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Measurement of the Z boson mass using Z decays in $e^+e^- \to Z\gamma$ events at LEP 2 energies

The L3 Collaboration

Abstract

Data taken with the L3 detector at centre-of-mass energies between 189 GeV and 208 GeV are used to measure the Z boson mass from Z decays. The data correspond to a total integrated luminosity of 627.33 pb^{-1}. The Z mass is determined to be 91.266 ± 0.034 (stat.) ± 0.072 (sys.) GeV, which is in good agreement with the LEP I measurement.

Introduction

At LEP centre-of-mass energies $\sqrt{s}$ above the Z peak, the process $e^+e^- \rightarrow Z\gamma \rightarrow q\bar{q}\gamma$ frequently occurs with hard initial state radiation (ISR). Due to the strong resonance in the non-radiative reaction $e^+e^- \rightarrow Z \rightarrow q\bar{q}$ the energy of the ISR photon is such that the invariant mass of the quark pair is close to the mass of the Z boson, M_Z.

Data recorded with the L3 detector [1] at centre-of-mass energies $\sqrt{s} = 189 - 208$ GeV are used to reconstruct the invariant mass spectrum of the hadronic Z decay products and to extract M_Z. The Z mass is then compared to the precision measurements of $m_Z = 91.190 \pm 0.003$ [2], derived from cross-section measurements at centre-of-mass energies around the Z-pole. This comparison can also be interpreted as a cross-check of the LEP beam energy as determined by the LEP energy working group [3].

Data and Monte Carlo Sample

The event selection has been optimized to obtain a clean event sample of hadronic Z decays. To remove pure leptonic final states we require more than 12 clusters measured in the calorimeters. In addition the sum of the cluster energies should be greater than 30% of the centre-of-mass energy and the tranverse energy imbalance should be less than 25% of the visible energy. The distributions are shown in Figures 1, 2, and 3, respectively.

Hadronic final states from two-photon collisions are boosted along the beam direction. To reject these events we therefore require the sum of the transverse energies of the hadronic clusters to be greater than 15% of the centre-of-mass energy. Additionally the longitudinal energy imbalance must not exceed 80% of the visible energy. These distributions are shown in Figures 4 and 5.

Four-jet final states from W- and Z-Pair Production show a total hadronic energy around the centre-of-mass energy. On the other hand the clusters are distributed more spherically than in the case of 2-jet-events. For events with the sum of the hadronic energy greater than 70% of the centre-of-mass energy, we boost the particles to the rest frame of the decaying Z and calculate the thrust of the boosted event. To remove 4-jet-events we apply a cut on the thrust at 0.85. The thrust distribution is shown in Figure 6. The cut on the total hadronic energy is already applied for this plot.

Extraction of the two-jet invariant mass

In each selected event we search for an isolated photon. The remaining part of the event is forced into two jets using the DURHAM algorithm. If no photon was found we assume that a photon escaped along the beam direction. Using three-particle kinematics the photon energy can be calculated with the following formula

$$E_\gamma = p_{\mathrm{tot}} \cdot \frac{|\sin(\theta_1 + \theta_2)|}{\sin\theta_1 + \sin\theta_2 + |\sin(\theta_1 + \theta_2)|} \; . \tag{1}$$

with

$$p_{\mathrm{tot}} = p_1 + p_2 + p_\gamma \; , \tag{2}$$

where p_1 and p_2 are the momenta of the two jets and p_γ is the photon momentum. The θ_i are the angles between the jets and the photon (respectively the beampipe).

If the jets are assumed to be massless, the sum over the momenta, $p_{\rm tot}$, equals the centre-of-mass energy, $\sqrt{s}$. We found that this approximation results in too high values of E_γ and accordingly too low $\sqrt{s'}$-values. Therefore the momentum sum, $p_{\rm tot}$, is expanded to

$$p_{tot} = \sqrt{s} - \sum_{Jets} \frac{1}{2}\left(\frac{m_j^2}{E_j}\right) + \mathcal{O}\left(\frac{m^4}{E^4}\right) \tag{3}$$

to take the jet masses into account.

The effective centre-of-mass energy of the quark-pair production, $\sqrt{s'}$, is calculated using

$$\sqrt{s'} = \sqrt{s} \cdot \sqrt{1 - \frac{2 \cdot E_\gamma}{\sqrt{s}}}. \tag{4}$$

The reconstructed $\sqrt{s'}$ distributions for data and Monte Carlo are compared in Figure 7. Position and shape of the Z peak are very well reproduced by the Monte Carlo.

Measurement of the Z mass

To measure the Z mass we use events in the $\sqrt{s'}$ range of

$$70 \text{ GeV} < \sqrt{s'} < 110 \text{ GeV}.$$

Three different methods have been applied.

Averaging method

We cut off the lower part of the $\sqrt{s'}$ histograms at half of the maximum peak height to remove the tails of the distribution (see Figure 8). For the remaining part of the histogram we calculate the mean $\sqrt{s'}$ value for data, $\langle\sqrt{s'}_{\rm Data}\rangle$, and signal Monte Carlo, $\langle\sqrt{s'}_{\rm MC}\rangle$. The Z mass extracted with this method, $M_{\rm Z}^{\rm mean}$, is calculated from

$$M_{\rm Z}^{\rm mean} = M_{\rm Z}^{\rm MC} + (\langle\sqrt{s'}_{\rm Data}\rangle - \langle\sqrt{s'}_{\rm MC}\rangle) , \tag{5}$$

where $M_{\rm Z}^{\rm MC}$ is the value of the Z mass that was used in the Monte Carlo generation process.

Binned χ^2 fit

For each selected signal Monte Carlo event we calculate the event weight

$$w_j(M_{\rm Z}^{\rm fit}) = \frac{\frac{{\rm d}\sigma}{{\rm d}\sqrt{s'}}(\sqrt{s'}_{\rm gen}, M_{\rm Z}^{\rm fit})}{\frac{{\rm d}\sigma}{{\rm d}\sqrt{s'}}(\sqrt{s'}_{\rm gen}, M_{\rm Z}^{\rm MC})} \tag{6}$$

where $\sqrt{s'}_{\rm gen}$ is the generated effective centre-of-mass energy, $M_{\rm Z}^{\rm MC}$ is the value of the Z mass that was used in the Monte Carlo generation process, and $M_{\rm Z}^{\rm fit}$ is the mass to be tested. A calculation of the differential cross section, $\frac{{\rm d}\sigma}{{\rm d}\sqrt{s'}}$, up to $\mathcal{O}(\alpha^2)$ can be found in [4].

We calculate the χ^2 value

$$\chi^2 = \sum_i^{N_{\rm bins}} \frac{(N_i^{\rm Data} - N_i^{\rm MC})^2}{\sigma_i^2} \tag{7}$$

in bins of $\sqrt{s'}$, where

$$\begin{aligned} N_i^{\mathrm{MC}} &= N_i^{\mathrm{Signal}} + N_i^{\mathrm{Background}} \\ N_i^{\mathrm{Signal}} &= \sum_j^{N_i^{\mathrm{bin}}} w_j(M_{\mathrm{Z}}^{\mathrm{fit}}) \\ \sigma_i^2 &= N_i^{\mathrm{MC}}. \end{aligned}$$

$N_i^{\mathrm{Background}}$ is the number of background events and N_i^{bin} is the number of signal Monte Carlo events in the corresponding bin.

The χ^2-value is minimized with respect to the test mass $M_{\mathrm{Z}}^{\mathrm{fit}}$ using the program package MINUIT.

Unbinned likelihood fit

The third method is an unbinned likelihood fit which is also used for the determination of the W boson mass [5]. The probability density to measure an event with invariant mass M_{inv} is given by

$$\mathrm{d}P = \omega \; \mathrm{d}M_{\mathrm{inv}} \; . \tag{8}$$

ω is the normalised differential cross section $\mathrm{d}\sigma/\mathrm{d}M_{\mathrm{inv}}$ at the measured value M_{inv} after selection cuts. In the realistic case where background is present, ω is given by

$$\omega(M_{\mathrm{inv}}, M_{\mathrm{Z}}) = \frac{f \cdot \frac{\mathrm{d}\sigma}{\mathrm{d}M_{\mathrm{inv}}}(M_{\mathrm{inv}}, M_{\mathrm{Z}}) + \frac{\mathrm{d}\sigma_{\mathrm{BG}}}{\mathrm{d}M_{\mathrm{inv}}}(M_{\mathrm{inv}})}{f \cdot \sigma(M_{\mathrm{Z}}) + \sigma_{\mathrm{BG}}}, \tag{9}$$

where σ and $\mathrm{d}\sigma/\mathrm{d}M_{\mathrm{inv}}$ are the total and differential accepted cross sections of the signal and σ_{BG} and $\mathrm{d}\sigma_{BG}/\mathrm{d}M_{\mathrm{inv}}$ are the total and differential cross section of the background, respectively.

The factor f is calculated such that $f \cdot \sigma(M_{\mathrm{Z}})$ agrees with the measured cross section. In other words, the following equation has to be fulfilled:

$$f \cdot \sigma(M_{\mathrm{Z}}) + \sigma_{\mathrm{BG}} = \frac{N_{\mathrm{data}}}{\mathcal{L}_{\mathrm{data}}}, \tag{10}$$

where N_{data} is the number of selected data events and $\mathcal{L}_{\mathrm{data}}$ denotes the integrated luminosity. This ansatz assumes that the background is known much better than the signal process and fluctuations in the number of data events are assigned to the signal.

The total likelihood L for measuring a set of invariant masses M_{inv}^i in data is the product of the individual event probabilities $\omega(M_{\mathrm{inv}}^i, M_{\mathrm{Z}})$ for each selected event i:

$$L(M_{\mathrm{Z}}) = \prod_{i=1}^{N_{\mathrm{data}}} \omega(M_{\mathrm{inv}}^i, M_{\mathrm{Z}}) \; . \tag{11}$$

Box Method

The calculation of the accepted cross-sections can not be done analytically because all selection cuts and the detector response have to be taken into account. Therefore, a Monte Carlo method is used for the calculation where the effects of event selection and detector simulation are considered.

The differential cross-section $\mathrm{d}\sigma/\mathrm{d}M_{\mathrm{inv}}(M_{\mathrm{inv}}^i)$ for data event i is determined from the number of Monte Carlo events which are lying in a small mass interval B_i around the measured mass M_{inv}^i. The endpoints of B_i are chosen such that the number of Monte Carlo events with a reconstructed mass less than M_{inv}^i is equal to the number of events with a mass greater than M_{inv}^i.

In the one-dimensional case the volume of the such a box, V_{box}^i, is simply the size of the interval. Then the differential cross-section can be calculated as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{\mathrm{inv}}}(M_{\mathrm{inv}}^i) \approx \frac{\Delta\sigma}{\Delta M_{\mathrm{inv}}}(M_{\mathrm{inv}}^i) = \frac{1}{\mathcal{L}_{\mathrm{MC}}} \cdot \frac{N_{\mathrm{box}}^i}{V_{\mathrm{box}}^i}, \tag{12}$$

where N_{box}^i is the number of Monte Carlo events in the box B_i. Also needed is the Monte Carlo luminosity which is the ratio of the total number of Monte Carlo events to the total process cross section without cuts:

$$\mathcal{L}_{\mathrm{MC}} = \frac{N_{\mathrm{MC}}}{\sigma^{\mathrm{tot}}} \ . \tag{13}$$

Expression 12 is not an exact calculation of $\mathrm{d}\sigma/\mathrm{d}M_{\mathrm{inv}}$ because it is an average over a mass interval. Only in the limit of infinite Monte Carlo statistics and zero box size the exact value is obtained. But the advantage of the box method is the automatic inclusion of selection and detector effects, and the size of the boxes is chosen to be as small as possible to reduce the difference between the approximate equation 12 and the true value of $\mathrm{d}\sigma_{\mathrm{acc}}/\mathrm{d}M_{\mathrm{inv}}$. On the other hand the number of Monte Carlo events in the box has to be large enough to reduce the statistical error. A good performance of the cross-section calculation is reached with box sizes chosen such that 0.4% of the selected Monte Carlo events in the fit range are contained in each box.

Monte Carlo reweighting

The Monte Carlo events found in the box are reweighted using the weights w of Equation 6. The differential and total cross sections have to be recalculated using the weights:

$$\sigma(M_{\mathrm{Z}}) = \frac{1}{\mathcal{L}_{\mathrm{MC}}} \cdot \sum_{j=1}^{N_{\mathrm{MC}}} w_j(M_{\mathrm{Z}}^{\mathrm{fit}}) \tag{14}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}M_{\mathrm{inv}}}(M_{\mathrm{inv}}(i)) = \frac{1}{V_{\mathrm{box}}^i} \times \frac{1}{\mathcal{L}_{\mathrm{MC}}} \cdot \sum_{j=1}^{N_{\mathrm{box}}^i} w_j(M_{\mathrm{Z}}^{\mathrm{fit}}) \ , \tag{15}$$

where N_{MC} ist the total number of selected signal Monte Carlo events in the chosen $\sqrt{s'}$ interval and V_{box}^i and N_{box}^i are the volume of box i and the number of selected signal Monte Carlo events in box i, respectively.

The probability density ω (eq. 9) is calculated using the cross sections 14 and 15 and inserted into the likelihood function, which is maximised using MINUIT.

All three mass determination methods give consistent results as seen in tables 3, 4, and 5.

Linearity of the methods

To check the behaviour of the three fitting methods, four Monte Carlo files were generated with a different Z mass. These Monte Carlo samples were treated like data and a Z mass was determined for each sample to check the linearity of the extraction methods. The results can be found in table 1 and are displayed together with a linear fit in Figure 9 for the likelihood method.

Monte Carlo	Likelihood	χ^2	Averaging
90.787	90.781 ± 0.026	90.777 ± 0.025	90.812 ± 0.026
90.987	90.979 ± 0.026	90.978 ± 0.025	90.947 ± 0.026
91.387	91.358 ± 0.026	91.359 ± 0.026	91.347 ± 0.026
91.587	91.535 ± 0.026	91.545 ± 0.025	91.605 ± 0.026

Table 1: The results obtained by fitting Monte Carlos with four different Z masses $M_{\mathrm{Z}}^{\mathrm{MC}}$. In general there is a good agreement between the generator mass and the result of the mass determination. All masses are given in GeV. The errors are only statistical.

Systematic uncertainties

The following systematic effects have been studied:

- Initial State Radiation
  Different approaches to model ISR are used in the Monte Carlo generators PYTHIA and KK2F. The KK2F Monte Carlo has the most complete modelling of the ISR process, corresponding to $\mathcal{O}(\alpha^2)$ with coherent exclusive exponentiation (CEEX). A more simple structure function approach is used in the PYTHIA program. We therefore use the result obtained with KK2F and take 50% of the difference between the two generators, that is 21 MeV, as systematic uncertainty.

- Jet energies and angles
  The effect of two different sets of energy calibration factors is studied. A difference of 46 MeV is found and taken as systematic error. During the reconstruction of the events, a vertex correction to the nominal LEP fill-vertex is applied. To this we assign 40 MeV uncertainty, which corresponds to 20% of the mass difference between the mass determined with and without the vertex correction.

- Monte Carlo statistics
  The uncertainty resulting from limited Monte Carlo statistics is found to be 6 MeV.

- Hadronisation
  Different tunings of the PYTHIA hadronisation model have been tested. We estimate 12 MeV to be the contribution to the systematic error. It is foreseen to study other models like ARIADNE and HERWIG in future.

- Background
  By scaling the background Monte Carlos by ±50% a 20 MeV mass difference is observed and taken as systematic uncertainty.

- Detector ineffencies
  To estimate the effect of the detector simulation, the mass has also been determined using a signal Monte Carlo without any detector inefficencies. We find a difference in the reconstructed Z mass of 18 MeV and take 50%, that is 9 MeV, as systematic error.

- LEP energy
  The beam energy measured by the LEP machine has an uncertainty of 20 – 25 MeV depending on the centre-of-mass energy. This leads to an error of the Z mass of 20 MeV.

Source	Effect [MeV]
Initial State Radiation	21
Jet energies	46
Jet angles	40
MC statistics	6
Hadronisation	12
Background	20
Detector simulation	9
LEP energy	20
Total	72

Table 2: Systematic uncertainties for the combined result

We obtain 72 MeV as total systematic uncertainty for the mean mass determined from the whole data sample.

Results

We summarize the results from the averaging method, the χ^2 fit, and the likelihood fit in Tables 3, 4, and 5, respectively. Figure 11 displays the spread of the individual measurements for each energy bin. The results of the three fitting methods are in good agreement with each other.

Figure 10 shows the $\sqrt{s'}$ distribution in the fitting range for the likelihood method. Also shown is the Monte Carlo distribution, which is weighted accordingly. We find a Z mass of 91.266 ± 0.034 (stat.) $\pm$ 0.072 (sys.) GeV which is in good agreement with the LEP I value of $M_Z = 91.190 \pm 0.003$ GeV. The spread of the measurements with the likelihood method for each energy bin is shown in Figure 12.

If the measured mass, M_Z^{fit}, differs from the "true" mass, M_Z, this can be due to a shift in the beam energy, ΔE_{beam}:

$$\Delta E_{\text{beam}} = -E_{\text{beam}} \cdot \frac{M_Z^{\text{fit}} - M_Z}{M_Z} \, . \tag{16}$$

The value obtained with the Z mass determined with the likelihood method,

$$\Delta E_{\text{beam}} = -0.083 \pm 0.037 \text{ (stat.)} \pm 0.075 \text{ (sys.) GeV} \, ,$$

is in agreement with no shift.

$\sqrt{s}$ GeV	M_Z^{Mean} GeV
189	$91.234 \pm 0.059 \pm 0.074$
192	$91.343 \pm 0.146 \pm 0.080$
196	$91.339 \pm 0.094 \pm 0.076$
200	$91.201 \pm 0.102 \pm 0.076$
202	$91.444 \pm 0.149 \pm 0.080$
205	$91.345 \pm 0.106 \pm 0.073$
207	$91.221 \pm 0.083 \pm 0.073$
208	$91.562 \pm 0.350 \pm 0.073$
Mean	$91.275 \pm 0.035 \pm 0.072$

Table 3: Results of the averaging method for each energy bin.

$\sqrt{s}$ [GeV]	$M_{\mathrm{Z}}^{\chi^2}$ [GeV]
189	$91.261 \pm 0.058 \pm 0.074$
192	$91.295 \pm 0.139 \pm 0.080$
196	$91.371 \pm 0.090 \pm 0.076$
200	$91.066 \pm 0.098 \pm 0.076$
202	$91.321 \pm 0.142 \pm 0.080$
205	$91.290 \pm 0.101 \pm 0.073$
207	$91.185 \pm 0.083 \pm 0.073$
208	$91.636 \pm 0.328 \pm 0.073$
Mean	$91.253 \pm 0.034 \pm 0.072$

Table 4: Results of the χ^2 method for each energy bin

$\sqrt{s}$ [GeV]	M_{Z}^{LL} [GeV]
189	$91.270 \pm 0.059 \pm 0.074$
192	$91.292 \pm 0.138 \pm 0.080$
196	$91.380 \pm 0.092 \pm 0.076$
200	$91.104 \pm 0.100 \pm 0.076$
202	$91.286 \pm 0.134 \pm 0.080$
205	$91.325 \pm 0.103 \pm 0.073$
207	$91.200 \pm 0.084 \pm 0.073$
208	$91.606 \pm 0.340 \pm 0.073$
Mean	$91.266 \pm 0.034 \pm 0.072$

Table 5: Results of the likelihood fit for each energy bin

References

[1] L3 Collab., B. Adeva *et al.*, Nucl. Inst. Meth. **A 289** (1990) 35;
M. Chemarin *et al.*, Nucl. Inst. Meth. **A 349** (1994) 345;
M. Acciarri *et al.*, Nucl. Inst. Meth. **A 351** (1994) 300;
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[2] L3 Collab., M. Acciarri *et al.*, E. Phys. J. **C 16** (2000) 1.

[3] The LEP Energy Working Group, *Evaluation for the LEP centre-of-mass energy for data taken in 2000*, LEP Energy Working Group 01/01.

[4] F.A. Berends *et al.*, in "Z Physics at LEP 1", CERN Report CERN 89-08, eds G. Altarelli,R. Kleiss and C. Verzegnassi (CERN, Geneva, 1989) Vol. 1, p. 89..

[5] The W-Physics Analysis Group of the L3 Collaboration, *Preliminary Results on the Measurement of Mass and Width of the W boson at LEP*, L3 Note 2637, February 2001.

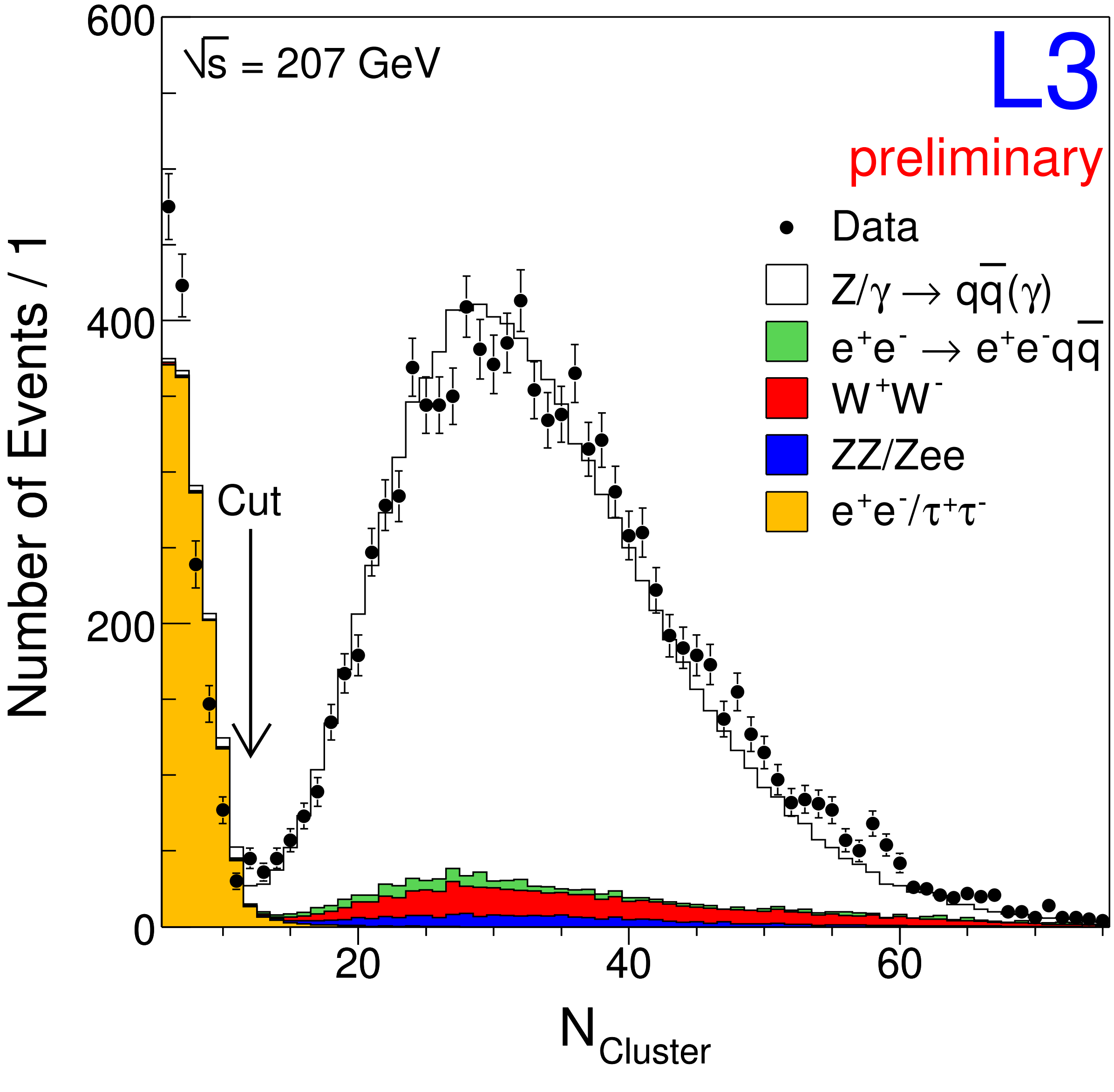

Figure 1: Distribution of the number of clusters for the 207 GeV data set.

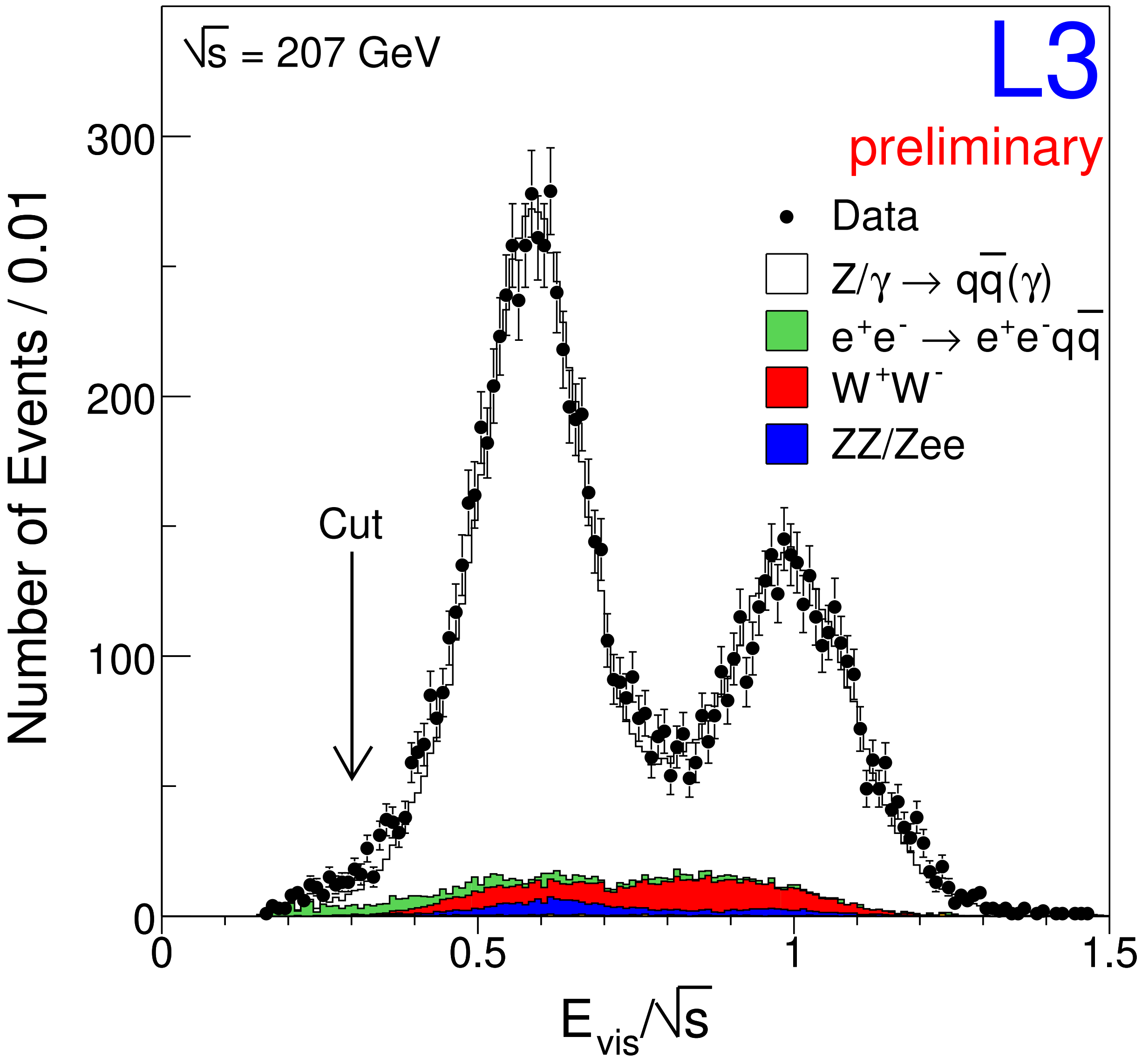

Figure 2: Distribution of the visible energy, normalised to the centre-of-mass energy, for the 207 GeV data set.

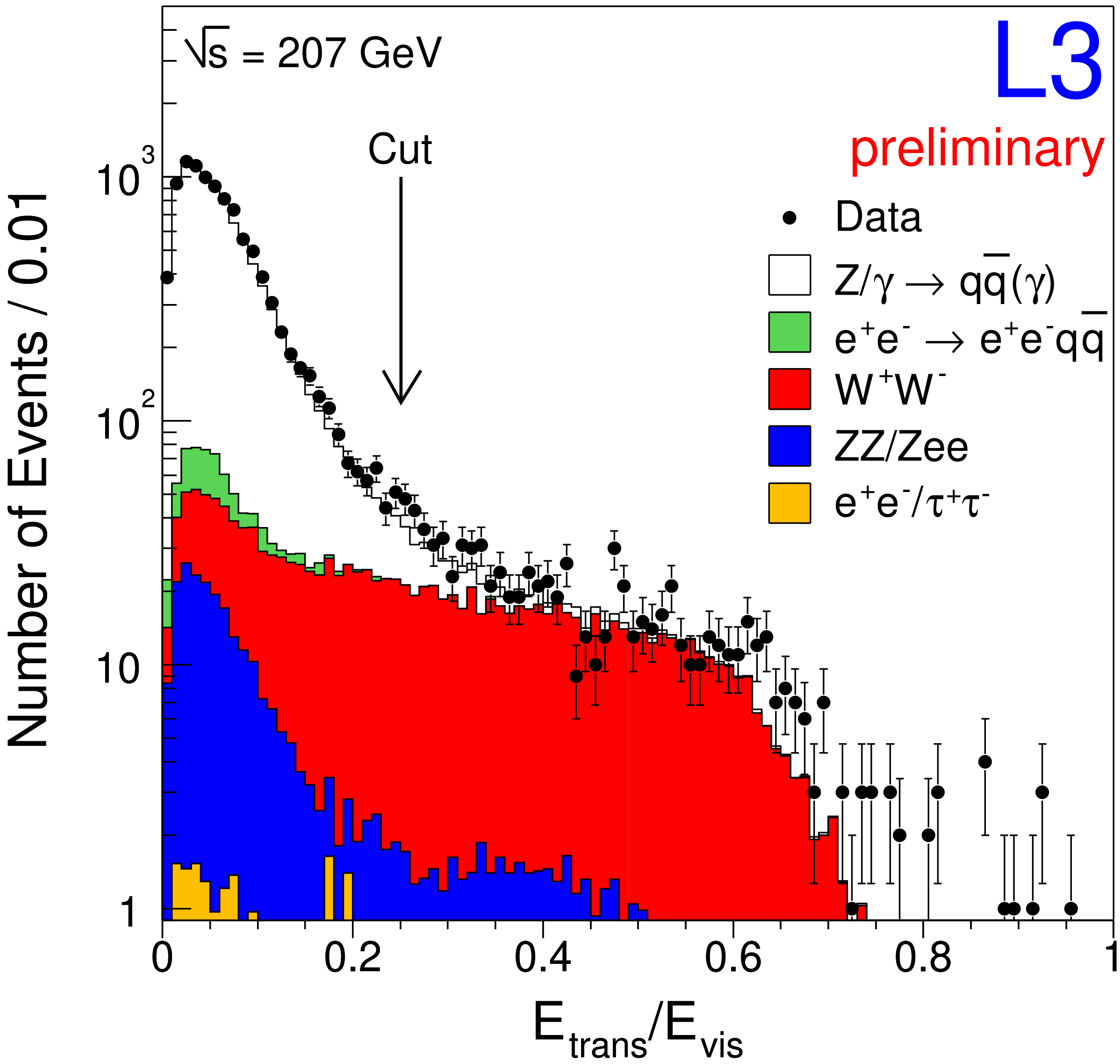

Figure 3: Distribution of the transverse energy imbalance, normalised to the visible energy, for the 207 GeV data set.

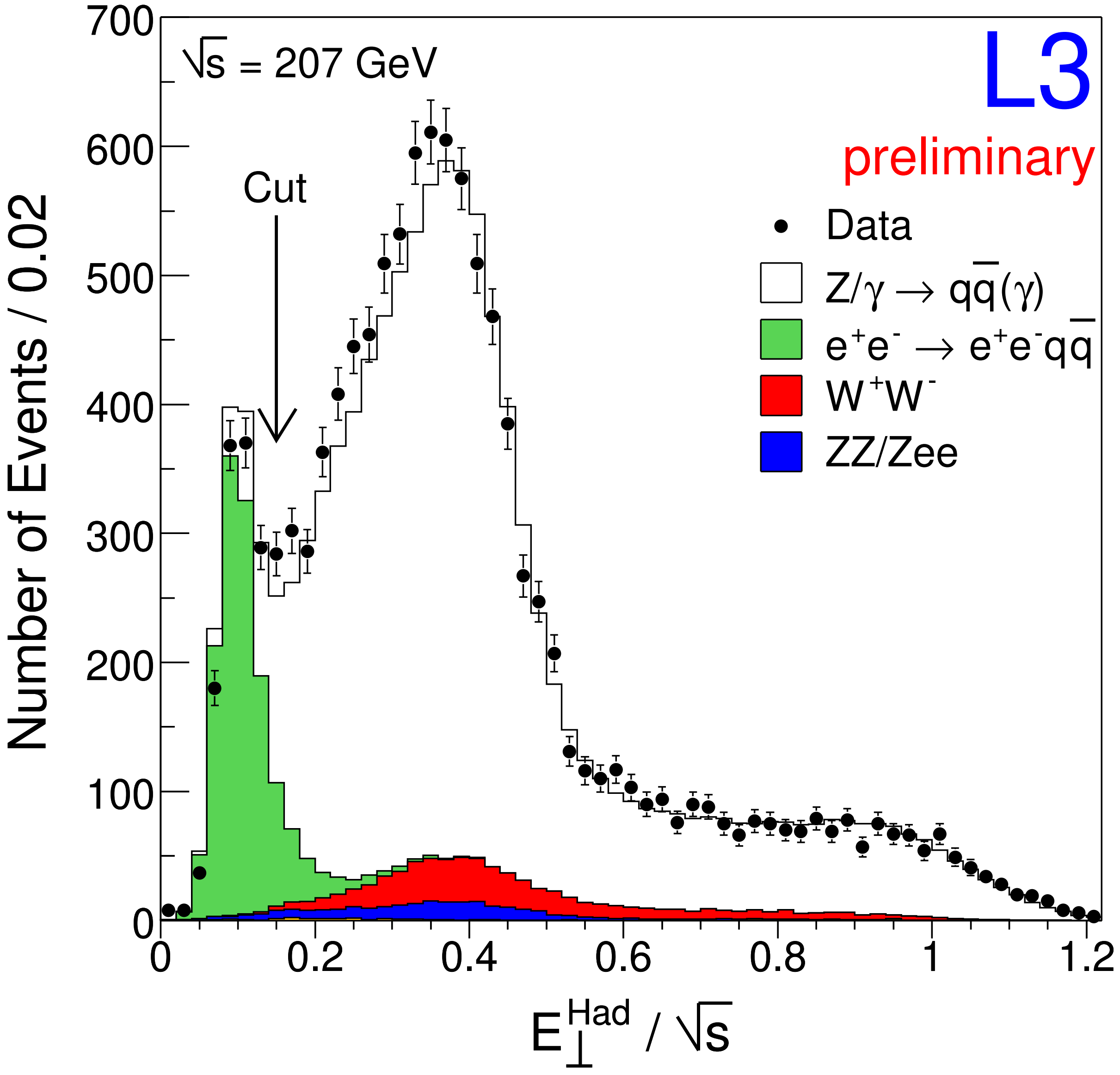

Figure 4: Distribution of the transverse energy of the hadronic clusters, normalised to the centre-of-mass energy, for the 207 GeV data set.

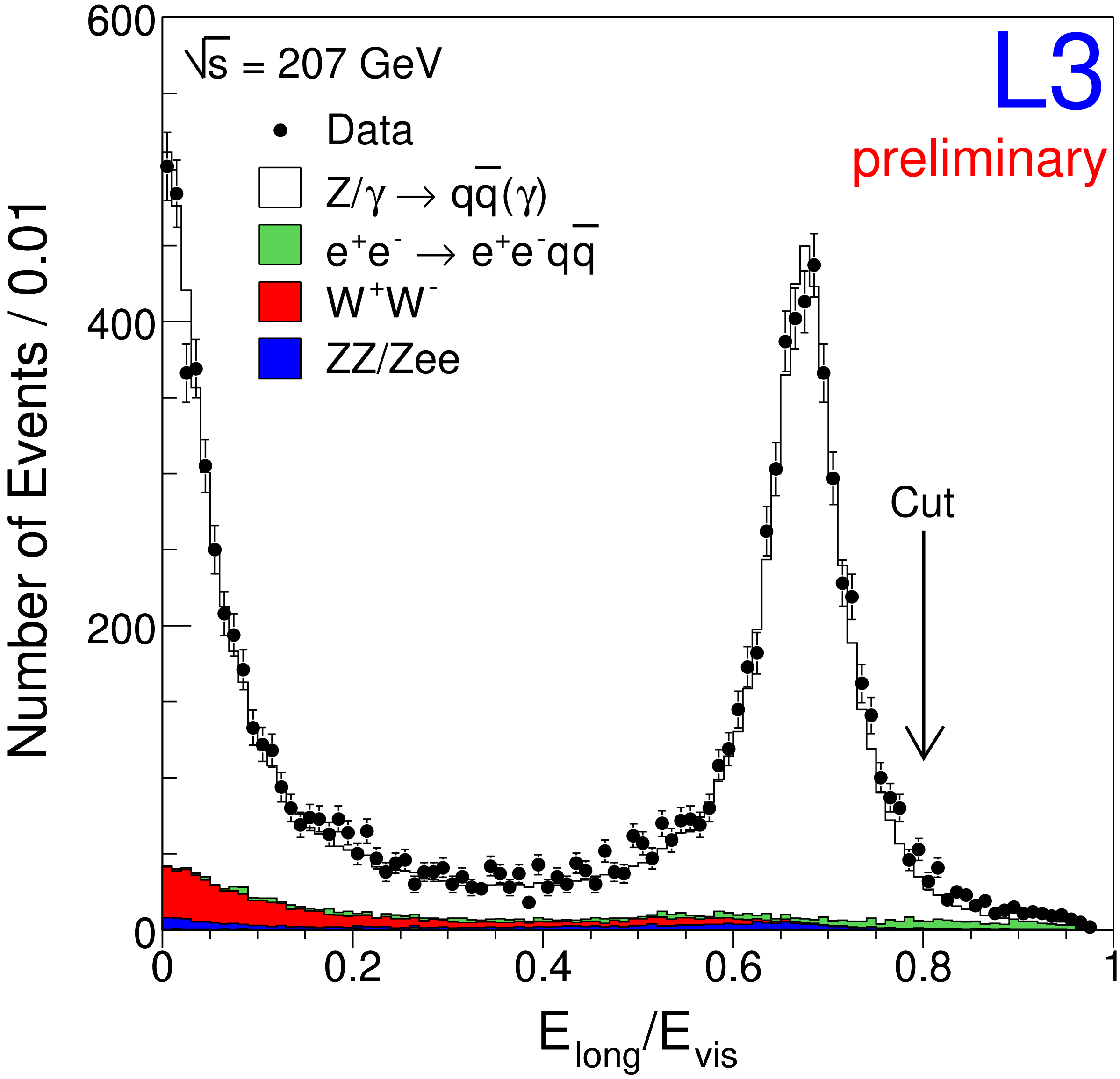

Figure 5: Distribution of the longitudinal energy imbalance, normalised to the visible energy, for the 207 GeV data set.

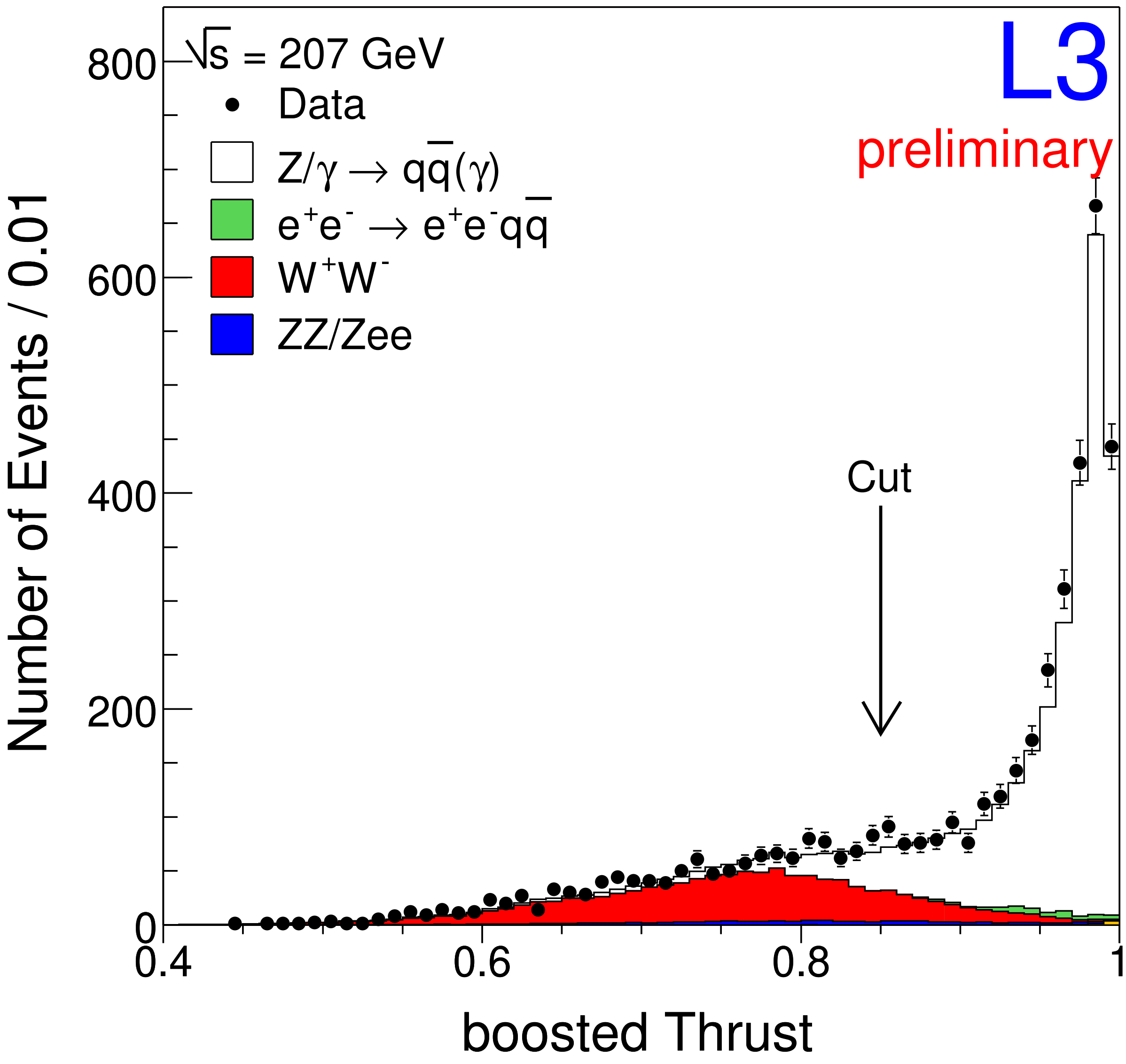

Figure 6: Distribution of the thrust in the rest frame of the decaying Z for the 207 GeV data set. A cut on the hadronic energy is already applied ($E_{Had} > 0.7 \cdot \sqrt{s}$).

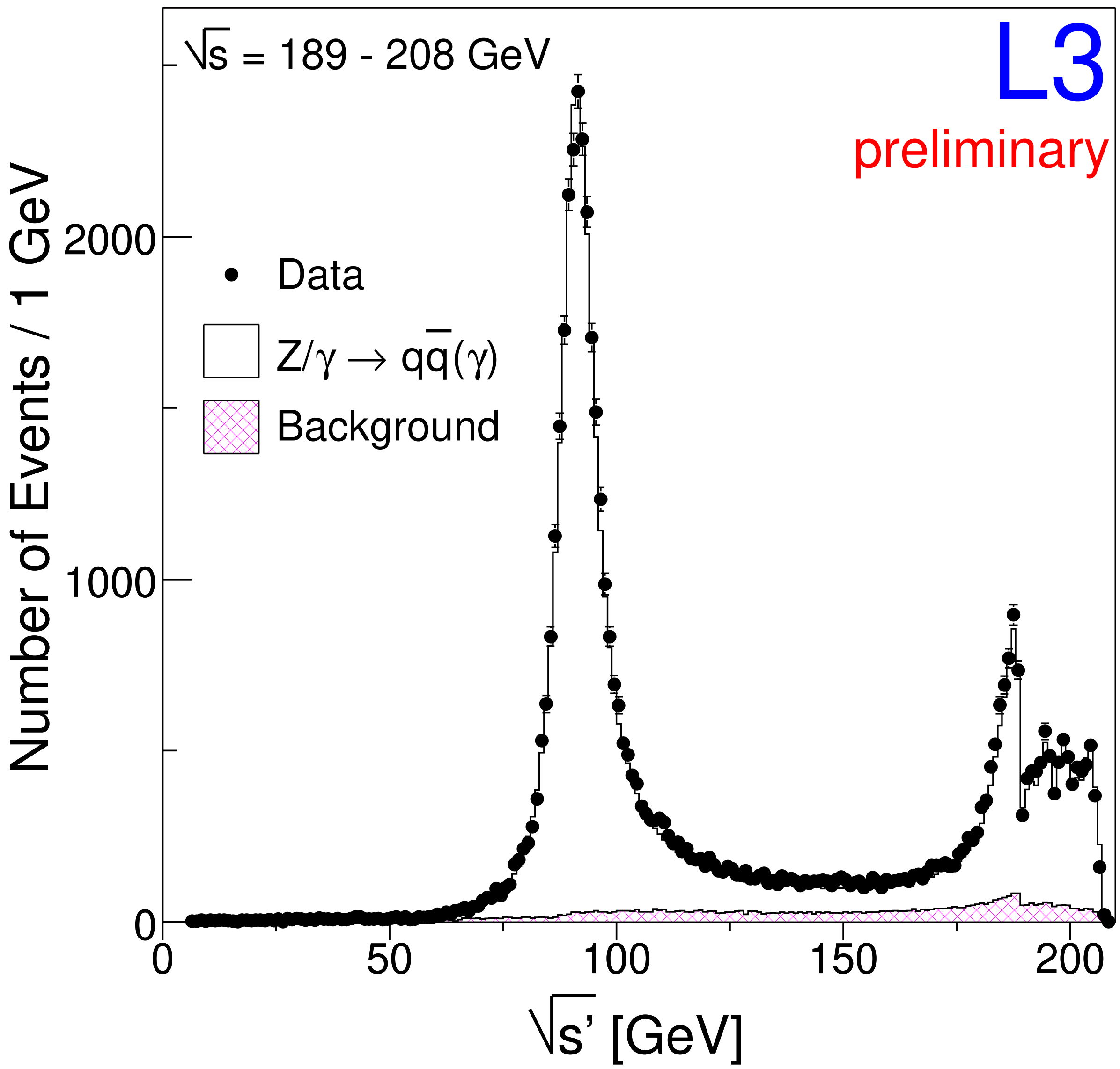

Figure 7: The $\sqrt{s'}$ distribution summed over all energy bins from 189 GeV to 208 GeV for the hadronic final state. The return to the Z peak is clearly visible.

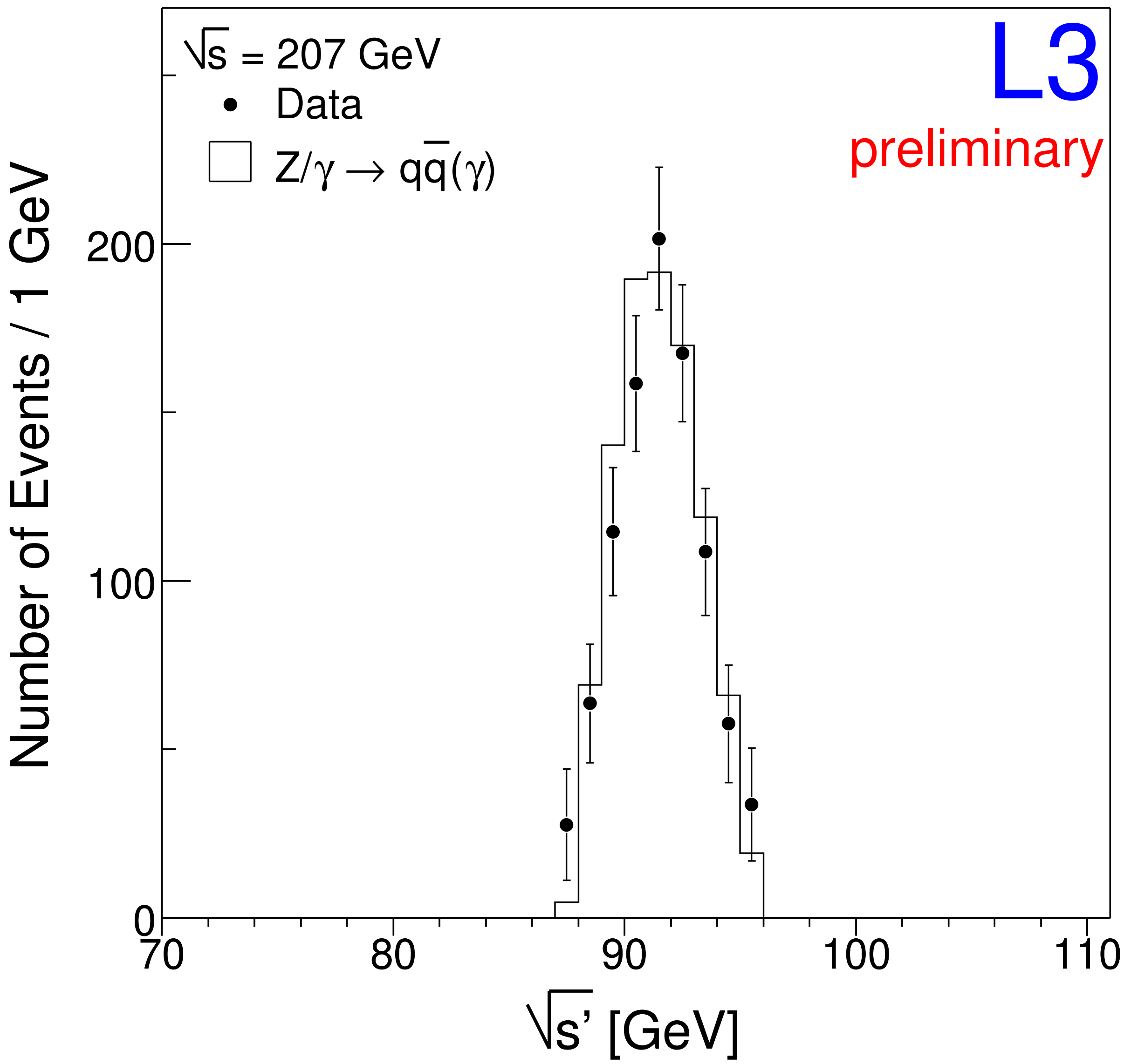

Figure 8: The distribution which is used for the averaging method. The lower half of the $\sqrt{s'}$ histogram has been cut off. The remaining part is averaged.

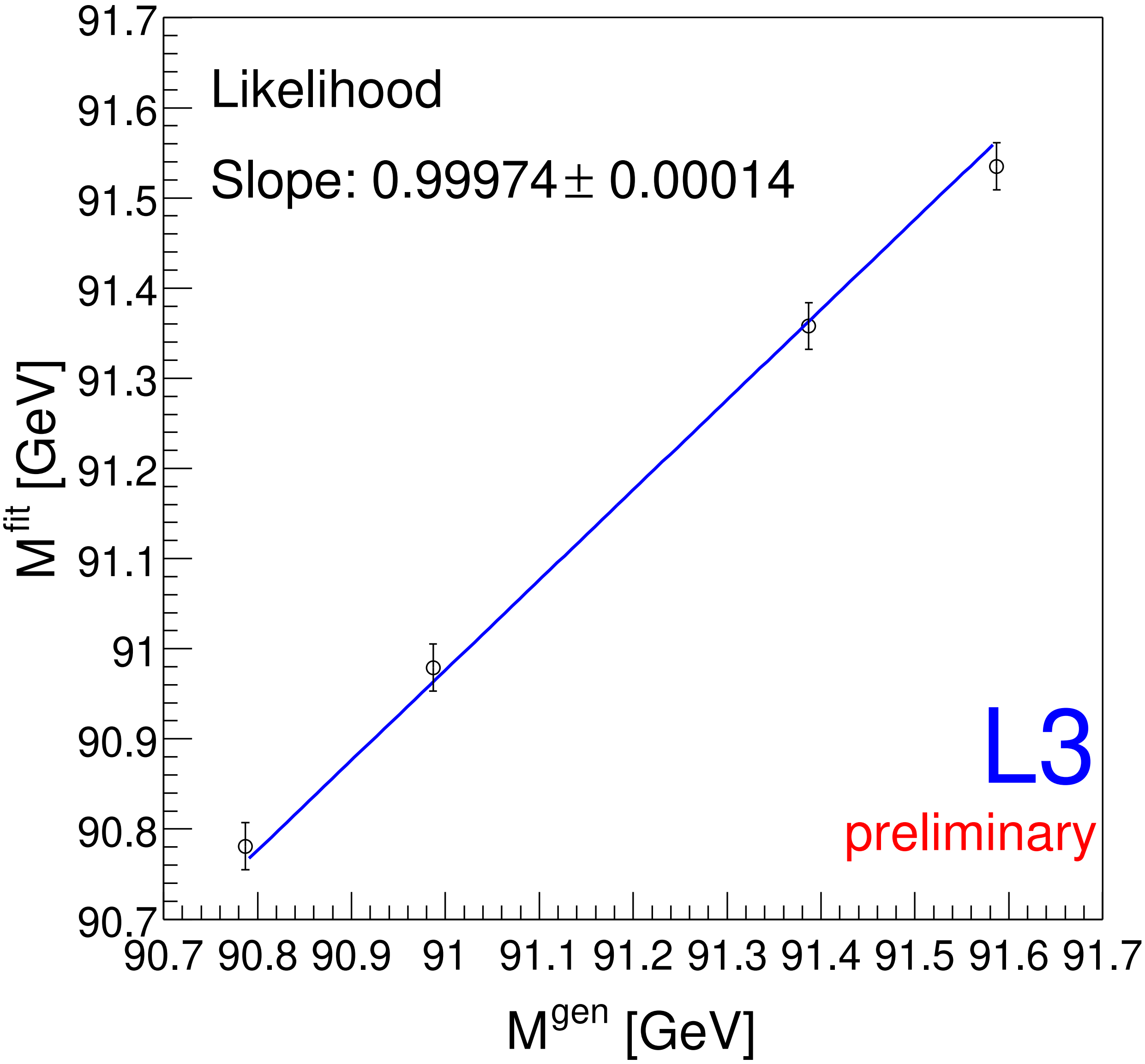

Figure 9: The plot shows the results of the likelihood method, M^{fit}, for four test Monte Carlos with different Z masses, M^{gen}. A linear fit is applied to distribution and also shown in the plot.

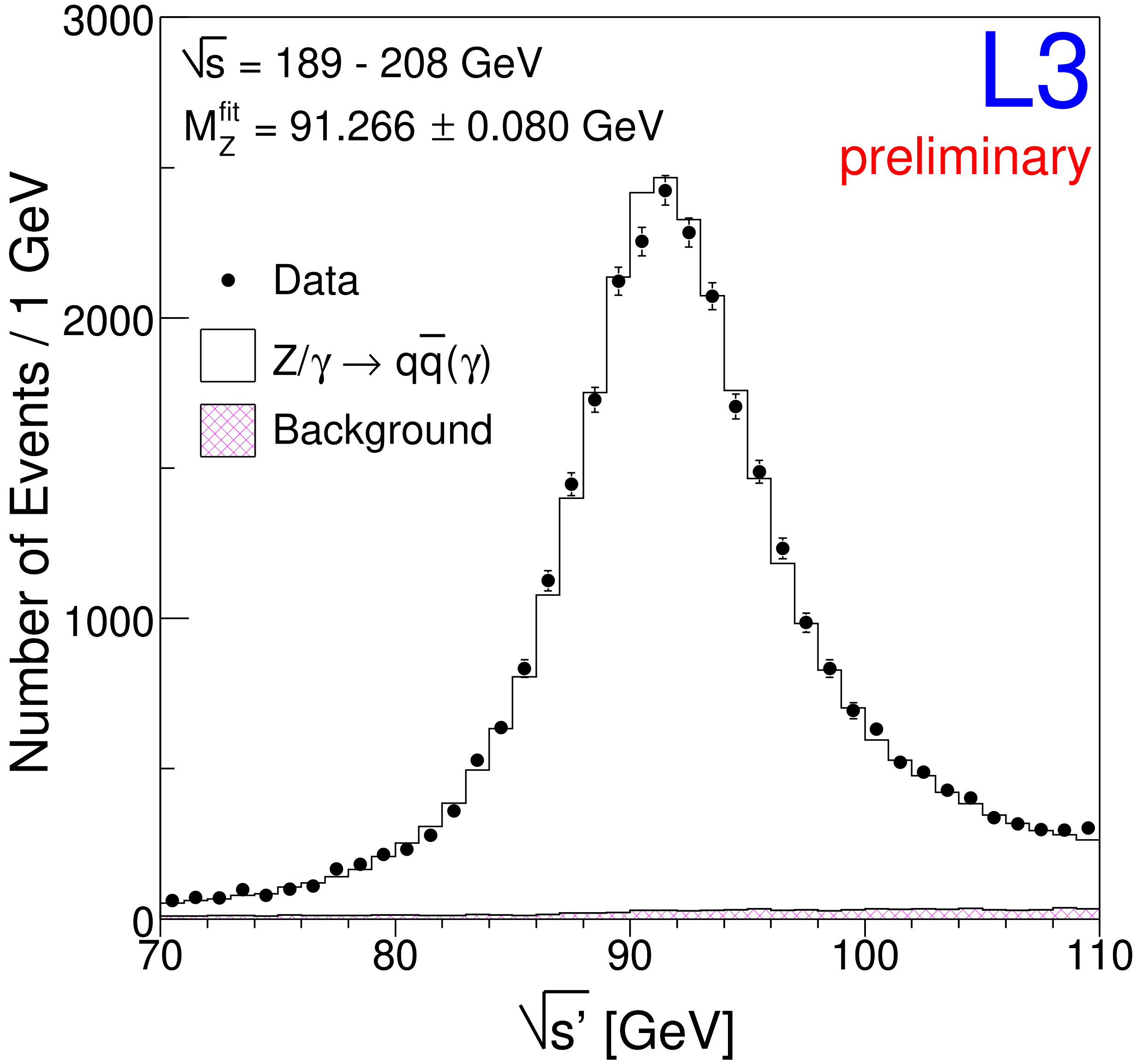

Figure 10: The $\sqrt{s'}$ distribution, summed over all energy bins from 189 GeV to 208 GeV. Also shown is the reweighted signal Monte Carlo and the background expectation.

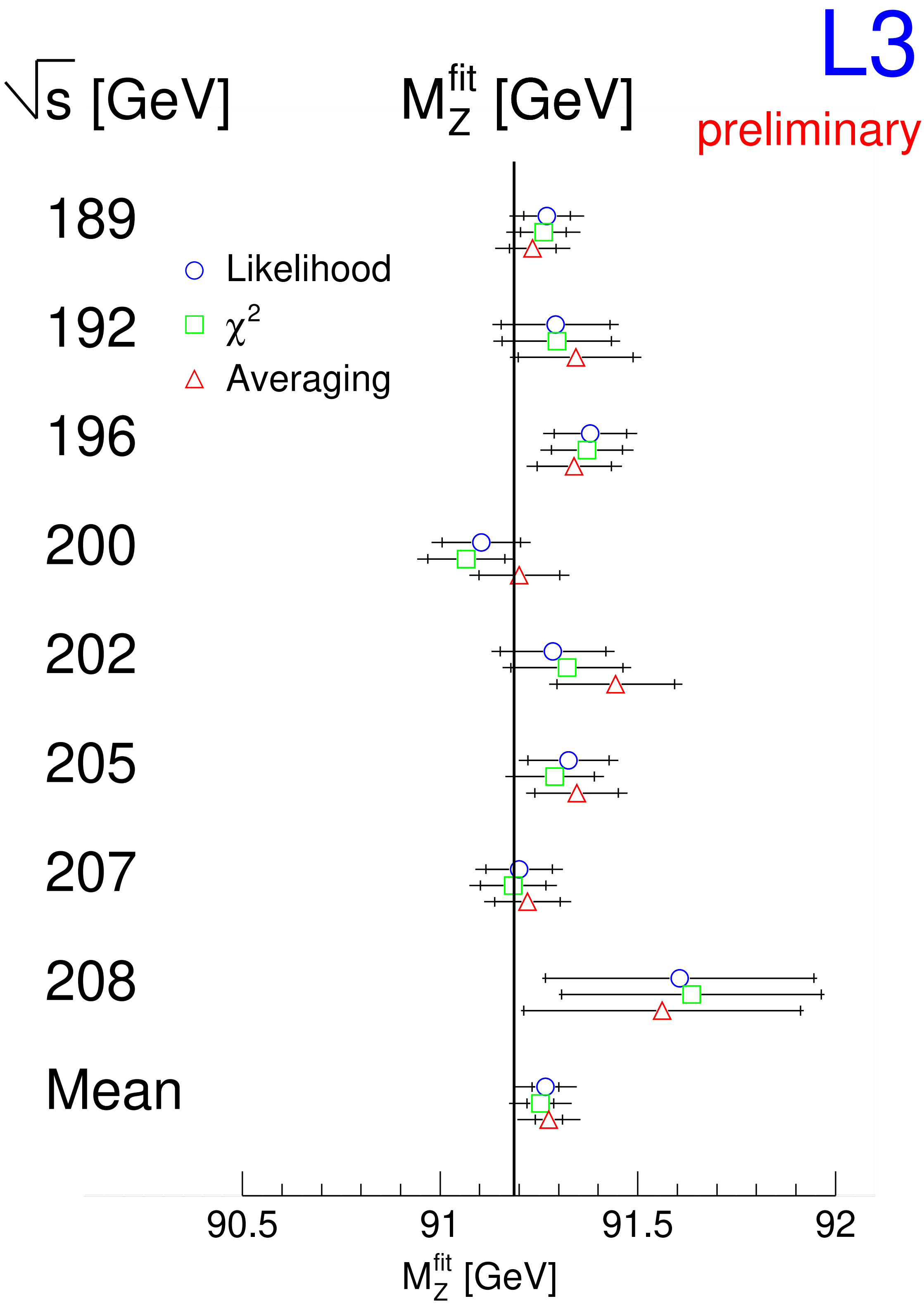

Figure 11: This graph displays the results of the likelihood fit, the χ^2 fit, and the averaging method for each energy bin. The mean results are in good agreement.

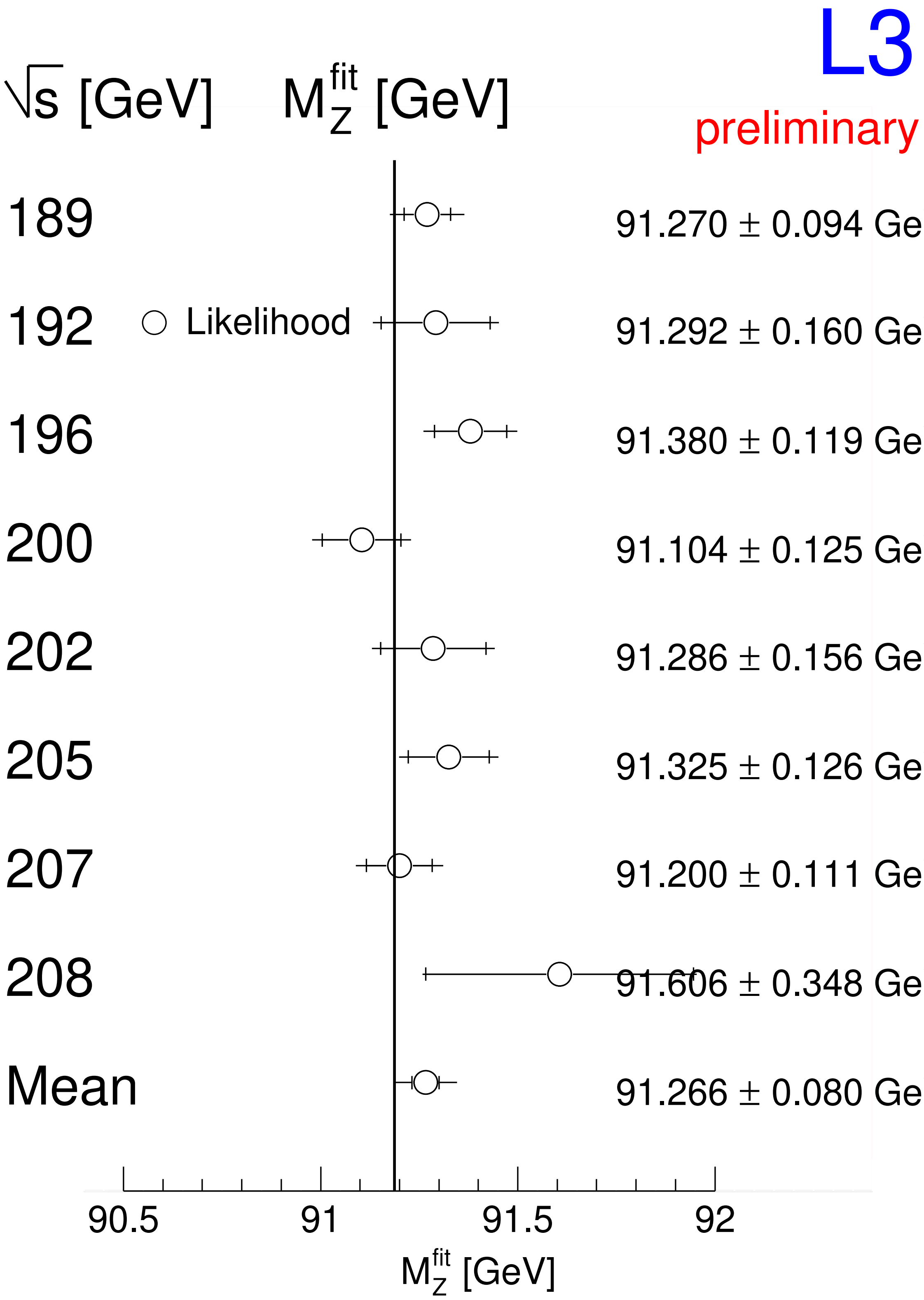

Figure 12: This graph displays the results of the likelihood fit for each energy bin. Also shown are the numerical values. The mean result is in agreement with the LEP I value of the Z mass.