OSCILLATIONS OF THE B_d^0 MESONS

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The status of the time-dependent $B_d^0 - \bar{B}_d^0$ mixing measurements is reviewed. The new precise world average value of the B_d^0 meson oscillation frequency corresponds to a mass difference Δm_d between the two B_d^0 mass eigenstates of

$$\Delta m_d^{\mathrm{world}} = 0.471 \pm 0.016 \ \mathrm{ps}^{-1}.$$

1 Introduction

As in the case of K-mesons, oscillations between particle-antiparticle states are expected in neutral B-mesons. In the Standard Model, the mechanism causing mixing is a second-order weak interaction through box diagrams (Figure 1).

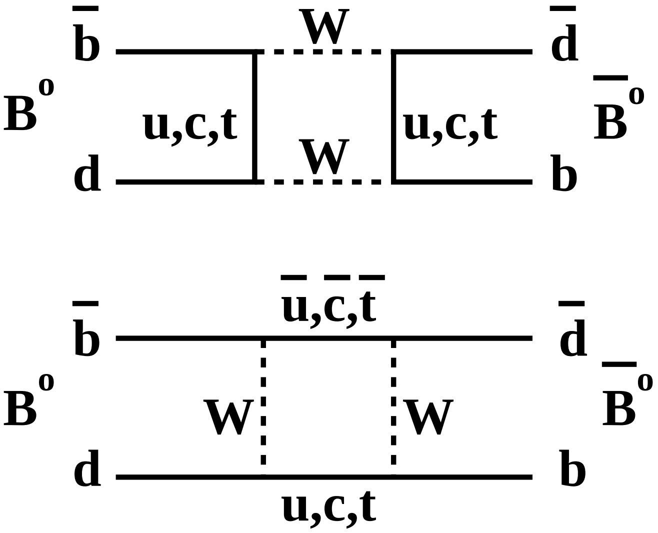

Figure 1: The Standard Model mechanism of mixing in the B_d^0 meson system.

The flavour eigenstates B_d^0 ($\bar{\mathrm{b}}$d) and $\bar{B}_d^0$ (b$\bar{\mathrm{d}}$) are linear combinations of the mass eigenstates B_1 and B_2. Neglecting the decay width difference between B_1 and B_2 and the effects of CP violation, both expected to be small, the probability P to find a B_d^0 decaying at proper time t, provided it was produced as $\bar{B}_d^0$ at $t = 0$, is given by

$$P(\bar{B}_d^0 \to B_d^0) = \frac{1}{\tau} e^{-\frac{t}{\tau}} \left(\frac{1 - \cos \Delta m_d\, t}{2} \right)$$

where τ is the lifetime of the B_d^0 meson. A measurement of the oscillation frequency thus gives a direct measurement of the mass difference Δm_d between the two mass eigenstates.

The phenomenon of B^0-$\bar{B}^0$ mixing is well established by experiment [1,2,3]. The time dependence of mixing for B_d^0 mesons has been measured at LEP, SLC and Tevatron using different techniques [4,5,6,7,8,9].

2 Method

In order to tag a B candidate as mixed or unmixed, it is necessary to determine its flavour state both at production (initial state) and at decay (final state). Several techniques are applied for flavour tagging. One can use the charge of the lepton from semileptonic B decay; a negatively charged lepton is produced in b flavoured meson decay. The charge of $D^{*\pm}$ mesons from B_d^0 decays is used as well in semi-exclusive analyses. For fully inclusive analyses, final state tagging techniques include jet charge (the momentum- or rapidity-weighted sum of charges of the particles) and charge dipole methods. SLD benefits from the high SLC beam polarization. The large forward backward asymmetry for $Z \to b\bar{b}$ decays is used as an efficient ($\epsilon \sim 100$ %) tag of the initial state flavour based on the polar angle of the B candidate. The initial state can also be tagged by the charge of a track from the primary vertex; it is correlated with the production state of the B if that track is a decay product of a B^{**} state, or it is the first particle in the fragmentation chain (leading particle) [5]. The combinations of those tags from the same side to the decay of a B meson candidate or from the opposite side are used to classify events as mixed or unmixed.

To study the time dependence of mixing one needs to estimate the production and the decay vertex positions of the B hadron candidate. The proper time t is related to the decay length in the laboratory system l by $t = l m_{\mathrm{B}}/p_{\mathrm{B}}$ where m_{B} and p_{B} are the mass and momen-

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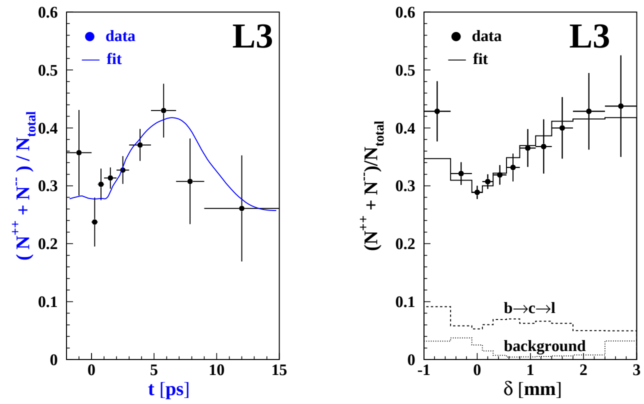
tum of the b-hadron. The resolution can be parametrized as $\sigma_t = \sqrt{a_1^2 + a_2^2 t^2}$, where a_1 depends on the error on decay length l (typically 100-300 μm) and a_2 on the error on the Lorentz boost factor (typically 10-20%). Impact parameters of leptons are also used for the mixing time dependence measurements [6,7]. In this method the time of decay is measured indirectly and statistically from the impact parameters of the reconstructed lepton trajectories with respect to the beam axis.

The maximum likelihood fit is performed on the selected sample. The likelihood function is constructed for the like- and unlike-sign events and normally has very complicated structure.

3 Results

A total of 24 different analyses have been performed to measure Δm_d, of which 2 are new and 2 have been updated since summer 1997. The different measurements are illustrated on the example of the contributed papers to the ICHEP98 conference [Abstracts 371,372,552,727]. In Figure 2 the measurement [7] by the L3 collaboration of the fraction of like-sign dilepton events is plotted as a function of proper time (Figure 1a) or the lepton impact parameter (Figure 1b) and compared to the fit result. The expected variation with proper time is clearly observed. The flavour of the b quark at production time is determined from the charge of the lepton in the opposite hemisphere (lepton - lepton combination).



(a) (b)

Figure 2: The L3 lepton-lepton analysis. Ratio of same-sign dilepton events to the total number of dilepton events versus the measured proper time (a) or versus the lepton impact parameter (b) compared to the fit result.

The measurement [8] by the OPAL collaboration using lepton -jet charge combination is shown in Figure 3. Jet charge technique gives much bigger statistics than lepton tag but tagging purity is smaller. The purity of the jet

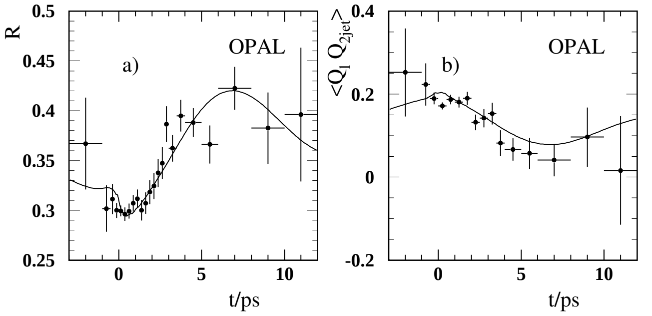


Figure 3: The OPAL lepton - jet charge analysis. a) the right-sign fraction R versus proper time t for $|Q_{2jet}| > 2$ and b) $< Q_l Q_{2jet} >$ product versus t for $|Q_{2jet}| < 2$.

charge tagging is the major systematic uncertainty for this method.

A flavour tagging technique based on the correlations of B meson flavour with the charge of the leading particle from a primary vertex (same side tag) [12] is pioneered in the CDF measurement [5]. The B flavour at decay is determined by reconstructing the semileptonic decays $B^0 \to l^+ D^{(*)-} X$ and $B^+ \to l^+ \bar{D}^0 X$. In Figure 4 the asymmetry between right-sign initial-final state flavour combinations and wrong-sign combinations $A(t) = (N_{RS}(t) - N_{WS}(t))/(N_{RS}(t) + N_{WS}(t))$ is plotted for decay signatures: $l^+\bar{D}^0$ (dominated by B^+), l^+D^- and $l^+D^{(*)-}$ (dominated by B^0). The dashed line is the result of the fit. The time dependence (oscillation) in the neutral B signatures is present, giving the Δm_d measurement.

In Fig. 5 the individual measurements of Δm_d and their combined value are shown. The procedure for combining of the available Δm_d measurements has been developed by the LEP B oscillations working group [10]. The measurements of Δm_d are scaled to a common set of physics input parameters and then combined taking into account the common systematic uncertainties among the analyses. Following the procedure discussed in [10], a combined value of

$$\Delta m_d^{\mathrm{LEP+CDF+SLD}} = 0.477 \pm 0.010\,(\mathrm{stat}) \pm 0.013\,(\mathrm{syst})\mathrm{ps}^{-1}$$

is found. The dominant uncertainty on Δm_d is now the systematics. This combined value of Δm_d from time dependent mixing measurements, and the values of $\tau_{B_d^0}$ and $\chi_d^{\Upsilon(4S)}$ [11], may be used to determine $\chi_d^{\mathrm{LEP+CDF+SLD+\Upsilon(4S)}} = 0.175 \pm 0.009$, or equivalently $\Delta m_d^{\mathrm{world}} = 0.471 \pm 0.016\ \mathrm{ps}^{-1}$.

The Δm_d average can be used to improve the knowledge of the fraction of B_s^0 mesons in a sample of weakly decaying b-hadrons. Assuming $f_{\mathrm{B_d}} = f_{\mathrm{B_+}}$ and $f_{\mathrm{B_d}} + f_{\mathrm{B+}} + f_{\mathrm{B_s}} + f_{\mathrm{b-baryon}} = 1$, the values of χ_s, $f_{\mathrm{b-baryon}}$ and the b-lifetimes from [11] and $\overline{\chi}$ from [13], the world

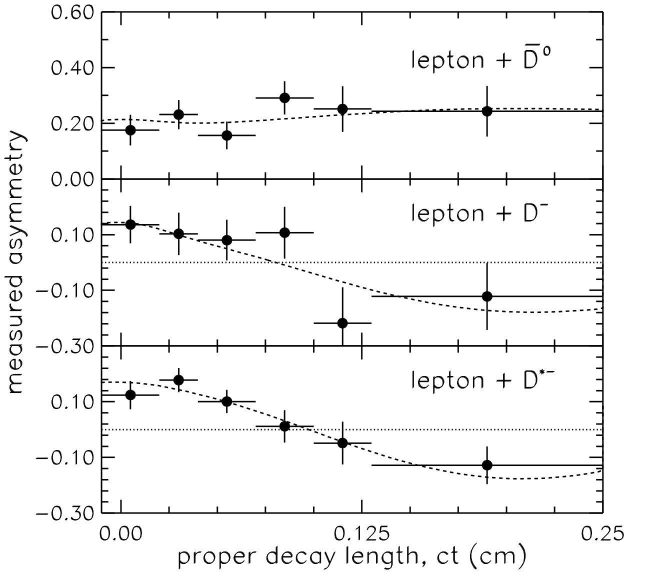


Figure 4: The CDF leading pion - lepton-D^*/D analysis. The top plot, dominated by B^+, shows no time dependence. The other two plots are mostly due to B_d^0 contribution, giving the Δm_d measurement.

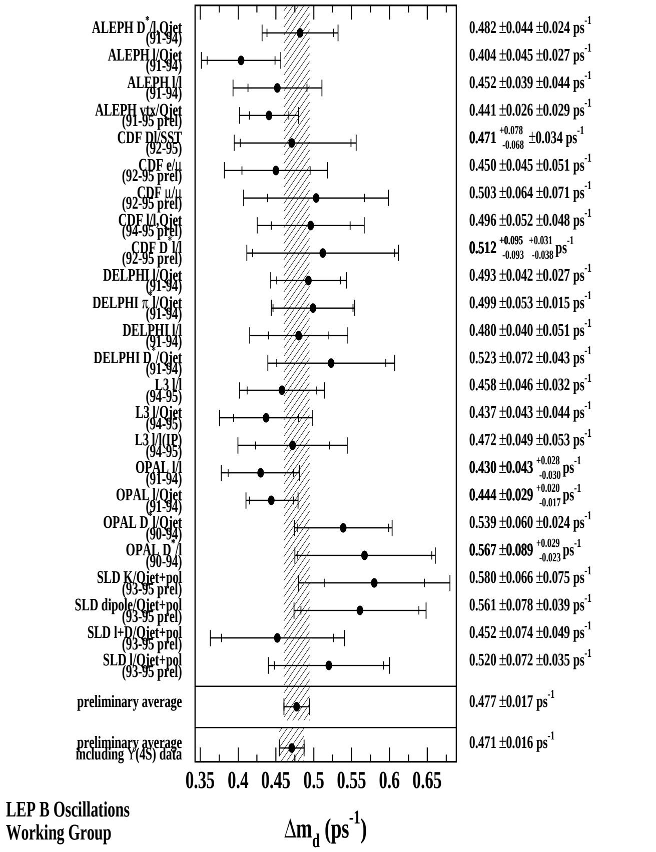


Figure 5: Summary of the Δm_d measurements.

average of Δm_d leads to $f_{B_s} = (10.6 \pm 1.6)\%$. Averaging this value of f_{B_s} from mixing with that from branching ratio measurements $f_{B_s} = (11.6^{+3.4}_{-2.9})\%$ [10], the fraction of B_s^0 mesons in a sample of weakly decaying b-hadrons is found to be $f_{B_s} = (10.8 \pm 1.4)\%$. This further implies that $f_{B_d} = f_{B^+} = (39.5 \pm^{1.3}_{1.4})\%$.

4 Constraint on CKM mixing matrix and new physics

The result of Δm_d measurement may be used to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{td}. Assuming the dominant contribution to the mass difference arises from the matrix element of a box diagram of Figure 1 with W bosons and top quark as sides, B_d^0 meson oscillation frequency can be expressed[14] as

$$\Delta m_d = |V_{td} V_{tb}^*|^2 \, \frac{G_F^2}{6\pi^2} \, m_{B^0} \, m_W^2 \, F(\frac{m_t^2}{m_W^2}) \, f_{B^0}^2 \, B_{B^0} \, \eta_B$$

where F is the known function of m_t^2/m_W^2 ratio. The product $f_{B^0}^2 B_{B^0} \eta_B$ is coming from QCD involvement in the weak decay of the B_d^0 meson and consists of a B^0 decay constant, a non-perturbative correction factor (bag factor) and a perturbative QCD correction, respectively. The V_{tb} is assumed to be equal to 1. The oscillation frequency Δm_d is now measured to a precision of 3%, but the uncertainty on QCD factors limits the extracted value of V_{td} to an accuracy of 20% [11]. Nevertheless, this may be used to constrain the nature of a possible CP violation in B decays. In the Wolfenstein parametrization[15] of the CKM flavour mixing matrix, the V_{td} can be expressed as $V_{td} = A\,\lambda^3\,(1-\rho-i\eta)$, where λ is equal to the sine of the Cabibbo angle and the complex parameter $i\eta$ accounts for the CP violating phase. Unitarity condition on CKM matrix, involving the V_{td} matrix element, is geometrically represented in the complex plane $(\eta - \rho)$ by the "unitarity triangle". In Figure 6a this triangle [‡] is shown[16] with constraint bands ($\pm\sigma$) from the measurements of Δm_d, the CP violation in the K^0 system ϵ_K and, the $|V_{ub}|/|V_{cb}|$ ratio. The power of the co nstraint from the measurement of the mixing in the B meson system can be increased by measuring the ratio of the oscillation frequencies of B_d^0 and B_s^0 mesons. In the ratio many common factors cancel, and lattice QCD pre-

[‡] $\bar{\rho} = \rho(1-\frac{\lambda^2}{2}), \bar{\eta} = \eta(1-\frac{\lambda^2}{2})$

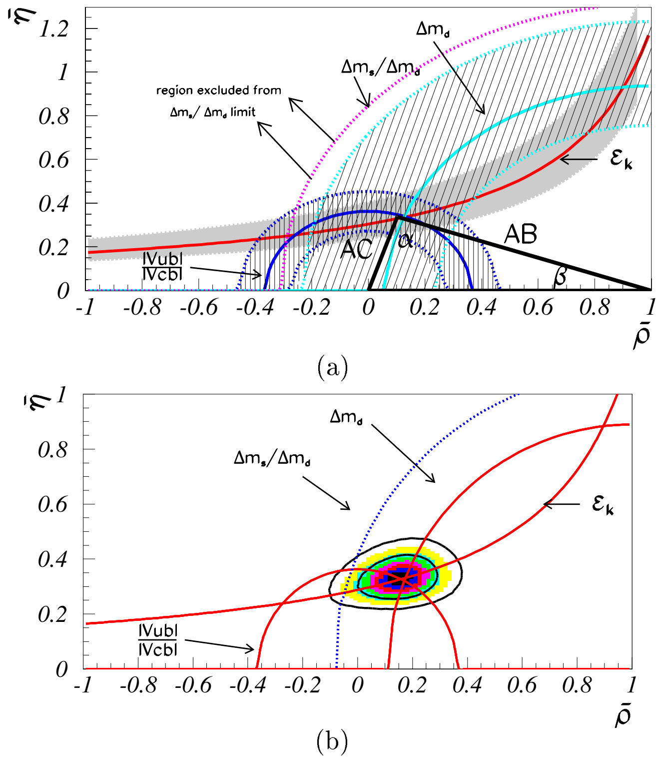


Figure 6: The unitarity triangle in the $(bar\eta - \bar{\rho})$ plane. a) The constraints on Δm_d, ϵ_K and $|V_{ub}|/|V_{cb}|$ are shown as bands corresponding to $\pm\sigma$. b) The allowed region for vertex A. The contours at 68% and 95% CL are shown.

dicts then a QCD correction factor to the precision of the order of 10%. There is no measurement up to now of the mass difference Δm_s in the system of B_s^0 mesons, a lower limit $\Delta m_s > 12.4\text{ps}^{-1}$ at 95% CL is set [17]. The limit on Δm_s allows to restrict further the possible domain for the vertex position, A in Figure 6, of the unitarity triangle. This is shown in Figure 6a as the region excluded at 95% CL by the $\Delta m_s/\Delta m_d$ ratio limit. The comparison of the calculated CKM matrix elements with the values based on direct measurements is used to place restrictions on new physics [16]. Those three constraints were used to fit the position of the vertex A. The result of the fit is presented in Figure 6b, where the vertex position and the 68% and 95% CL contours are shown. It is clear from the figure that the measurements of such different physical quantities as the mixing of the B_d^0 mesons, the CP violation in K^0 mesons and the ratio of charmless decays to decays with charm, $|V_{ub}|/|V_{cb}|$, are very compatible with each other (the curves cross at the same point). It means there is no big change of corresponding CKM matrix elements due to new physics.

5 Impact on CP-violation measurements

The importance of a precise measurement of the B_d^0 oscillation frequency Δm_d can be appreciated because of the impact it has for the searches of CP-violation in the neutral B meson decays. CP-violation is expected in the B decays [18]. In the system of neutral B mesons, CP-violation can arise from interference between the two paths to a given final state f, that is without or with mixing $B \to f$ or $B \to \bar{B} \to f$. Large effects may be seen in the difference in decay rates of B_d^0 and $\bar{B}_d^0$ to the CP eigenstate $J/\psi K_s^0$ [18], which corresponds to a decay asymmetry

$$A_{CP}(t) \equiv \frac{\bar{B}_d^0(t) - B_d^0(t)}{\bar{B}_d^0(t) + B_d^0(t)} = \sin(2\beta)\sin(\Delta m_d t).$$

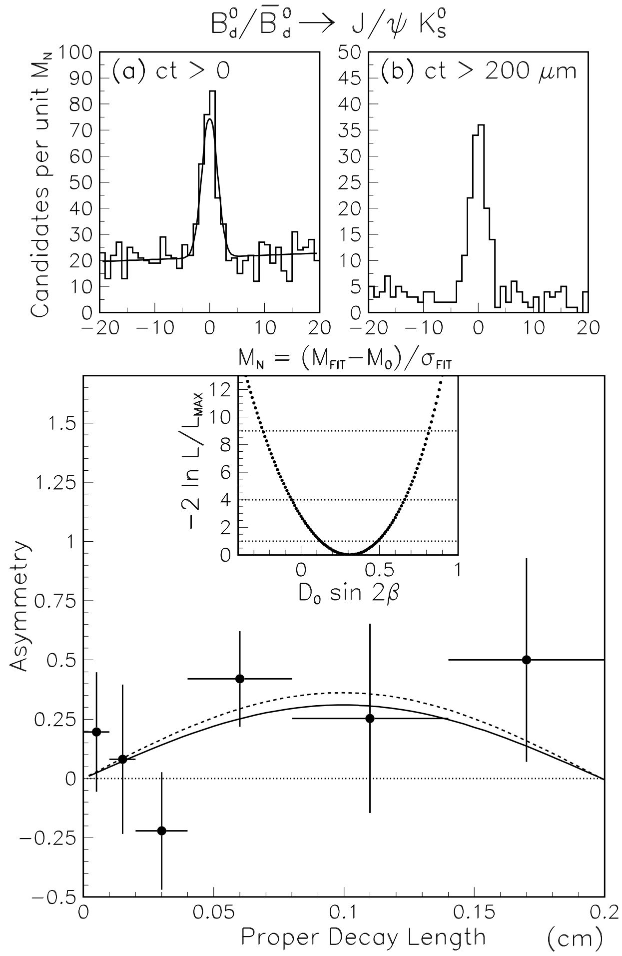
This relationship shows that CP-violation effect, $\sin(2\beta)$, has to be measured on top of the time dependence from the mixing, $\sin(\Delta m_d t)$. The results of the first attempts to measure time dependent asymmetries of $B_d^0/\bar{B}_d^0 \to J/\psi K_s^0$ have been presented at the ICHEP98 conference by the CDF [19] and the OPAL [20] collaborations. For the time being, the world pool of such events consists of 198 events by CDF and 24 events with a purity of about 60% by OPAL. In Figure 7a the CDF measurement is shown for the mass distribution of $J/\psi K_s^0$ candidates, where one can see clearly identified $B_d^0/\bar{B}_d^0 \to J/\psi K_s^0$ candidates. The background is highly suppressed for large decay lengths (Figure 7b). The same side tag (leading "pion") method has been applied for initial state tagging. The asymmetry as a function of proper decay length is shown in Figure 7c together with the result of the fit to the function $D_0 \sin(2\beta)\sin(\Delta m_d t)$, where D_0 is dilution factor. For $\sin(2\beta)$ CDF finds $1.8 \pm 1.1 \pm 0.3$. The central value is unphysical since the amplitude of the measured asymmetry is larger than D_0 (the statistics are not big enough yet), but for this example we would like to underline that the largest systematic uncertainty is due to Δm_d. There will be some improvement on the statistical error on Δm_d in the future, but the major improvement on Δm_d precision will come from an improved knowledge of the Δm_d fit input parameters (b-hadron fractions, b-lifetimes, decay branching ratios, fragmentation, etc.)

6 Conclusion

The new precise world average value of the B_d^0 meson oscillation frequency corresponds to a mass difference Δm_d between the two B_d^0 mass eigenstates of

$$\Delta m_d^{\text{world}} = 0.471 \pm 0.016 \text{ ps}^{-1}.$$

This result provides an important constraint on the CKM mixing matrix and will serve as the basic input parameter for future CP-violation studies. This measurement is



(c)

Figure 7: The normalized mass distribution of the $J/\psi K^0_s$ candidates of CDF experiment with $ct > 0$ (a) and $ct > 200\mu$m (b). (c) shows the sideband-subtracted tagging asymmetry as function of the reconstructed $J/\psi K^0_s$ proper decay length versus fit to $A_0 \sin(\Delta m_d t)$.

used to improve the knowledge of the fraction of weakly decaying B^0_s mesons in a sample of weakly decaying b-hadrons, giving a result of $f_{\mathrm{B_s}} = (10.8 \pm 1.4)\%$.

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