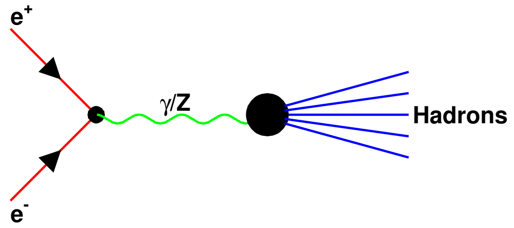
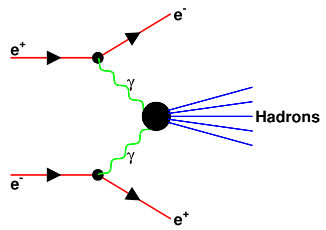
QCD at LEP2

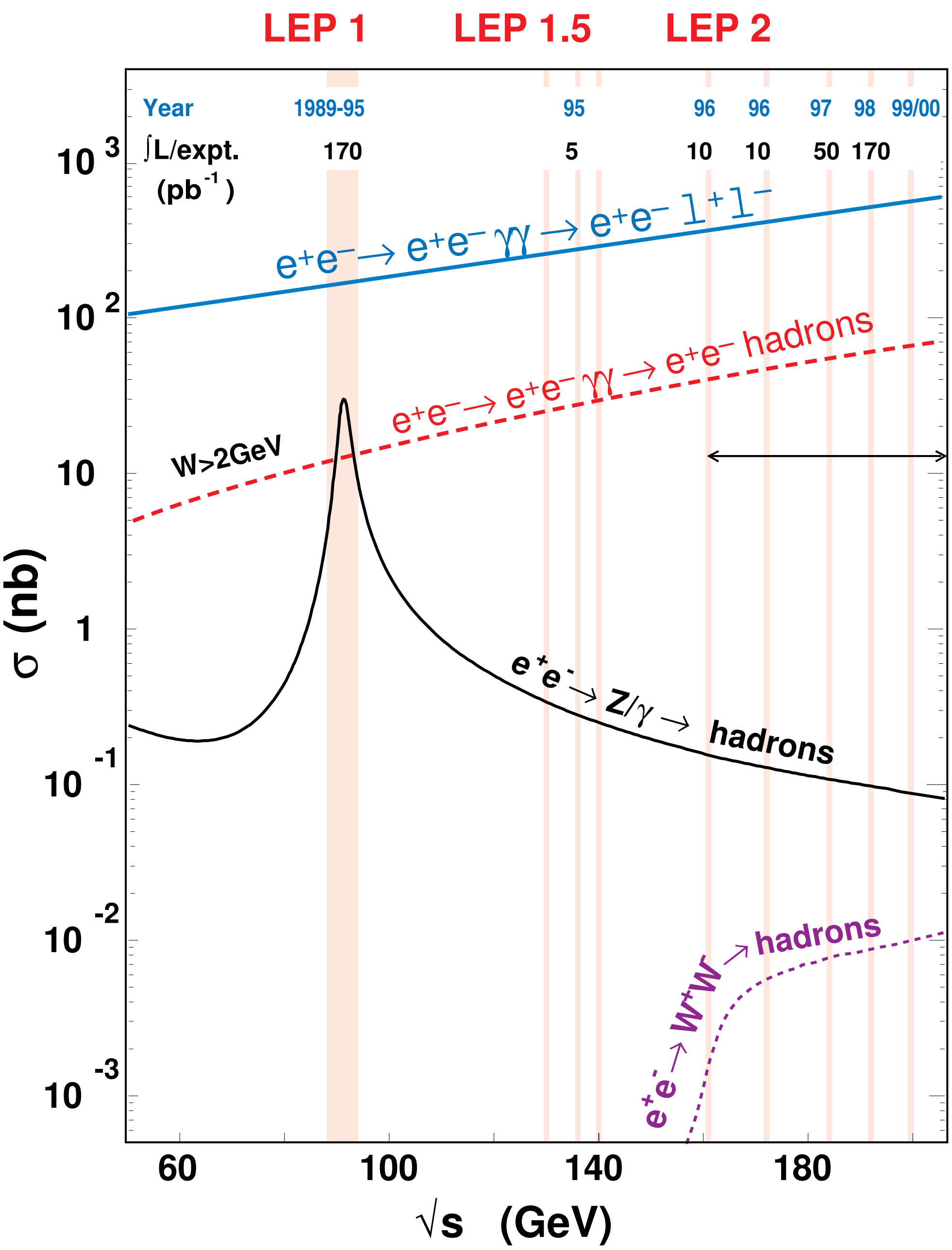


- **Annihilation**
  - Jet Rate
  - Event Shape
  - Power Correction
  - α_{s}
  - Heavy Flavour
  - Inclusive Spectra
  - B-E and Colour Reconnection



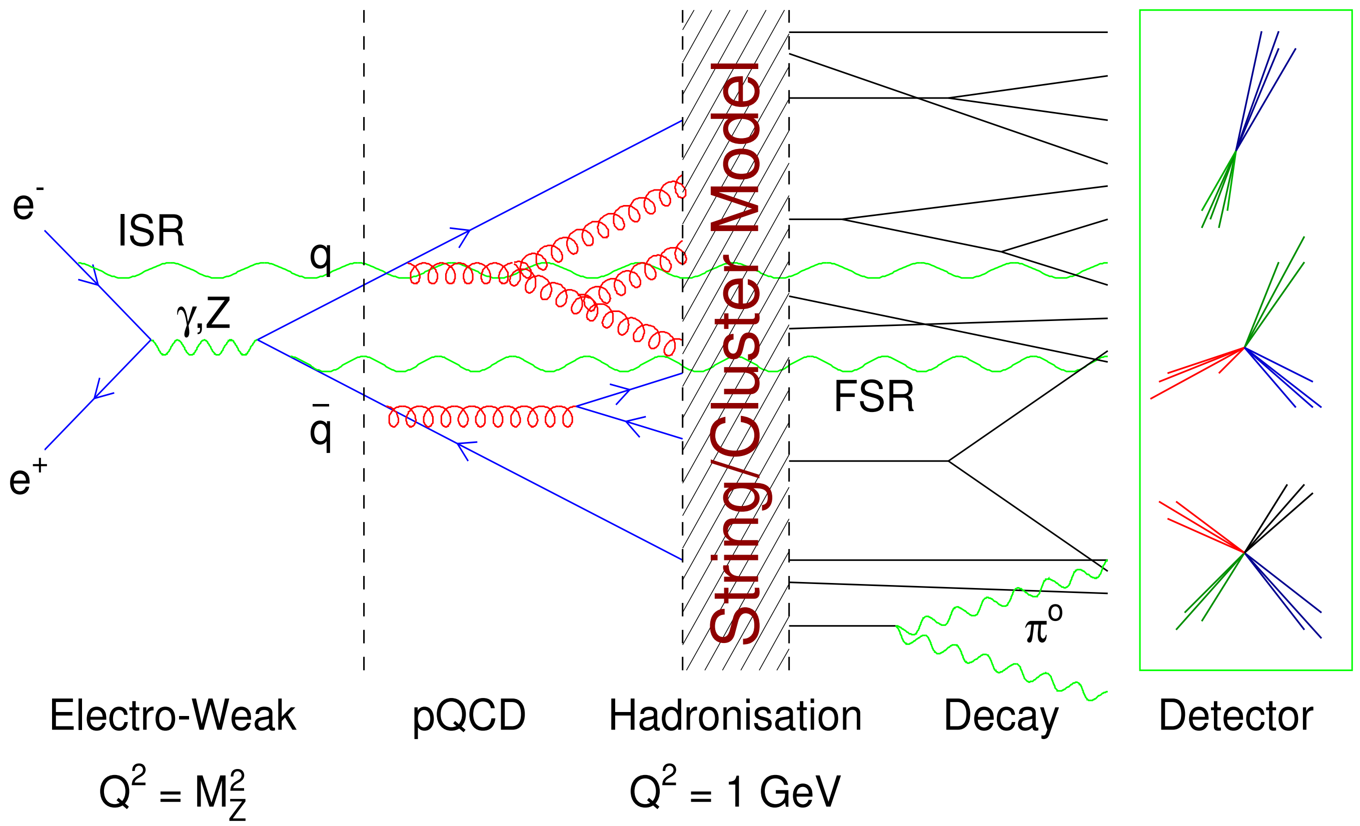
- **2-Photon**
  - $\gamma\gamma$ Cross Section
  - Heavy Flavour
  - Jet Production
  - γ Structure Function
  - $\gamma^{\star}\gamma^{\star}$

Sunanda Banerjee
TIFR (Mumbai) & L3



2-photon processes and hadron production through annihilation are dominant processes at LEP 2 energies

⇒ These processes have low background and high statistics
(They can be studied with small systematic error)

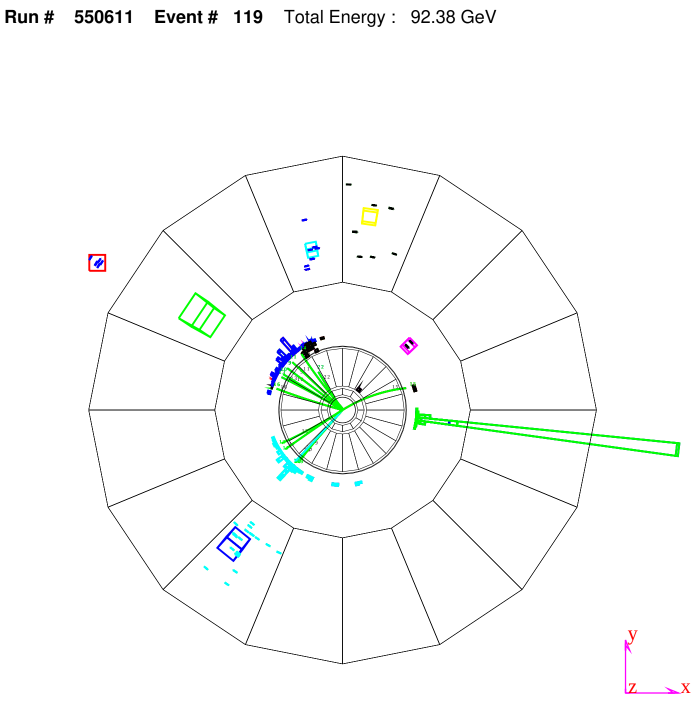


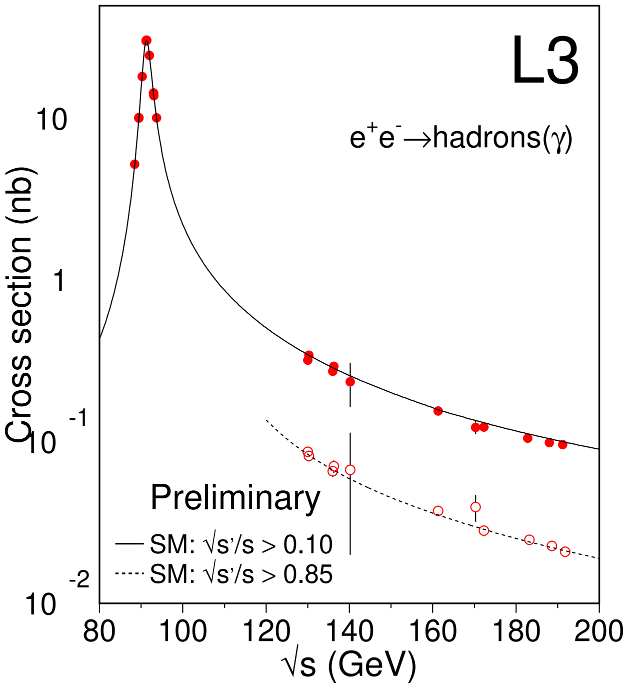
- Production processes well defined
  Hadronic activity only in the final state
- Effect of hadronisation decreases with increasing centre of mass energies
  Jets retain initial parton direction

$\Rightarrow$ Allows clean test of QCD with minimum assumptions

New particle searches, W pair production studies require a thorough understanding of QCD backgrounds

Annihilation





Transverse Imbalance :	.0839		Longitudinal Imbalance :	-.0132	
Thrust :	.7534	Major :	.4539	Minor :	.0782
Event DAQ Time :	940627 75112				

Three sources of data:

- Reduced $\sqrt{s'}$ from hadronic Z decays with an energetic isolated photon (∼ Few 1000)

$$\sqrt{s'} = \sqrt{\mathrm{s}\left(1 - 2\frac{\mathrm{E}_\gamma}{\sqrt{\mathrm{s}}}\right)}$$

  L3: 6 bins between 30-86 GeV; DELPHI: 3 bins between 32-80 GeV

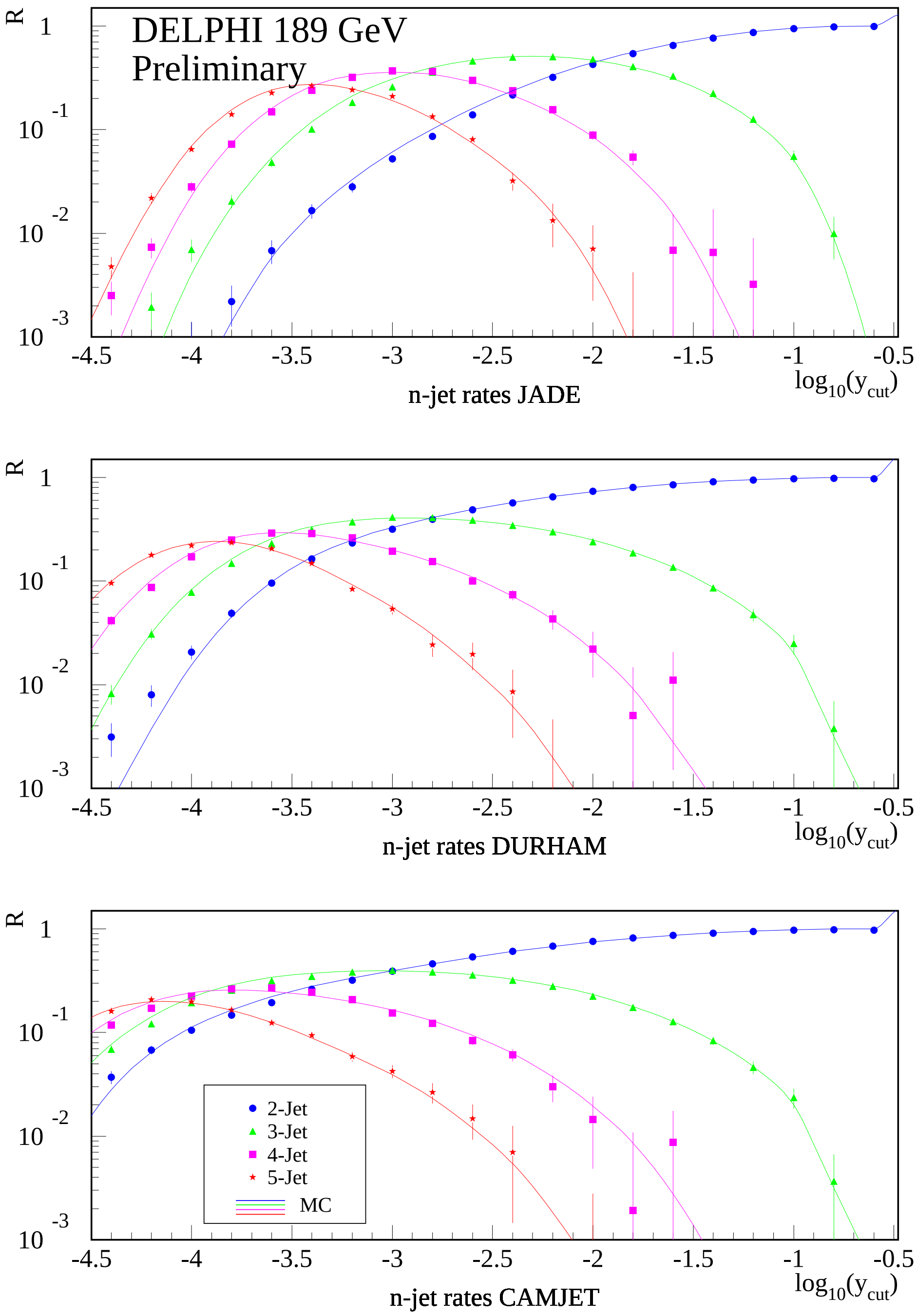
  Background due to π^0/η^0 decays

- Hadronic Z decays at $\sqrt{s} \approx m_Z$ ALEPH, DELPHI, L3, OPAL (∼ Few 100K)

- High energy data at $\sqrt{s}$ = 130, 136, 161, 172, 183, 189, 192–202 GeV
  ALEPH, DELPHI, L3, OPAL (∼ Few 1000)

  Background: ISR γ - radiative return to Z, $\mathrm{W}^+\mathrm{W}^-$, ZZ production

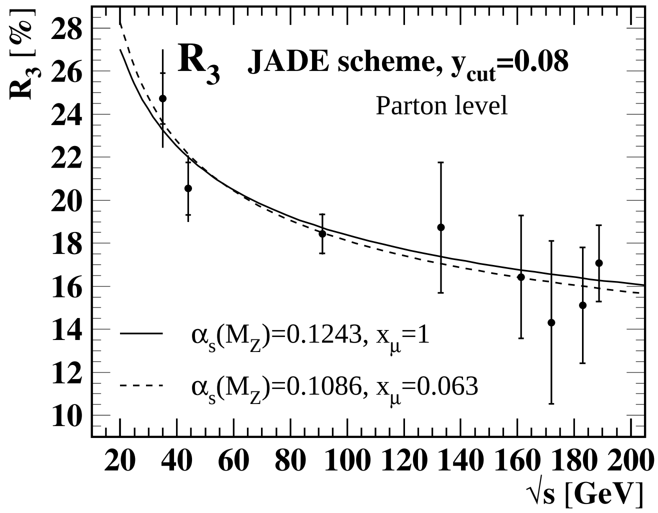
Jets are reconstructed with Jade, Durham, Cambridge algorithms

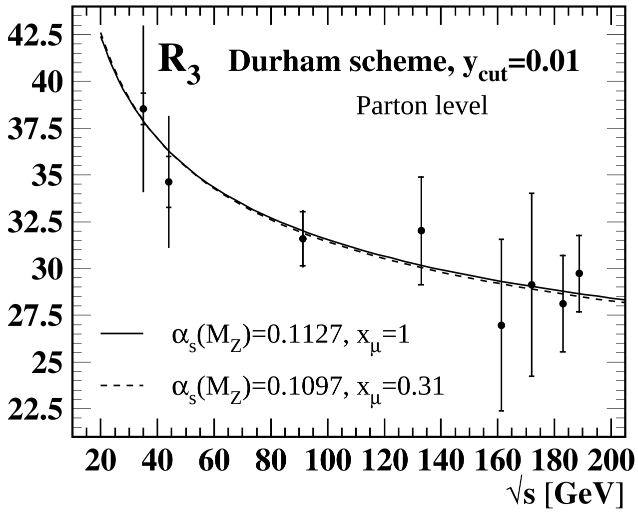


Jet rates as a function of y_{cut} well described by Parton Shower Monte Carlo

JADE data have been re-analysed in view of improved theoretical understandings

JADE and OPAL data analysed in a coherent approach



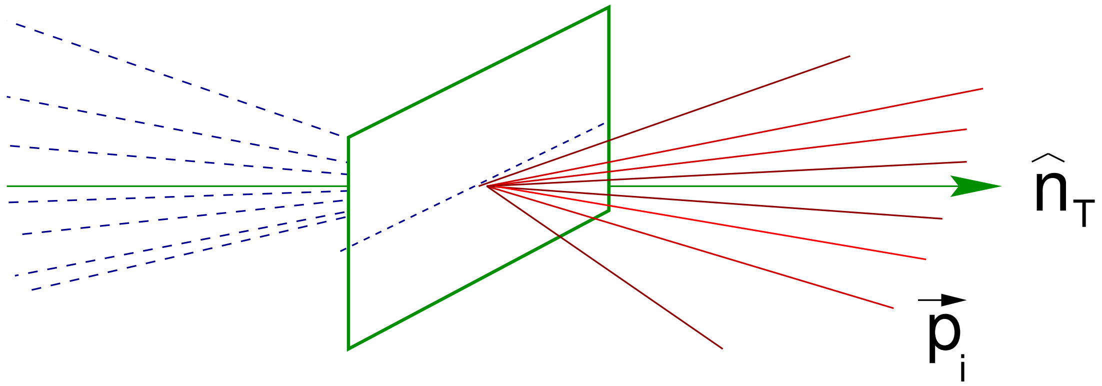


Fits performed with QCD evolution of α_s
(Fixed scale of 1 or with floating x_μ)

	R_3^J ($y_{cut} = 0.08$)		R_3^D ($y_{cut} = 0.01$)	
α_s (m_Z)	0.1243	0.1086	0.1127	0.1097
x_μ	1.0	0.063 ± 0.025	1.0	0.310 ± 0.148
χ^2/d.o.f.	3.63/7	2.98/6	5.18/7	5.03/6
$\Delta\alpha_s$ (Stat)	$\pm$ 0.0024		$\pm$ 0.0011	
$\Delta\alpha_s$ (Had.)	$\pm$ 0.0032		$\pm$ 0.0082	
$\Delta\alpha_s$ (H.O.)	$^{+0.0074}_{-0.0061}$		$^{+0.0037}_{-0.0024}$	
$\Delta\alpha_s$ (Total)	$^{+0.0085}_{-0.0073}$		$^{+0.0091}_{-0.0086}$	

Optimised scale does not improve fit quality

$$\alpha_s(m_Z) = 0.1181 \, ^{+0.0066}_{-0.0056}$$



Thrust:

$$T = \max_{\hat{n}_T} \frac{\sum_a |\vec{p}_a \cdot \hat{n}_T|}{\sum_a |\vec{p}_a|}$$

Scaled heavy jet mass:

$$M_\pm^2 = \left(\sum_{a \in S_\pm} p_a \right)^2$$

$$\rho = \frac{\max\left[M_+^2, M_-^2\right]}{s}$$

C Parameter:

$$C = \frac{3}{s} \sum_{i<j} |p_i| \cdot |p_j| \cdot \sin^2 \theta_{ij}$$

$\Rightarrow$ Describe longitudinal shower development

Jet broadenings variables:

$$B_\pm = \frac{\sum_{a \in S_\pm} |\vec{p}_a \times \hat{n}_T|}{2 \sum_a |\vec{p}_a|}$$

$$B_T = B_+ + B_- \qquad \text{Total}$$

$$B_W = \max(B_+, B_-) \qquad \text{Wide}$$

$\Rightarrow$ Describe transverse shower development

Sensitive to hard gluon emission describing multi-jet topology

- Select hadronic events from characteristic properties:

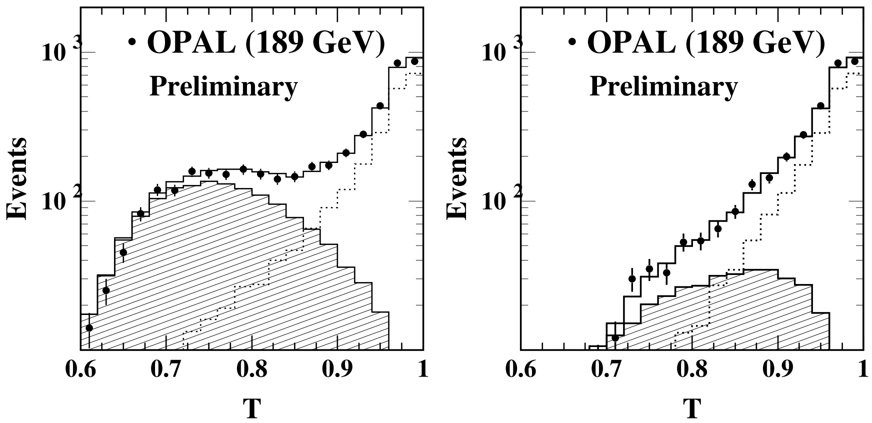
  Large visible energy, high multiplicity

- Actively reject background due to 2-photon, ISR, WW/ZZ

  Smaller systematic error

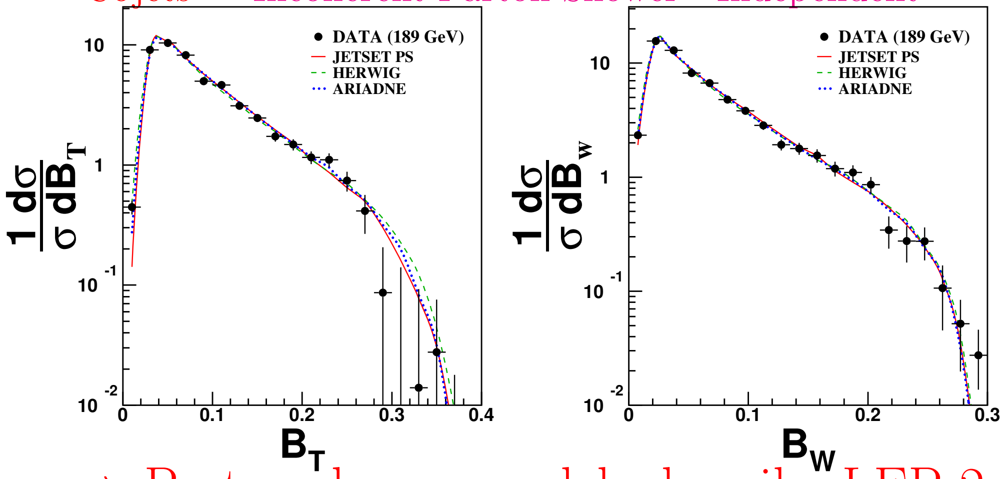
- Use Monte Carlo for detector corrections:

  Resolution, acceptance, $\cdots$



Compare corrected distributions to QCD models tuned at LEP 1

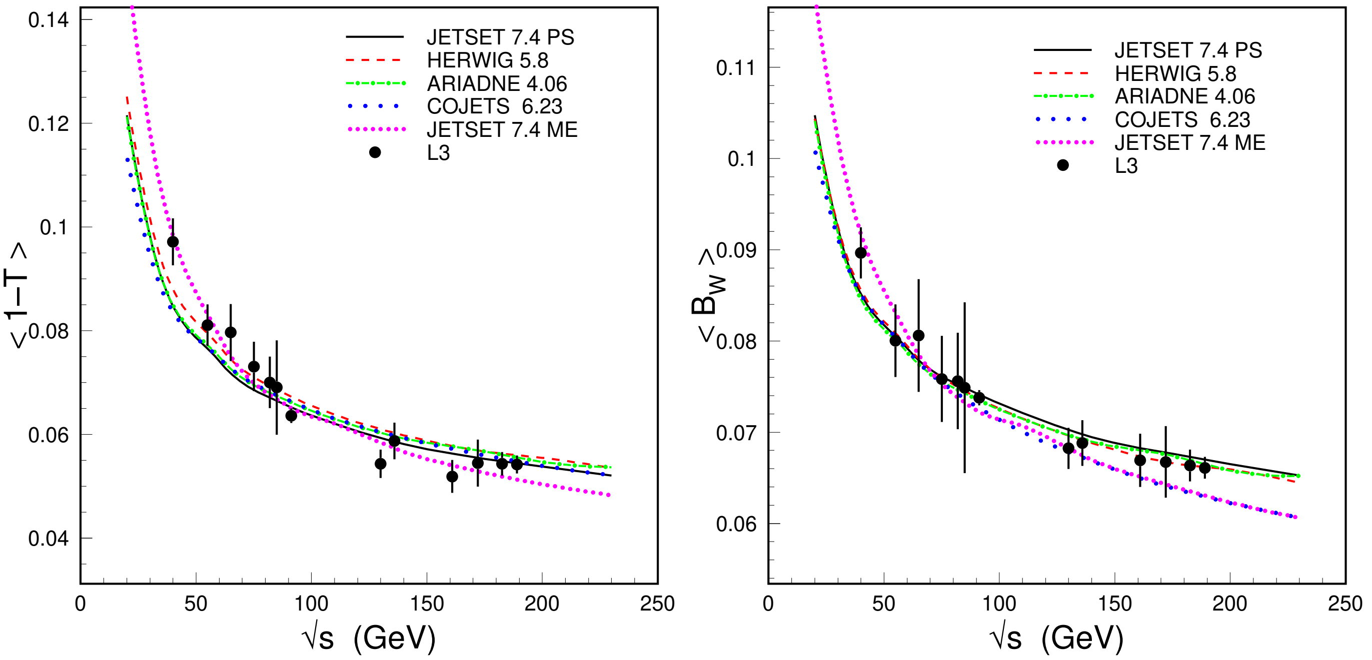
Jetset	Coherent Parton Shower	String
Herwig	Coherent Parton Shower	Cluster
Ariadne	Colour dipole	String
Cojets	Incoherent Parton Shower	Independent



$\Rightarrow$ Parton shower models describe LEP 2 data well

Energy evolution of event shape variables:

- ✧ Logarithmic dependence of α_s on energy
- ✧ Fragmentation having $1/Q^n$ dependence



All models with parton shower describe energy dependence well

Matrix element model uses calculations up to $\mathcal{O}(\alpha_s^2)$

Too few partons – compensated by retuning at each CM energy

Not expected to explain energy evolution with one point tuning

First moment of an event shape variable f

$$\langle f \rangle \;=\; \langle f_{\rm pert} \rangle \;+\; \langle f_{\rm pow} \rangle$$

Perturbative contribution can be estimated using ERT matrix:

$$\langle f_{\rm pert} \rangle \;=\; A\frac{\alpha_{\rm s}(Q)}{2\pi} \;+\; (B + A \cdot (\beta_0 \ln \mu - 2)) \left(\frac{\alpha_{\rm s}(Q)}{2\pi}\right)^2$$

A, B are known constants; $\beta_0 = (11 \cdot {\rm N_c} - 2{\rm N_f})/3$; $\mu = Q/\sqrt{s}$

Power correction term is given by

$$\langle f_{\rm pow} \rangle \;=\; c_f \cdot \mathcal{P}$$

$c_f = 2, 2, 1, \frac{1}{2}, 3\pi$ for $1 - T$, ρ, B_T, B_W and C

$\mathcal{P}$ universal for all these variables

$$\mathcal{P} \;=\; \mathcal{P}\,(\sqrt{s},\, \alpha_{\rm s}(\sqrt{s}),\, \mu_I,\, \alpha_0(\mu_I))$$

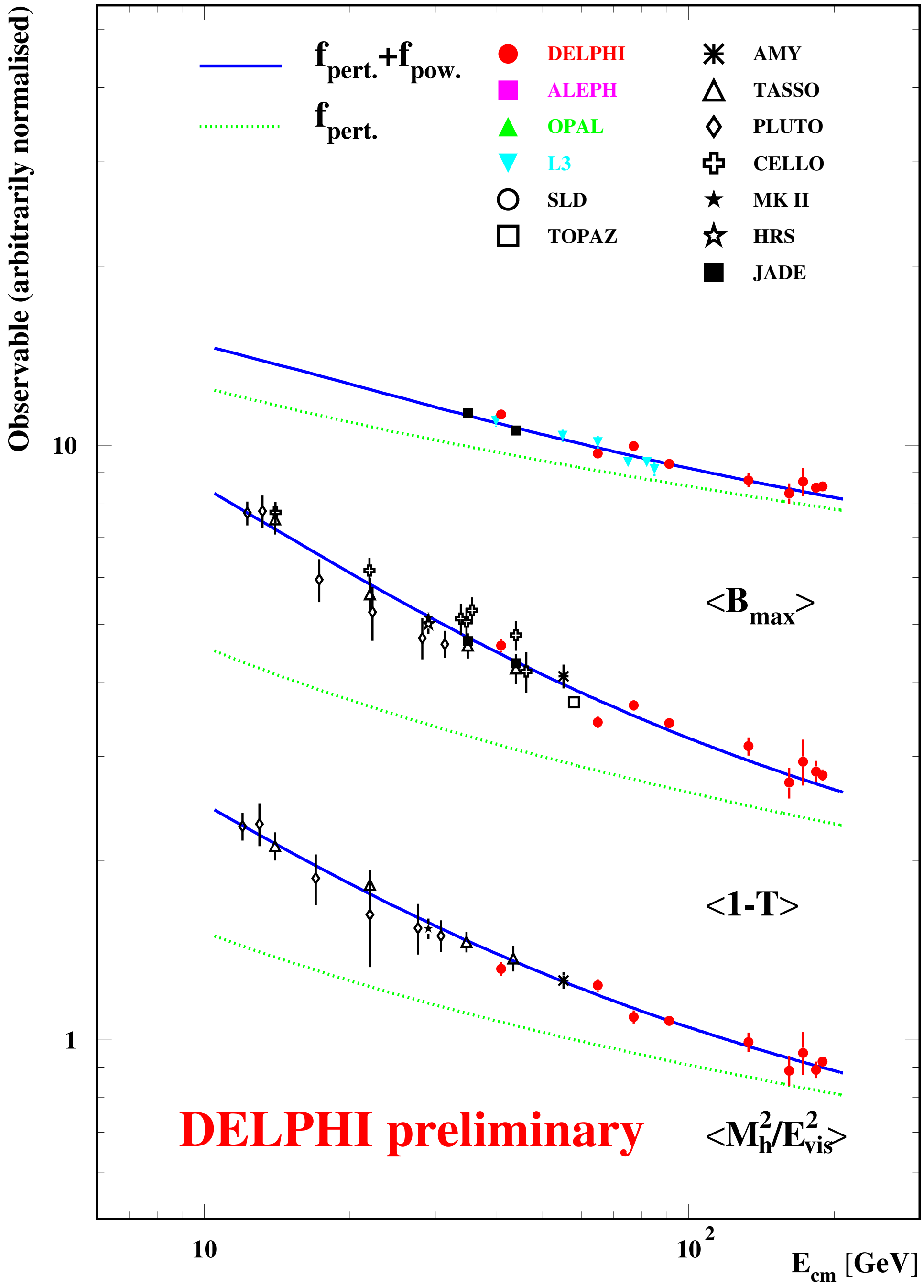
α_0 is a non-perturbative parameter accounting for contributions below an infrared matching scale μ_I $(= 2$ GeV$)$

For B_T, B_W an additional logarithmic skew term gets added

$\Rightarrow$ Two parameters to explain energy evolution of the five event shape variables

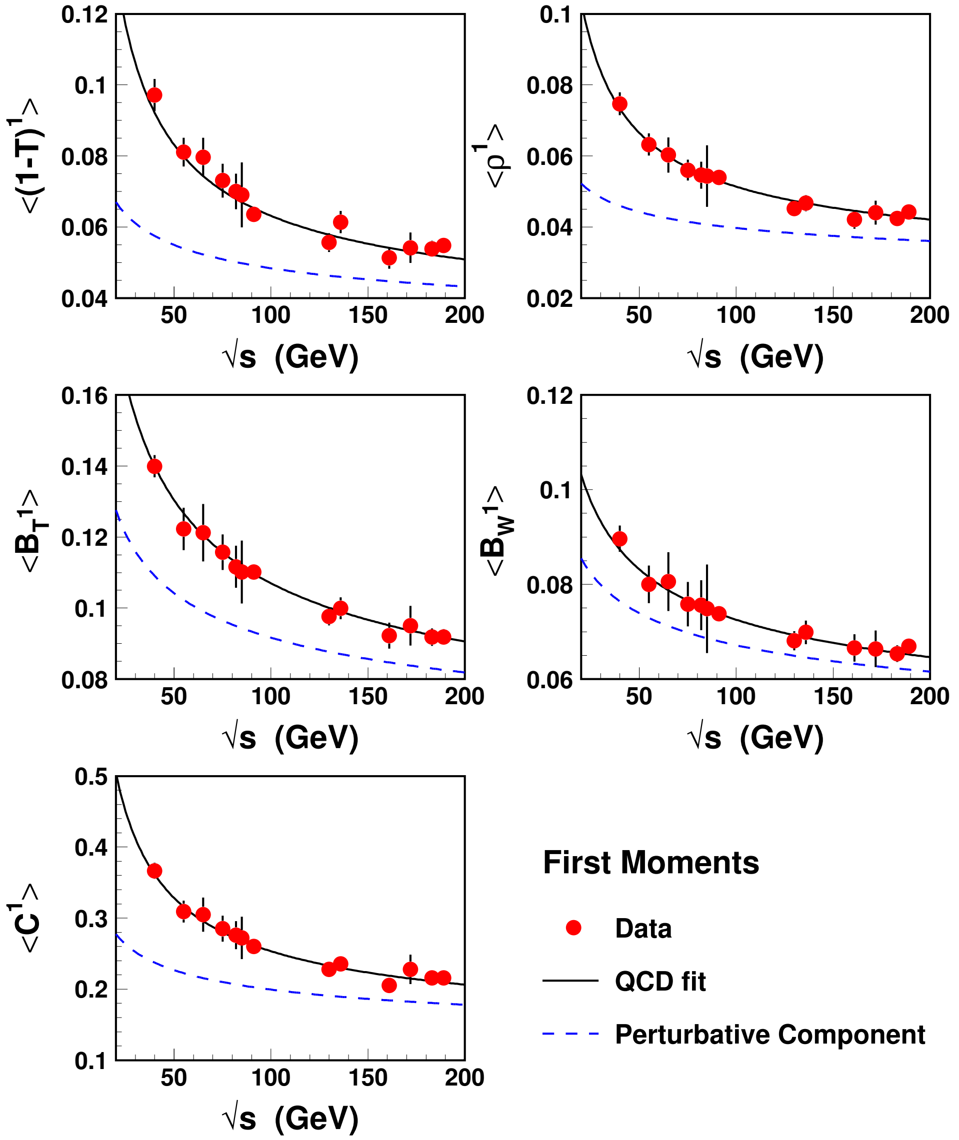
$\alpha_{\rm s}(m_Z)$, $\alpha_0(2\ {\rm GeV})$ universal for all distributions

Annihilation: Power Correction



- ➢ Perturbative component alone cannot describe energy evolution of $\langle f \rangle$
- ➢ At very high energies power correction term insignificant

Annihilation: Power Correction



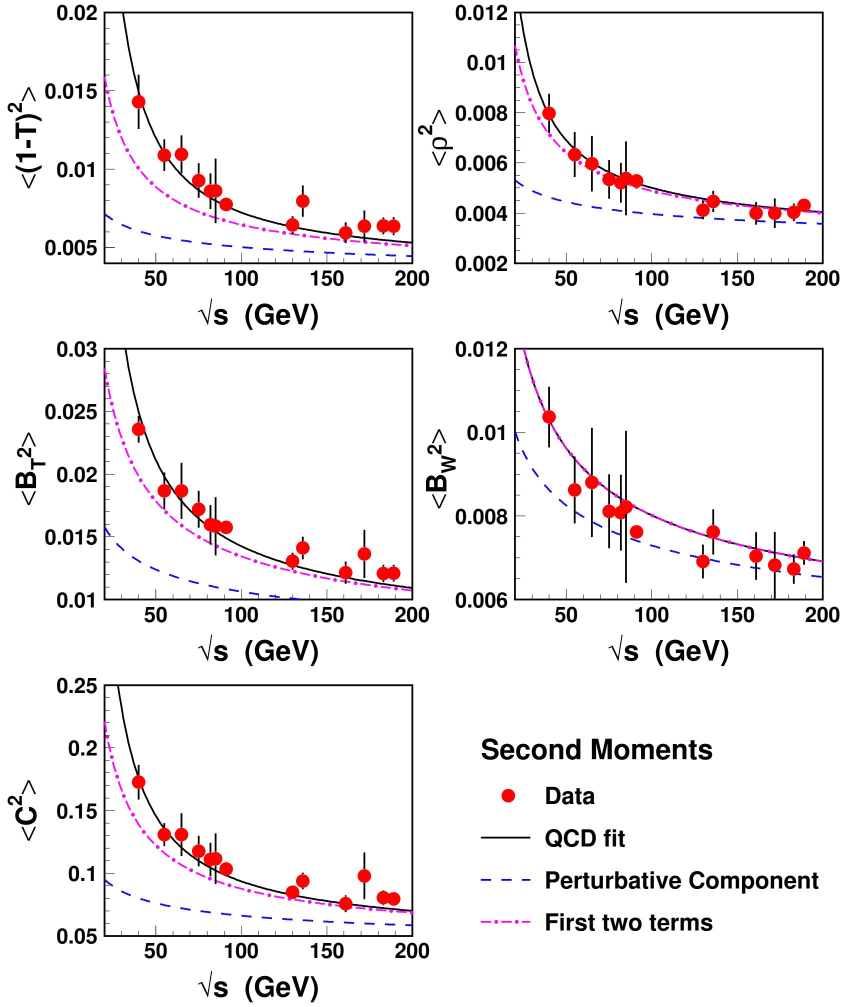
L3 has measurements from 40 – 189 GeV

	α_0 (2 GeV)	χ^2/dof
$1-T$	0.565 ± 0.054	11.6/11
ρ	0.468 ± 0.040	5.6/11
B_T	0.473 ± 0.036	3.5/11
B_W	0.434 ± 0.072	4.2/11
C	0.485 ± 0.030	6.3/11

$\Rightarrow$ Evidence of universality of power law correction

$$\langle f^2 \rangle = \langle f^2_{\text{pert}} \rangle + 2\langle f_{\text{pert}} \rangle \cdot c_f \mathcal{P} + \frac{A_2}{Q^2} \cdots$$

- Use fit results from first moments for the first two terms ($\alpha_s(m_Z)$, $\alpha_0(2\text{ GeV})$)
- The remaining term $\left(\frac{A_2}{Q^2}\right)$ is obtained from a fit for each variable (A_2 supposed to be small)



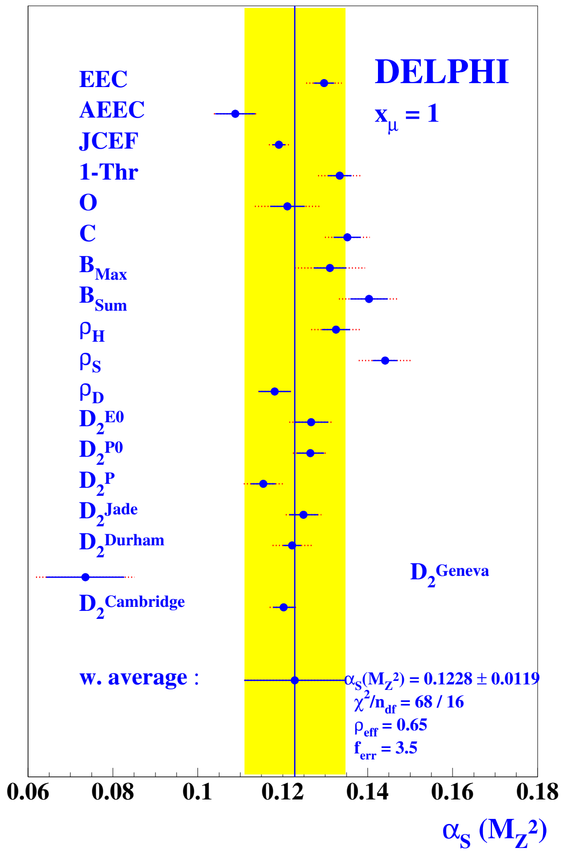
	$1-T$	ρ	B_T	B_W	C
A_2	4.60 ± 0.99	0.72 ± 0.63	9.68 ± 1.74	0.00 ± 1.95	7.34 ± 1.81

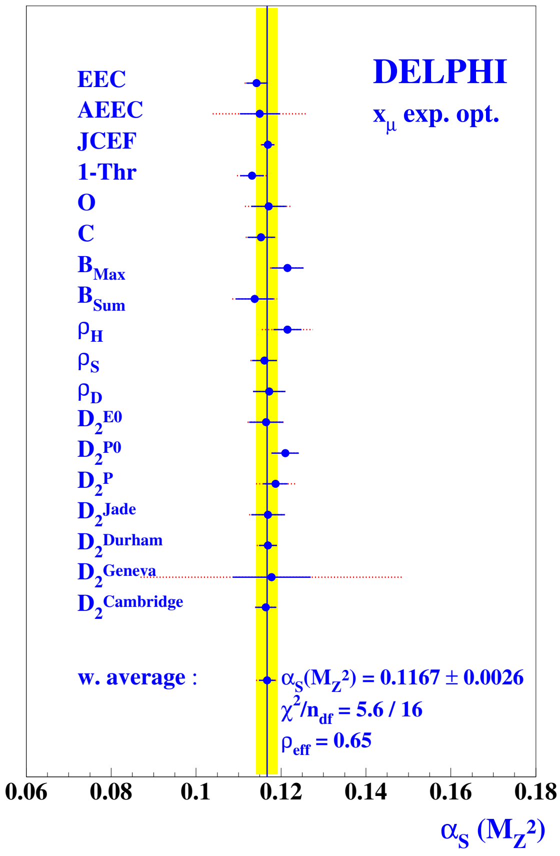
A_2 substantial for $1-T$, B_T, C

Annihilation: α_s

DELPHI

- Used 1.5 million hadronic Z decays from LEP 1
- Examined 18 event shape variables
- Fit to $\mathcal{O}(\alpha_s^2)$ calculations in a range where theory can describe data
- Fit oriented distribution $R(y, \cos\theta_T)$ with $\cos\theta_T < 0.84$





Two approaches used:

- Fix $\mu = 1 \Rightarrow$ fit quality poor, α_s values scattered over a large region
  $\alpha_s(m_Z) = 0.1228 \pm 0.0119$
- Fit α_s as well as $\mu \Rightarrow$ Good fits obtained ($\chi^2/\mathrm{dof} \approx 1$)
  μ varies from 0.0033 $(1-T)$ to 7.10 (D_2^{Geneva})
  $\alpha_s(m_Z) = 0.1167 \pm 0.0026$

Annihilation: α_s

Analytical calculations exist up to $\mathcal{O}(\alpha_s^2)$

$$\begin{aligned} R(\alpha_s, y) &\equiv \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy} \\ &= \left(\frac{\alpha_s}{2\pi}\right)\cdot A(y) + \left(\frac{\alpha_s}{2\pi}\right)^2 \cdot [B(y) + 4\pi\beta_0 \ \ln\mu \ A(y)] \end{aligned}$$

for an event shape variable y ($1-T$, ρ, B_T, B_W, C)
where $\mu = \frac{Q}{\sqrt{s}}$, A, B obtained using ERT matrix elements

- ❍ Describes data well in multi-jet region (large y)
- ❍ Fails in the two jet region (small y) where multiple soft/collinear gluon emissions become dominant

Resummed (to all orders) calculations of

Leading log	$L\sum(\alpha_s L)^n$	$[L \equiv \left(\ln\frac{1}{y}\right)]$
Next-to-leading log	$\sum(\alpha_s L)^n$	

terms exist for certain variables

- ❍ Describes data well at small y values

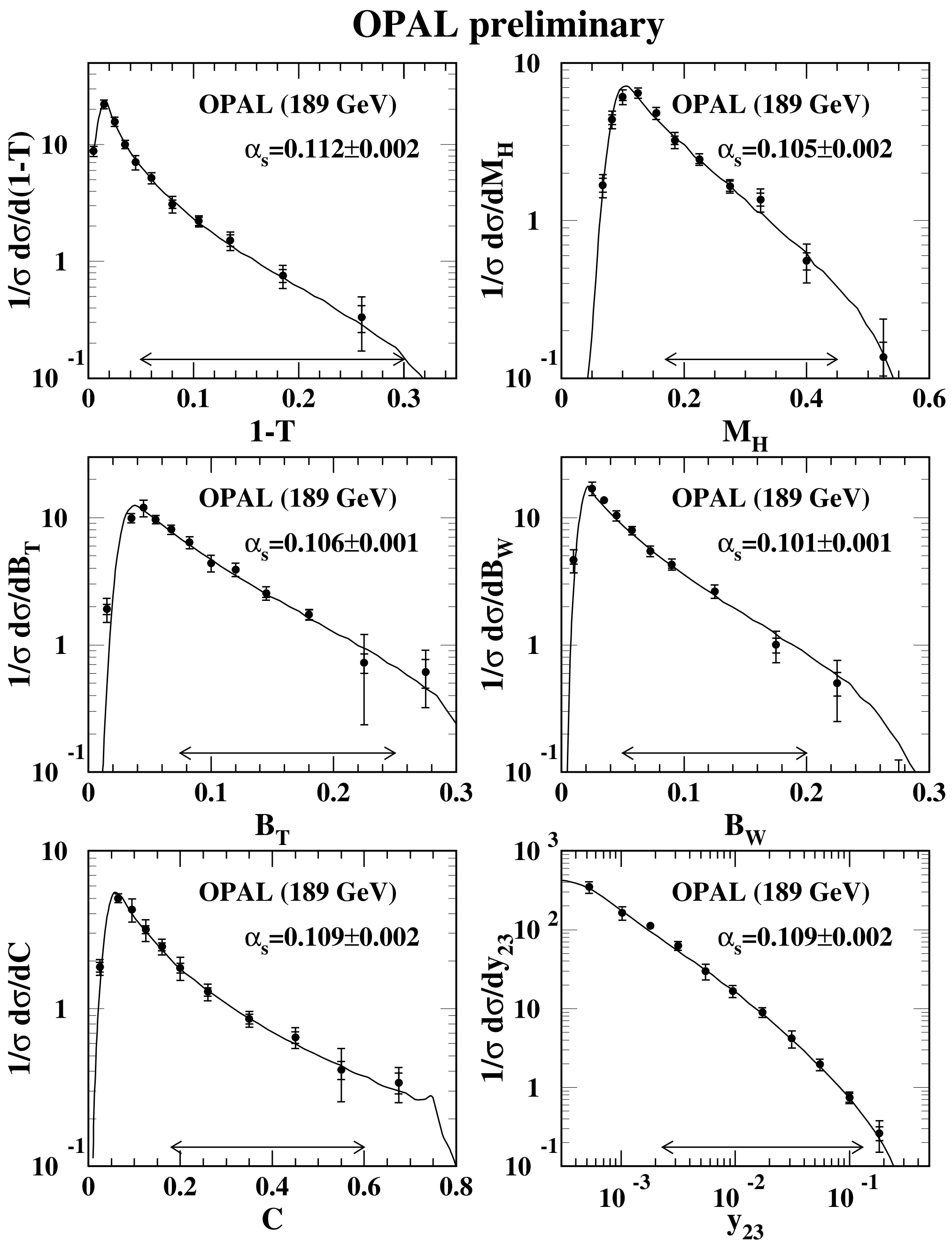
⇒ Combine the two calculations

- ✔ Taking care of common terms
- ✔ Fulfilling kinematic constraints at $y = y_{\mathrm{max}}$

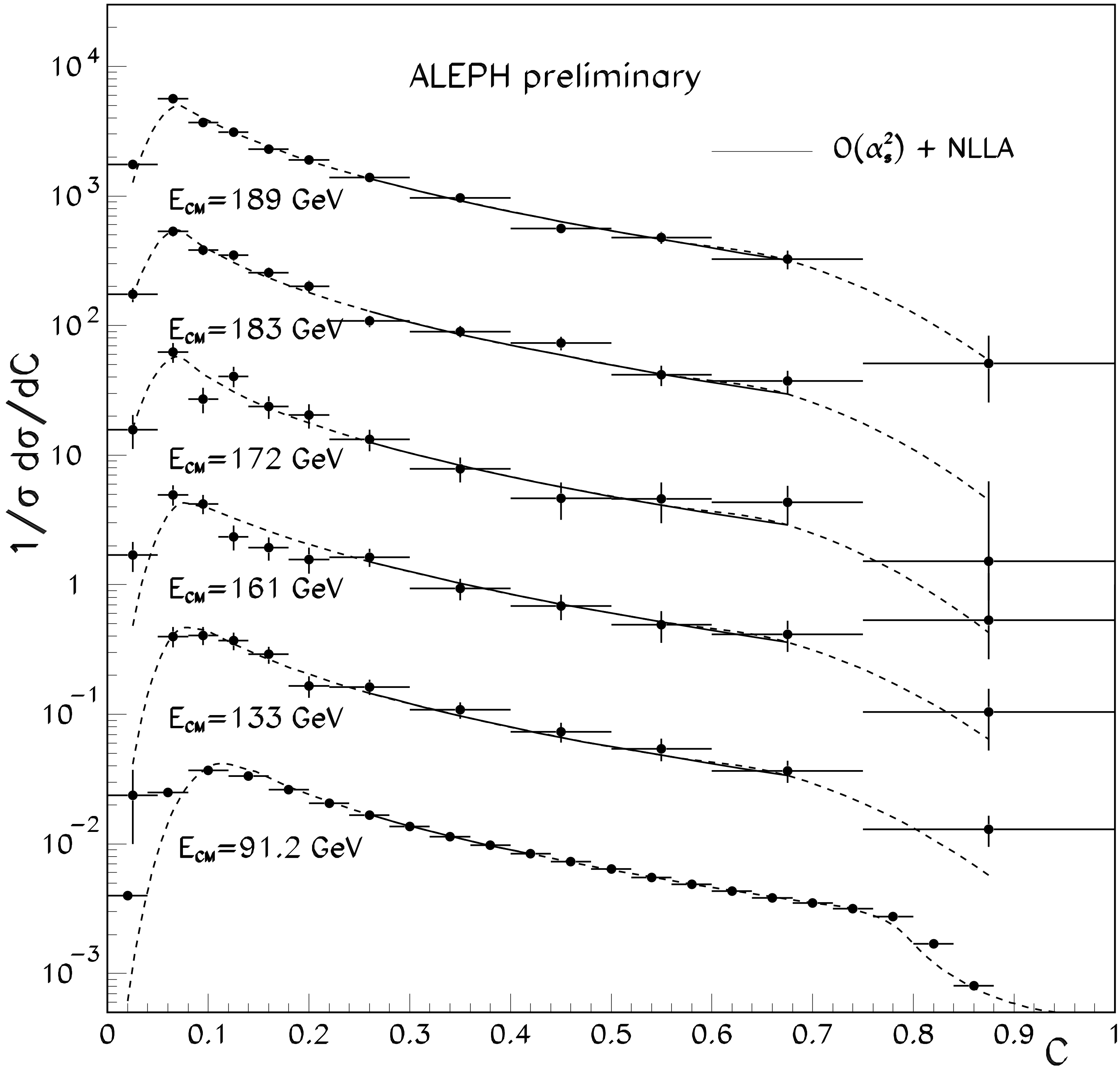
Describe data well over a wide kinematic region

Fold in hadronisation effect using Parton Shower Monte Carlo (small correction $\sim$ 10% at $\sqrt{s} \simeq m_Z$)

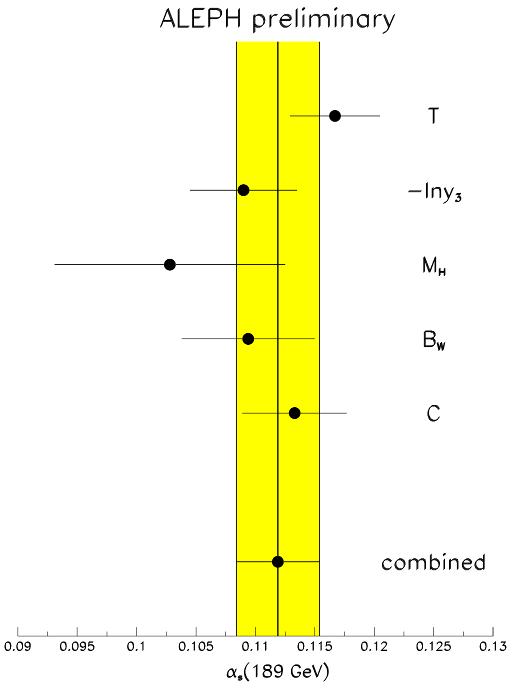
$\mathcal{O}(\alpha_s^2)$ + NLLA

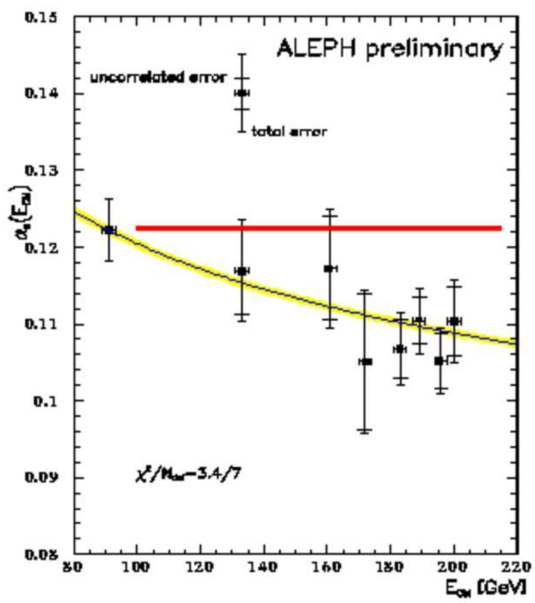


- Fit range narrower than region of agreement
- L3 uses wider fit range (probing small y)

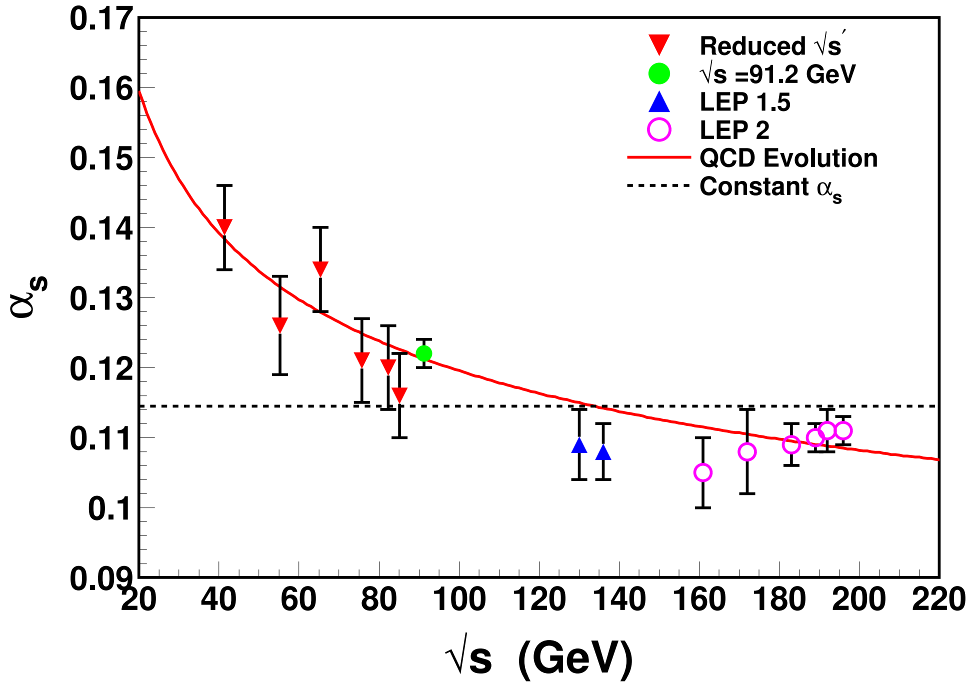


Good fit is obtained over a wide range of centre of mass energies



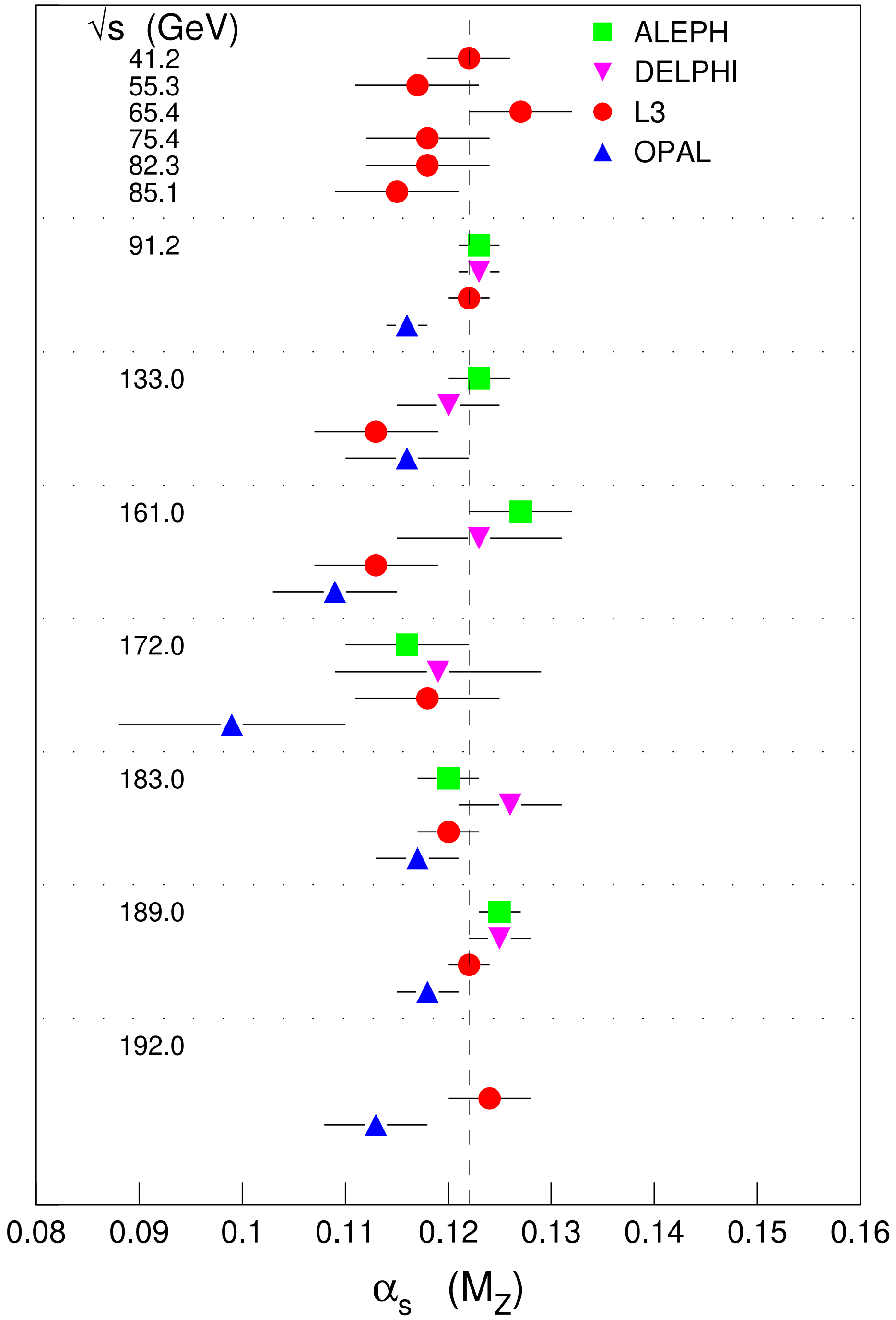


From LEP 2 data ALEPH $\rightarrow \alpha_s(m_Z) = 0.1209 \pm 0.0041$



$$\alpha_s(m_Z) = 0.1217 \pm 0.0013 \pm 0.0061$$
$$N_f = 5.0 \pm 1.3 \pm 2.0$$

Annihilation: α_s



ALEPH	DELPHI	L3	OPAL
$T,\ -\ln y_3,\ m_H,\ B_W,\ C$	$T,\ \rho$	$T,\ \rho,\ B_T,\ B_W,\ C$	$T,\ y_3,\ \rho,\ B_T,\ B_W,\ C$

$$\alpha_s(m_Z) \ = \ 0.122 \pm 0.004$$

OPAL examined 5 event shape variables (y_{23}, $1-T$, m_H, B_W, C) in 4.4 M hadronic Z decays

Classify the event sample by quark flavour

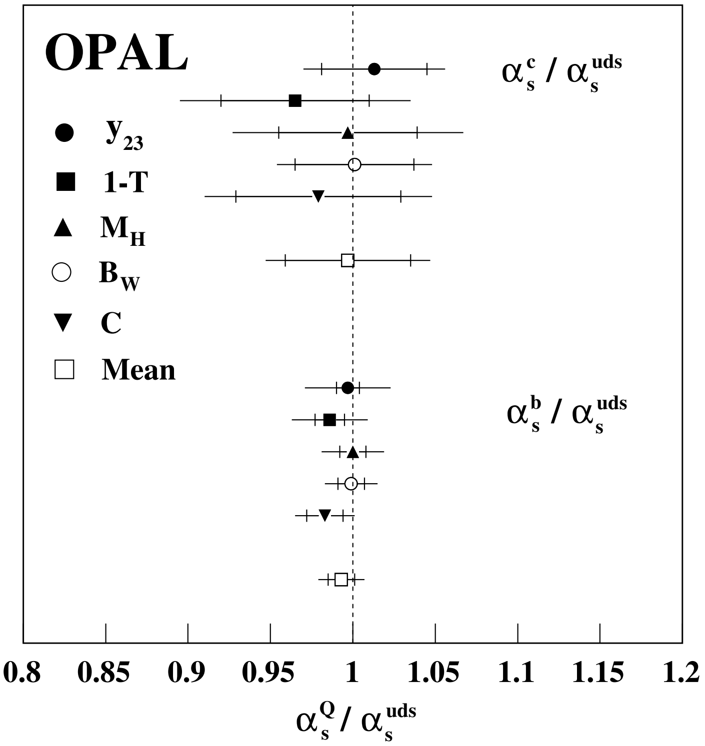
- Secondary vertex tagging is used with the help of silicon vertex detector:

  b events $\epsilon \sim 23$ % $\Pi \sim 86\%$

  u,d,s events $\epsilon \sim 35$ % $\Pi \sim 96\%$

- Select D* events using a cut on $\Delta M = M_{D^\star} - M_{D^0}$

  c events $\epsilon \sim 2$ % $\Pi \sim 55\%$



➢ Fit tagged samples to combinations of uds, c, b components with 3 α_s

➢ Take care of quark mass effect

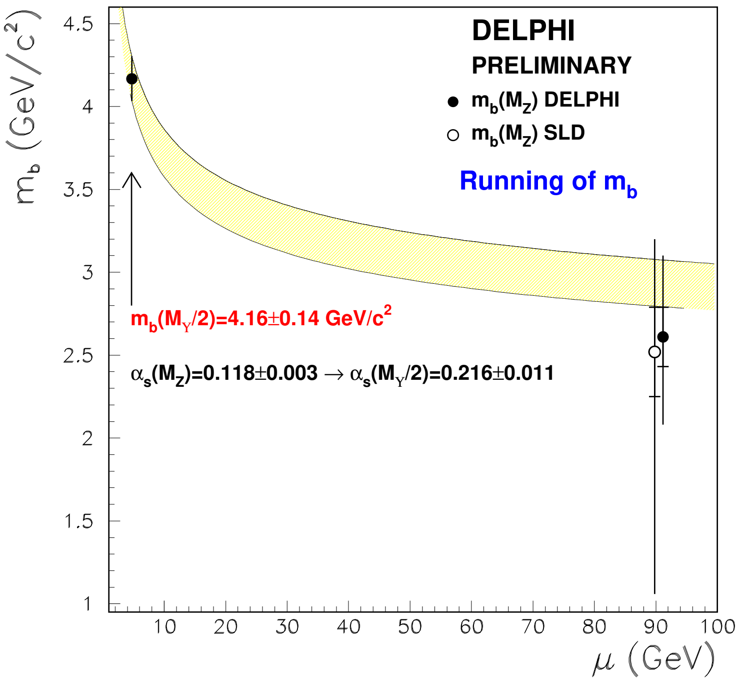
$$\alpha_s^c/\alpha_s^{uds} = 0.997 \pm 0.038 \pm 0.030 \pm 0.012$$
$$\alpha_s^b/\alpha_s^{uds} = 0.993 \pm 0.008 \pm 0.006 \pm 0.011$$

DELPHI measure 3- and 4-jet rates in hadronic Z decays with quark flavour tagged as b-quark or light (u,d,s) quark

Two different tagging methods using (a) signed impact parameter; (b) multiple characteristic variables give 85

ALEPH use flavour tagging in jet rates or event shape variables

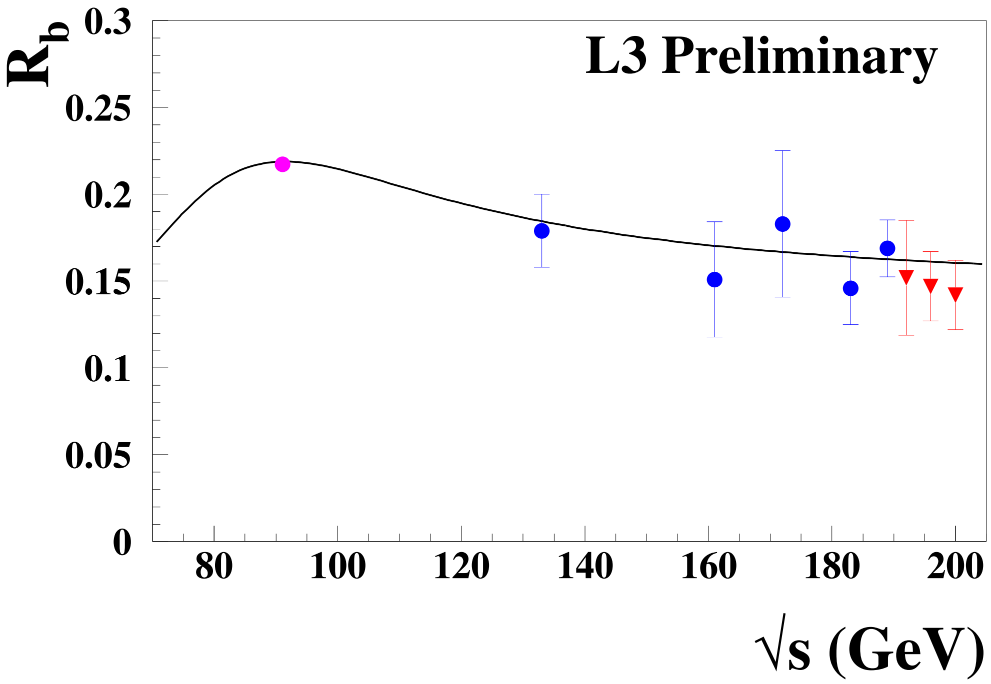
Gluon branching rate depends on quark mass

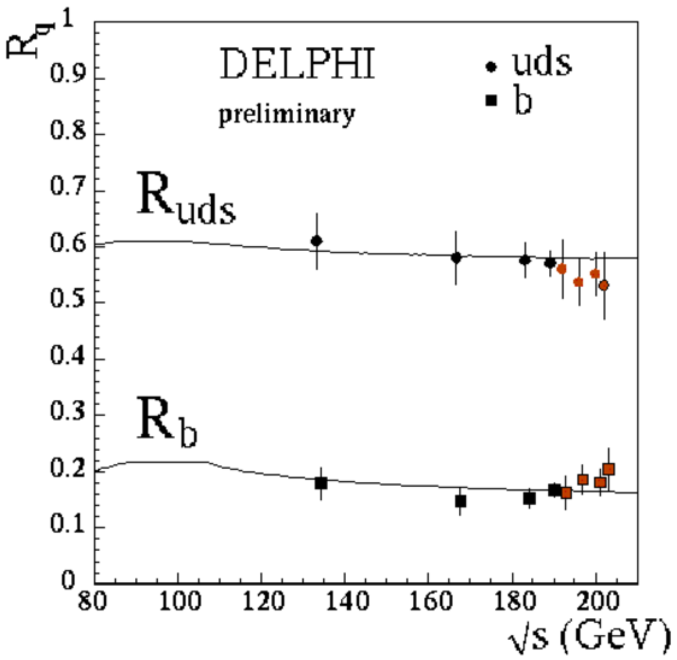


	$m_b(m_Z)$	Δm_b (GeV)			
	(GeV)	(stat)	(syst)	(had.)	(theory)
ALEPH	3.78	0.14	0.17	0.10	$^{+0.12}_{-0.13}$
DELPHI	2.61	0.18	0.04	$^{+0.45}_{-0.49}$	0.07
SLD	2.52	0.27		$^{+0.33}_{-0.47}$	$^{+0.54}_{-1.46}$

To be compared with $m_b(m_Z) = 4.20 \pm 0.08$ GeV from QCD sum rule and NNLO in $\Upsilon(1S)$

Continue flavour tagging in hadronic data at LEP2



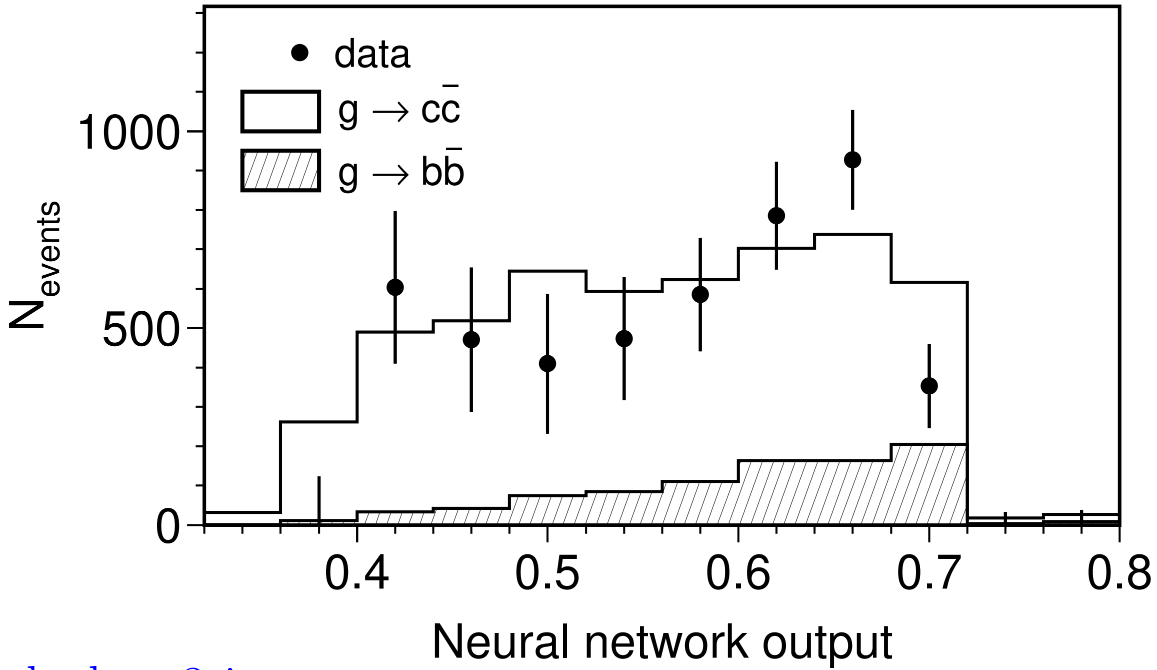


L3 and DELPHI use vertex tagging to obtain R_b and R_{uds}

$\rightarrow$ in agreement with Electroweak theory

Pre 99 summary from Z data

$$n_{g\to c\bar{c}} = 2.2\%$$
$$n_{g\to b\bar{b}} = 0.2\%$$



L3 looked at 3-jet events

- Lepton from least energetic jet
- Neural network analysis with energy flow variables
  $\epsilon_c = 0.4\% - 4.4\%$ $\epsilon_b = 1\% - 13\%$

$$n_{g\to c\bar{c}} = \left[2.45 \pm 0.35 \pm 0.45 - 3.74(n_{g\to b\bar{b}} - 0.26)\right]\%$$

OPAL made an analysis of 4-jet events

$$n_{g\to c\bar{c}} = [3.20 \pm 0.21 \pm 0.38]\%$$
$$n_{g\to b\bar{b}} = [0.215 \pm 0.043 \pm 0.080]\%$$

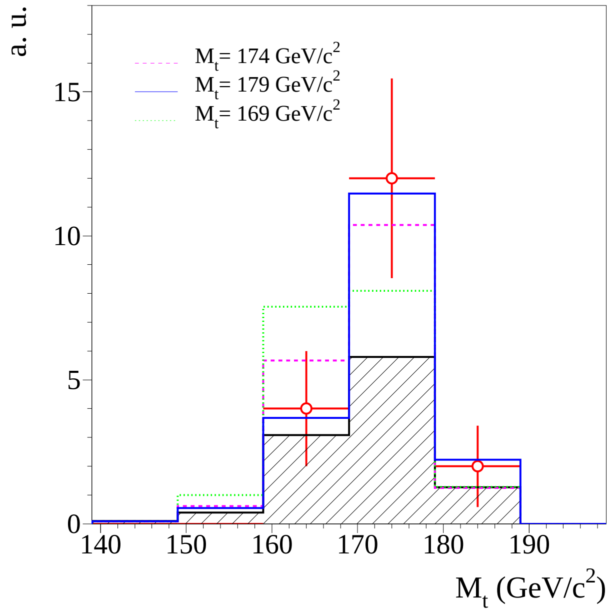
DELPHI analysed 4-b events

$$n_{g\to b\bar{b}} = [0.33 \pm 0.10 \pm 0.08]\%$$

Look for $e^+e^- \longrightarrow t\bar{c}\ (\bar{u});\quad t \longrightarrow bW;\quad W \longrightarrow f\bar{f}'$

Standard Model expectation $\leq 10^{-9}$ pb

ALEPH Preliminary



Search strategy:

- Top is almost at rest (near threshold)
- Accompanying light quark will have little energy
- b, W from top decay will be almost monochromatic
- W can decay leptonically (missing energy) or to jets [$\ell\nu$, $q\bar{q}'$ cluster around m_W]
- Wb system will cluster around top mass

L3 examined data at $\sqrt{s} = 183$ GeV

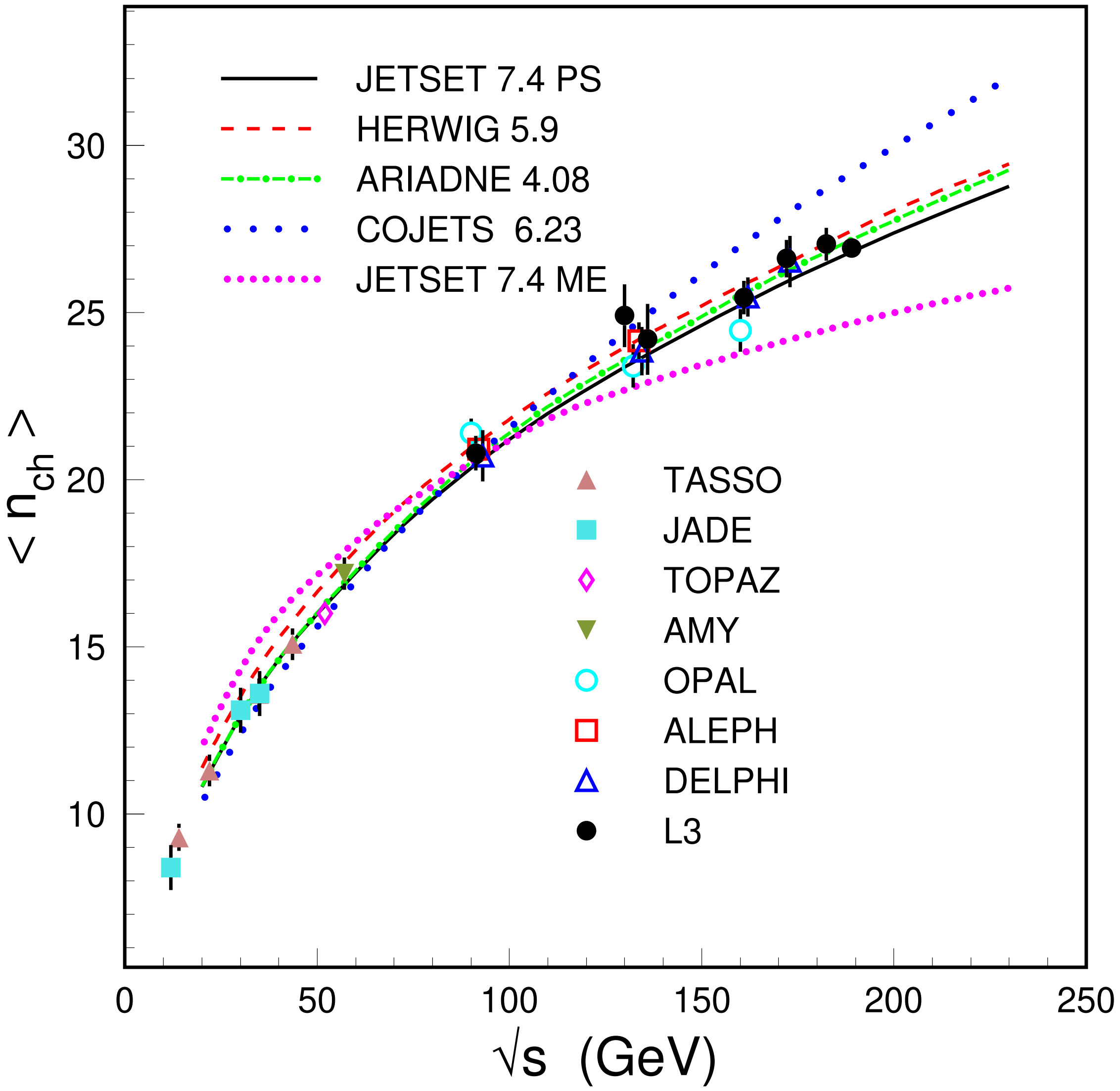
	# Observed	# Expected	95% CL Upper Limit
$\ell\nu$	0	0.3	575 pb
$q\bar{q}'$	1	3.8	1610 pb

ALEPH looked at $\sqrt{s} = 189$ GeV data

	# Observed	# Expected
$\ell\nu$	2	1
$q\bar{q}'$	16	10.5

The small excess not confirmed with 1999 data: 30 (26.3) Obs. (expect)

First moment of charged particle multiplicity distribution:



Coherent parton shower $\Rightarrow$ soft gluon suppression

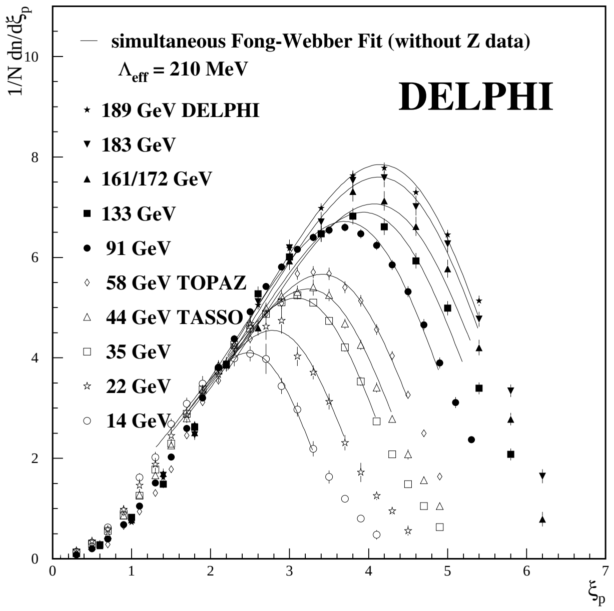
Explains energy evolution of $\langle n_{ch} \rangle$

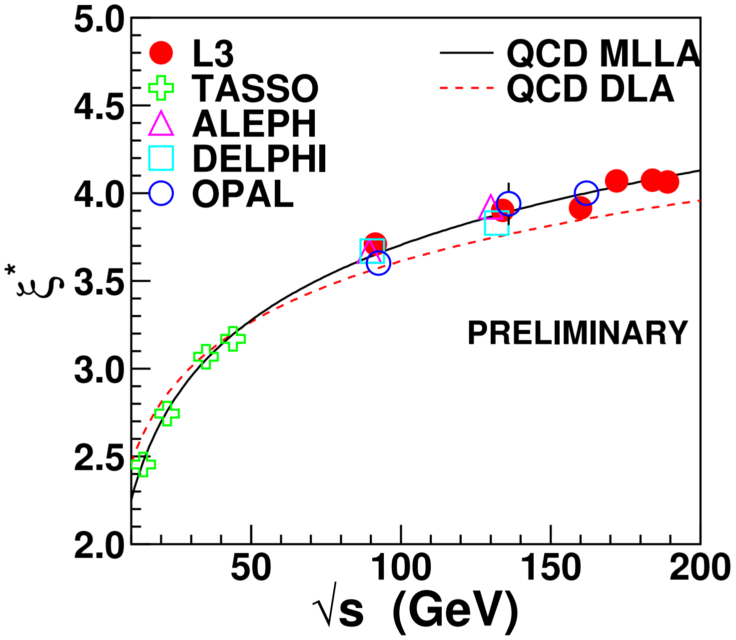
Cojets with incoherent parton shower predicts too high multiplicity at high $\sqrt{s}$

Soft gluon coherence will deplete particles at low x_p ($\equiv 2p/\sqrt{s}$) values

$\Rightarrow$ Bell like structure in ξ_p ($\equiv \ln\left(\frac{1}{x_p}\right)$)

Asymptotic form is Gaussian and in MLLA ξ distribution is a skewed Gaussian





$\xi^\star$ (peak position in the ξ_p distribution) shifts with $\sqrt{s}$ and QCD + LPHD predicts a specific energy dependence of $\xi^\star$

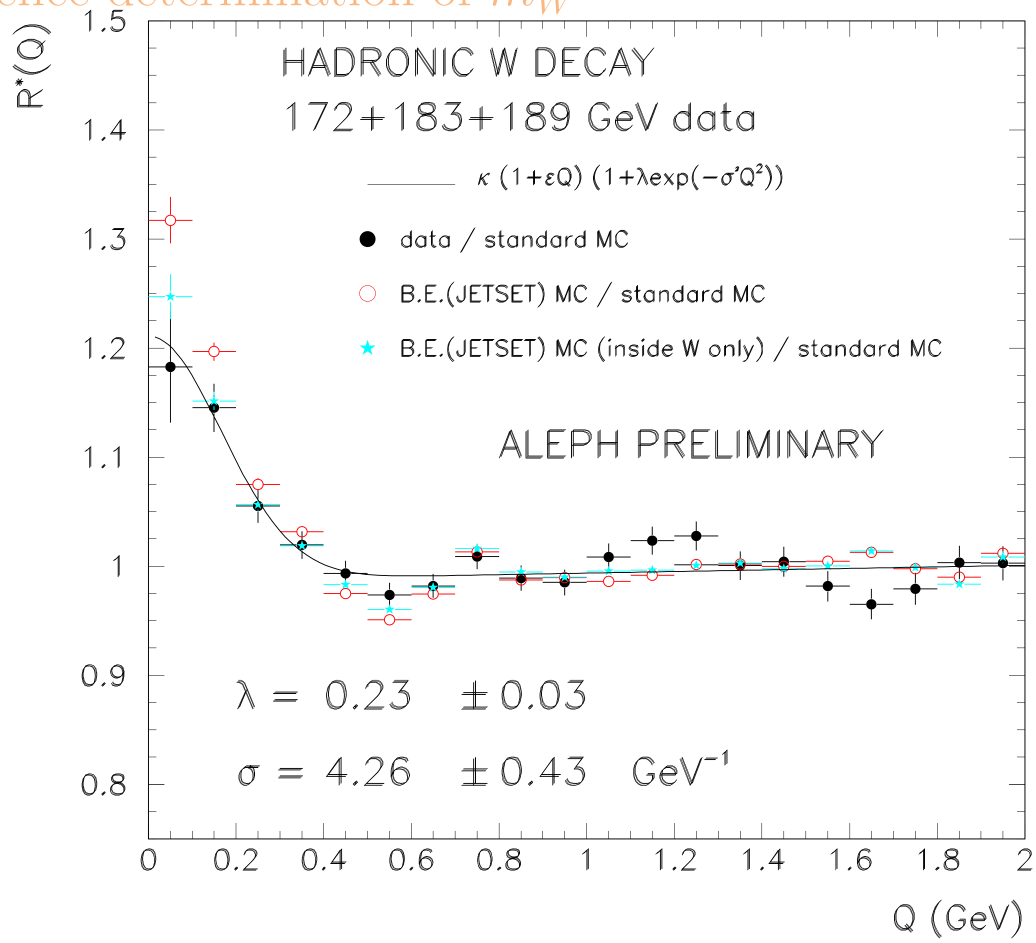
$$\xi^\star = 0.5Y + \sqrt{C}\cdot\sqrt{Y} - C + \mathcal{O}(Y^{-\frac{3}{2}})$$
$$Y = \ln\left(\frac{\mathrm{E}_{\mathrm{beam}}}{\Lambda_{\mathrm{eff}}}\right)$$

DLA [$\xi^\star \equiv 0.5Y$] gives a poor description of the energy evolution

$\Rightarrow$ MLLA calculation with $\Lambda_{\mathrm{eff}} \approx 200$ MeV describe data well

BE correlation between like sign pions observed in Z and W decays at LEP

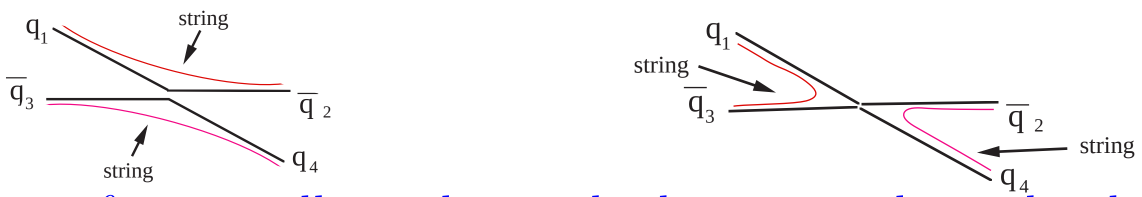
Does one have correlation among decay products of two W's?
$\Rightarrow$ influence determination of m_W



- BE correlation observed in $q\bar{q}'\ell\nu$ mode of W-production well reproduced by MC tuned to BE correlation data at LEP1
- Better agreement between data and MC with BE correlation implemented only for π's of the same W. Models with correlation also among π's of different W's disfavoured at 2.7σ level

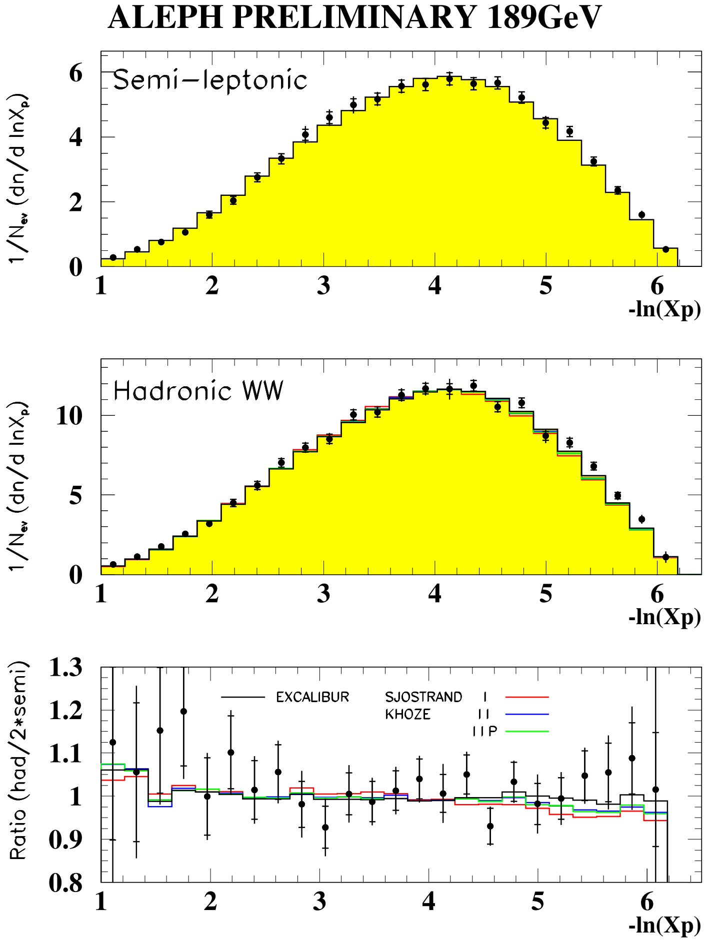
Annihilation: Colour Reconnection

For the process $e^+e^- \rightarrow W^+W^-$; $W^+ \rightarrow q_1\bar{q}_2$; $W^- \rightarrow q_3\bar{q}_4$;



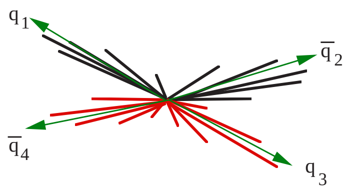
In absence of cross talk, each pair hadronises independently $\Rightarrow$ hadrons associate to a W along a string

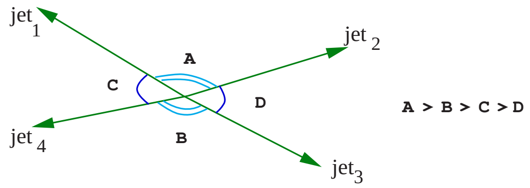
QCD interference causes gluon exchange between 2 initial $q\bar{q}'$ or immediate reconnection $\Rightarrow$ large effects in particle distributions



Compare multiplicities in $q\bar{q}'\ell\nu$ and 4-quark final states:

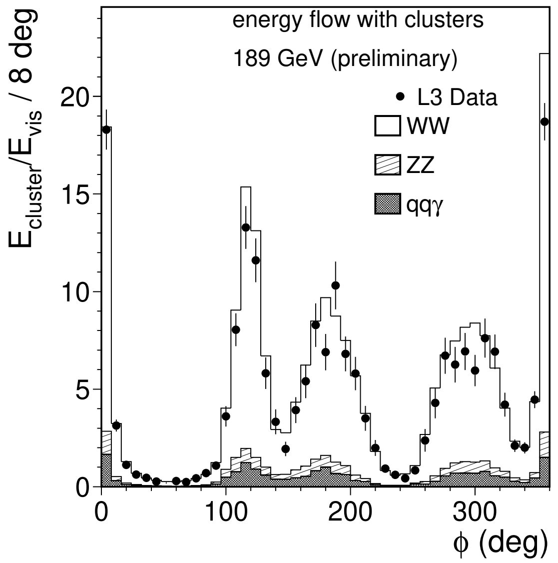
- No appreciable difference
- Not sensitive enough to discriminate colour reconnection

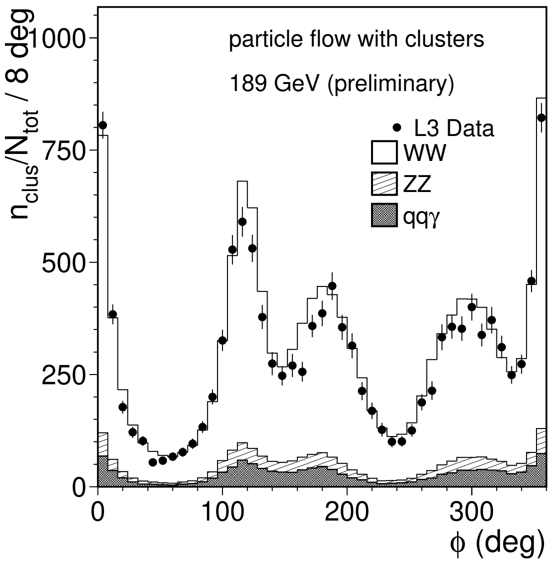




Examine energy and particle flow in interjet region:

- Even with colour reconnection jets will correspond to initial quark direction
- Use topological criteria to define jets A > B > C > D with A, B between 100° & 140°; C, D < 100°
- Define a plane spanned by 2 jets; project particle momenta in this plane and get angle ϕ with respect to a reference direction





To compare different inter-jet region, rescale ϕ with respect to angle between jets

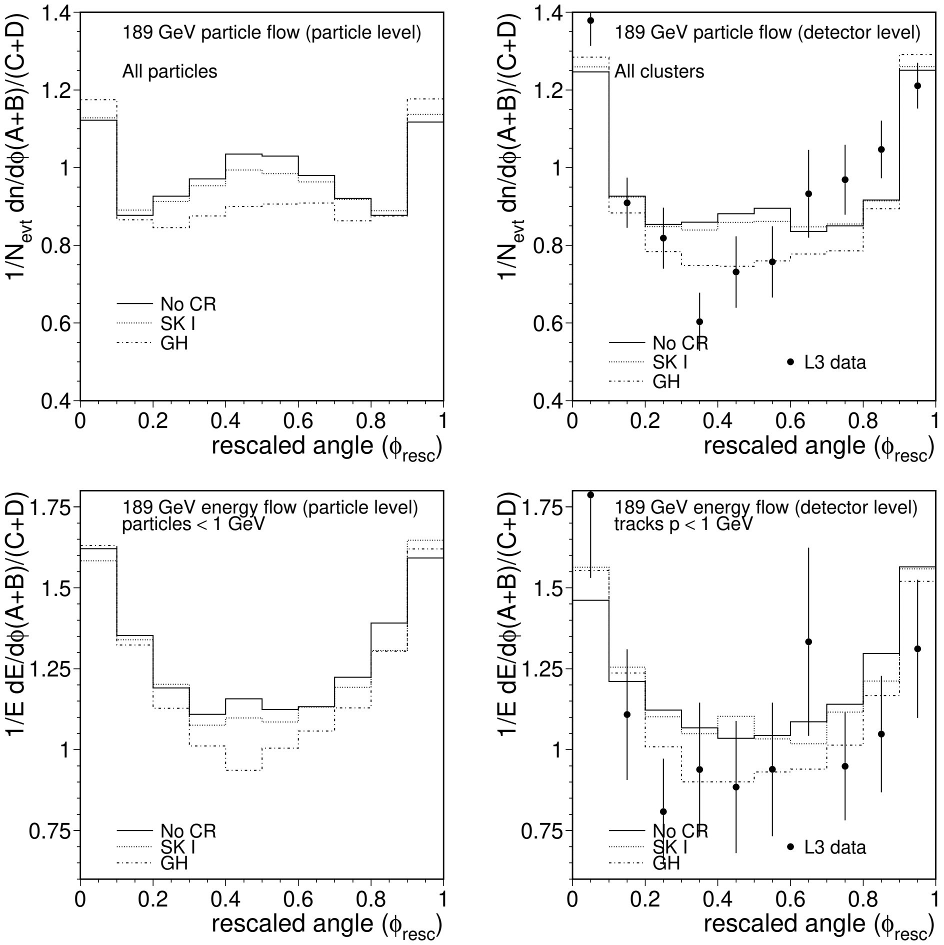
W-pair events are not coplanar ⇒ use 4 planes rather than 1 defined by adjacent jets and choose appropriate plane for a given particle

Annihilation: Colour Reconnection

Particle density or energy flow between q's from the same W are expected to be different from those from different W's. So use variables:

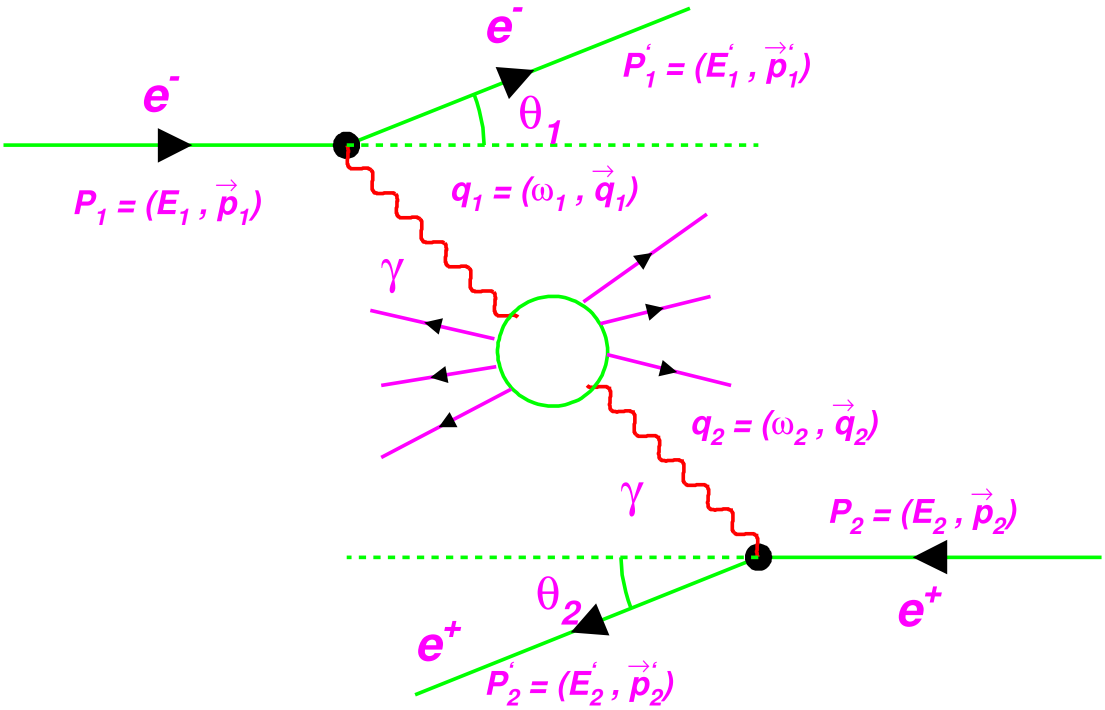
$$R_n = \frac{\frac{1}{N}\frac{dn}{d\phi}(\text{Regions A + B})}{\frac{1}{N}\frac{dn}{d\phi}(\text{Regions C + D})}$$

$$R_E = \frac{\frac{1}{E}\frac{dE}{d\phi}(\text{Regions A + B})}{\frac{1}{E}\frac{dE}{d\phi}(\text{Regions C + D})}$$



- ➤ Some reconnection models show large differences ($\sim$ 10%)
- ➤ Data not yet sensitive to discriminate models – need to wait
- ➤ Asymmetries in energy, particle flow can also be considered

$\Rightarrow$ Narrow down on colour reconnection effect by end of LEP2

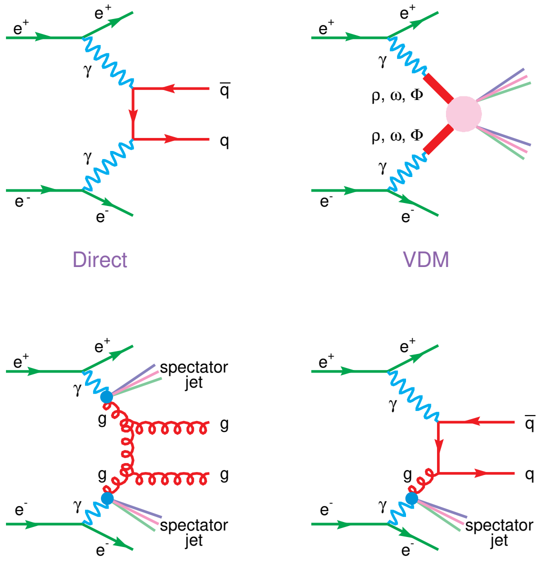


$$
\begin{aligned}
Q^2 &= -q_1^2 = 2E_1E_1'(1-\cos\theta_1) \\
P^2 &= -q_2^2 \\
W_{vis}^2 &= \left(\sum_h E_h\right)^2 - \left(\sum_h \vec{p_h}\right)^2 \le W_{\gamma\gamma}^2
\end{aligned}
$$

Depending on detection of scattered $e^{\pm}$:

- Untagged events $Q^2 \approx 0;\ P^2 \approx 0$
  - $\gamma\gamma$ total cross section
  - Inclusive c and b production
  - Jet production
- Single tag events $Q^2 > P^2 \approx 0$
  - Photon structure function
- Double tag events $Q^2 \approx P^2 > 0$
  - $\gamma^\star\gamma^\star$ cross section

Diagrams contributing to: $e^+e^- \to e^+e^-\gamma\gamma \to e^+e^-$ hadrons

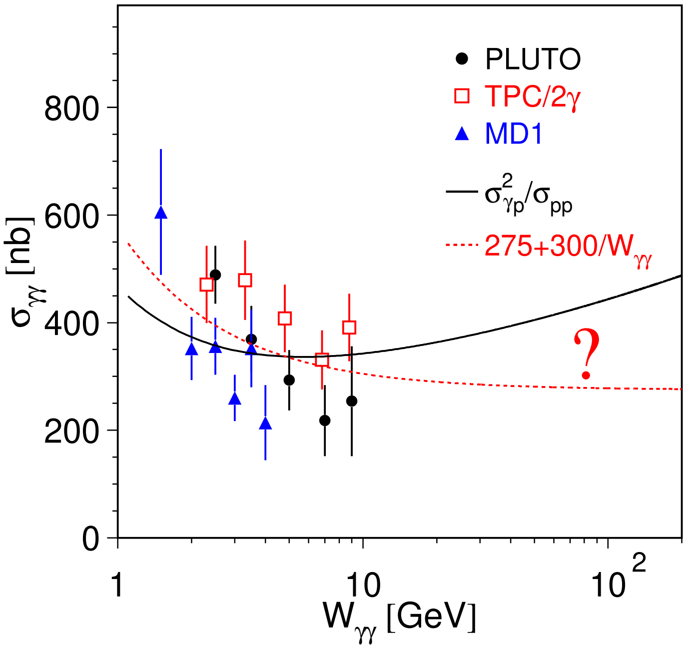


Direct VDM

Double Resolved Single Resolved

The VDM term dominates

Measured cross sections at PEP/PETRA are inconclusive:

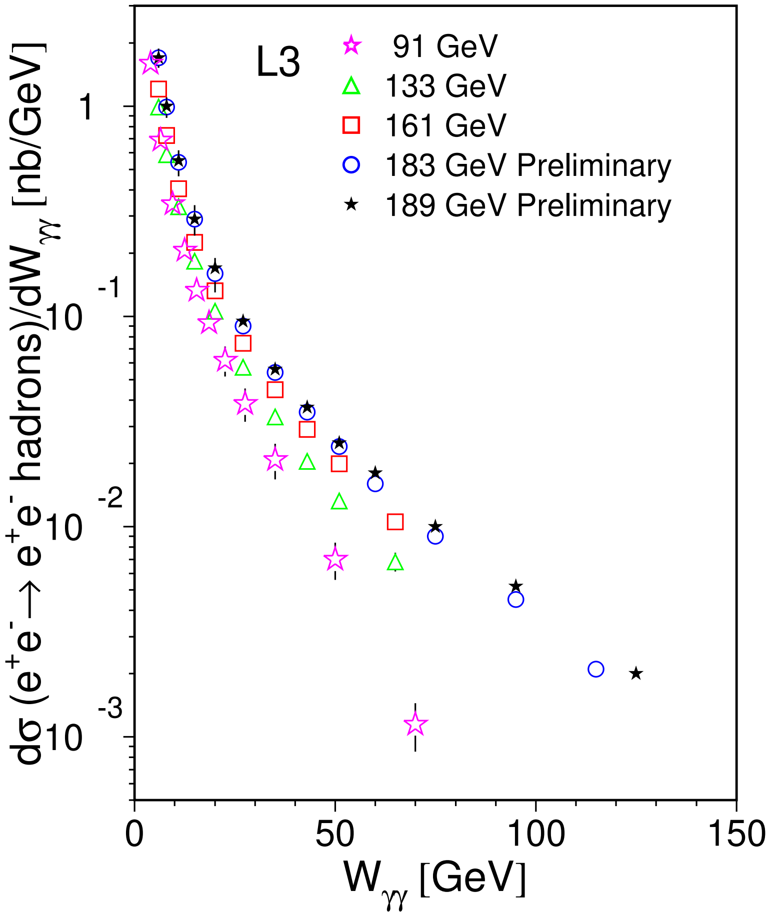


- ➢ Wild spread in the measurements
- ➢ $W_{\gamma\gamma}$ span is limited to $W_{\gamma\gamma} < 10$ GeV

Use unfolding methods to convert measured W_{vis} to $W_{\gamma\gamma}$

Depends in Monte Carlo model used: PHOJET and PYTHIA agree at 20-30% level

Use unfolded $W_{\gamma\gamma}$ to determine experimentally measured cross section $\sigma(e^+e^- \to e^+e^- + \text{hadrons})$



σ_{eeh} increases with increasing E_{beam} and decreasing $W_{\gamma\gamma}$

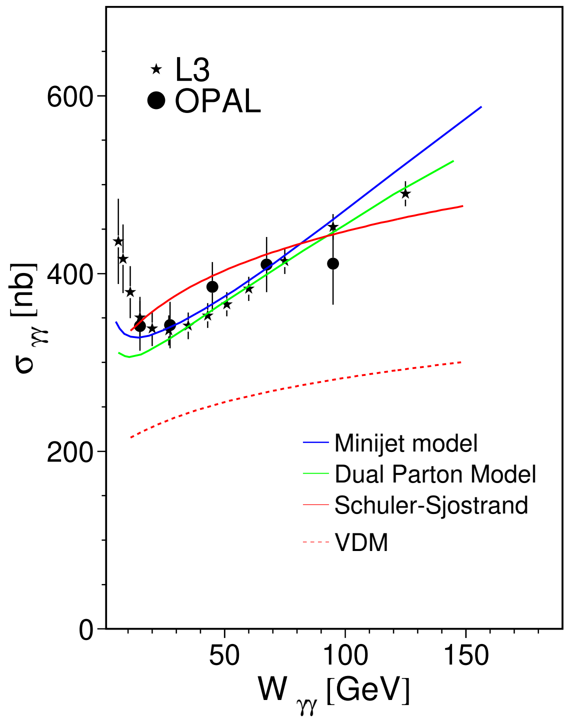
Take out $\gamma\gamma$ luminosity effect

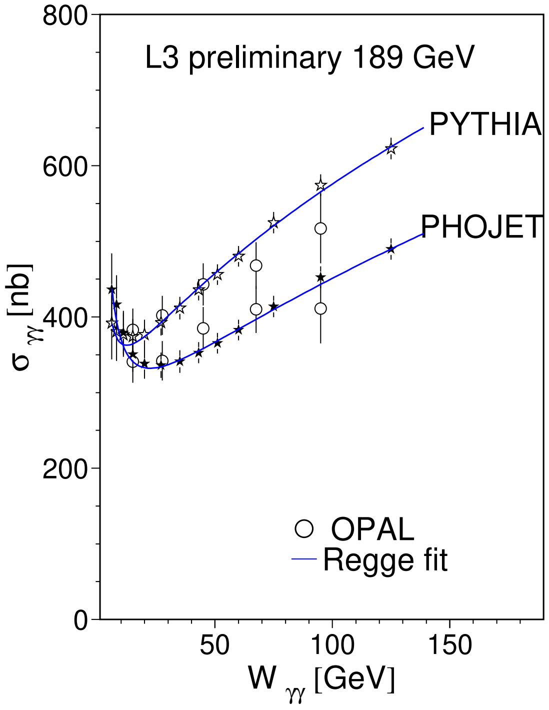
$$\frac{d\sigma_{eeh}}{d(W_{\gamma\gamma}^2/s)} = \int \frac{dQ^2}{Q^2}\frac{dP^2}{P^2} \sum_{a,b=T,S} \mathcal{L}_{ab}\sigma_{ab}^{\gamma\gamma}(W_{\gamma\gamma}, P^2, Q^2)$$

$$\sigma_{ab}^{\gamma\gamma}(W_{\gamma\gamma}, P^2, Q^2) = F_a(Q^2)F_b(P^2)\sigma_{ab}^{\gamma\gamma}(W_{\gamma\gamma})$$

summing over photon helicity states and taking care of γ form factors

2-Photon: Total Cross Section





- ❑ L3 and OPAL compatible if same generator used in unfolding
- ❑ Both experiments show clear rising cross section with energy

At high $W_{\gamma\gamma}$, Pomeron exchange dominates (Donachie, Landshoff)

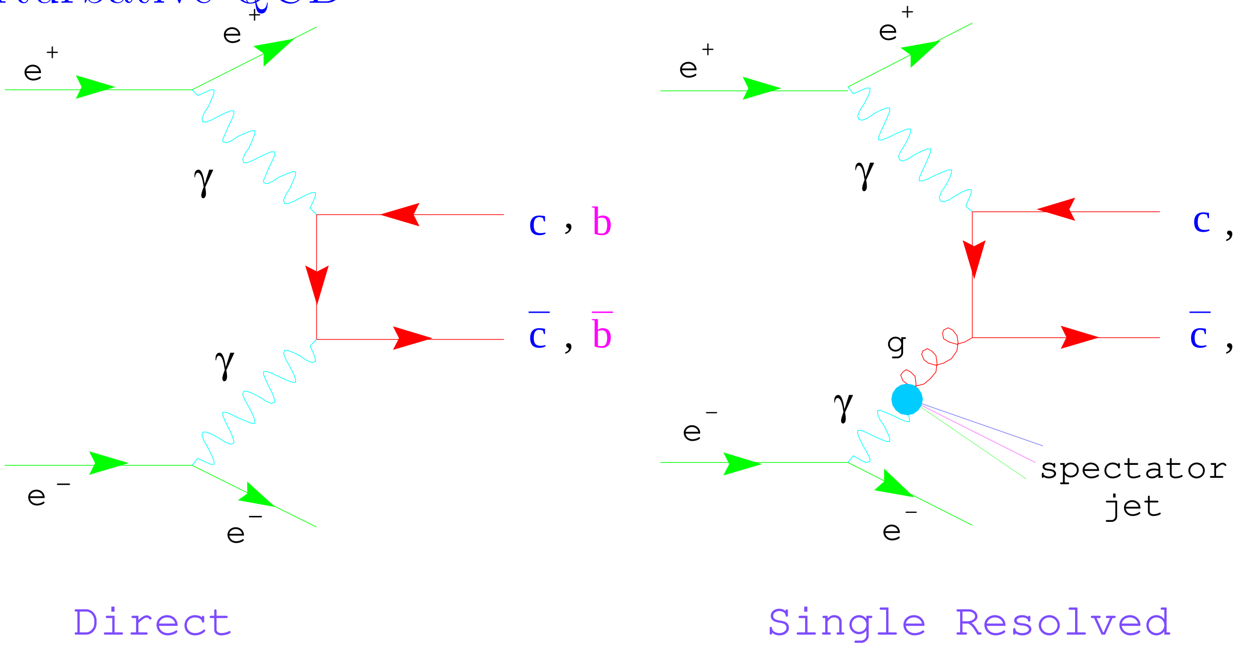
$$\sigma_{tot} \; = \; A \cdot s^{\epsilon} \; + \; B \cdot S^{-\eta}$$

Fixing $\eta = 0.34$ (from fit to all hadronic σ_{tot}), $\epsilon = 0.22 \pm 0.02$

– 7σ different from universal fit ($\epsilon = 0.095 \pm 0.002$)

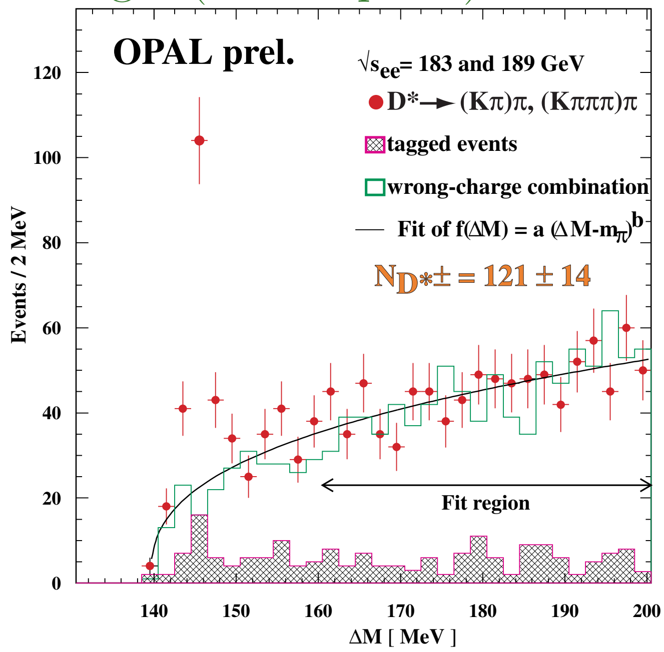
Two Pomeron model: $\alpha_{soft}(0) \sim 1.08$; $\alpha_{hard}(0) \sim 1.4$	Donachie *et al.*
Smooth suppression of hadron-like and point-like γ interaction	Schuler *et al.*
Dual parton model with unitarisation constraint	Engel *et al.*
Mini-jet model using QCD to calculate # of hard collisions	Godbole *et al.*

Heavy flavour production cross section are calculable in perturbative QCD

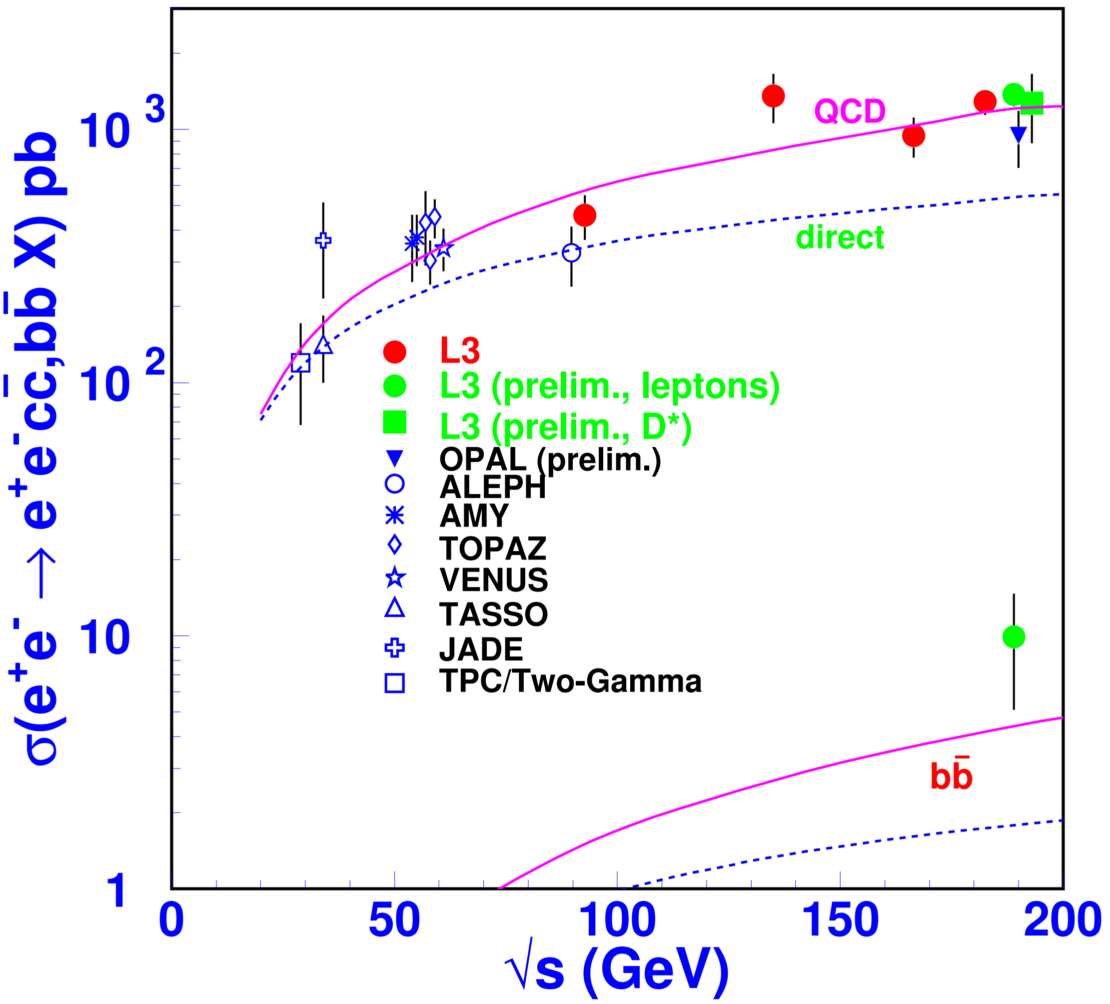


Charm Identification:

- $D^{\star\pm}$ tag : (ALEPH 73pb^{-1}, L3 165pb^{-1}, OPAL 220pb^{-1})
- High $p_\perp$ lepton tag : (L3 176pb^{-1}) – also for b



$$\sigma_{b\bar{b}} = 9.9 \pm 2.9\ (\mathrm{stat}) \pm 3.8\ (\mathrm{syst})\ \mathrm{pb}$$



☆ $\sigma(e^+e^- \to e^+e^- c\bar{c} + X)$ increases with $\sqrt{s}$

☆ Direct process insufficient to describe data

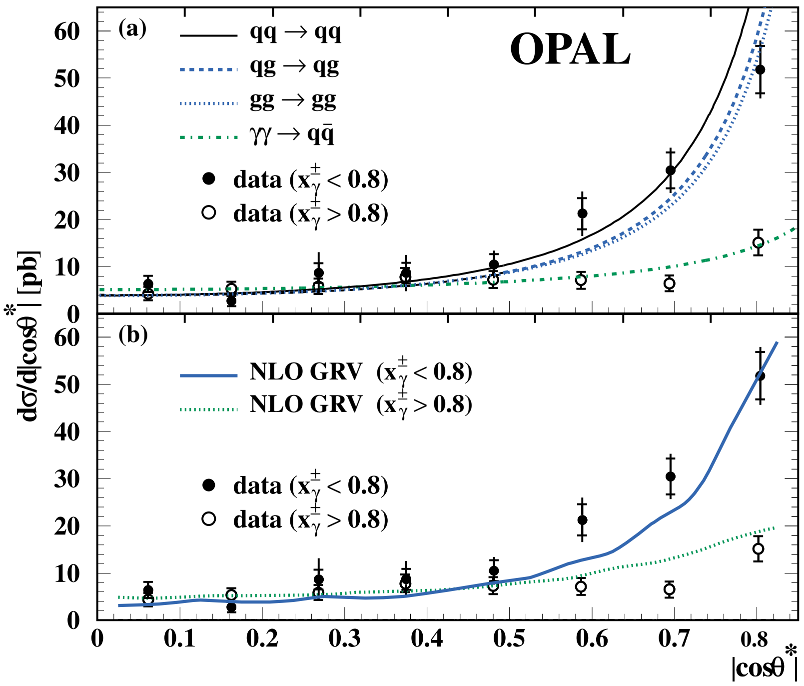
$\Rightarrow$ Evidence of gluon component in photon

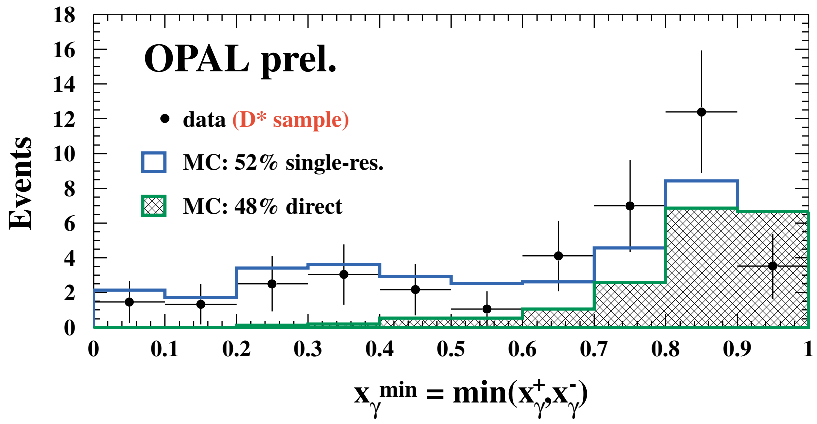
Differential distributions ($p_\perp^{D^\star}/W_{vis}$, x_γ, $\cdots$) have better discrimination between direct and resolved process

$\Rightarrow$ OPAL data demonstrates inadequacy of direct process alone

Separate direct and resolved processes in two jet sample using

$$x_\gamma^\pm = \frac{\sum_{\rm jet}(\rm E \pm p_z)}{\sum_{\rm had}(\rm E \pm p_z)}$$





$x_\gamma^\pm < 0.8$ Resolved (Gluon exchange) $\propto (1 - |\cos\theta^\star|)^{-2}$

$x_\gamma^\pm > 0.8$ Direct (Quark exchange) $\propto (1 - |\cos\theta^\star|)^{-1}$

Agrees with NLO QCD calculations

Quark constituents dominant at large x

Gluon constituents dominant at small x

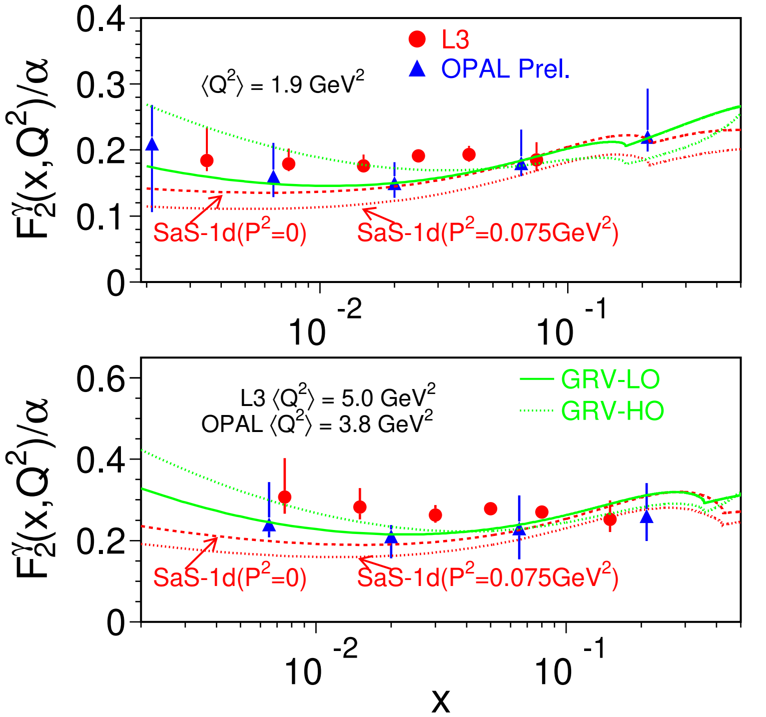
Use the tagged $e^{\pm}$ to evaluate:

$$
\begin{aligned}
Q^2 &= 2\mathrm{E_b}\mathrm{E_{Tag}}(1-\cos\theta_{\mathrm{Tag}}) \\
x &= \frac{Q^2}{Q^2+W^2+P^2} \\
y &\simeq 1-\frac{\mathrm{E_{Tag}}}{\mathrm{E_b}}\cos^2\frac{\theta_{\mathrm{Tag}}}{2} \\
\frac{d^2\sigma_{e\gamma\to eX}}{dx\,dQ^2} &= \frac{2\pi\alpha^2}{xQ^4}\left[\left(1+(1-y)^2\right)F_2^{\gamma}(x,Q^2) \; - \; y^2 F_L^{\gamma}(x,Q^2)\right]
\end{aligned}
$$

Data from 4 LEP experiments with Q^2 from 1 to 3000 GeV^2 and x down to 0.001

	$\sqrt{s}$ (GeV)	θ_{Tag} (mrad)	Q^2 (GeV^2)	x
ALEPH	91	65 – 150	6 – 44	0.005 – 0.90
	91	318 – 927	35 – 3000	0.03 – 0.97
	183	35 – 155	7 – 200	0.003 – 0.97
DELPHI	91	43 – 135	4 – 30	0.005 – 0.977
	161, $\cdots$ 189	42 – 610	10 – 1000	0.01 – 0.8
L3	91	26 – 66	1.2 – 9	0.002 – 0.2
	91	200 – 600	40 – 500	0.05 – 0.98
	183	30 – 66	9 – 30	0.01 – 0.3
OPAL	91	27 – 55	1.1 – 6.6	0.0025 – 0.2
	161, 172	33 – 120	6 – 100	0.02 – 0.6
	91, 189	25 – 60	1.5 – 30	0.001 – 0.32

Low Q^2



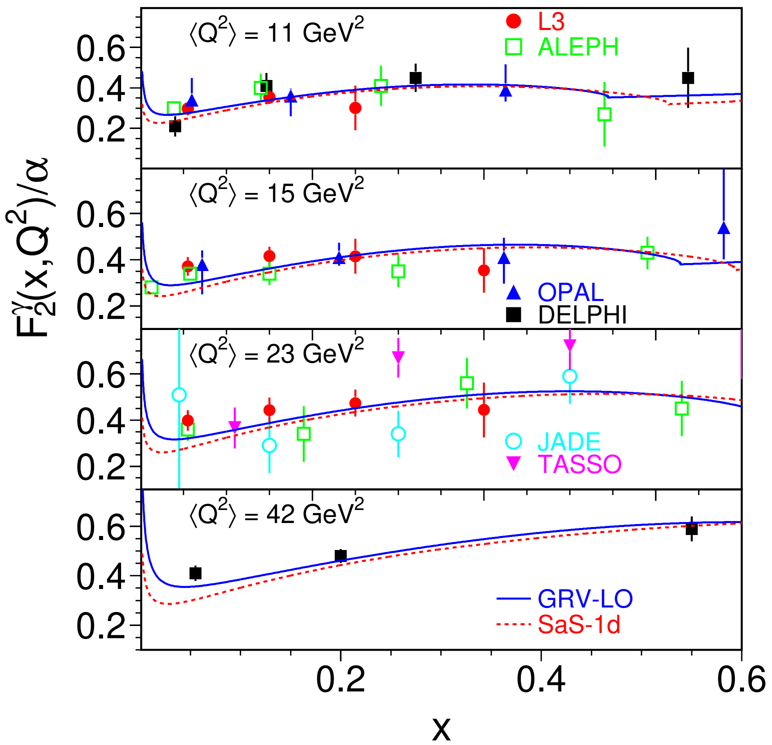
Needs unfolding to take care of $W_{vis} \rightarrow W_{\gamma\gamma}$, $P^2 \neq 0$

- Good agreement among experiments
- Need better MC's
- Access low x region

SAS-1d with evolution starting at $Q_0^2 = 0.36$ GeV^2 lies below data

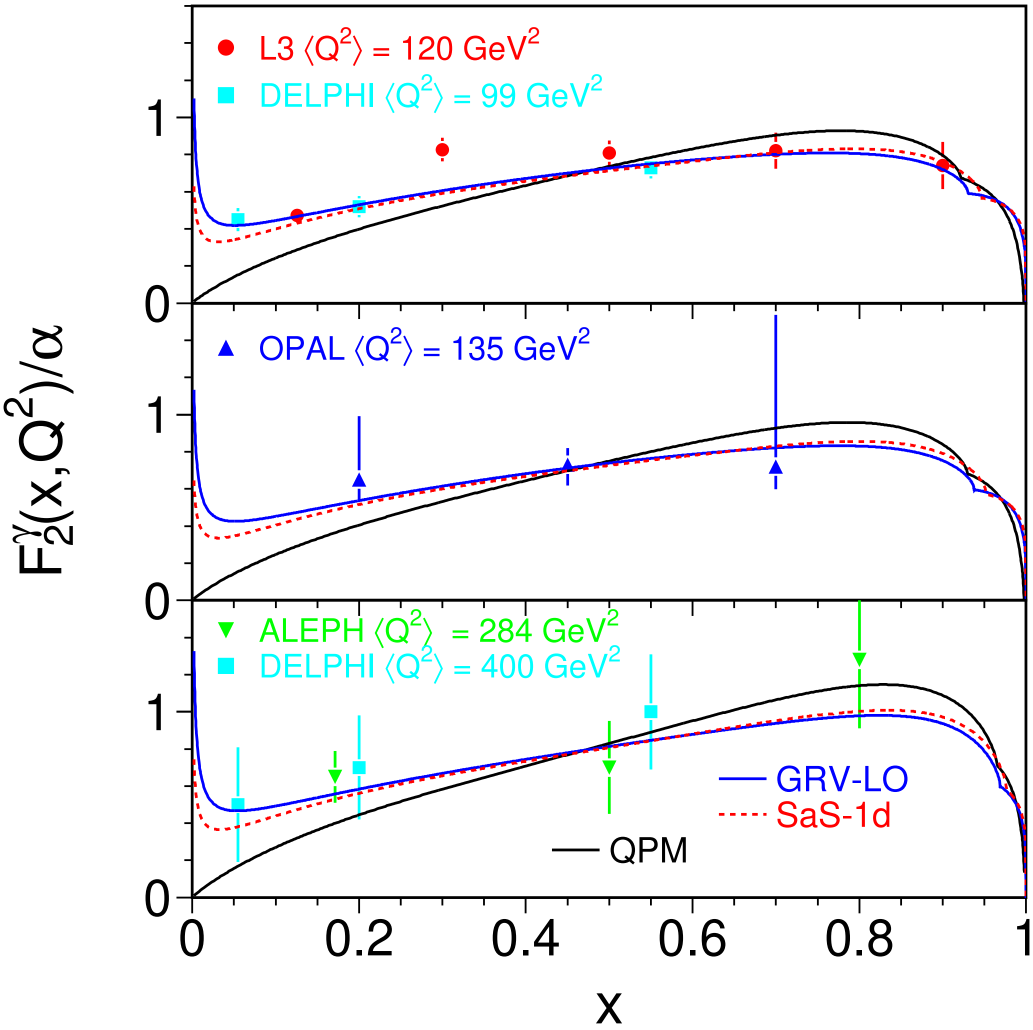
GRV with evolution starting at $Q_0^2 = 0.25$ GeV^2 better at low x

Medium Q^2



- Difference among models small
- Data in agreement with models

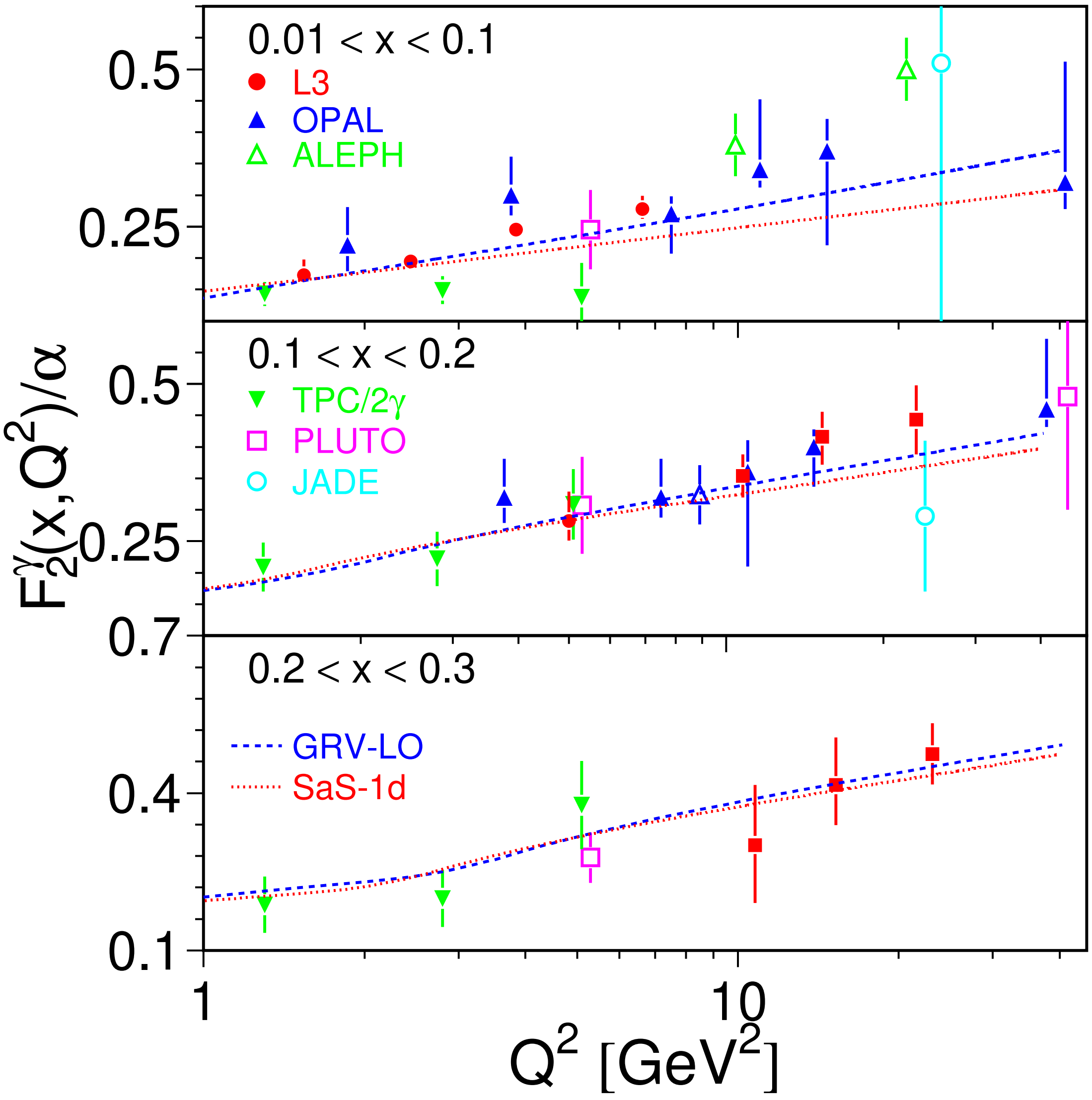
High Q^2



QPM insufficient to describe data below $x < 0.3$

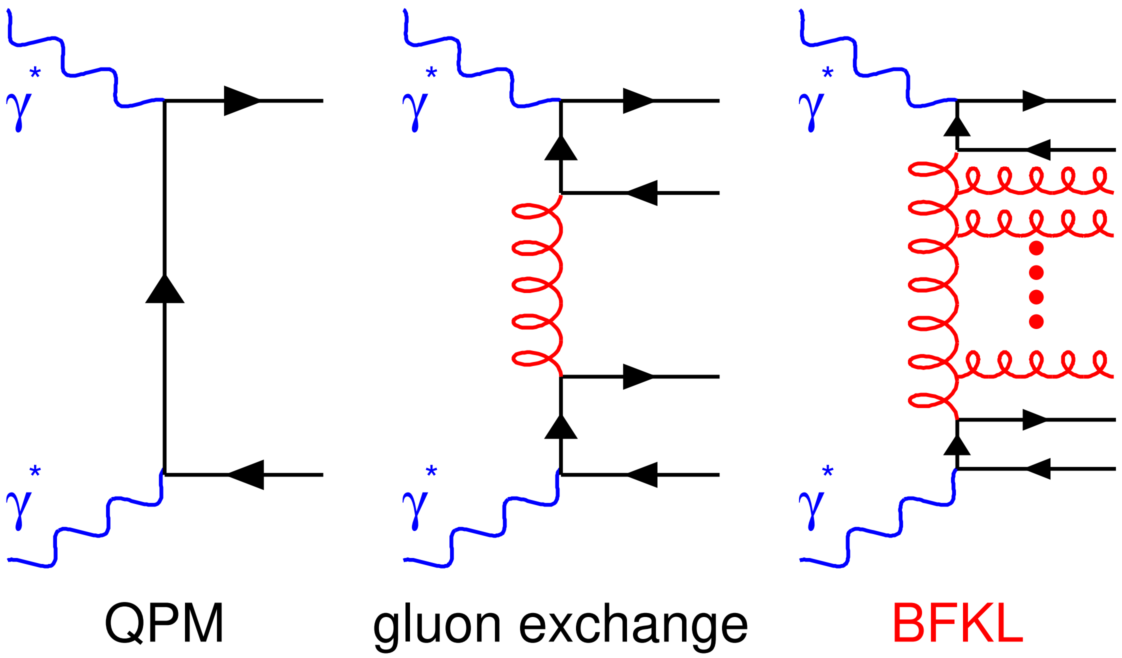
$\Rightarrow$ need resolved process

GRV-LO and SAS-1d agree well with data



- ➢ $\ln Q^2$ evolution of F_2^γ well established
- ➢ At high x region all QCD models agree among themselves and with data
- ➢ At low x models diverge and a better agreement with data is required

Double tag $\longrightarrow$ both γ's virtual



BFKL (Balitski-Fadin-Kuraev-Lipatov) diagram predicted to be dominant in $\gamma^\star\gamma^\star$ collision for

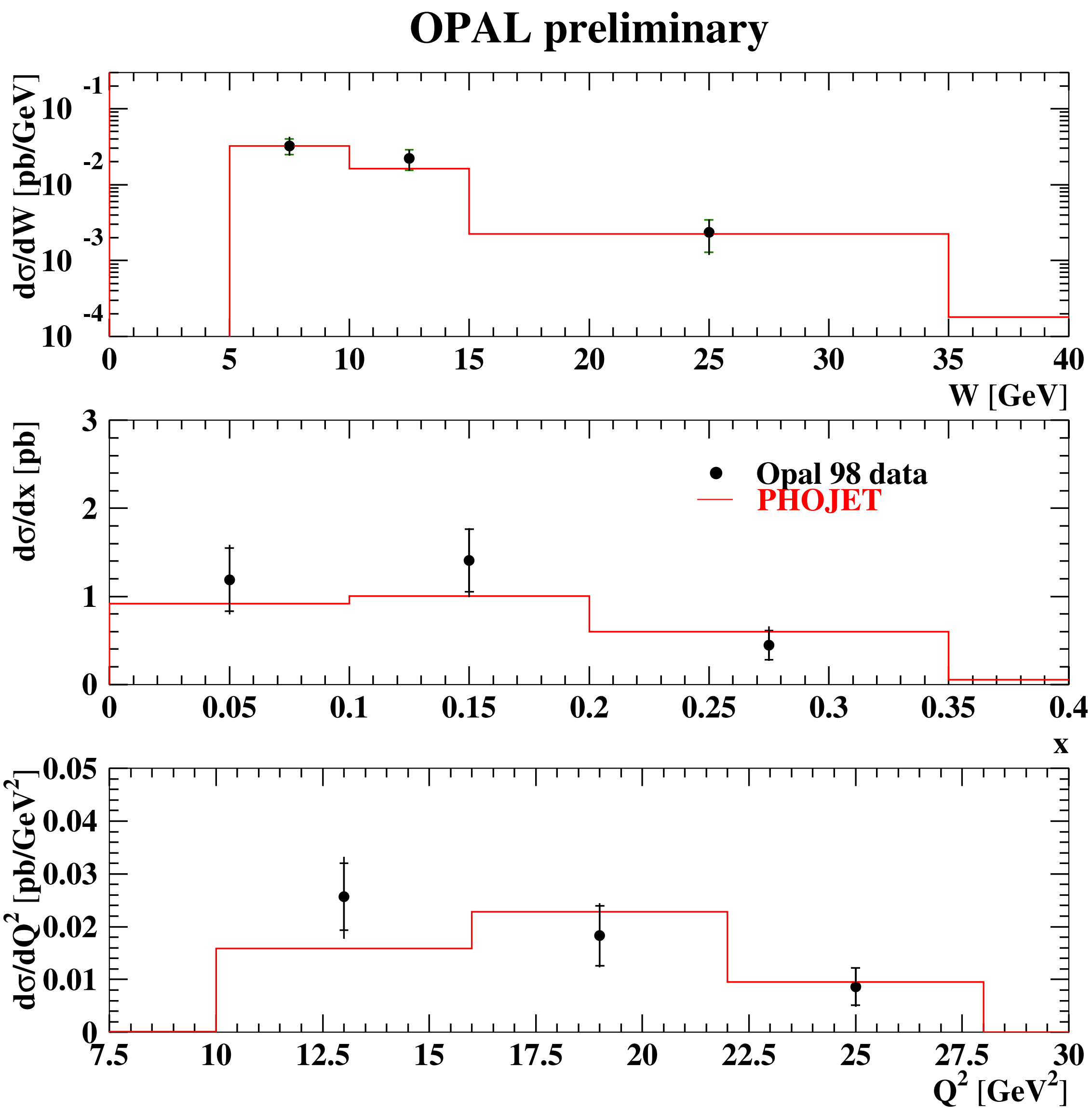
$$y \;=\; \ln\left(\frac{W^2_{\gamma^\star\gamma^\star}}{\sqrt{Q^2P^2}}\right) \;>>\; 1$$

In leading order approximation

$$\begin{aligned}
\sigma_{\gamma^\star\gamma^\star} &= \sigma_0\left(Q^2, Y\right)\left(\frac{s}{s_0}\right)^{\alpha_P - 1} \\
&= \sigma_0\left(Q^2, Y\right)\,\exp\left[Y\cdot(\alpha_P - 1)\right] \\
\alpha_P - 1 &= 4\ln\frac{2\mathrm{N_c}\alpha_\mathrm{s}}{\pi}
\end{aligned}$$

Using $\mathrm{N_c} = 3$, $\alpha_\mathrm{s} = 0.2$ $\longrightarrow$ $\alpha_P - 1 \approx 0.53$

Next to leading order calculations give $\alpha_P - 1 \leq 0.175$



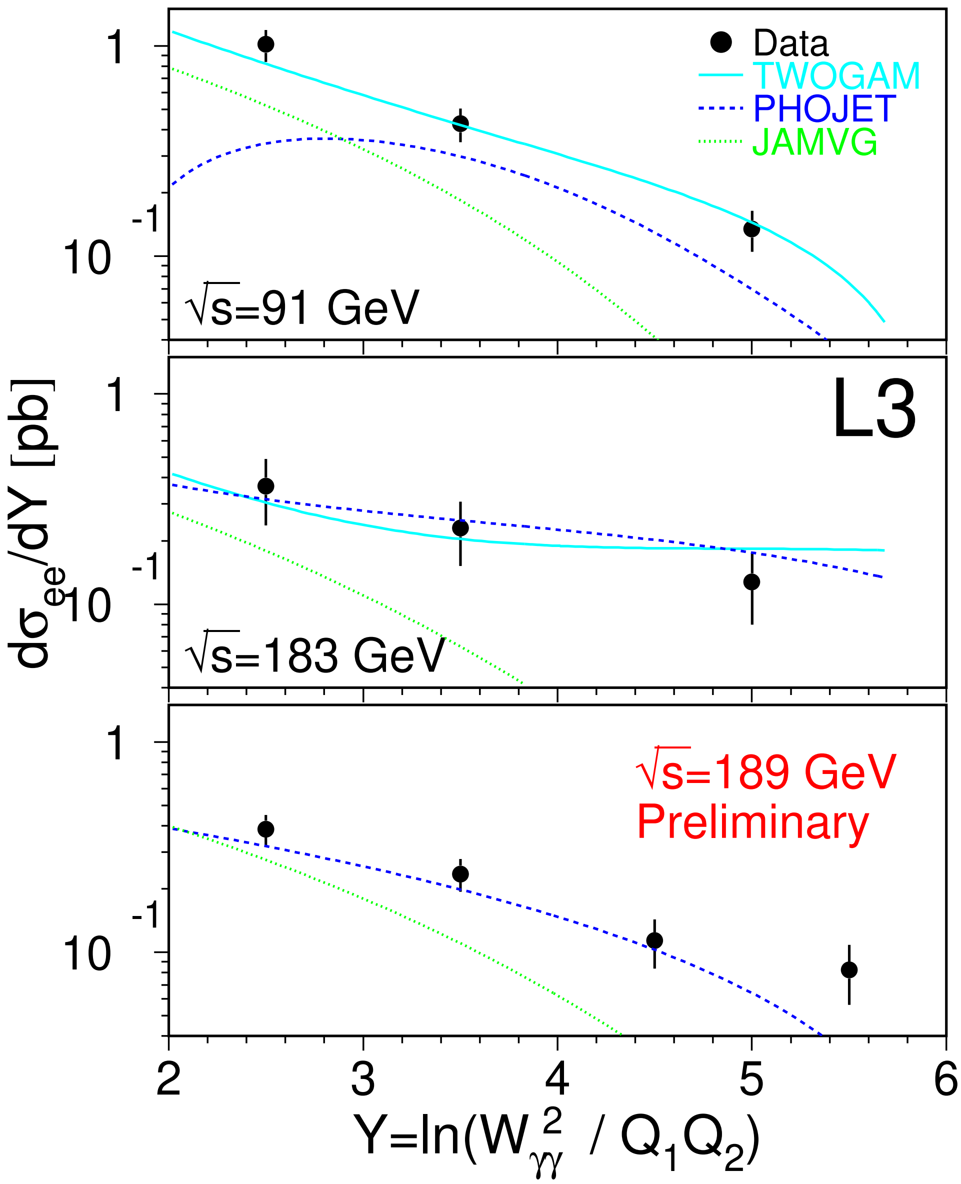
OPAL has 44 events from 189 GeV data with $\langle Q^2 \rangle = 17$ GeV^2
for $2 < Y < 6$, $E_e > 33$ GeV, $34 < \theta_e < 55$ mrad, $W_{\gamma\gamma} > 5$ GeV

$\sigma_{\gamma^\star\gamma^\star}$ (pb)

Measured	PHOJET	BFKL (LO)	BFKL (HO)
0.21 ± 0.06 (stat) ± 0.03 (syst)	0.25	5.7	0.5

BFKL not favoured

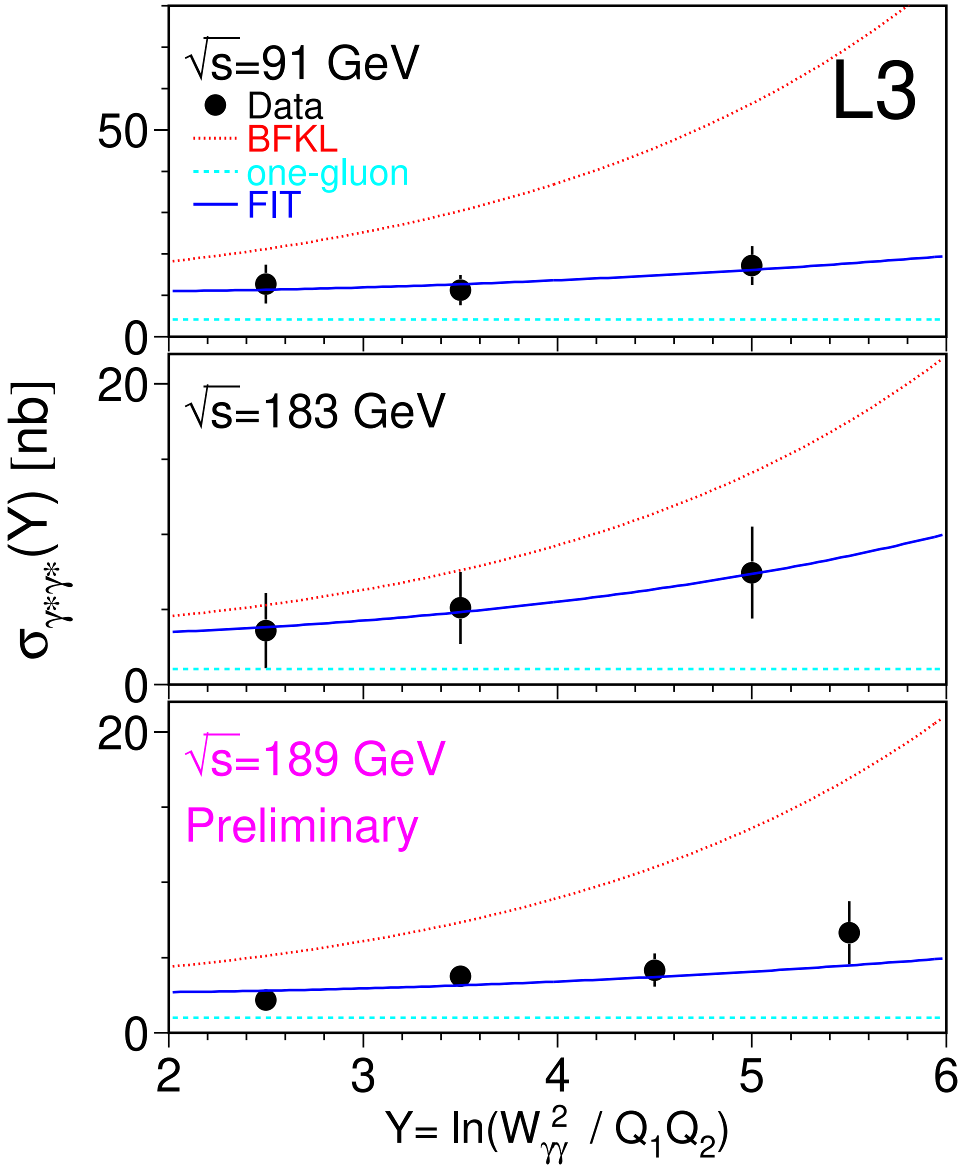
$\sqrt{s}$ (GeV)	# Events	$\langle Q^2 \rangle$ (GeV^2)
91	137	3.5
183	34	14
189	108	14.5



- ▲ Direct process insufficient
- ▲ For large Q^2, Monte Carlo predictions of TWOGAM and PHOJET agree well

Subtract direct part and use luminosity function

$$\sigma_{ee} = \mathcal{L}_{\gamma\gamma} \times \sigma_{\gamma^\star\gamma^\star}$$



Measured σ lower than LO BFKL and higher than NLO BFKL

Fit gives: $\alpha_P - 1 = 0.29 \pm 0.025$ with $\chi^2/\mathrm{dof} = 7/10$

	BFKL-LO	BFKL-NLO
χ^2/dof	263/10	21/10

- e^+e^- annihilation data to hadrons in the high energy LEP runs have given rise to many tests to QCD
  - QCD models which include soft gluon coherence effects are in good agreement with the data
  - The accuracy in the extraction of α_s is reaching its limits. All α_s values extracted from event shape distributions are consistent with $\alpha_s(m_Z) = 0.122 \pm 0.004$
  - Substantial progress has been made in understanding the power correction to the perturbative component for event shape variables. This could eventually lead to a precise determination of α_s
  - LEP data has been used in measuring running mass of heavy quark, gluon splitting probabilities, $\cdots$
  - One begins to understand QCD effects in W-mass measurements
- LEP is providing excellent results in the field of 2-photon physics
  - Evidence of gluon component in photon is observed
  - QCD is being tested in the hadronic photon structure function
  - BFKL is inadequate in explaining $\gamma^\star\gamma^\star$ cross section

Require more theoretical works to match good precision of the data