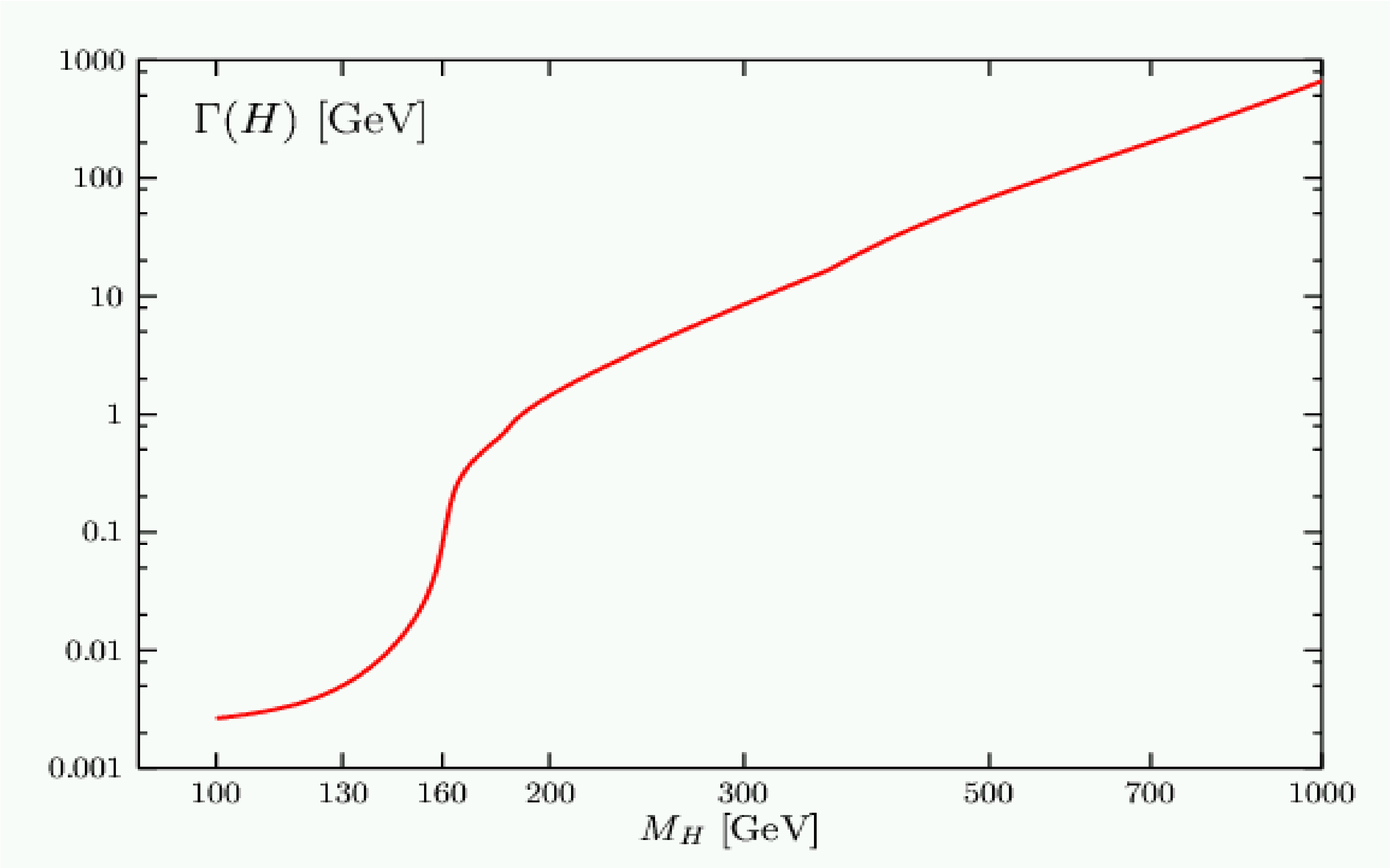
Overview of the branching ratios:

[*taken from hep-ph/0503172*]



The total SM Higgs boson width:

[*taken from hep-ph/0503172*]



Discovering the Higgs boson

What has to be done?

1. Find the new particle

Discovering the Higgs boson

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2. measure its mass ($\Rightarrow$ ok?)

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2. measure its mass ($\Rightarrow$ ok?)
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4. measure couplings to fermions
5. measure self-couplings
6. measure spin, . . .

Discovering the Higgs boson

What has to be done?

1. Find the new particle — T
2. measure its mass ($\Rightarrow$ ok?) — T
3. measure coupling to gauge bosons
4. measure couplings to fermions
5. measure self-couplings
6. measure spin, . . .

T = Tevatron,

Discovering the Higgs boson

What has to be done?

1. Find the new particle	T	L
2. measure its mass ($\Rightarrow$ ok?)	T	L
3. measure coupling to gauge bosons		L
4. measure couplings to fermions		L
5. measure self-couplings		
6. measure spin, . . .		

T = Tevatron, L = LHC,

Discovering the Higgs boson

What has to be done?

	T	L	I
1. Find the new particle	T	L	I
2. measure its mass ($\Rightarrow$ ok?)	T	L	I
3. measure coupling to gauge bosons		L	I
4. measure couplings to fermions		L	I
5. measure self-couplings			I
6. measure spin, . . .			I

T = Tevatron, L = LHC, I = ILC

We need the LHC and the ILC to find the Higgs
and to establish the Higgs mechanism!

2. Motivation for SUSY

The Standard Model (SM) cannot be the ultimate theory

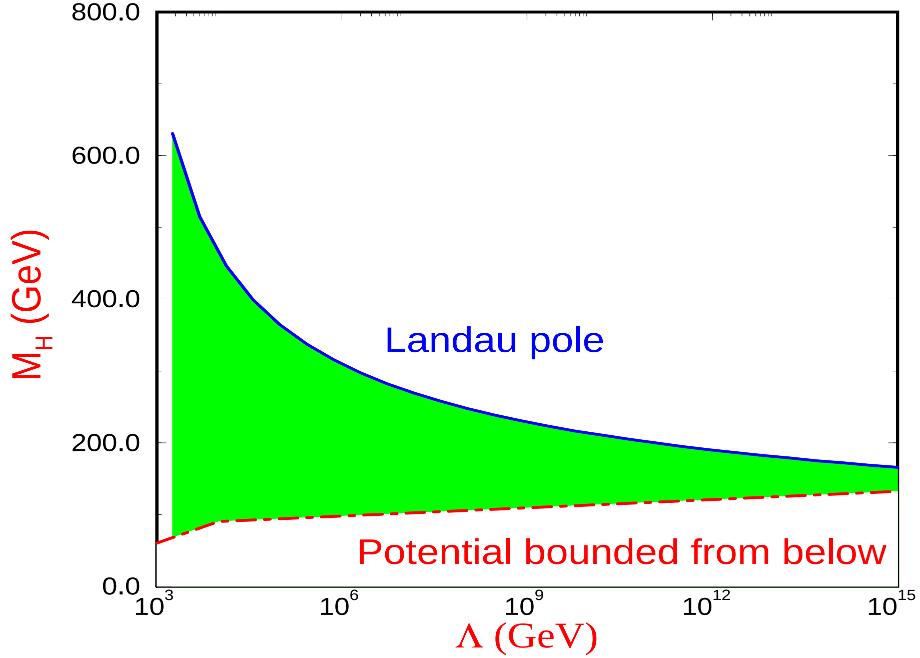
– The SM does not contain gravity
– Further problems: Hierarchy problem
– And another one: SM does not provide Cold Dark Matter candidate

Up to which energy scale Λ can the SM be valid?

– $\Lambda < M_{\rm Pl}$: inclusion of gravity effects necessary

– stability of Higgs potential: $\Rightarrow$

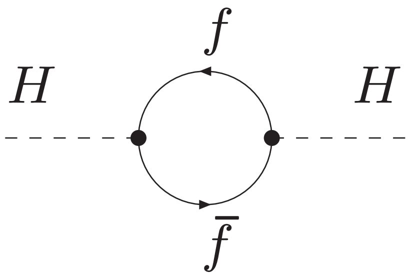
– Hierarchy problem :
Higgs mass unstable
w.r.t. quantum corrections
$\delta M_H^2 \sim \Lambda^2$



Mass is what determines the properties of the free propagation of a particle

Free propagation: 

inverse propagator: $i(p^2 - M_H^2)$

Loop corrections: 

inverse propagator: $i(p^2 - M_H^2 + \Sigma_H^f)$

QM: integration over all possible loop momenta k

dimensional analysis:

$$\Sigma_H^f \;\sim\; N_f\, \lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

$$\text{for } \Lambda \to \infty : \qquad \Sigma_H^f \;\sim\; N_f\, \lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim \Lambda^2} + 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim \ln \Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_{\text{Pl}}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\text{Pl}}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30}\, M_H^2$$

(for $M_H \lesssim 1$ TeV)

– no additional symmetry for $M_H = 0$

– no protection against large corrections

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\text{GUT}}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

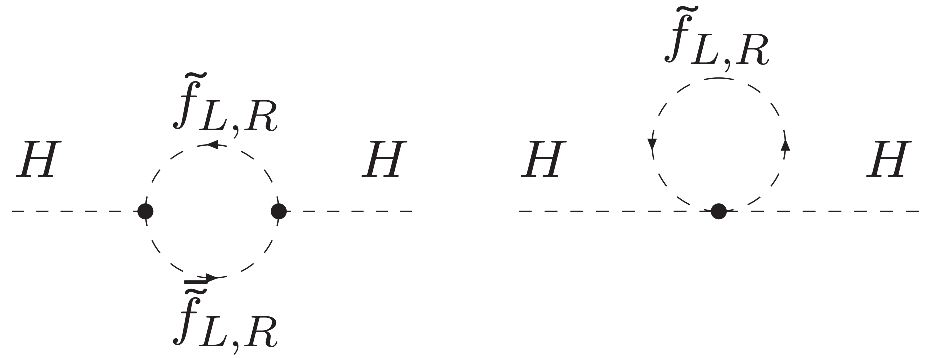
Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{ terms without quadratic div.}$$

for $\Lambda \to \infty$: $\quad \Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2\, \Lambda^2$

$\Rightarrow$ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda^2_{\tilde{f}} = \lambda^2_f$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \lambda^2_{\tilde{f}} = \lambda^2_f$

$$\Rightarrow \; \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda^2_f \, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\rm SUSY} \lesssim 1\,\text{TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Supersymmetry (SUSY) : Symmetry between

$$\text{Bosons} \leftrightarrow \text{Fermions}$$

$$Q\ |\text{Fermion}\rangle \rightarrow |\text{Boson}\rangle$$
$$Q\ |\text{Boson}\rangle \rightarrow |\text{Fermion}\rangle$$

Simplified examples:

$$Q\ |\text{top},\ t\rangle \rightarrow |\text{scalar top},\ \tilde{t}\rangle$$
$$Q\ |\text{gluon},\ g\rangle \rightarrow |\text{gluino},\ \tilde{g}\rangle$$

$\Rightarrow$ each SM multiplet is enlarged to its double size

Unbroken SUSY: All particles in a multiplet have the same mass

Reality: $m_e \neq m_{\tilde{e}} \Rightarrow$ SUSY is broken . . .

. . . via soft SUSY-breaking terms in the Lagrangian (added by hand)

SUSY particles are made heavy: $\boxed{M_{\text{SUSY}} = \mathcal{O}(1\ \text{TeV})}$

More than four reasons as a SUSY motivation

The SM is in a pretty good shape.

Why MSSM? (Is it worth to double the particle spectrum?)

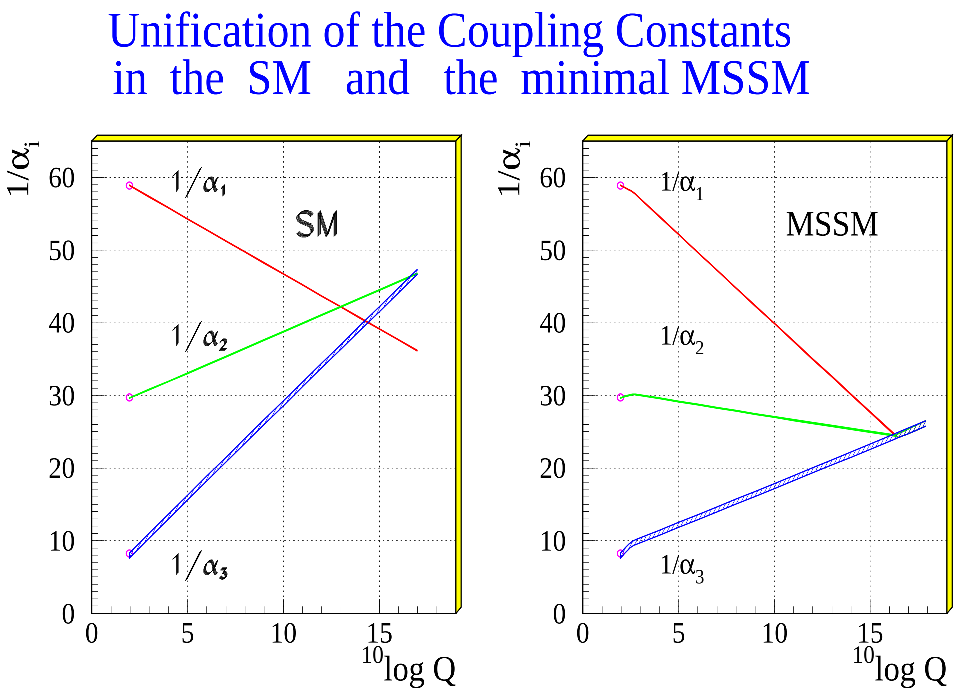
1.) Stability of the Higgs mass against higher-order corr.

2.) Unification of gauge couplings: Not possible in the SM, but in the MSSM (although it was not designed for it.)

3.) SUSY provides CDM candidate

4.) Spontaneous symmetry breaking via Higgs mechanism is automatic in SUSY GUTs

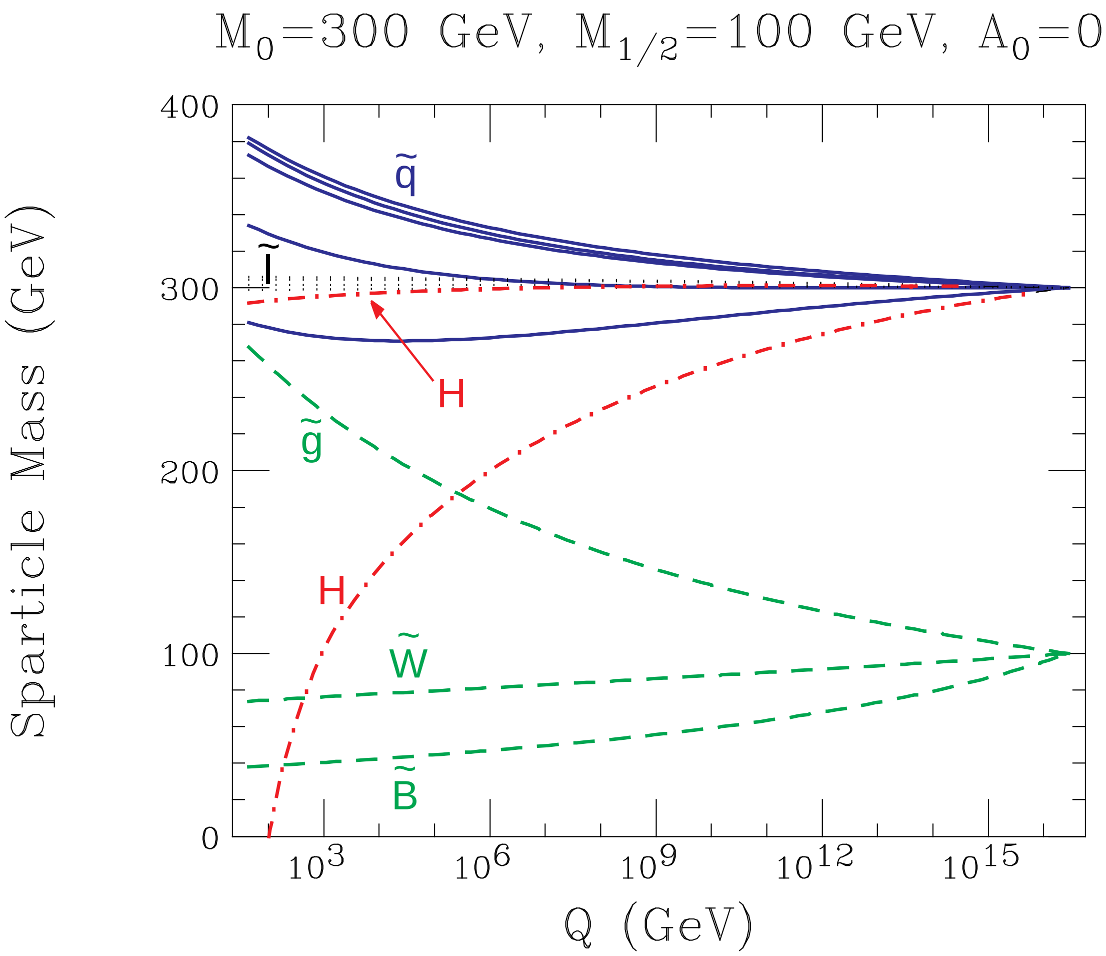
5.) . . .



[*Amaldi, de Boer, Fürstenau '92*]

Take the most simple MSSM version: CMSSM/mSUGRA
→ just three GUT scale parameters + $\tan\beta$

⇒ particle spectra from renormalization group running to weak scale



⇒ one parameter turns negative ⇒ Higgs mechanism for free

The Minimal Supersymmetric Standard Model (MSSM)

Superpartners for Standard Model particles

$$\left[u, d, c, s, t, b\right]_{L,R} \quad \left[e, \mu, \tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u}, \tilde{d}, \tilde{c}, \tilde{s}, \tilde{t}, \tilde{b}\right]_{L,R} \quad \left[\tilde{e}, \tilde{\mu}, \tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^{\pm}, H^{\pm}} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}^{\pm}_{1,2} \quad \tilde{\chi}^{0}_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector: Two Higgs doublets

Problem in the MSSM: many scales

3. MSSM Higgs Theory

Comparison with SM case:

$$\mathcal{L}_{\rm SM} = \underbrace{m_d \bar{Q}_L \Phi d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \Phi_c u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \Phi_c = i\sigma_2 \Phi^\dagger, \quad \Phi \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \Phi_c \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L \Phi^\dagger$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ $H_d (\equiv H_1)$ and $H_u (\equiv H_2)$ needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

Enlarged Higgs sector: Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$

$$+ \underbrace{\frac{g'^2 + g^2}{8}}(H_1\bar{H}_1 - H_2\bar{H}_2)^2 + \underbrace{\frac{g^2}{2}} |H_1\bar{H}_2|^2$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$

Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$$

Predictions for m_h, m_H from diagonalization of tree-level mass matrix:

$\phi_1 - \phi_2$ basis:

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} m_{\phi_1}^2 & m_{\phi_1\phi_2}^2 \\ m_{\phi_1\phi_2}^2 & m_{\phi_2}^2 \end{pmatrix} =$$

$$\begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

$$\Big\Downarrow \quad \leftarrow \text{Diagonalization}, \alpha$$

$$\begin{pmatrix} m_H^{2,\text{tree}} & 0 \\ 0 & m_h^{2,\text{tree}} \end{pmatrix}$$

Tree-level result for m_h, m_H:

$$m_{H,h}^2 = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow$ $m_h \leq M_Z$ at tree level

$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_h, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

Higgs couplings, tree level:

$$g_{hVV} = \sin(\beta - \alpha)\, g_{HVV}^{\text{SM}}, \quad V = W^\pm, Z$$

$$g_{HVV} = \cos(\beta - \alpha)\, g_{HVV}^{\text{SM}}$$

$$g_{hAZ} = \cos(\beta - \alpha)\, \frac{g'}{2\cos\theta_W}$$

$$g_{hb\bar{b}}, g_{h\tau^+\tau^-} = -\frac{\sin\alpha}{\cos\beta}\, g_{Hb\bar{b},H\tau^+\tau^-}^{\text{SM}}$$

$$g_{ht\bar{t}} = \frac{\cos\alpha}{\sin\beta}\, g_{Ht\bar{t}}^{\text{SM}}$$

$$g_{Ab\bar{b}}, g_{A\tau^+\tau^-} = \gamma_5 \tan\beta\, g_{Hb\bar{b}}^{\text{SM}}$$

$\Rightarrow$ $g_{hVV} \le g_{HVV}^{\text{SM}}$, g_{hVV}, g_{HVV}, g_{hAZ} cannot all be small

$g_{hb\bar{b}}, g_{h\tau^+\tau^-}$: significant suppression or enhancement w.r.t. SM coupling possible

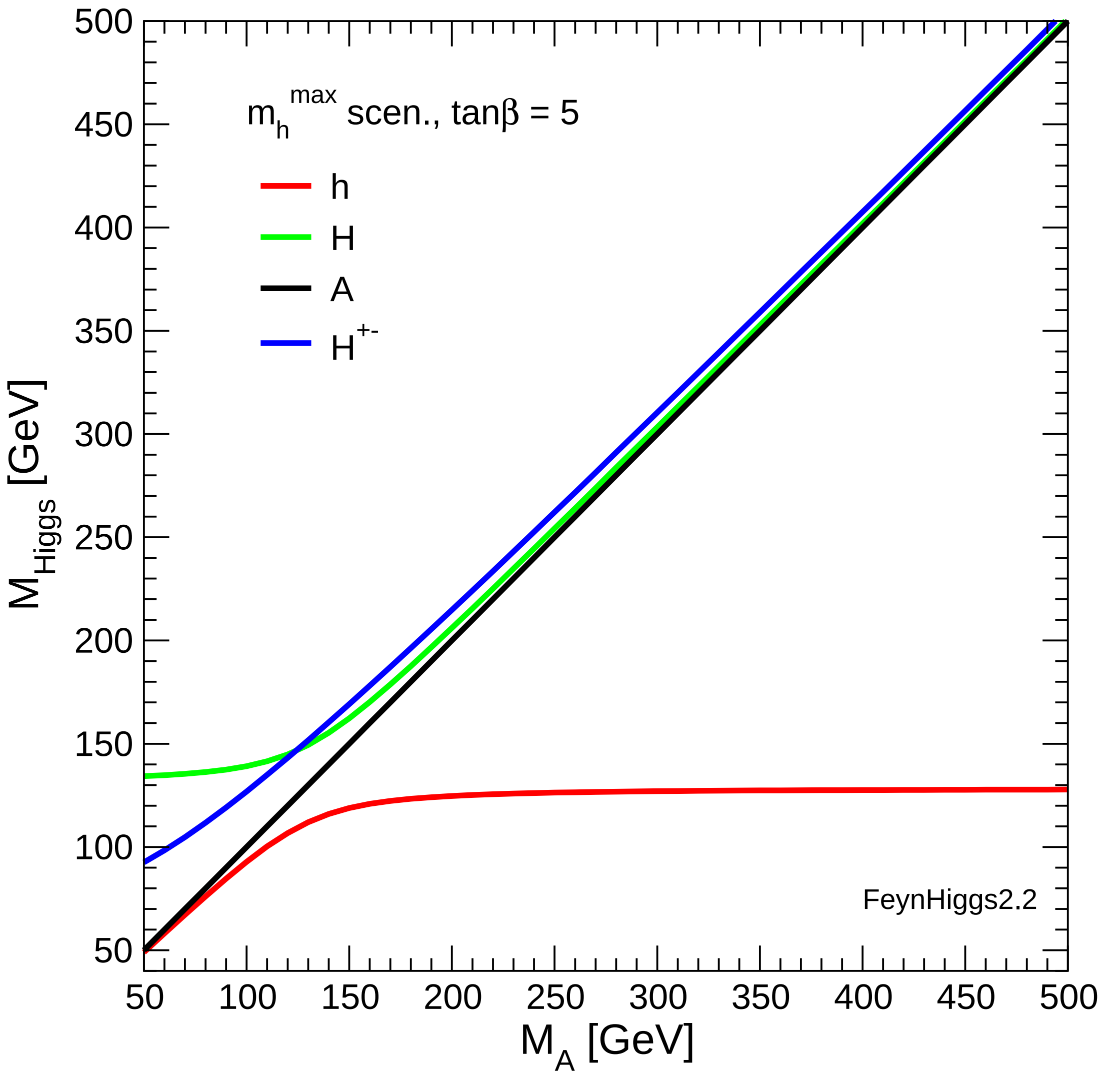
The decoupling limit:

For $M_A \gtrsim 150$ GeV:

The lightest MSSM Higgs is SM-like

The heavy MSSM Higgses:
$M_A \approx M_H \approx M_{H^\pm}$

of course there are exceptions . . .



The lightest MSSM Higgs boson

MSSM predicts upper bound on M_h:

tree-level bound: $m_h < M_Z$, excluded by LEP Higgs searches!

Large radiative corrections:

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}, \frac{e\, m_t^2}{M_W s_W}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $\Delta M_h^2 \sim G_\mu m_t^4 \log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$

> The MSSM Higgs sector is connected to all other sector via loop corrections (especially to the scalar top sector)

Present status of M_h prediction in the MSSM:

Complete one-loop and 'almost complete' two-loop result available

Upper bound on M_h in the MSSM:

"Unconstrained MSSM" :

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$M_h \lesssim 135 \text{ GeV}$$

for $m_t = 170.9 \pm 1.8\,\text{GeV}$

(including theoretical uncertainties from unknown higher orders)
$\Rightarrow$ observable at the LHC

Obtained with:

FeynHiggs

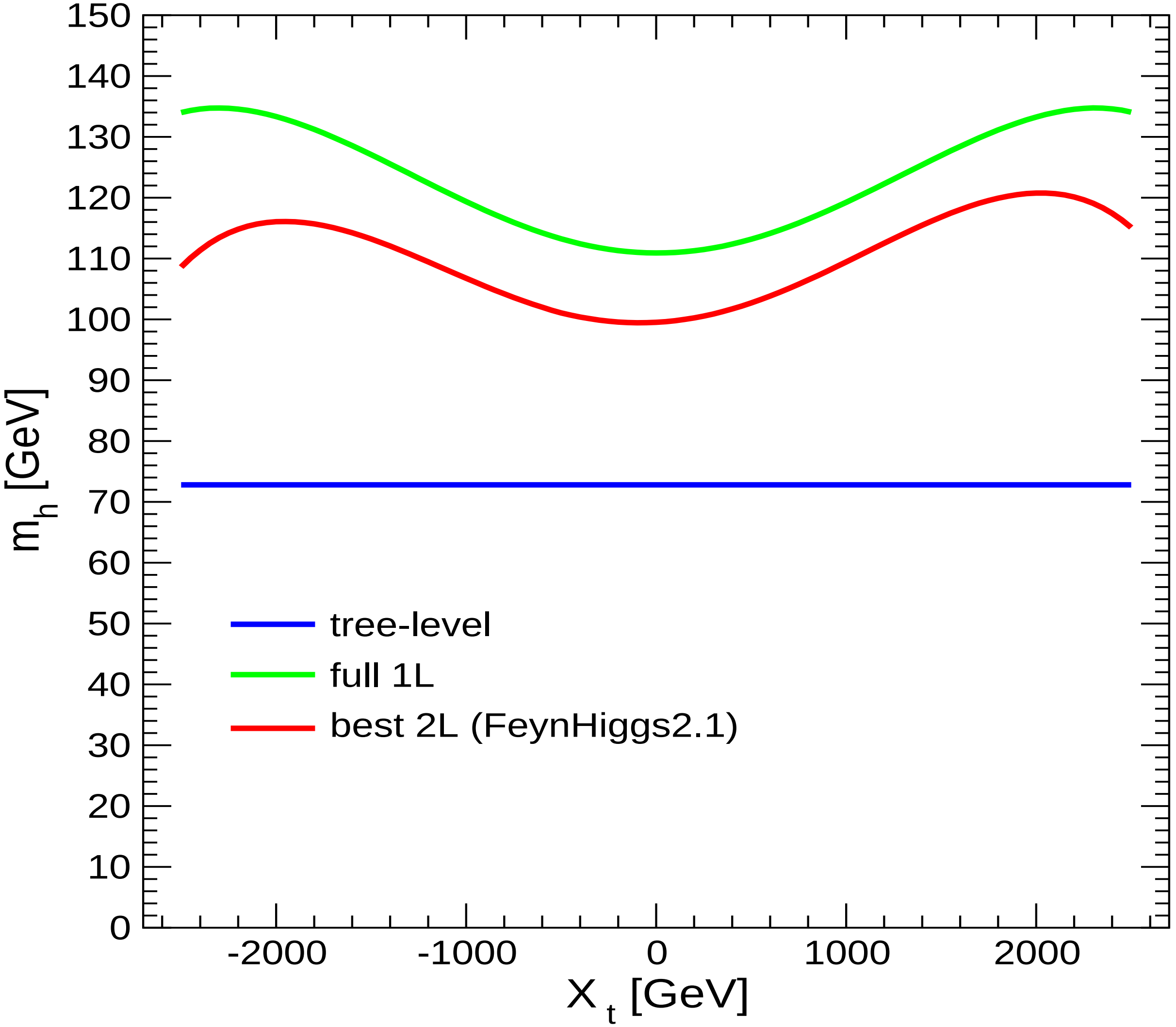
[*S.H., W. Hollik, G. Weiglein '98 – '02*]
[*T. Hahn, S.H., W. Hollik, H. Rzehak, G. Weiglein '03 – '08*]

`www.feynhiggs.de`

$\to$ all Higgs masses, couplings, BRs (easy to link, easy to use :-)

Effects of the two-loop corrections to the lightest Higgs mass:

Example for one set of MSSM parameters



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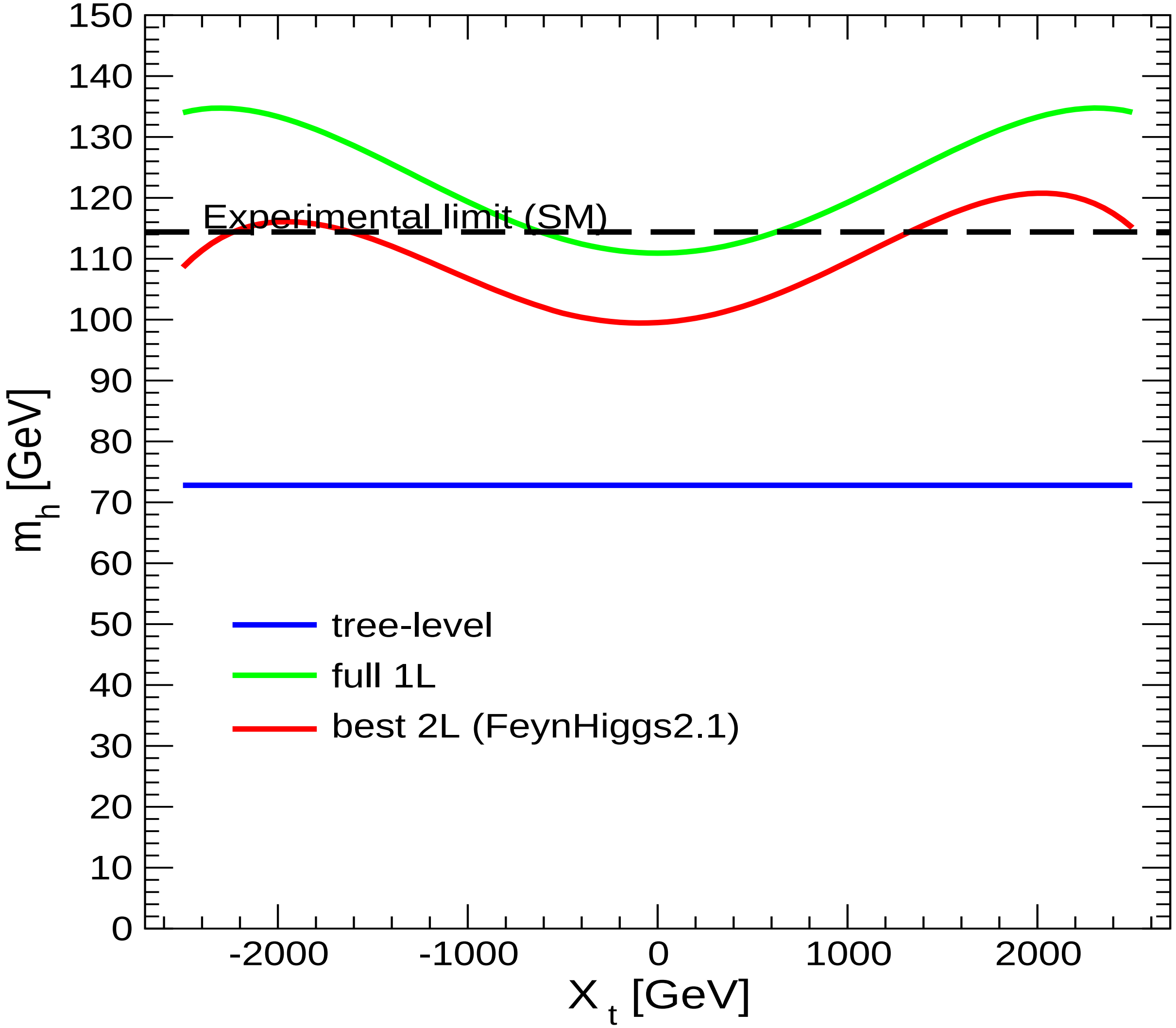


Comparison with experimental limits

⇒ strong impact on bound on SUSY parameters

Remaining theoretical uncertainties in prediction for M_h in the MSSM:

[*G. Degrassi, S.H., W. Hollik, P. Slavich, G. Weiglein '02*]

– From unknown higher-order corrections:
  $\Rightarrow \Delta M_h \approx 3$ GeV

– From uncertainties in input parameters

  $m_t, \; \ldots, \; M_A, \; \tan\beta, \; m_{\tilde{t}_1}, \; m_{\tilde{t}_2}, \; \theta_{\tilde{t}}, \; m_{\tilde{g}}, \; \ldots$

  $\Delta m_t \approx 2$ GeV $\Rightarrow \Delta M_h \approx 2$ GeV

Higgs couplings, production cross sections

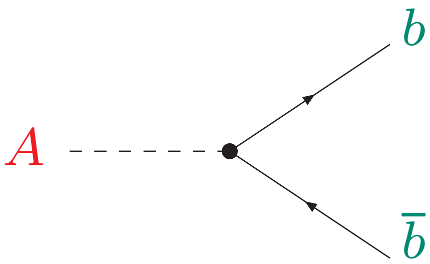
$\Rightarrow$ also affected by large SUSY loop corrections

Extreme example: $\Gamma(h \to b\bar{b}) \to 0$ via loop corrections possible

The heavy MSSM Higgs bosons

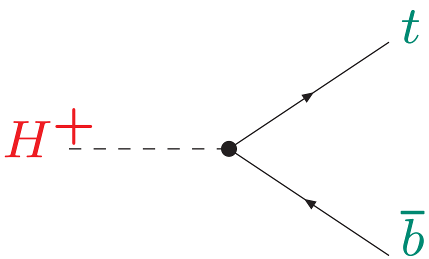
Differences compared to the SM Higgs:

Additional enhancement factors compared to the SM case:



$$y_b \quad \longrightarrow \quad y_b \, \frac{\tan\beta}{1 + \Delta_b}$$

At large $\tan\beta$: either $H \approx A$ or $h \approx A$



$$y_b \, \frac{\tan\beta}{1 + \Delta_b}$$

$$\Delta_b = \frac{2\alpha_s}{3\,\pi} m_{\tilde{g}}\, \mu \, \tan\beta \times I(m_{\tilde{b}_1}, m_{\tilde{b}_2}, m_{\tilde{g}}) + \frac{\alpha_t}{4\,\pi} A_t\, \mu \, \tan\beta \times I(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \mu)$$

$\Rightarrow$ other parameters enter $\Rightarrow$ strong μ dependence

Most powerful LHC search modes for heavy MSSM Higgs bosons:

$$b\bar{b} \to H/A \to \tau^+\tau^- + X$$
$$gb \to tH^\pm + X,\ \ H^\pm \to \tau\nu_\tau$$
$$pp \to t\bar{t} \to H^\pm + X,\ \ H^\pm \to \tau\nu_\tau$$

Enhancement factors compared to the SM case:

$$H/A \ : \ \frac{\tan^2\beta}{(1+\Delta_b)^2} \times \frac{\mathrm{BR}(H \to \tau^+\tau^-) + \mathrm{BR}(A \to \tau^+\tau^-)}{\mathrm{BR}(H \to \tau^+\tau^-)_{\mathrm{SM}}}$$

$$H^\pm \ : \ \frac{\tan^2\beta}{(1+\Delta_b)^2} \times \mathrm{BR}(H^\pm \to \tau\nu_\tau)$$

$\Rightarrow$ Δ_b effects so far neglected by ATLAS/CMS

also relevant for $\mathrm{BR}(H/A \to \tau^+\tau^-)$, $\mathrm{BR}(H^\pm \to \tau\nu_\tau)$

also relevant: correct evaluation of $\Gamma(H/A/H^\pm \to \mathrm{SUSY})$

$\Rightarrow$ additional effects on $\mathrm{BR}(H/A \to \tau^+\tau^-)$, $\mathrm{BR}(H^\pm \to \tau\nu_\tau)$

Prediction of M_h in the CMSSM

[*Buchmüller, Cavanaugh, de Roeck, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07*]

General idea:

Take the most simple MSSM version: CMSSM/mSUGRA
$\rightarrow$ just three GUT scale parameters + $\tan\beta$

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– combine all electroweak precision data as in the SM
– combine with B physics observables
– combine with CDM and $(g-2)_\mu$
– include SM parameters with their errors: m_t, ...

– scan over the full CMSSM parameter space

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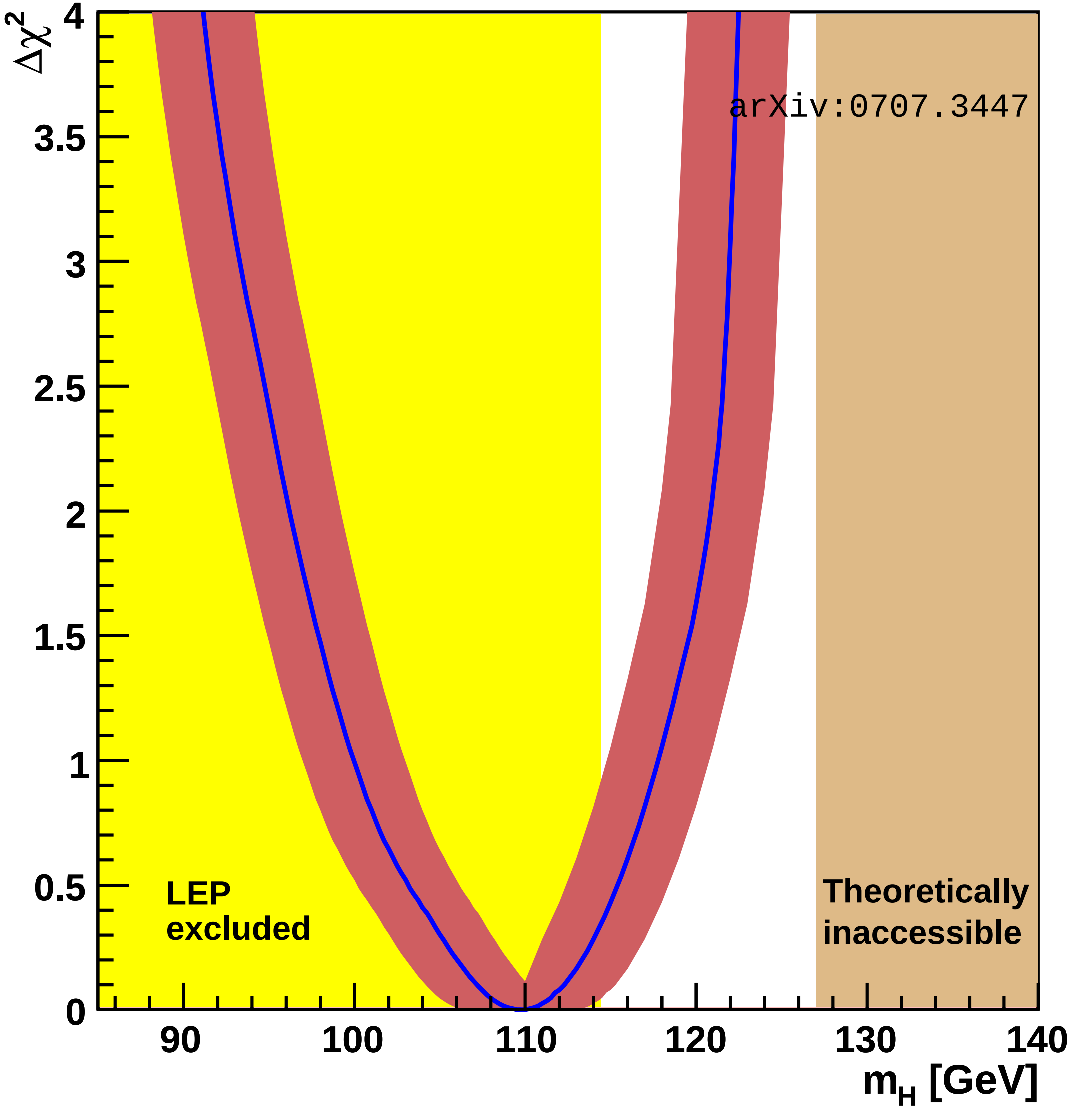
$\Rightarrow$ preferred M_h values

Red band plot:

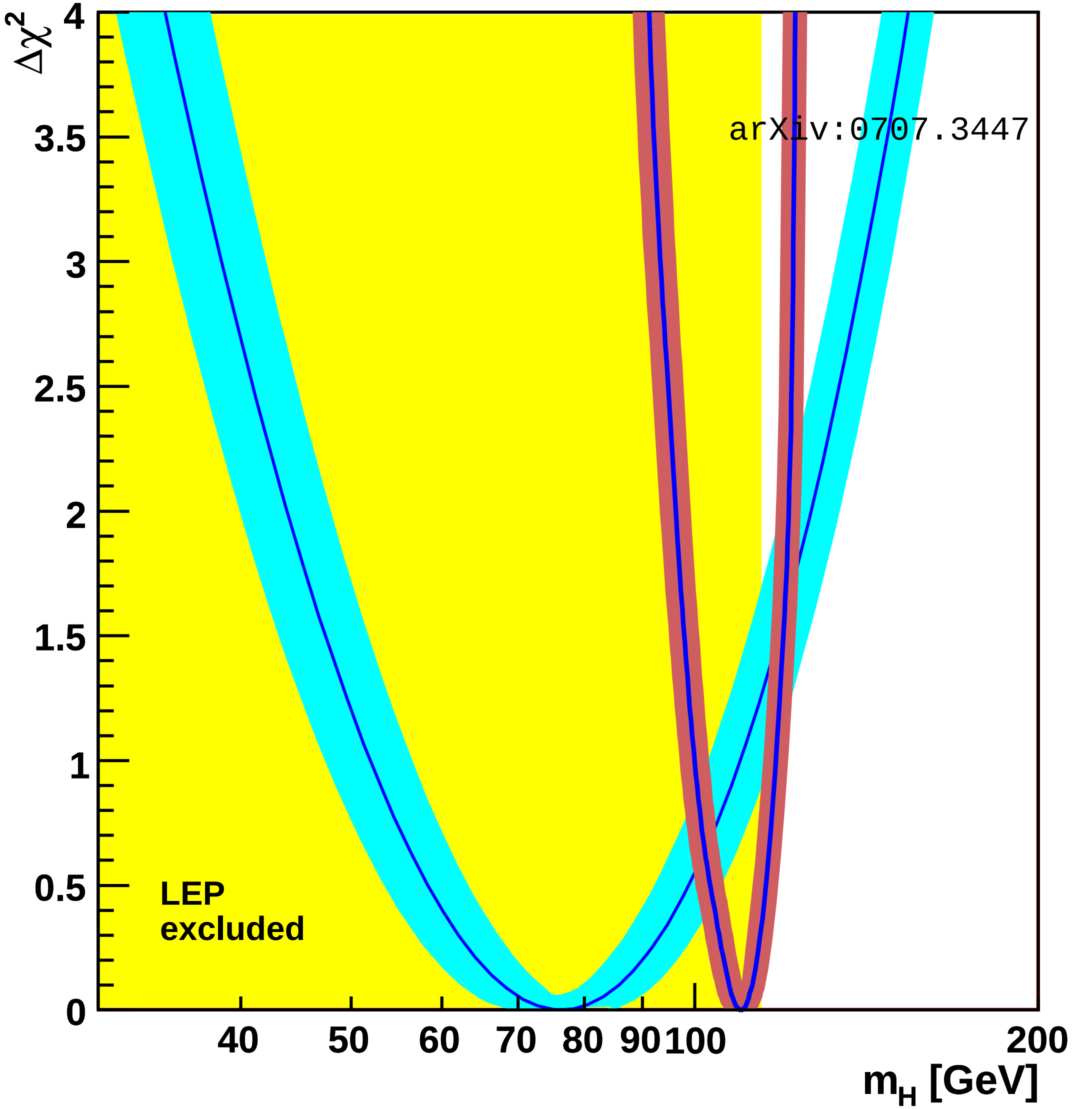
[*Buchmüller, Cavanaugh, de Roeck, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07*]



$M_h = 110^{+8}_{-10}\,(\text{exp}) \pm 3(\text{theo})$ GeV

Blue/Red band plot:

[*Buchmüller, Cavanaugh, de Roeck, S.H., Isidori, Paradisi, Ronga, Weber, Weiglein '07*]



CMSSM (despite its simplicity) is better than the SM

4. Conclusions

- The Higgs boson is the last missing particle of the SM

- Establishing the Higgs mechanism:
  Finding the particle, measure mass, couplings, quantum numbers

- SM: Blue band plot: $\Rightarrow M_H^{\text{SM}} = 76^{+33}_{-24}$ GeV (too light for LEP bounds?)

- SUSY remains to be the best bet for physics beyond the SM

- One Higgs is light, $M_h \lesssim 135$ GeV "often" SM-like (decoupling limit)
  but exceptions are possible
  $\Rightarrow$ drastically different couplings
  $\Rightarrow$ strong changes in phenomenology possible

- Two neutral heavy Higgs bosons, two charged Higgs bosons
  important enhancement/suppression factors as compared to the SM

- CMSSM/mSUGRA: Red band plot: $\Rightarrow M_h^{\text{CMSSM}} = 110^{+8}_{-10} \pm 3$ GeV