L3 note # 1805
July 10, 1995

Measurement of QCD color factors with four jet events

The L3 Collaboration

Abstract

A study of four jet events selected from 2.7M hadronic Z decays collected during 1991, 1992, 1993 and 1994 by the L3 detector is presented. The measured jet configurations allow to test the underlying gauge group of the strong interactions. A fit to the differential cross section yields the ratios of the *color factors*, C_A/C_F and T_R/C_F, which label the gauge group. The unexpected dependence of C_A/C_F on the jet resolution parameter y_{cut} is discussed.

submitted to the EPS and the lepton-photon Conference, july 1995

Introduction

The gluon self-coupling is a fundamental property of QCD arising from the non Abelian nature of the theory which describes the strong interactions. Up to second order in perturbation theory, three constants, C_F, T_R and C_A, so called *color factors*, determine the structure of the gauge group. These constants are related to the fundamental couplings of the theory: C_F determines the coupling strength of a gluon to a quark, T_R the strength of the splitting of a gluon into a quark pair and C_A is related to the gluon self-coupling. Their values for $SU(3)$, the gauge group of QCD, are $C_F = 4/3$, $T_R = n_f \cdot T_F = 5/2$, $C_A = 3$, where $n_f = 5$ is the number of active flavors at LEP energies.

Using the analytical expression of the differential cross section for four parton processes at Born level [1], a fit of the *color factors* can be performed on a four jet sample, the experimental signature of four parton events. The procedure that has been followed is an unbinned likelihood fit to the jet pair invariant mass distributions of the ratios $C_1 = C_A/C_F$ and $C_2 = T_R/C_F$, since C_F together with the QCD coupling constant, α_s, can be absorbed into an overall normalization of the total four jet cross section.

Selection of the Sample

We have selected hadronic events on the Z peak from 91, 92, 93 and 94 running periods.

Two different experimental methods for measuring the energy flow have been used. The first one is based on pure calorimetric criteria while the second one further includes charged tracks and correlations among different detector regions [2]. The visible energy resolution ($\simeq 13\%$ for the first method) is improved down to $\simeq 9\%$ with the second method. After applying standard cuts, we select 2.7M hadronic events.

The four jet sample has been obtained from the former applying the JADE [3] jet-finder algorithm on calorimetric clusters with four different values of the jet resolution parameter, $y_{cut} = 0.02, 0.03, 0.04, 0.05$. In order to improve the jet angular resolution of the clustering algorithm, three preclustering schemes have been used: DURHAM [4] algorithm with $y_\perp = 0.001$ and LUCLUS [5] in the invariant mass option with $y_{cut} = 0.005$ and in the relative transverse momentum option with $d_{join} = 2$ GeV. With these procedures, the jet angular resolution is improved by $\simeq 10\%$.

The final four jet sample is selected for each preclustering scheme and value of the jet resolution parameter from the events which are reconstructed as four jets with both experimental methods. This requirement enhances the purity of the sample in four parton events. The number of selected events for each preclustering algorithm are shown in table 1.

y_{cut}	Durham ($y_\perp$)	Luclus (y_{cut})	Luclus (d_{join})
0.02	105611	99518	109979
0.03	49545	48067	51910
0.04	24334	24311	25939
0.05	12055	12389	13085

Table 1: Number of selected four jet events on data for the three preclustering schemes.

Hadronization and Detector Correction

The analytical differential cross section has been used as the theoretical probability density function both on hadron and on detector level.

Fragmentation and detector effects change the value of the fitted *color factors*. This effect can be viewed as follows:

$$\left(C_1^H, C_2^H\right) = g^H\left(C_1^P, C_2^P\right)$$

$$\left(C_1^D, C_2^D\right) = g^D\left(C_1^H, C_2^H\right)$$

where the indices P, H, D label parton, hadron and detector levels and g^H and g^D are respectively the hadronization and detector response functions which have been calculated from simulated event samples.

For the fragmentation effects, 6M JETSET matrix element events have been generated while for detector effects 250K four parton events from JETSET matrix element have been fully simulated and reconstructed. The misidentified non four parton events are taken from 1.3M JETSET parton shower events.

The inversion of the *color factor* mapping is implemented by reweighting the Monte Carlo events on a grid of C_1, C_2 values and using a linear interpolation between the points of the grid. The unfolding has been carried out in two steps, mapping the measured *color factors* back to hadron level and futher mapping the corrected values to parton level.

Systematic Studies

We have checked the following possible sources of systematic error in the analysis:

- **Experimental uncertainties**
  The values of the kinematical variables of the four jet events depend on the experimental method for measuring the calorimetric energy flow. Since the *color factor* fit depends on the kinematics of the event, different energy flow methods can produce unlike results. We have chosen for the central values of the *color factors* the method which includes charged tracks and correlation among different detector regions because it takes into account more detector effects.

  The full difference of the values obtained with both methods is quoted as systematic experimental uncertainty. The average over the precluster algorithms is quoted in table 2.

	$y_{cut} = 0.02$	$y_{cut} = 0.03$	$y_{cut} = 0.04$	$y_{cut} = 0.05$
$\Delta C_1(exp)$	0.08	0.13	0.33	0.90
$\Delta C_2(exp)$	0.24	0.33	0.66	0.88

Table 2: Experimental uncertainties of the color factors averaged over the precluster algorithms.

- **Uncertainties of the correction method**
  The addition of JETSET P.S. events for simulating the misidentified non four parton events introduces two possible sources of error. First, the fraction of background events is uncertain since the *parton* definition in P.S. events is unclear[1)]; secondly, the topology of the P.S. events differs from the topologies predicted by the full matrix element. The former has been estimated varying the *parton* definition in P.S. events and therefore changing the background fraction. The latter has been estimated comparing the values obtained from the two-step unfolding with the values obtained with a straight unfolding from detector to parton level, which makes no use of the full matrix element generated sample.

  Both errors have been treated independently and added quadratically to determine the uncertainty of the detector correction method. The values averaged over the three preclustering algorithms are quoted in table 3.

	$y_{cut} = 0.02$	$y_{cut} = 0.03$	$y_{cut} = 0.04$	$y_{cut} = 0.05$
$\Delta C_1(bck)$	0.06	0.04	0.06	0.08
$\Delta C_1(unf)$	0.09	0.22	0.22	0.21
$\Delta C_2(bck)$	0.84	0.20	0.07	0.04
$\Delta C_2(unf)$	0.95	0.22	0.19	0.33

Table 3: Systematic errors of the unfolding method due to the fraction of misidentified non four parton events (labelled *bck*) and the unfolding path as discussed in the text (labelled *unf*), averaged over the three preclustering algorithms.

- **Hadronization uncertainties**
  The most relevant parameters in the JETSET M.E. for this analysis are the QCD scale parameter Λ_{ME}, the transverse fragmentation width σ_q and the longitudinal fragmentation parameter b. The L3 tuned values [6] have been varied within their errors and, since they are correlated, the largest deviation has been taken as systematic uncertainty. The values are given in table 4.

	$y_{cut} = 0.02$	$y_{cut} = 0.03$	$y_{cut} = 0.04$
$\Delta C_1(had)$	0.04	0.09	0.09
$\Delta C_1(BE)$	0.11	0.16	0.16
$\Delta C_2(had)$	0.37	0.30	0.39
$\Delta C_2(BE)$	0.10	0.08	0.39

Table 4: Systematic error due to uncertainties in hadronization modeling (labelled *had*) and shift of the color factors on parton level when turning Bose-Einstein effects off (labelled *BE*), averaged over the precluster algorithms. The hadronization uncertaintes corresponding to $y_{cut} = 0.05$ have not been included.

  Another possible source of systematic error is the parametrization of Bose-Einstein effects in JETSET. They have been turned off and the observed shifts have been mapped to parton level. The values are also shown in table 4.

[1)] We define a parton shower three jet event on parton level as a three *parton* event. The value of the jet resolution parameter corresponds to the y_{min} value of the matrix element.

Since Bose-Einstein parametrization is correlated with the fragmentation parameters the maximum observed deviation is taken as systematic uncertainty in the hadronization.

- **Quark mass effects**
The analytical calculation for the differential cross section implemented in the matrix element option of JETSET assumes massless quarks. Mass effects only appear as corrections on the phase space. We have compared the results of the fit with the ones obtained on a sample of events generated with all the quark mass effects included in leading order [7]. The differences are small and can be neglected [2].

- **Higher orders**
A possible source of systematic errors is the misidentification of five parton processes as four jet events. Although the matrix element option in JETSET does not include these processes, the parton shower option can be used as an estimation of this contribution. The effect on the *color factors* has been estimated in two different ways. First, adding to the JETSET M.E. sample parton shower events which have been reconstructed as five jets on parton level and four jets after fragmentation. Secondly, the misidentified events have been simulated with a five parton matrix element generator [8]. Since both samples are normalized to the rates predicted by the parton shower, these two methods can not be considered as completely independent. The effect on the *color factors* due to higher order inclusion is compatible for both approaches. The observed shifts can only be an estimation of the higher order effects because the virtual corrections to four parton processes have not been yet calculated. Morover, for a full matrix element generator up to third order in perturbation theory, the fragmentation parameters should be retuned. This shows the correlation between fragmentation and higher order effects and thus, the latter, estimated as the smallest of them, has not been included in the full systematic error.

Results and Discussion

The results for each value of the jet resolution parameter y_{cut} have been obtained using the average over the three different preclustering algorithms. The *color factors* thus determined are

y_{cut}	$C_1 \pm \Delta C_1(stat) \pm \Delta C_1(sys)$	$C_2 \pm \Delta C_2(stat) \pm \Delta C_2(sys)$
0.02	$1.32 \pm 0.06 \pm 0.20$	$2.21 \pm 0.15 \pm 1.38$
0.03	$1.85 \pm 0.12 \pm 0.36$	$1.41 \pm 0.24 \pm 0.67$
0.04	$2.43 \pm 0.23 \pm 0.58$	$2.12 \pm 0.44 \pm 1.09$
0.05	$2.91 \pm 0.49 \pm 1.21$	$2.14 \pm 0.71 \pm 1.52$

The 68% C.L. contour plot has been drawn in fig 1. The correlation between C_1 and C_2 is $\rho = 0.09, 0.07, 0.02, -0.06$ for $y_{cut} = 0.02, 0.03, 0.04$ and 0.05.

The unexpected variation of C_1 with y_{cut} can not be explained with the systematic errors taken into account at least for $y_{cut} = 0.02$. This effect might be due either to an unknown physical effect or to the non included terms in the Monte Carlo simulation.

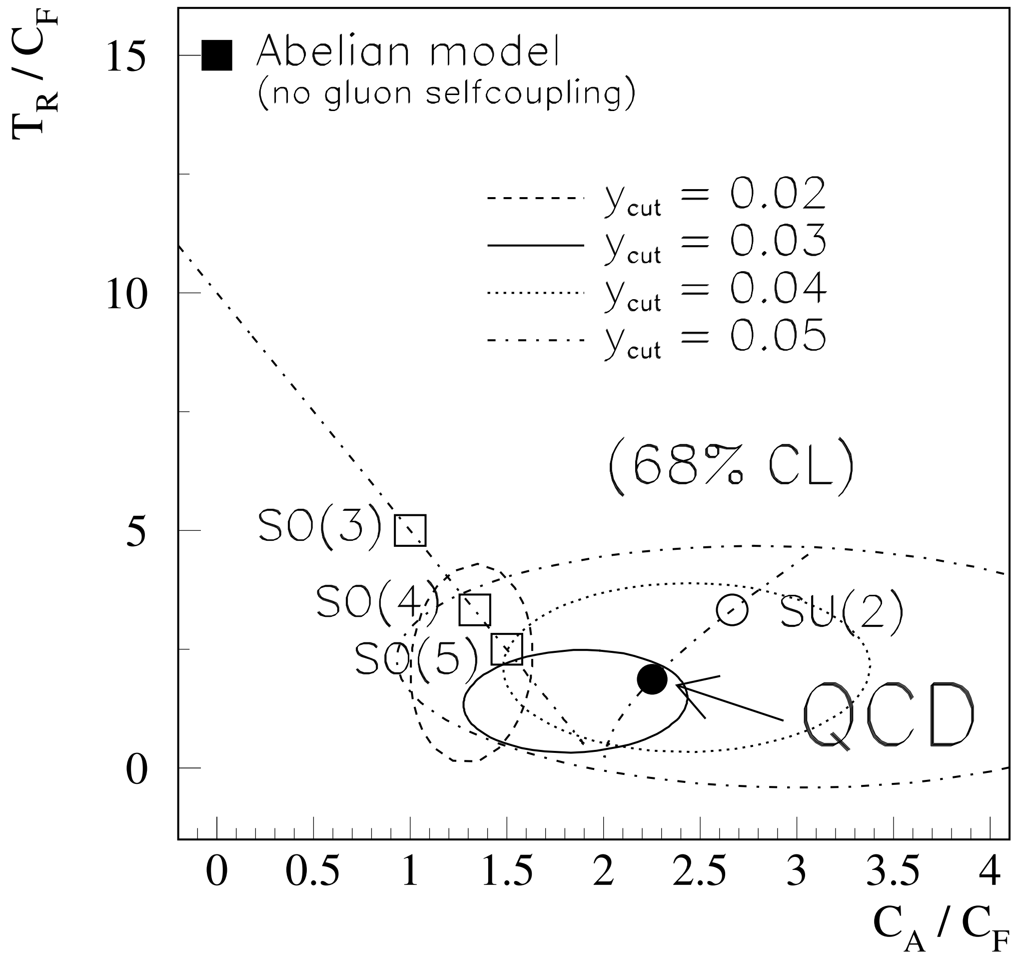

Figure 1: Color factors as determined in this analysis for different values of the jet resolution parameter y_{cut}. Their value for other Lie groups have been also included.

Following the latter explanation of the effect, it could be argued that the fraction of misidentified events is relevant for the lowest y_{cut} value. Since the knowledge of the kinematics of these events and their rate in the data sample is not complete due to the lack of non-leading contributions of the parton shower generator, the effect on the *color factors* could be large.

We have checked the estimation of the misidentified event rate in our data sample fitting this fraction on it. As in the detector correction procedure described before this task is technically hard to implement, we have developed a new correction method also suitable for cross-checking the analysis.

The analytical expression of the differential cross section can be viewed as a linear combination of kinematical functions, which depends on the parton pair invariant masses and do not rely on the gauge group structure. The coefficients of this linear combination are the *color factors*. The aim is to parametrize the fragmentation and detector effects on the kinematical part. The *color factors* can be thus fitted on detector level and no unfolding is required. Within this scheme we can also include the misidentified event distribution on detector level and fit its rate on data.

The kinematical functions, depending on the jet invariant masses, have been parametrized by a binning of the allowed phase-space. Assuming a linear dependence of the detector level differential cross section on the *color factors*, the kinematical part of it can be extracted by reweighting the Monte Carlo sample to different values of the *color factors*. The linearity holds almost prefectly for a wide range of C_1 and C_2 values and the slight non linearities can be absorbed with a correction of the observed deviations when the Monte Carlo sample is fitted.

The misidentified fraction fitted with this method agrees with the parton shower estimation and the observed shift on C_1 is confirmed with this independent procedure. Nevertheless, this effect could still be due to the inclusion of parton shower events because their kinematics is not expected to completely reproduce the real misidentified events. Since most of the misidentification comes from detector effects, the distribution of such events should be compared to the predictions of full JETSET M.E. simulated and reconstructed. On the other hand, higher order contributions could play an important role and are not taken into account in second order matrix element.

In order to find out whether this *effect* depends on the quark flavor and mass, we have performed the fit on a heavy quark enriched sample obtained with high p_t lepton tag. The results are compatible with the previously shown values.

We finally remark that although an unknown physical effect can not be excluded, the lack of a consistent simulation, extremely important at low y_{cut}, does not allow a definitive conclusion. Therefore discarding the results at low y_{cut}, the *color factors* measured at $y_{cut} = 0.03$ can be chosen as best value, on the basis of the smallest statistical error keeping the fraction of two and three parton missidentified events small.

$$\begin{aligned} \frac{C_A}{C_F} &= 1.85 \pm 0.38 \\ \frac{T_R}{C_F} &= 1.41 \pm 0.71 \end{aligned}$$

From the T_R/C_F measurement we can derive the number of light strongly interacting fermions. We obtain $n_f = 3.8 \pm 1.9$, compatible with five active flavors and more than two standard deviations away from the QCD + light gluino hypothesis which, in the approximation of a massless gluino [9], leads to an effective $n_f = 8$. On the other hand, the Abelian gluon model is completely ruled out by this measurement as can be seen in fig 1.

References

[1] R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. **B 178** (1981) 421.

[2] A. Ricker,"Testing the Gluon Self-Coupling in 4-jet Events from Z decays" PhD. thesis, RWTH Aachen, 1995.

[3] JADE Collab., W. Bartel et al., Z. Phys. **C 33** (1986) 23;
JADE Collab., S. Bethke et al., Phys. Lett. **B 213** (1988) 235.

[4] N. Brown and W.J. Stirling, Z. Phys. **C 53** (1992) 629;
S. Catani et al., Phys. Lett. **B 269** (1991) 432; S. Bethke et al., Nucl. Phys. **B 370** (1992) 310.

[5] T. Sjöstrand, Comp. Phys. Comm. **28** (1883) 229.

[6] L3 Collaboration, B. Adeva et al., Z. Phys. **C 55** (1992) 39.

[7] A. Ballestrero, R. Maina and S. Moretti, Phys. Lett. **B 294** (1992) 425.

[8] F. Wäckerle, "Studie von Fünf-Parton-Zuständen in der Elektron-Positron-Annihilation $e^+e^- \rightarrow Z^0, \gamma \rightarrow Partonen$", Karlsruhe Preprint IKEP-KA/93-19.

[9] J. Ellis, D.V. Nanopoulos and D.A. Ross, Phys. Lett. **B 305** (1993) 375.