EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

Search for oscillations in the ratio of factorial to cumulant moments in Z decays at LEP

The L3 Collaboration

Abstract

The ratio of cumulant factorial to factorial moments of the charged particle multiplicity shows a quasi-oscillatory behaviour which is predicted by the NNLLA of perturbative QCD. However, a similar analysis of the sub-jet multiplicity distribution at perturbative scales shows that these oscillations cannot be related to the NNLLA prediction. We investigate also how it is possible to reproduce the oscillations within the framework of Monte Carlo models.

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Introduction

The shape of a distribution can be analyzed in terms of its moments. For studying the shape of the charged particle multiplicity distribution, the normalized factorial moments are often used. The normalised factorial moment of order q is defined in terms of the multiplicity distribution, P_n, by:

$$F_q = \frac{\sum\limits_{n=q}^{\infty} n(n-1)....(n-q+1)P_n}{\left(\sum\limits_{n=1}^{\infty} nP_n\right)^q}$$

The factorial moments reflect correlations in particle production. If particles are produced independently the particle distribution is poissonian, for which all factorial moments are equal to one. If the particles are correlated, the distribution is broader than Poisson and the factorial moments are greater than one. If the particles are anti-correlated, the distribution is narrower than Poisson and the factorial moments are smaller than one.

The cumulant factorial moments are obtained from the factorial moments by:

$$K_q = F_q - \sum_{m=0}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m$$

The cumulant factorial moment, K_q expresses the genuine q-particle correlation function, i.e., the function which describes the correlations between q particles. The behaviour of F_q and H_q with moment order q is shown in fig. 1 for the Poisson distribution and for the negative binomial distribution, which has often been used to fit the charged particle multiplicity distribution.

Since $|K_q|$ and F_q increase rapidly with their moment order, it is useful to define the ratio $H_q = \frac{K_q}{F_q}$, which has the same order of magnitude over a large range of q. One can view this ratio as a cumulant factorial moment normalised to the factorial moment. It reflects the genuine q-particle correlation function relative to the global correlation function.

The H_q have been calculated for the soft gluon multiplicity distribution at different orders of perturbative QCD (pQCD) [1]:

- For the Double Leading Logarithm Approximation (DLLA), H_q decreases to 0 as q^{-2}.
- For the Modified Leading Logarithm Approximation (MLLA), H_q decreases to a negative minimum at $q = 5$, and then rises to approach 0 asymptotically.
- For the Next to Leading Logarithm Approximation (NLLA), to H_q decreases to a positive minimum at $q = 5$ and goes to a positive constant value for large moment rank.
- For the Next to Next to Leading logarithm Approximation (NNLLA), H_q decreases to a negative first minimum for $q = 5$, and for $q > 5$, H_q shows quasi-oscillation around 0.

These behaviours are illustrated in figure 2.

Assuming the validity of the Local Parton Hadron Duality hypothesis (LPHD), such behaviour is also expected for the charged particle multiplicity distribution.

The H_q of the charged particle multiplicity distribution have been measured by the SLD collaboration [2] for $e^+e^- \longrightarrow$ hadrons at $\sqrt{s} = M_Z$. They observed (figure 3) a negative minimum for $q = 5$ followed by quasi-oscillations about 0 for higher q. This result has been interpreted as confirmation of the NNLLA prediction.

The H_q moments have also been calculated from published charged particle multiplicity distributions of ALEPH, DELPHI, OPAL [3, 4] and in each case, similar behaviour with a negative first minimum followed by quasi-oscillation about 0, has been seen. Furthermore, this result is not limited to e^+e^- physics; this oscillatory behaviour of the H_q moments has been observed also in hadron collisions [5].

While it is tempting to conclude that the observed behaviour is a confirmation of the NNLLA prediction, it must be realized that such a conclusion rests strongly on the validity of LPHD. Although the LPHD hypothesis is accepted for single-particle distributions and mean multiplicity, difficulties have been detected when resonances are involved [6, 7]. Further, heavy quark decays will cause a modification of the multiplicity distribution which is unlikely to be accounted by LPHD [4]. For these reasons, we consider the assumption of LPHD as highly questionable.

Therefore in this paper we present measurements of the ratio of cumulant to factorial moments not only of the charged particle multiplicity distribution but also of the sub-jet multiplicity distributions. By considering sub-jets, i.e., jets obtained with small value of jet resolution parameter, we can, in some sense, follow the development of parton to hadron by varying the jet resolution parameter (and hence the energy scale). The variation of the mean sub-jet multiplicity has been found [8] to agree well with the analytic perturbative QCD calculation in the perturbative region (energy scale greater than about $1 - 2$ GeV) and with Monte Carlo in both the perturbative and non perturbative region. These sub-jet multiplicity distributions should therefore enable us to extend this analysis to the energy scales of partons and to that of perturbative QCD, avoiding (or at least lessening) dependence on the assumption of LPHD.

Selection procedure

This analysis is based on data collected by the L3 detector [9] in 1994 at the energy of Z, which corresponds to approximately 1500k produced hadronic Z decays.

Events are selected using a procedure based on the track momenta measured in the Central Tracking Detector including the Silicon Micro-vertex Detector. In order to select well-measured tracks, the distance of closest approach of the tracks to the interaction point is required to be less than 5 mm and tracks with a transverse momentum less than 100 MeV are rejected. Hadronic events are then selected using track momenta by requiring

$$0.15 < \frac{\sum|\vec{p}|}{\sqrt{s}} < 0.8\ ,\ \frac{|\sum p_{\parallel}|}{\sum|\vec{p}|} < 0.75 \text{ and } \frac{|\sum \vec{p}_{\perp}|}{\sum|\vec{p}|} < 0.75$$

In addition, events must have a thrust axis direction within $|\cos(\theta_{\mathrm{th}})| \leq 0.70$. Further, a cut on low charged multiplicities ($N_{\mathrm{ch}} > 5$) was applied to remove background events coming from $e^+e^- \longrightarrow \tau^+\tau^-$.

About 850k events survive this selection.

In a first step, the charged particle multiplicity distribution has been corrected for the detector acceptance and efficiency using a matrix [10] determined from Monte Carlo events generated by JETSET [11] which have been passed through detector simulation [12]. (This method is model dependent and should only be used if the simulated level reproduces adequately the data, which is the case in our analysis, as is shown in figure 4). The simulated events which survive the selection procedure are used to construct the matrix:

$$M(n_{\mathrm{sim}}, n_{\mathrm{gen}}) = \frac{N(n_{\mathrm{sim}}, n_{\mathrm{gen}})}{N(n_{\mathrm{sim}})}$$

where $N(n_{\mathrm{sim}}, n_{\mathrm{gen}})$ is the number of events with n_{sim} simulated tracks and n_{gen} generated tracks, and $N(n_{\mathrm{sim}})$ the number of events with n_{sim} simulated tracks.

Effects from selection procedure, Initial State Radiation, K^0_S and Λ decays has been corrected using the factor:

$$C_{\mathrm{corr}}(n) = \frac{P^{\overline{\mathrm{ISR}},\mathrm{K}^0_\mathrm{S},\Lambda}(n)}{P^{\mathrm{ISR},\overline{\mathrm{K}^0_\mathrm{S},\Lambda}}(n)} \times \frac{P^{\overline{\mathrm{sel}}}(n)}{P^{\mathrm{sel}}(n)}$$

where

- $P^{\overline{\mathrm{ISR}},\mathrm{K}^0_\mathrm{S},\Lambda}(n)$ is the normalised multiplicity distribution of generated tracks without ISR where K^0_S and Λ are allowed to decay.
- $P^{\mathrm{ISR},\overline{\mathrm{K}^0_\mathrm{S},\Lambda}}(n)$ is the normalised multiplicity distribution of generated tracks with ISR and without K^0_S and Λ decay.
- $P^{\overline{\mathrm{sel}}}(n)$ the normalised multiplicity distribution of generated tracks before selection, and
- $P^{\mathrm{sel}}(n)$ the normalised multiplicity distribution of generated tracks after the selection.

Both corrections were applied to obtain the fully corrected charged multiplicity distribution:

$$P^{\mathrm{corr}}_{\mathrm{data}}(n) = C_{\mathrm{corr}}(n) \times \sum_j M(n,j) P^{\mathrm{raw}}_{\mathrm{data}}(j)$$

The same procedure has been employed to correct sub-jet multiplicity distributions.

To evaluate the errors of the H_q, the following method has been applied: We have generated 10000 multiplicity distributions from the experimental one, by allowing a Gaussian fluctuation of each $P(n)$. The widths of the gaussians were given by the statistical errors of the $P(n)$. For all generated multiplicity distributions, we have calculated each of the H_q. The statistical error on the H_q is taken as the standard deviation of the 10000 values of H_q.

Charged particle multiplicity analysis

The corrected charged particle multiplicity distribution is shown in fig. 5 and compared to that of JETSET. Reasonably good agreement is found.

To be less sensitive to the large statistical fluctuations in the high multiplicity tail of the multiplicity distribution due to a finite data sample, the largest multiplicities ($N_{\mathrm{ch}} > 50$), which represents only a few events, have been removed from the experimental multiplicity distribution. The ratios of cumulant factorial to factorial moment are shown in fig. 6 for both the initial charged particle multiplicity distribution and the truncated one. These results are in qualitatively good agreement with those of the SLD collaboration.

In both the truncated and the non-truncated cases, the H_q have a negative first minimum at $q = 5$ and quasi-oscillations around 0 for greater q. But the values of H_5 as well as the amplitudes of the quasi-oscillations are somewhat different: The amplitudes are slightly greater in the truncated case and $|H_5|$ is larger. Furthermore, amplitudes and $|H_5|$ are found to increase if the multiplicity distribution is further truncated (not shown).

The H_q calculated from JETSET 7.4 PS agree well with the data both for the entire and the truncated distributions, when the generated charged multiplicities are truncated in the same way as the data. The H_q calculated from HERWIG 5.9 [13] do not agree with the data, but they do show an oscillatory behaviour with a negative first minimum at $q = 7$ and all minima and maxima shifted by 2 orders.

Sub-jet multiplicity analysis

Since the behaviour of the H_q agrees qualitatively with that predicted in NNLLA, it is natural to interpret this agreement as a confirmation of the NNLLA prediction, and SLD made this interpretation. However, JETSET agrees with the data and HERWIG shows qualitatively similar behaviour. But the parton shower of these programs are not at the NNLLA order of pQCD. Hence the H_q from JETSET and HERWIG should not be expected to oscillate. That they do, together with doubt about the LPHD assumption suggests that we view the NNLLA interpretation with scepticism.

The sub-jet analysis assumes that the sub-jet nultiplicity distribution is related to the parton multiplicity distribution at the energy scale corresponding to the jet resolution. By choosing a scale where perturbative QCD should be applicable ($\gtrsim$ 1 GeV), we should be able to test the pQCD predictions for the behaviour of the H_q. We define sub-jets using the Durham algorithm [14]. This algorithm uses $y_{ij} = \frac{2\min(E_i^2,E_j^2)}{E_{\rm vis}^2}(1-\cos^2\theta_{ij})$ as the distance between particles i and j. The pair of particles having the smallest y_{ij} is combined, by calculating the sum of their momenta, into a cluster which is further treated like a particle. This process is repeated until no pair of particles or clusters remains with $y_{ij} > y_{\rm cut}$. The Durham algorithm has the property of minimizing the hadronization effects, since it can associate particles with large angles and small transverse energies which is a specific effect of hadronization. For these reasons the Durham algorithm is commonly used for perturbative QCD analysis.

It has been shown [8, 15, 16] in previous papers (see for review [17]) that sub-jet mean multiplicities agree well with the NLLA prediction in the perturbative region, i.e., for values of $y_{\rm cut}$ greater than 10^{-3} which corresponds to transverse energies greater than ~ 1 GeV.

Since this study starts with the charged track multiplicity distribution, we have also constructed sub-jets using only charged tracks. The charged particles and sub-jets multiplicities are, of course, identical for sufficiently small values of $y_{\rm cut}$ ($y_{\rm cut} \lesssim 10^{-8}$).

Using both charged and neutral particles instead of only the charged particles gives similar results. We have checked that, for $y_{\rm cut} = 0.01$ the fractions of 2-jet and 3-jet events are comparable, 63% and 32% for charged and neutral particles and 61% and 32.5% for charged tracks only.

As a further check on the use of sub-jets constructed using only charged tracks, we have calculated the average sub-jet multiplicities in 2-jet and 3-jet events (M_2 and M_3) obtained for a set of values of y_0: $10^{-7} \leqslant y_0 < y_1$, where $y_1 = 0.01$ is the cut-off parameter which resolves 2-jet and 3-jet events. $M_2(y_0)$ and $M_3(y_0)$ correspond to the number of gluons radiated at different energy scales (at different places in the parton shower evolution) for the 2-jet and 3-jet events.

The NLLA prediction at fixed order of $\alpha_{\rm s}$ give for M_2 and M_3 [18]:

$$M_2 = 2 + \frac{\alpha_{\rm s}}{2\pi} C_F (L_0 - L_1)(L_0 + L_1 - 3)$$

$$\text{and } M_3 = 3 + \frac{1}{2}\frac{\alpha_s}{2\pi}\left[2C_F(L_0+L_1-3) + N_C\left(L_0 - \frac{1}{3}L_1 - \frac{14}{3}\right) + \frac{2}{3}N_f\right]$$

where $L_0 = \ln(\frac{1}{y_0})$, $L_1 = \ln(\frac{1}{y_1})$ and N_f is the number of flavours. We use $N_f = 3$, since only the light flavours are really active in this kind of process [17, 19].

We have calculated the ratio $\frac{M_3-3}{M_2-2}(y_0)$, which corresponds to the ratio of the amount of soft gluon production, when the primary quarks and hard gluon have been taken into account. This ratio, which has the advantage of being independent of α_s, depends only on the resolution parameters y_0 and y_1. The NLLA prediction is seen (figure 7) to be in good agreement with the data in the perturbative region. This result supports the assumption that we can perform the sub-jet analysis with sub-jets obtained with only charged particles.

The sub-jet multiplicity distributions used for the calculation of the H_q have been corrected using the same procedure as for the charged particle multiplicity distribution (i.e., correction matrix and correction for ISR, K^0_S and Λ decays). The resulting H_q are shown in figure 8 for three different values of y_{cut}.

A first minimum at $q = 4$ followed by quasi-oscillation about 0, is seen for $y_{cut} = 6 \times 10^{-6}$ (energy scale $\approx$ 100 MeV) (figure 8a). This behaviour is qualitatively similar to that for the charged particle multiplicity distribution, as well as for sub-jet multiplicity distributions with smaller y_{cut}, but both the position of the first minimum and of subsequent maxima and minima are shifted to lower values of q.

As y_{cut} is increased, the behaviour of the H_q changes further. The negative first minimum near 5 disappears. For $q \leqslant 5$ each H_q alternates between postive and negative values. For greater q, the oscillations begin to disappear. For $y_{cut} = 1.5 \times 10^{-5}$ (energy scale $\approx$ 160 MeV) these oscillations have disappeared, as seen in figure 8b.

For $y_{cut} > 1.5 \times 10^{-5}$ and in particular in the perturbative region $y_{cut} \geqslant 6 \times 10^{-4}$ (corresponding to an energy scale $\gtrsim$ 1 GeV), the oscillations and the negative first minimum near 5 have completely disappeared as seen in figure 8c, for a $y_{cut} \approx 10^{-3}$ (energy scale $\approx$ 1.3 GeV). Instead, H_q alternates between positive and negative values for each q. Furthermore $|H_q|$ increases sharply with q.

Although good agreement with NLLA is found for the production rates of soft gluon seen in the perturbative region, none of the predictions made at different orders of pQCD for the H_q is observed. Even though pQCD describes well collective processes (like the average sub-jet multiplicity) it seems to be unable to describe the shape of these distributions. Therefore, the minimum and oscillatory behaviour of the H_q seen for the charged particle multiplicity distribution and for very low values of y_{cut} appears unrelated to the behaviour of the H_q calculated in NNLLA.

However JETSET PS reproduces the observed behaviour at all values of y_{cut}, and HERWIG shows a qualitatively similar behaviour.

Monte Carlo study of the oscillatory behaviour of the H_q

Given the success of JETSET and HERWIG and the failure of the NNLLA prediction seen in the previous section, we attempted to find that aspect of the Monte Carlo generators which is responsible for the agreement.

We have varied several options in JETSET and studied their influence on the behaviour of the H_q. First we used different models of parton generation using in all case the Lund string fragmentation model. We tried the following:

- no angular ordering in the parton shower, which makes it essentially an LLA shower with the addition of energy-momentum conservation at each branching.
- $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ matrix elements instead of a parton shower.
- matrix element production of $q\bar{q}$ only.

Next we considered the possibility that this behaviour could come from the fragmentation model, which could simulate some hidden higher order aspects of pQCD. We used JETSET 7.4 with the various choices of parton generation used previously but with independent fragmentation for the hadronization process. We also used HERWIG 5.9 which uses cluster fragmentation.

In all cases the H_q have a negative first minimum near $q = 5$ and quasi-oscillations for greater q. Some examples are shown in figure 9

This investigation done with Monte Carlo generators shows that we can reproduce the oscillatory behaviour of the H_q without the need of the NNLLA of pQCD.

Conclusions

The ratio of cumulant factorial to factorial moments has been measured for both the charged particle and sub-jet multiplicity distributions obtained with the Durham algorithm. A perturbative QCD calculation at the next to next to leading logarithm order predicts H_q to have a negative first minimum at $q \approx 5$ and quasi-oscillation around 0 for higher q. This behaviour is observed for the charged particle multiplicity distribution, confirming previous results. However our analysis of the H_q of the sub-jet multiplicity distributions reveals that this behaviour appears only for very small $y_{\rm cut}$, corresponding to energy scales $\lesssim 100$ MeV, which is far away from the perturbative region. Furthermore an investigation performed on different models of parton generation, and for different fragmentation models has shown in each case the same oscillatory behaviour for H_q.

From this we conclude, contrary to the claim of ref. [2], that the oscillations of H_q observed in the charged particle multiplicity distribution are unrelated to the behaviour predicted by the NNLLA perturbative QCD calculations.

References

[1] I.M. Dremin and V.A. Nechitaĭlo, JETP Lett. **58** (1993) 881.

[2] SLD Collab., K. Abe *et al.*, Phys. Lett. **B 371** (1996) 149.

[3] A. Giovannini, S. Lupia, R. Ugoccioni, Phys. Lett. **B 374** (1996) 231.

[4] R. Ugoccioni, Structures in multiplicity distributions and oscillations of moments, Preprint LU TP 96-27, University of Lund, 1996, Invited talk presented at the XXVI International Symposium on Multiparticle Dynamics.

[5] N. Nakajima, M. Biyajima and N. Suzuki, Phys. Rev. **D 54(7)** (1996) 4333.

[6] DELPHI Collab., P. Abreu *et al.*, Nucl. Phys. **B 1** (1995) 3.

[7] OPAL Collaboration, R. Akers *et al.*, Z. Phys. **C 68** (1995) 1.

[8] OPAL Collab., R. Akers *et al.*, Z. Phys. **C 63** (1994) 363.

[9] L3 Collab., B. Adeva *et al.*, Nucl. Instr. Meth. **A 289** (1990) 35; construction of L3 J.A. Bakken *et al.*, Nucl. Instr. Meth. **A 275** (1989) 81; B. Adeva *et al.*, Nucl. Instr. Meth. **A 323** (1992) 109; *et al.*, Nucl. Instr. Meth. **A 323** (1992) 162; Nucl. Instr. Meth. **A 351** (1994) 300. Instr. Meth. **A 383** (1996) 342. .

[10] OPAL Collab, P.D. Acton *et al.*, Z. Phys. **C 53** (1992) 535.

[11] T. Sjöstrand, Comp. Phys. Comm. **39** (1986) 347;
T. Sjöstrand and M. Bengtsson, Comp. Phys. Comm. **43** (1987) 367..

[12] The L3 detector simulation is based on GEANT, see R. Brun *et al.*, CERN report CERN DD/EE/84-1 (Revised), 1987, and uses GHEISHA to simulate hadronic interactions, see 85/02, 1985..

[13] G. Marchesini and B. Webber, Nucl. Phys. **B 310** (1988) 461;
G. Marchesini *et al.*, Comp. Phys. Comm. **67** (1992) 465..

[14] S. Bethke, Z. Kunszt, D.E. Soper and W.J. Stirling, Nucl. Phys. **B 370** (1992) 310.

[15] L3 Collab., O. Adriani *et al.*, Phys. Rep. **236** (1993) 1.

[16] A. Buytenhuijs, QCD gluon radiation studies using the L3 detector, Ph.D. thesis, University of Nijmegen, 1996, The Netherland.

[17] V.A. Khoze and W. Ochs, Perturbative QCD approach to multiparticle production, Preprint MPI-PhT/96-29; Durham DTP/96/36; HEP-PH/97011421, 1996.

[18] S. Catani, B.R. Webber, Yu.L. Dokshitzer, F. Fiorani, Nucl. Phys. **B 383** (1992) 419.

[19] S. Lupia and W. Ochs, Phys. Lett. **B 365** (1996) 339.

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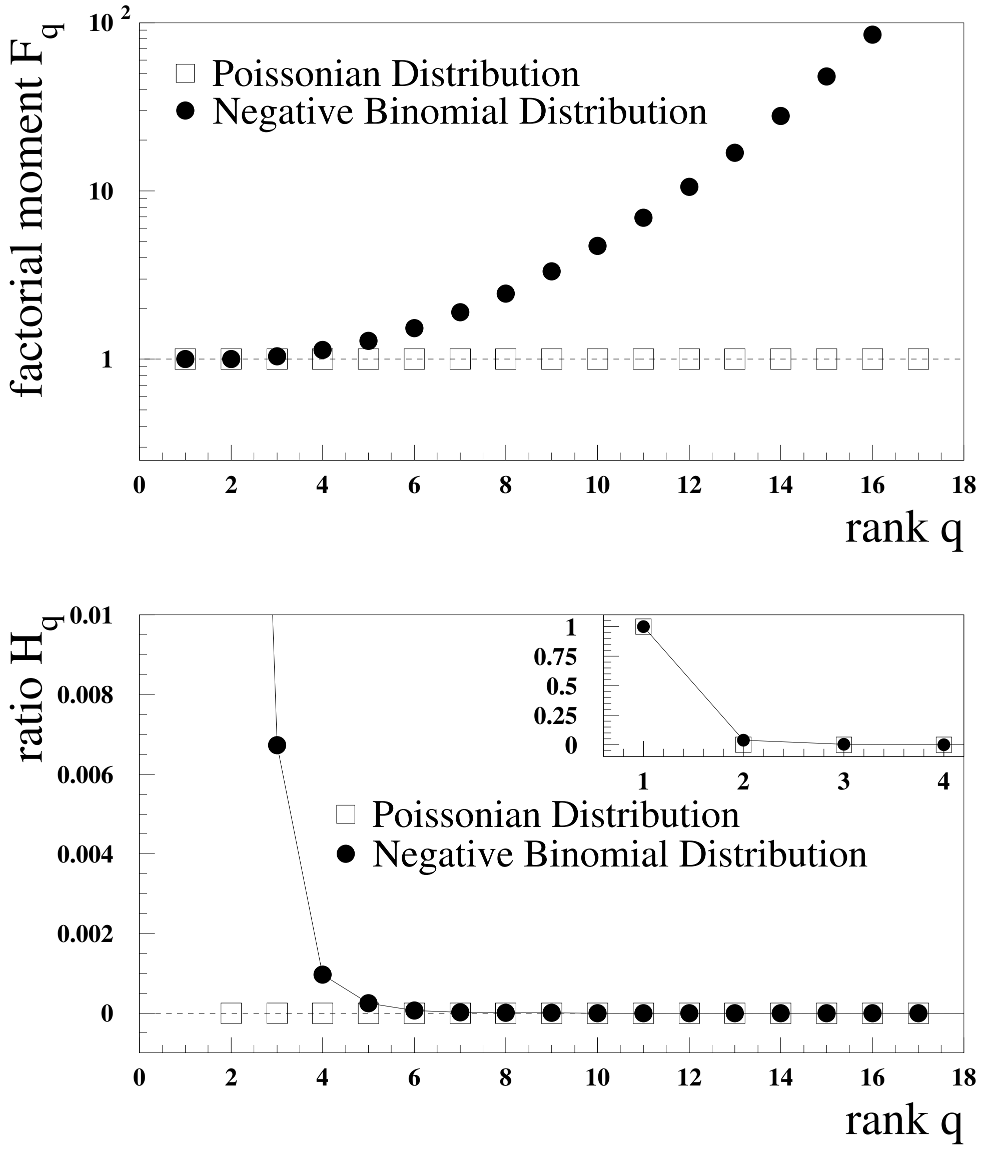

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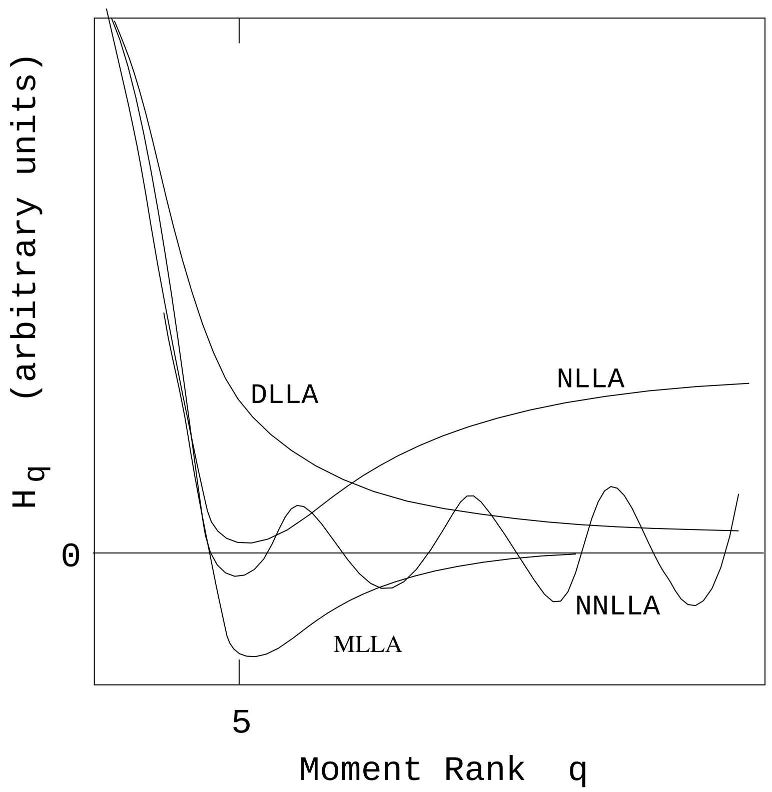

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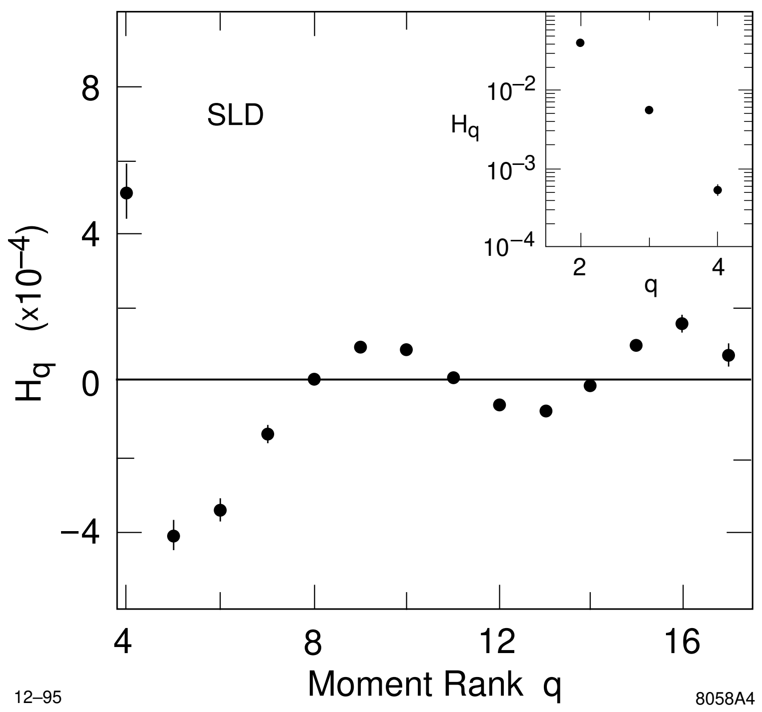

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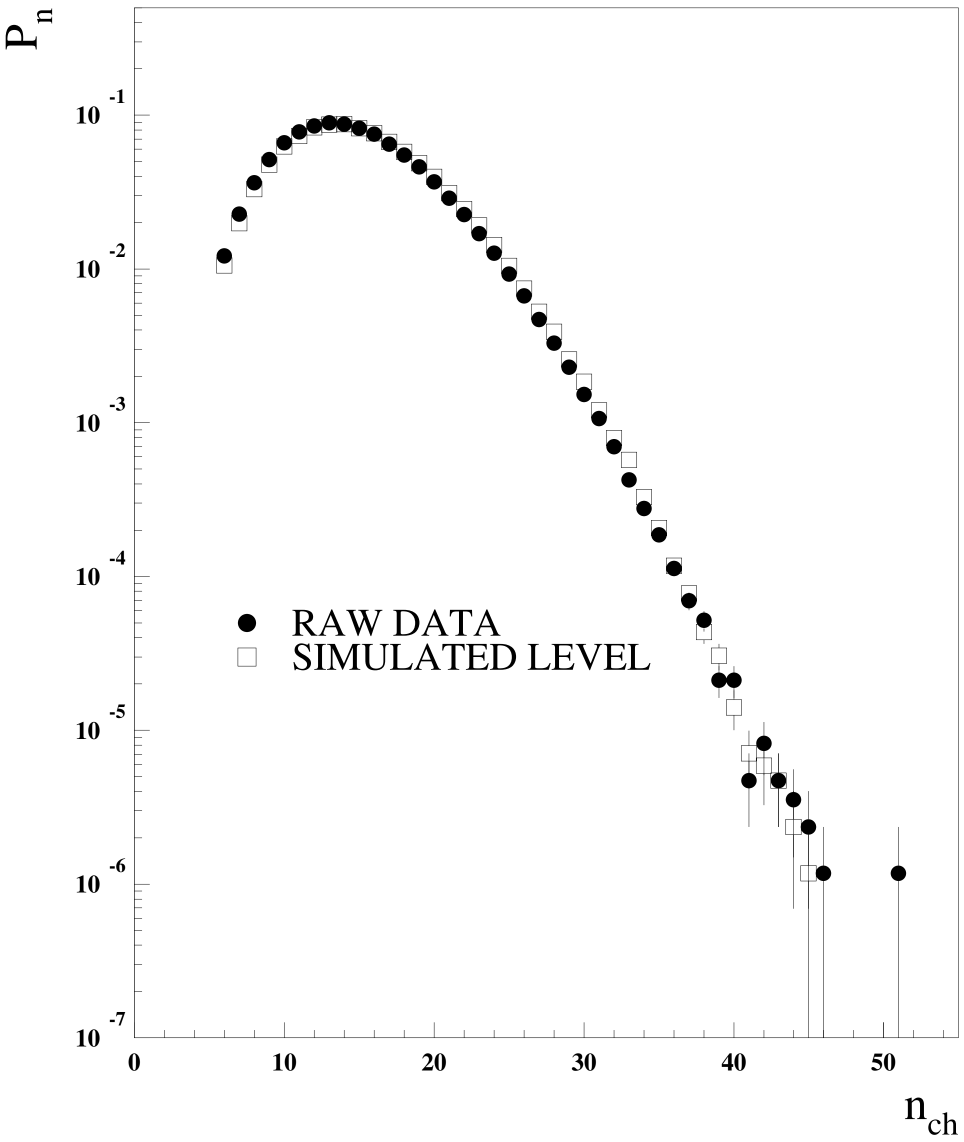

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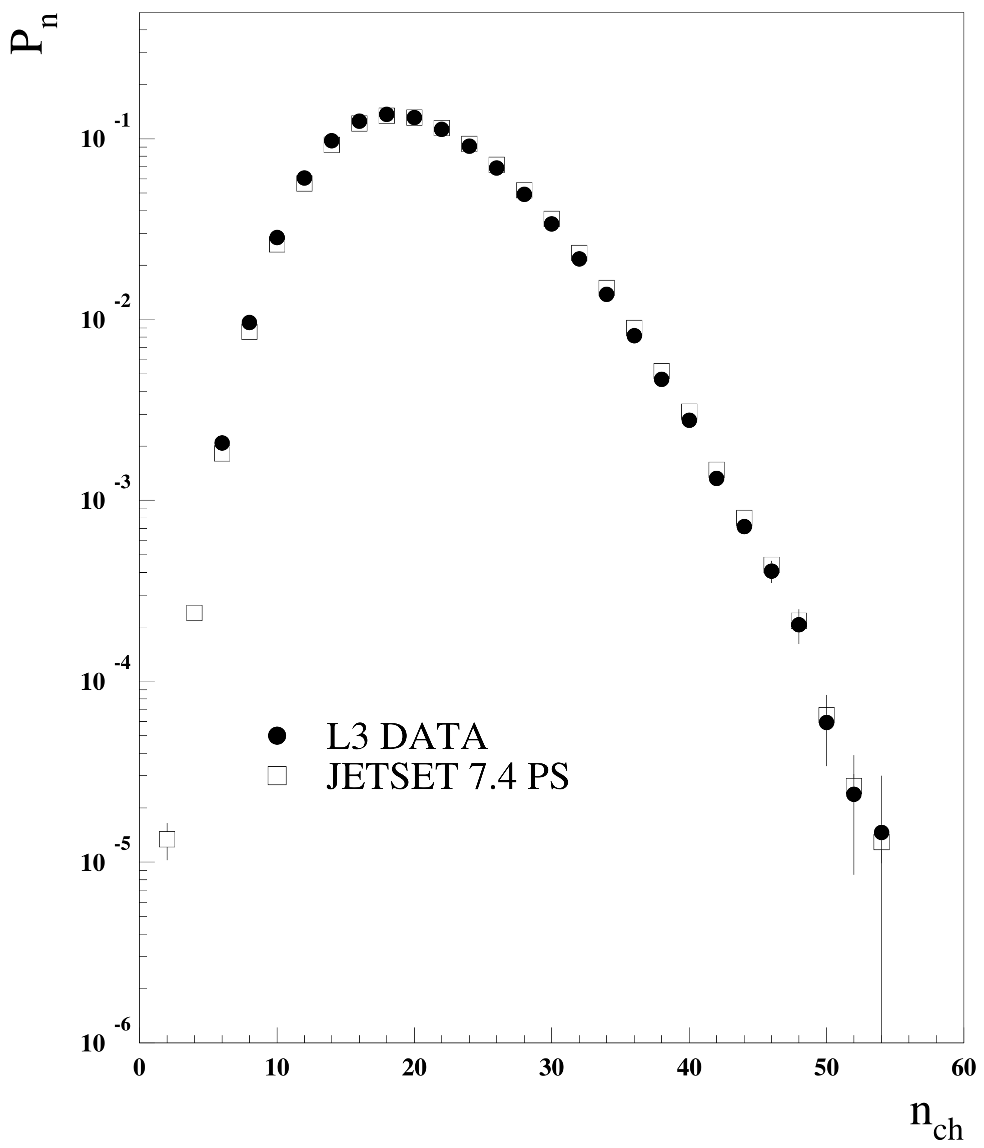

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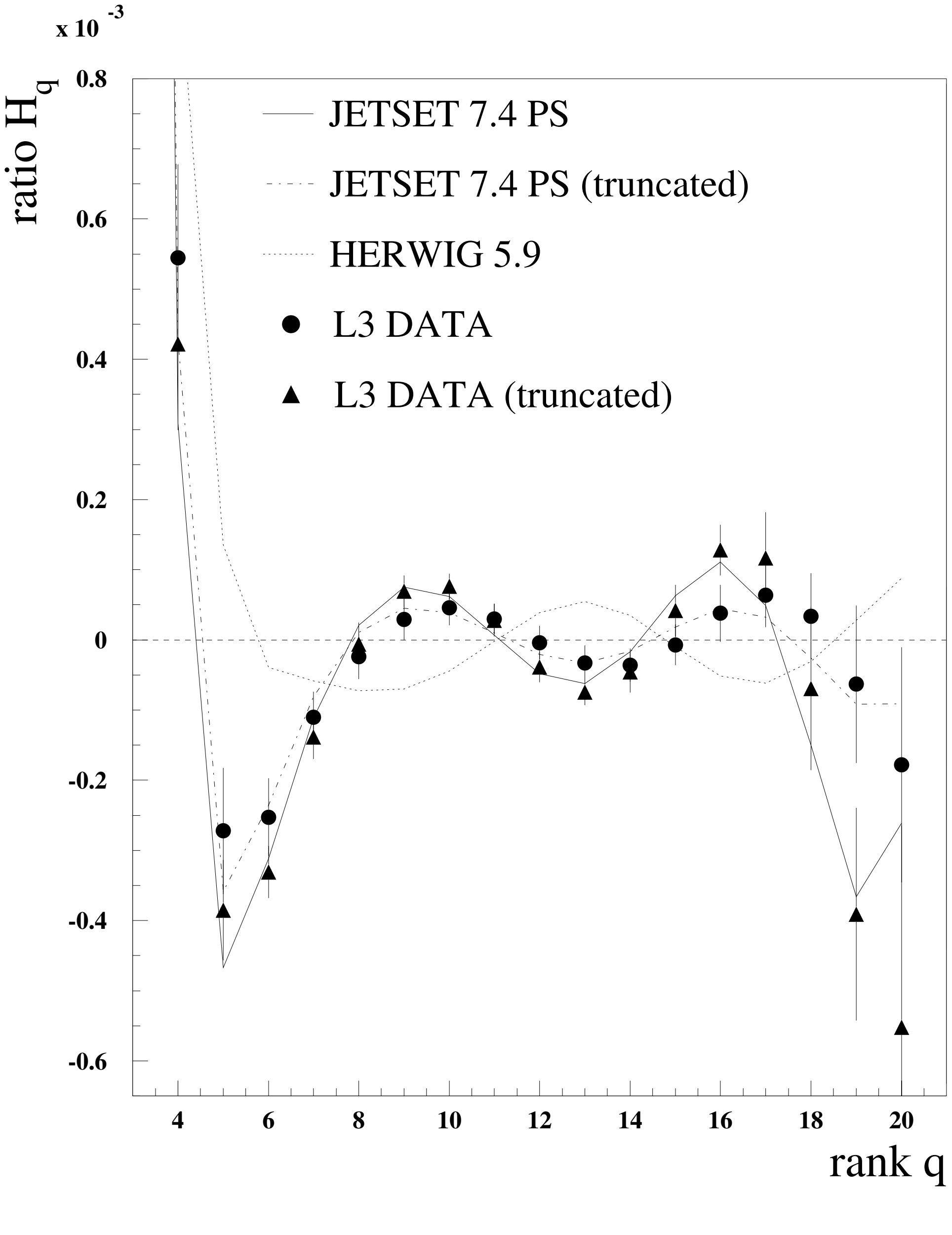

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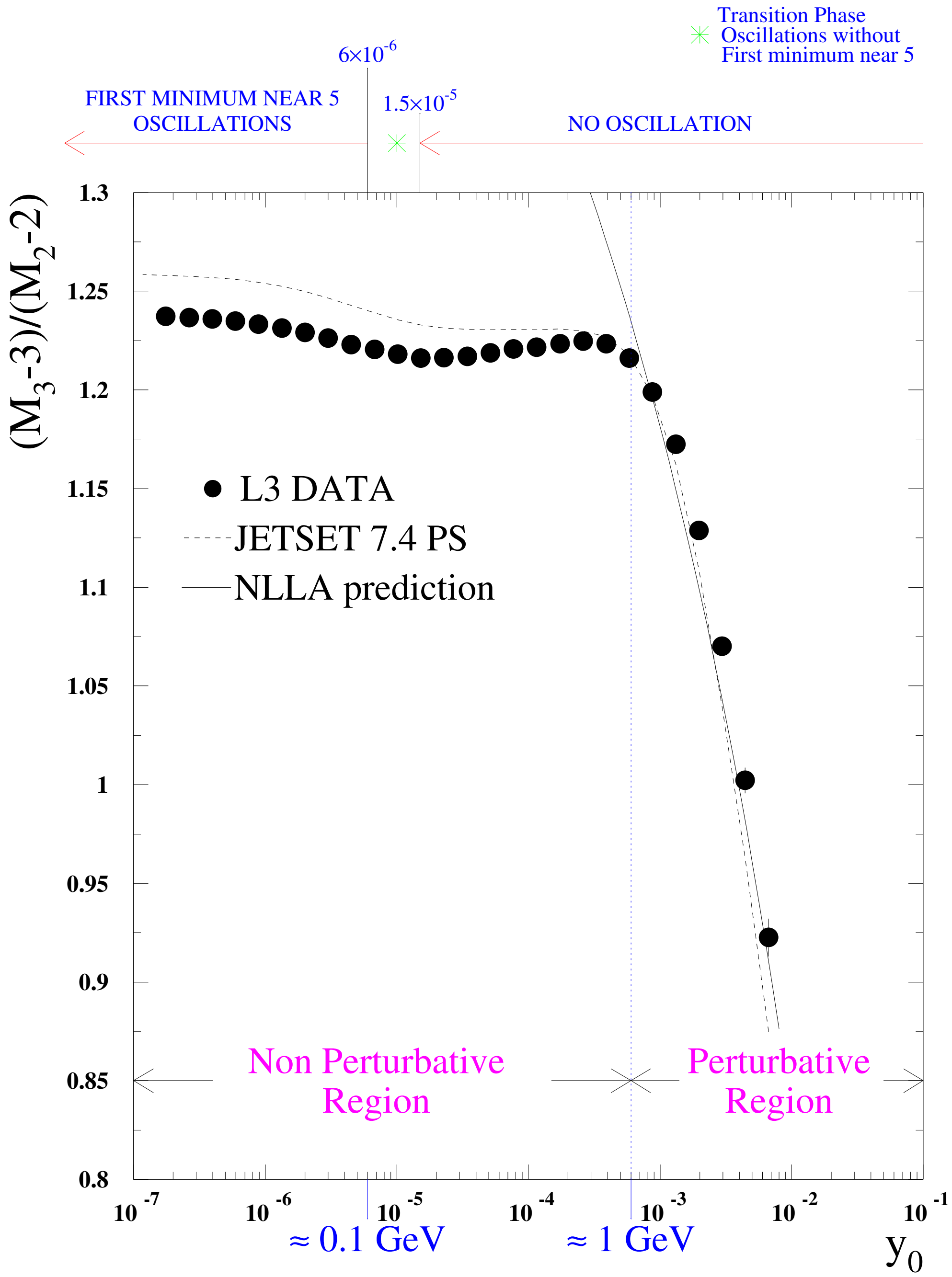

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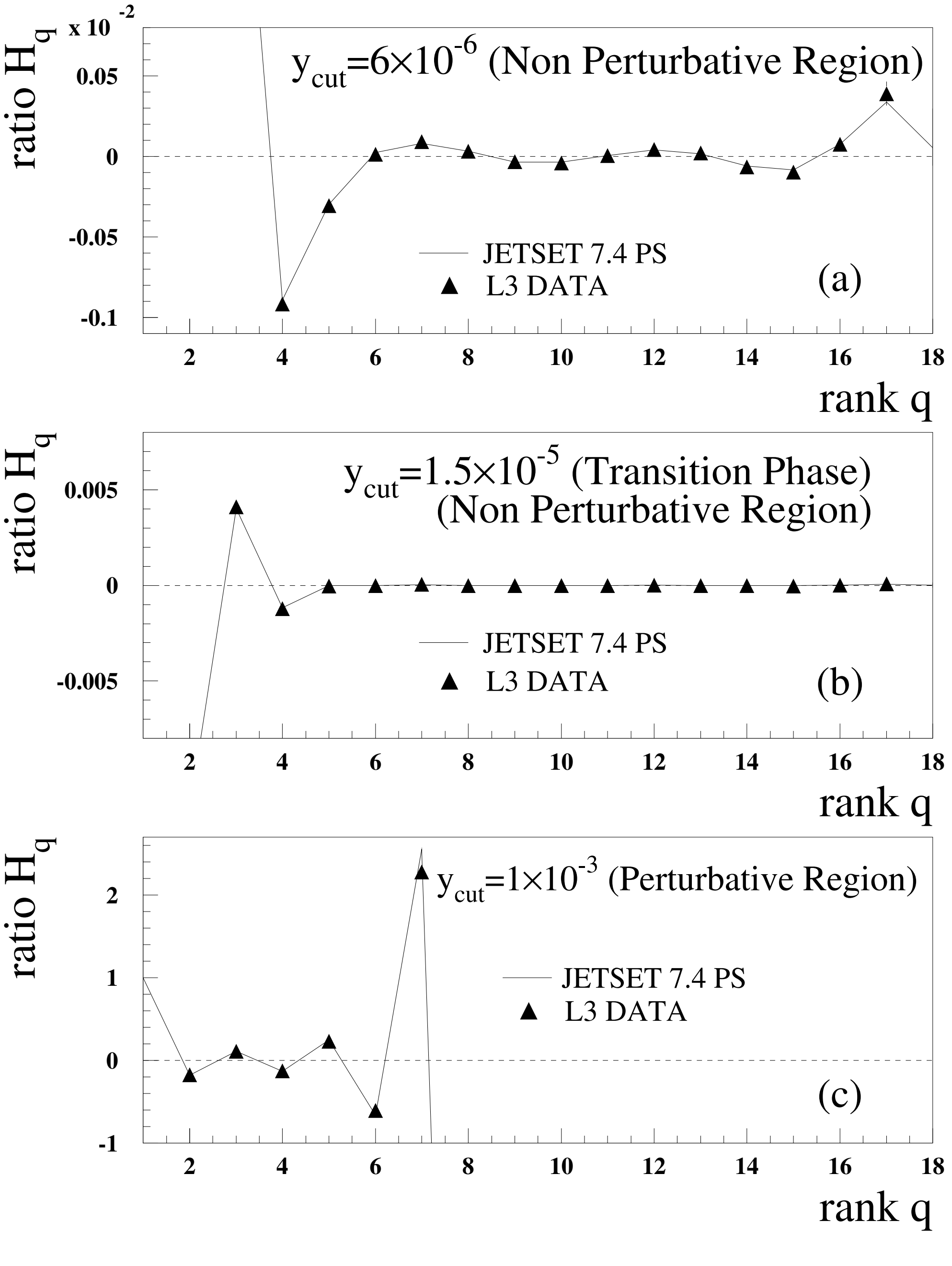

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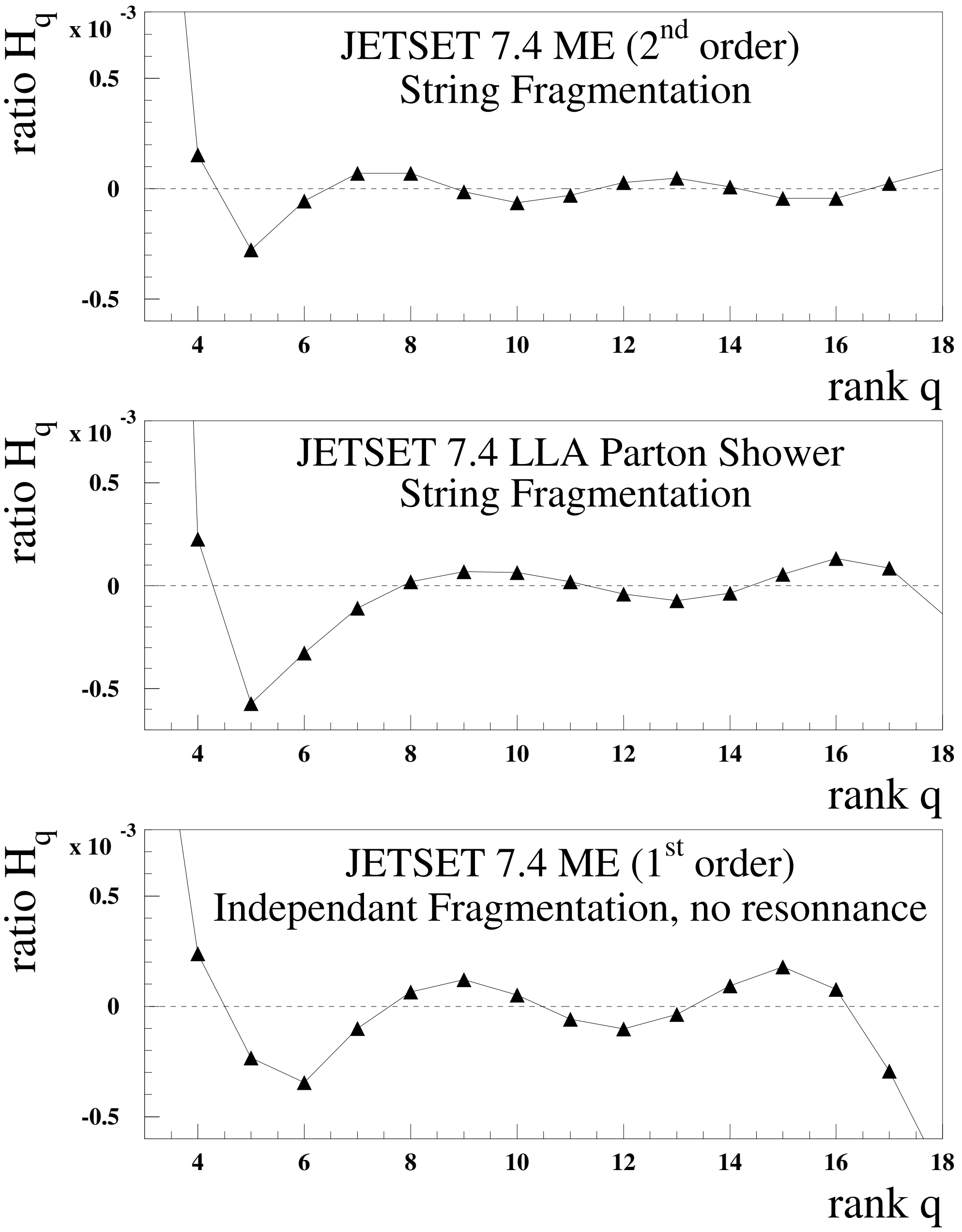

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