Combined Analysis of Precision Electroweak Results

Martin W. Grünewald

Humboldt University Berlin

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Outline

Measurements and Results

Combination and Consistency

Interpretation within the Standard Model

Summary and Outlook

Acknowledgements and reference:

The LEP electroweak working group:
http://www.cern.ch/LEPEWWG/
Averages of preliminary results

Z lineshape and forward-backward asymmetries

Each LEP experiment:

Analysis of $e^+e^- \rightarrow$ hadrons(γ), $\ell^+\ell^-(\gamma)$

Measurements of $\sigma_{had}(s)$, $\sigma_\ell(s)$, $A_{FB}^\ell(s)$

at 88 GeV $< \sqrt{s} <$ 94 GeV

χ^2 fits to measurements:

$\sqrt{s}$-dependence: Breit-Wigner resonance

$BW(s) \propto 1/(s - M_Z^2 + is\Gamma_Z/M_Z)$

Deconvolution of QED radiation

4 sets of Z parameters:		**Z lineshape**
M_Z		position
Γ_Z		width
$\sigma^0_{had} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e}{\Gamma_Z}\frac{\Gamma_{had}}{\Gamma_Z}$		height
$R_\ell = \frac{\sigma^0_{had}}{\sigma^0_\ell} = \frac{\Gamma_{had}}{\Gamma_\ell}$	$\ell = e, \mu, \tau$	
$A_{FB}^{0,\ell} = \frac{3}{4}A_e A_\ell$	$\ell = e, \mu, \tau$	

$+$ correlation matrix

Method of combination

Average based on the 4 Z parameter sets:
- Errors propagated to Z parameters
- Common errors correlated in error matrix
- Checked with fit to all $\sigma_{\mathrm{had}}(\mathrm{s})$

LEP $\sqrt{\mathrm{s}}$ energy (absolute, point-to-point):
- 1.7MeV on M_Z, 1.3MeV on Γ_Z, 0.03% on σ^0_{had}

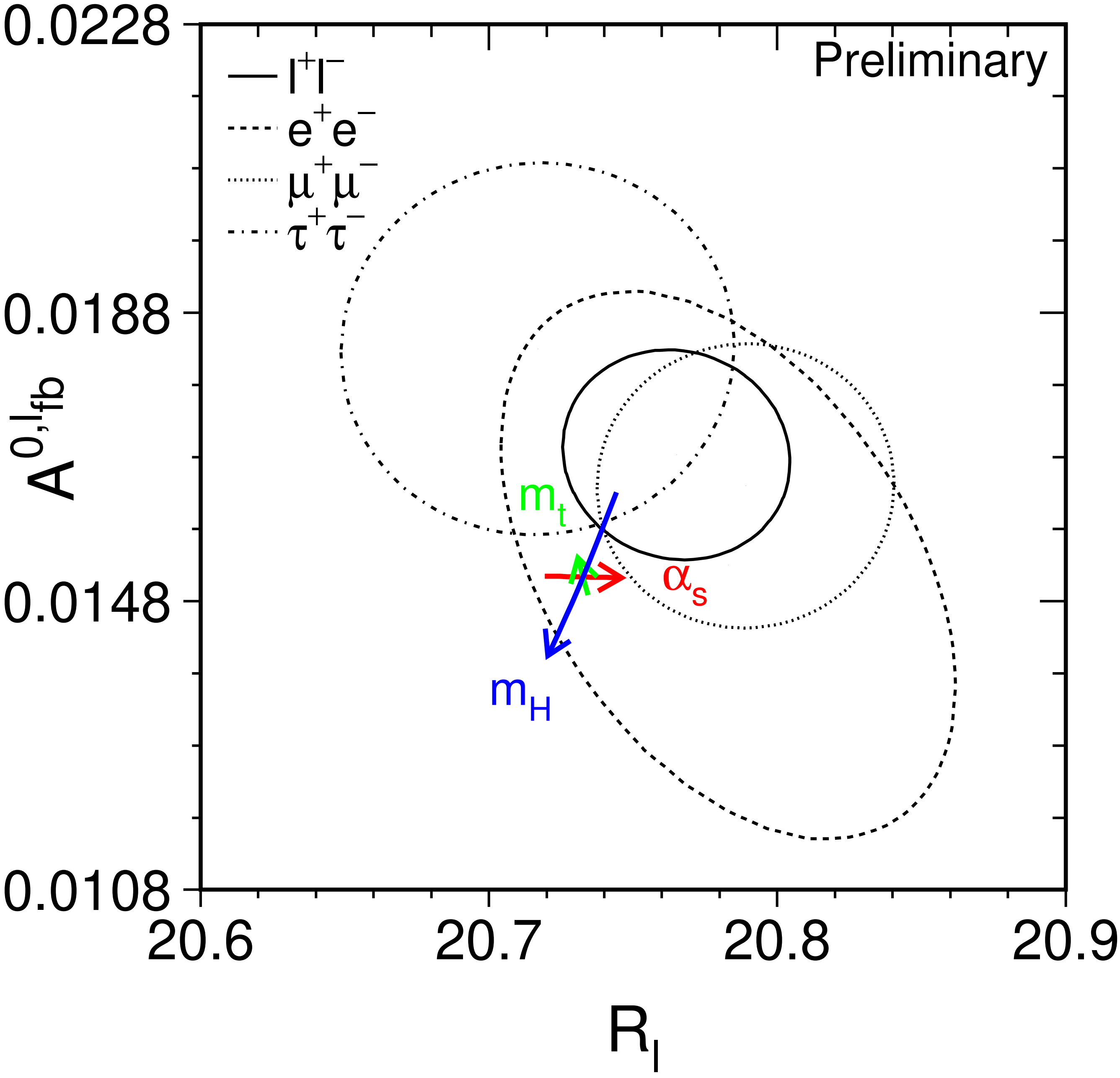
Luminosity theory error:
- Currently: $\pm 0.11\%$ on all Z cross sections
- Future: $\pm 0.06\%$ (c.f. B.Ward's talk)

QED convolution $\mathcal{O}(\alpha^3)$:
- Currently effect is treated as error:
- 0.5MeV on M_Z, 0.6MeV on Γ_Z, 0.05% on σ^0_{had}
- Future: correction with negligible error

Bhabha scattering (t-channel and interference):
- $\pm 0.1\%$ on σ_e, ± 0.001 on A^e_{FB}

Beenakker and Passarino, PLB425 (1998) 199

Z lineshape and forward-backward asymmetries



Standard-Model prediction shown for:

$$\alpha_S(M_Z^2) = 0.119 \pm 0.002 \qquad \text{(PDG98)}$$
$$M_t = 173.8 \pm 5.0 \text{ GeV}$$
$$M_H = 300^{+700}_{-220} \text{ GeV}$$

Z lineshape and forward-backward asymmetries

Results of combination:

$\chi^2/\mathrm{dof} = 28/27\ (42\%)$ no lepton universality
$\chi^2/\mathrm{dof} = 31/31\ (47\%)$ lepton universality

Neutral current lepton universality:

$$
\begin{aligned}
\mathrm{M_Z} &= 91186.7 \pm 2.1\ \mathrm{MeV} \\
\Gamma_\mathrm{Z} &= 2493.9 \pm 2.4\ \mathrm{MeV} \\
\sigma^\circ_\mathrm{had} &= 41.491 \pm 0.058\ \mathrm{nb} \\
\mathrm{R}_\ell &= 20.765 \pm 0.026 \\
\mathrm{A}^{\circ,\ell}_\mathrm{FB} &= 0.01683 \pm 0.00096
\end{aligned}
$$

Correlation matrix:

Offdiagonal elements small $(< 19\%)$

Strong coupling constant from R_ℓ:

$$\alpha_\mathrm{S}(\mathrm{M}^2_\mathrm{Z}) = 0.124 \pm 0.004 \pm 0.002(\mathrm{M_H})$$

Derived lineshape quantities

Partial decay widths of the Z boson:

$$\Gamma_{\text{had}} = 1742.3 \pm 2.3 \text{ MeV}$$

$$\Gamma_{\ell} = 83.90 \pm 0.10 \text{ MeV}$$

Invisible decay width:

$$\Gamma_{\text{inv}} = \Gamma_{\text{Z}} - \Gamma_{\text{had}} - (3 + \Delta_{\tau})\Gamma_{\ell}$$

$$= 500.1 \pm 1.9 \text{ MeV}$$

$$\Delta\Gamma_{\text{inv}} < 2.8 \text{ Mev (95\% C.L.)}$$

Number of light neutrino species:

$$\text{N}_{\nu} = \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell}} \left(\frac{\Gamma_{\ell}}{\Gamma_{\nu}}\right)_{\text{SM}}$$

$$= 2.994 \pm 0.011$$

Polarisation asymmetries

Tau polarisation at LEP:

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathrm{A}_\tau(1+\cos^2\theta)+2\mathrm{A_e}\cos\theta}{1+\cos^2\theta+2\mathrm{A}_\tau\mathrm{A_e}\cos\theta}$$

Linear in couplings parameters A_ℓ
Analysis dependent on τ decay mode

Correlated uncertainties:

Radiative corrections in $\tau\to\nu\pi$, $\tau\to\nu\rho$
Modelling of $\tau\to\nu\mathrm{a_1}$ decays
Branching fractions of τ decays
$\delta\mathrm{A_e} = \pm0.0001$, $\delta\mathrm{A}_\tau = \pm0.0010$

Weighted averages (LEP):

$$\mathrm{A_e} = 0.1479 \pm 0.0051$$
$$\mathrm{A}_\tau = 0.1431 \pm 0.0045$$

Left-right polarisation asymmetry (SLD):

$$\mathrm{A_{LR}} = 0.1510 \pm 0.0025 \qquad \equiv \mathrm{A_e}$$

Effective coupling constants

Using $A_{FB}^{o,\ell} = \frac{3}{4}A_\ell^2$ from LEP:

$$|A_\ell| = 0.1498 \pm 0.0043$$

Average of LEP+SLD:

$\chi^2/\mathrm{dof} = 2.4/3\ (49\%)$

$$A_\ell = 0.1491 \pm 0.0018$$

Access to effective coupling constants:

$$A_\ell = 2\frac{g_{V\ell}/g_{A\ell}}{1+(g_{V\ell}/g_{A\ell})^2}$$

$$\Gamma_\ell = \frac{G_F M_Z^3}{6\pi\sqrt{2}}\left[g_{V\ell}^2 + g_{A\ell}^2\right]\left(1+\Delta_\ell^{QED}\right)$$

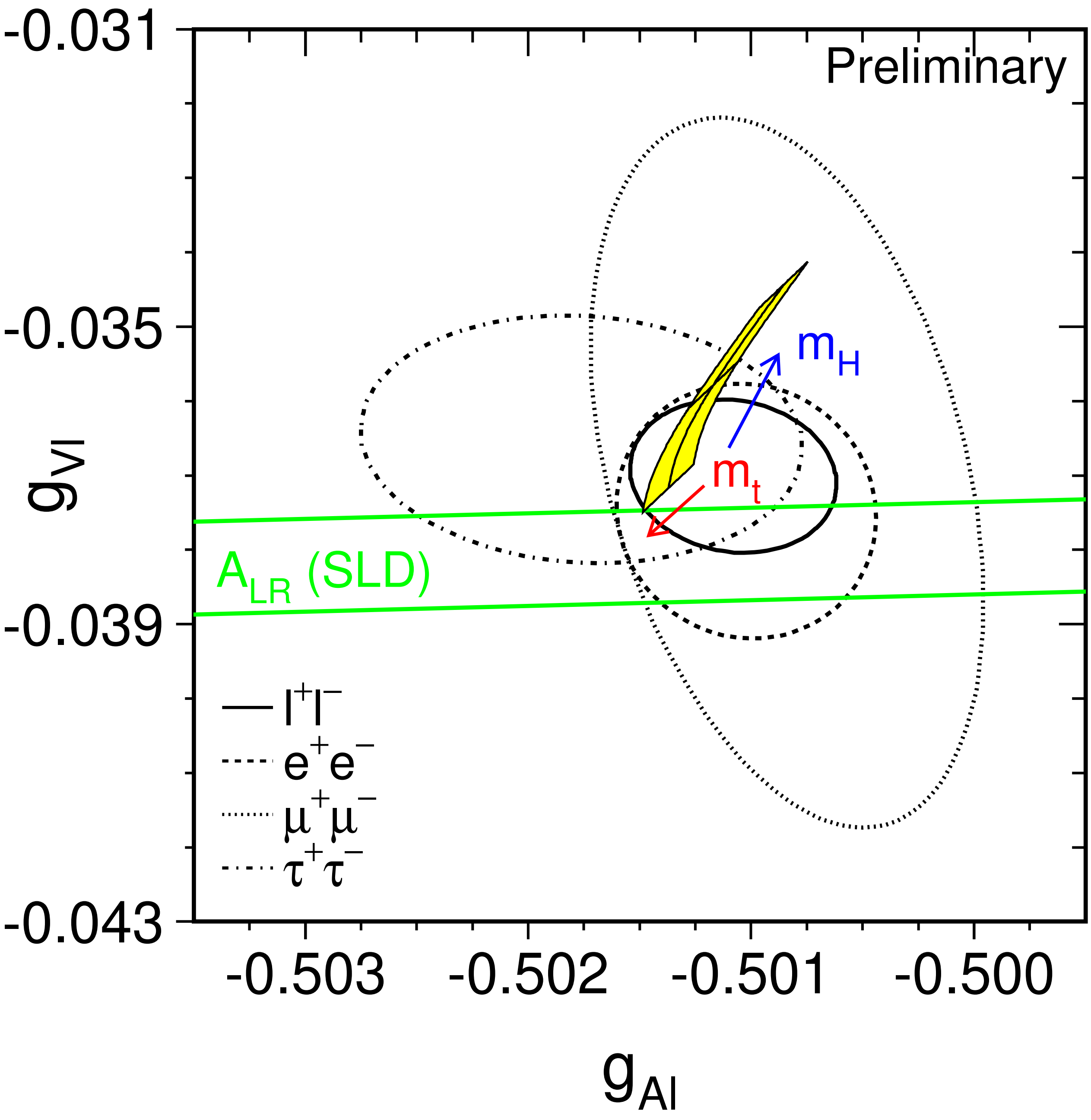
Sign of A_ℓ:

Relative sign between $g_{V\ell}$ and $g_{A\ell}$

Derived results (LEP+SLD):

$$g_{V\ell} = -0.03759 \pm 0.00046$$
$$g_{A\ell} = -0.50105 \pm 0.00030 \neq -1/2$$

Effective coupling constants



Standard-Model prediction shown for:

$M_t = 173.8 \pm 5.0$ GeV

$M_H = 300^{+700}_{-220}$ GeV

Negligible dependence on $\alpha_S(M_Z^2)$

Heavy flavour results

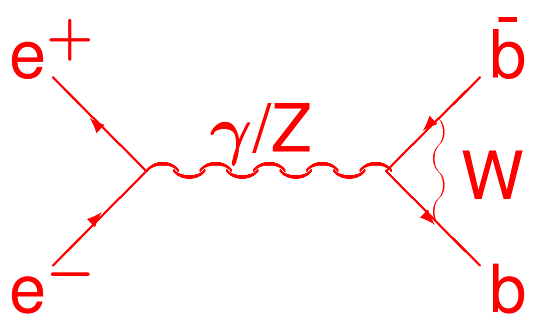
Analysis of $e^+e^- \to b\bar{b},\ c\bar{c}$

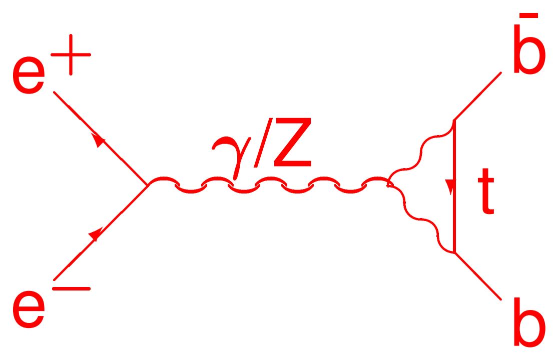
Electroweak parameters:

b quarks	c quarks	
$R_b = \frac{\Gamma_b}{\Gamma_{had}}$	$R_c = \frac{\Gamma_c}{\Gamma_{had}}$	
$A_{FB}^{o,b} = \frac{3}{4}A_e A_b$	$A_{FB}^{o,c} = \frac{3}{4}A_e A_c$	
$A_{LR/FB}^{o,b} = A_b$	$A_{LR/FB}^{o,c} = A_c$	(SLD)

Particular interest in $e^+e^- \to b\bar{b}$:

Special corrections at γ/Z-$b\bar{b}$ vertex





Loops with top quarks and W bosons

Sensitivity to triple-gauge-boson couplings

Sensitivity to new physics

Method of combination

Most complicated combination procedure:

Results are depended on each other

$\longrightarrow$ R_b depends on R_c

Results depend on additional assumptions

$\longrightarrow$ Decay models for heavy-quark mesons

Treatment of models:

Agree on common range of variation

Assign correlated systematic errors

Treatment of electroweak parameters:

Determine dependence of results on other electroweak parameters

Parametrise dependencies explicitly:

Example: update R_b measurement

$$R_b^{updated} = R_b^{measured} + a_{R_c} \frac{R_c^{fit} - R_c^{used}}{R_c^{fit}}$$

Perform χ^2 fit taking into account:

Explicit dependencies: $R_b^{fit} \leftrightarrow R_b^{updated}$

Correlated errors due to model parameters

Details: CERN-PPE/96-017, NIMA378 (1996) 378

Heavy flavour results

Result of combination (LEP+SLD):

Including late SLD results:

$$
\begin{aligned}
R_b &= 0.21656 \pm 0.00074 \\
R_c &= 0.1735 \pm 0.0044 \\
A_{FB}^{o,b} &= 0.0990 \pm 0.0021 \\
A_{FB}^{o,c} &= 0.0709 \pm 0.0044 \\
A_b &= 0.867 \pm 0.035 \\
A_c &= 0.647 \pm 0.040
\end{aligned}
$$

$+$ correlation matrix

Largest correlations:

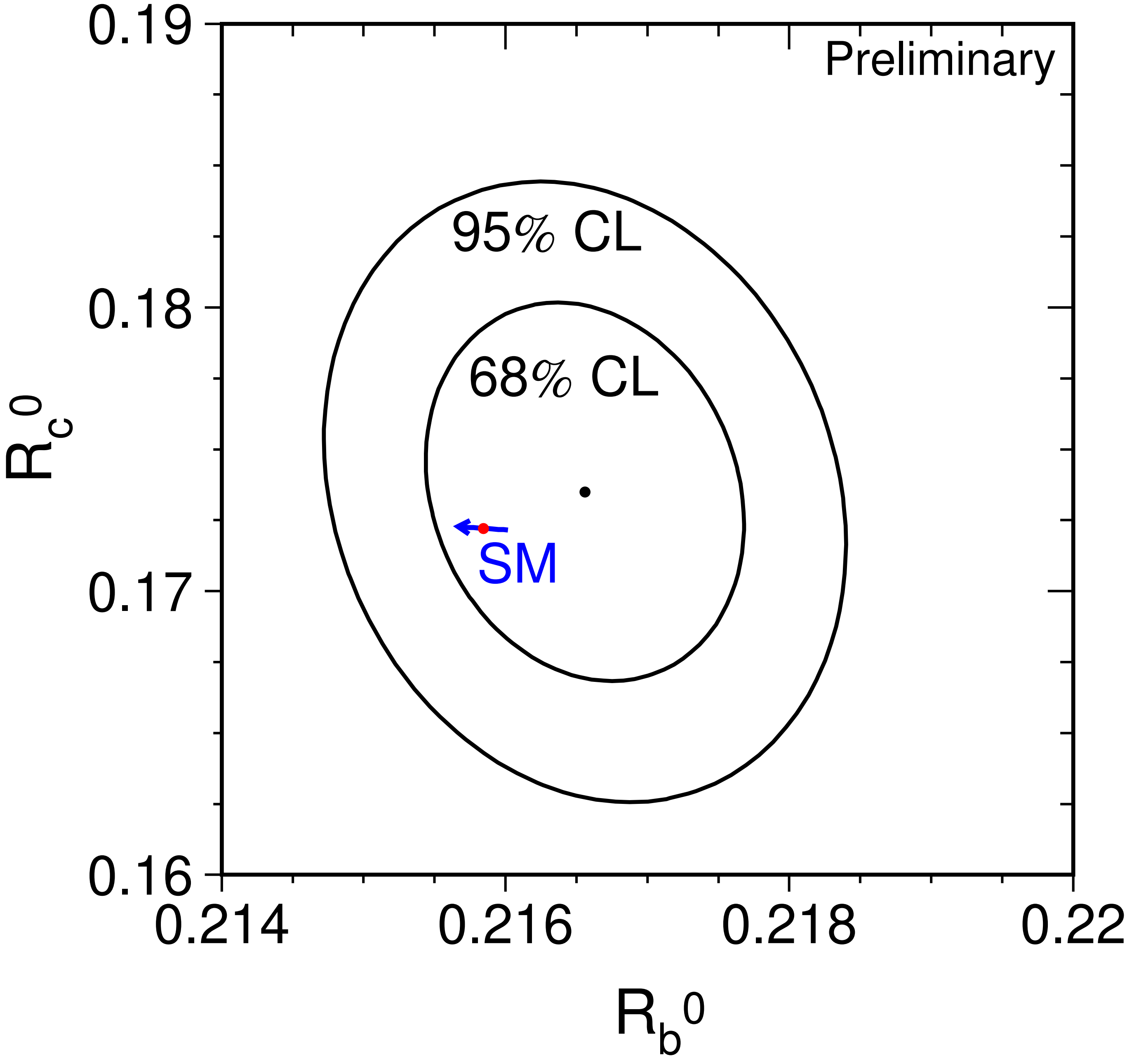
(R_b, R_c): -0.17, $(A_{FB}^{o,b}, A_{FB}^{o,c})$: $+0.13$

Experimental nature:

Separation $b\bar{b}$ versus $c\bar{c}$

Other correlations small ($\lesssim 7\%$)

Heavy flavour results



Standard-Model prediction shown for:

$M_t = 173.8 \pm 5.0$ GeV

Negligible dependence on M_H or $\alpha_S(M_Z^2)$

Hadronic charge asymmetry Q_{FB}

Measure mean difference in jet charges

Forward versus backward hemispheres

LEP charge asymmetry measurements:

Detector dependent effects

Hadronisation uncertainties

Perform average in $\sin^2\theta_{\text{eff}}^{\text{lept}}$ directly

Correlated errors:

Average not very sensitive

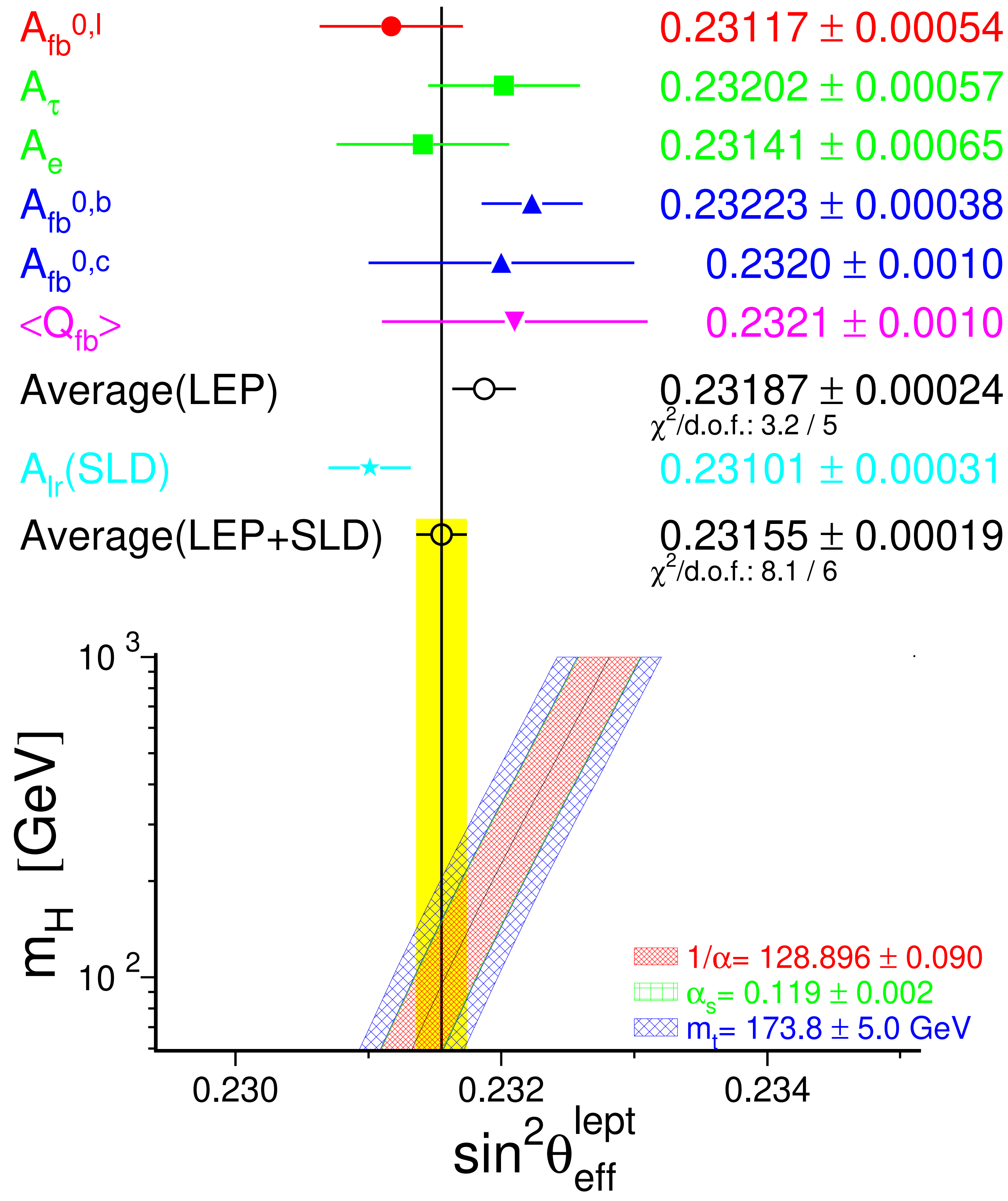
Average $\sin^2\theta_{\text{eff}}^{\text{lept}}$ derived from LEP Q_{FB}:

$$\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.2321 \pm 0.0010$$

Effective electroweak mixing angle

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = \frac{1}{4}\left(1 - \frac{g_{V\ell}}{g_{A\ell}}\right)$$

Preliminary



NuTeV's $\sin^2\theta_W$ measurement

Neutrino-nucleon scattering experiment:

Paschos-Wolfenstein relation (lowest order):

$$R^- = \frac{\sigma^{\nu}_{NC} - \sigma^{\bar\nu}_{NC}}{\sigma^{\nu}_{CC} - \sigma^{\bar\nu}_{CC}} = \frac{1}{2} - \sin^2\theta_W$$

Small electroweak radiative corrections:

R^- measures on-shell ew mixing angle:

$\sin^2\theta_W = 1 - M_W^2/M_Z^2$

NuTeV result combined with CCFR:

$$\sin^2\theta_W = 0.2255 \pm 0.0021$$

$$-0.00142\frac{M_t^2 - (175\ \mathrm{GeV})^2}{(100\ \mathrm{GeV})^2}$$

$$+0.00048\ln\frac{M_H}{150\ \mathrm{GeV}}$$

Use LEP M_Z to express as M_W measurement:

$$M_W = 80.25 \pm 0.11\ \mathrm{GeV}$$

Constraints on the Standard Model

Relevant Standard-Model input parameters:

Masses: M_t, M_Z, M_W, M_H

Couplings: $\alpha_S(M_Z^2)$, $\alpha(M_Z^2)$

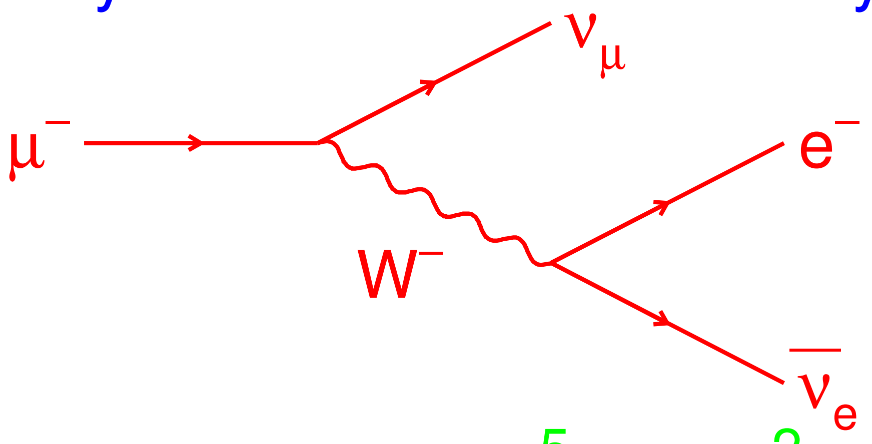
Fermi constant G_F used to replace M_W:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha(0)}{2} \cdot \frac{1}{M_W^2 \sin^2\theta_W} \cdot \frac{1}{1-\Delta r}$$

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$$\Delta r = \Delta\alpha - \cot^2\theta_W \Delta\rho + \ldots$$

G_F precisely measured in muon decay:

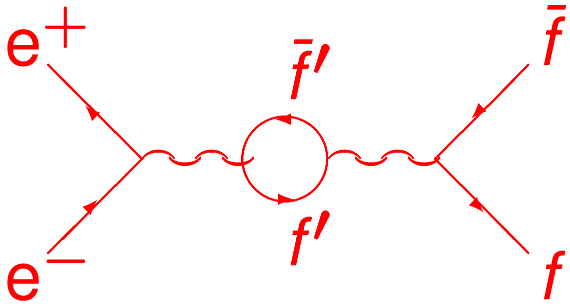


$$G_F = 1.16639(2) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$$

Error on G_F negligible for SM analysis

QED finestructure constant α

Fermion loops in photon propagator:



$$\alpha(0) = \frac{1}{137.0359895(61)} \longrightarrow$$

$$\alpha(M_Z^2) = \frac{\alpha(0)}{1-\Delta\alpha} = \frac{1}{128.896 \pm 0.090}$$

Eidelman and Jegerlehner, ZPC67 (1995) 585

No precise matrix-element calculation:

Strong QCD effects in case of quark loops

Use dispersion integral over σ_{had} at low $\sqrt{s}$

Large experimental errors on σ_{had}

Error on $\alpha(M_Z^2)$ important for SM analysis

Recently several new evaluations of $\alpha(M_Z^2)$:

Theory driven error reduction

Davier et al., Erler, Groote et al., Kühn et al., Krasnikov et al., Alemany et al.

Need σ_{had} measurement at $\sqrt{s} = 1$–7 GeV

Effective couplings of the neutral weak current:

$$
\begin{aligned}
g_{Vf} &= T_f^3 - 2q_f \sin^2\theta_W \\
&\to \sqrt{\rho}\cdot\left(T_f^3 - 2q_f \sin^2\bar{\theta}_W\right) \\
g_{Af} &= T_f^3 \;\to\; \sqrt{\rho}\cdot T_f^3
\end{aligned}
$$

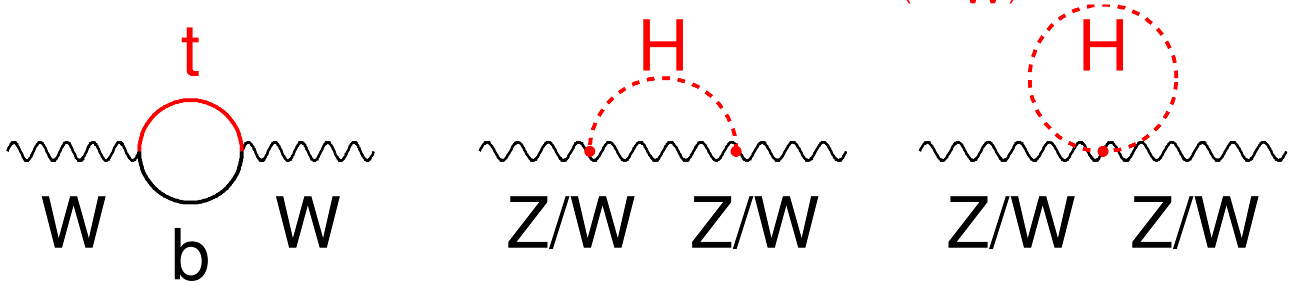
Electroweak radiative corrections:

$$
\rho = \frac{1}{1-\Delta\rho}
$$

$$
\frac{\sin^2\bar{\theta}_W}{\sin^2\theta_W} = 1 + \cot^2\theta_W \Delta\rho + \ldots
$$

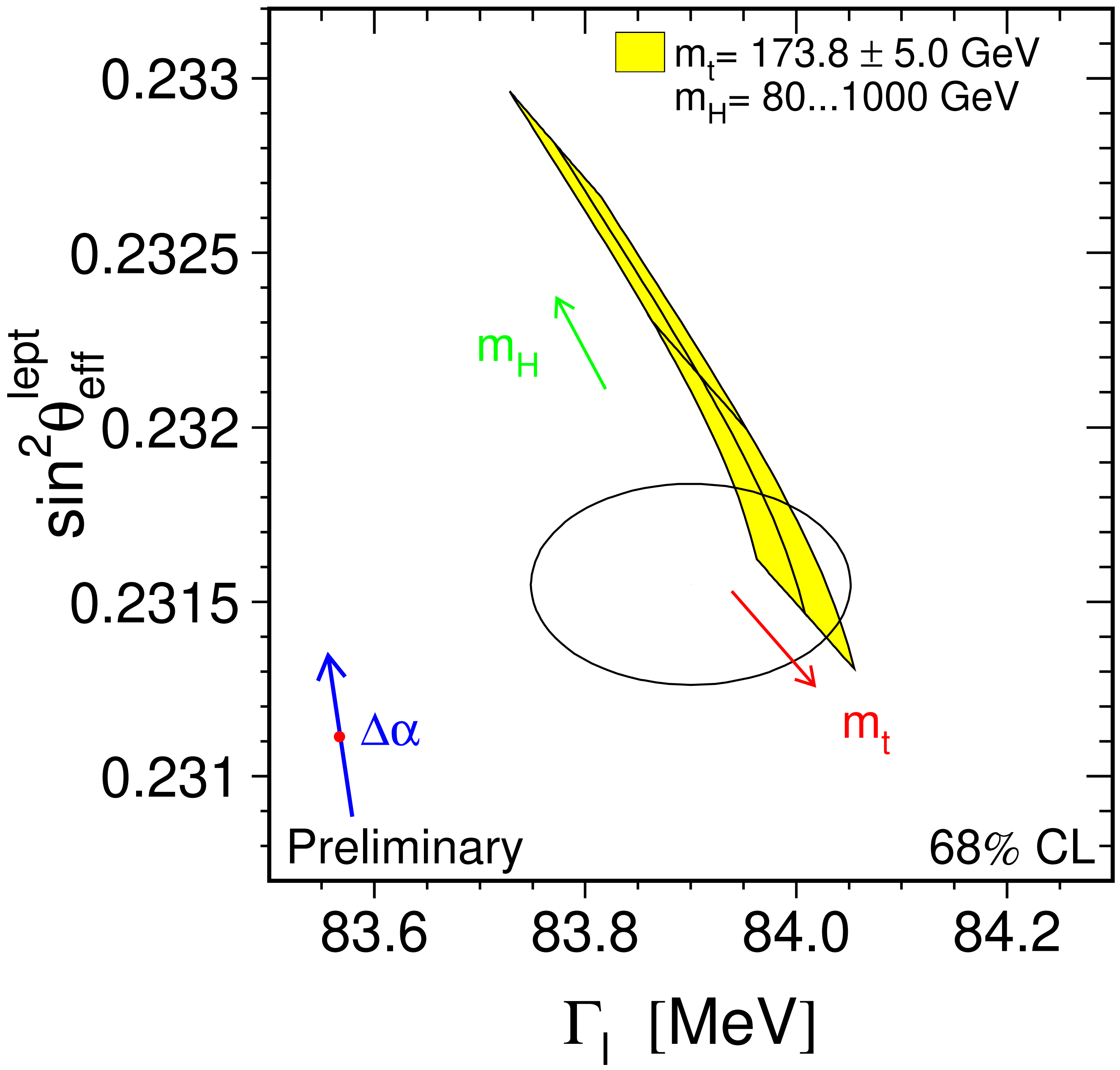
Dependence of weak corrections $\Delta\rho, \ldots$:

$\propto M_t^2$ $\qquad\qquad$ $\propto \ln\left(\frac{M_H}{M_W}\right)$



High sensitivity to M_t
Lower sensitivity to M_H

Running of α is not sufficient:



Also weak corrections (M_t, M_H) necessary

Constraints on the Standard Model

Precision electroweak measurements:

Sensitive to electroweak radiative corrections
Prediction of M_W, M_t, M_H etc.
$\longrightarrow$ Tests of the Standard Model
$\longrightarrow$ Constraints on SM parameters

Fits to measurements using programs:

TOPAZ0 (G.Passarino et al.)
ZFITTER (D.Bardin et al.)

Incorporating[1] recent theoretical calculations:

QED radiator to $\mathcal{O}(\alpha^3)$[2]
Four-loop QCD effects[3]
Nonfactorizable QCD/EW corrections[4]
Two-loop $\mathcal{O}(\alpha^2 M_t^2/M_W^2)$ corrections[5]

[1] Bardin and Passarino, hep-ph/9803425

[2] Montagna et al. PLB406 (1997) 243

[3] Larin et al., PLB400 (1997) 379, PLB405 (1997) 327;
Chetyrkin et al., PRL79 (1997) 2184, hep-ph/9708255

[4] Czarnecki et al. PRL77 (1996) 3955; Harlander et al. PLB426 (1998) 125

[5] Degrassi et al., PLB383 (1996) 219, PLB394 (1997) 188, PLB418 (1998) 209

Standard-Model analysis of results:

Without direct measurements of M_W and M_t

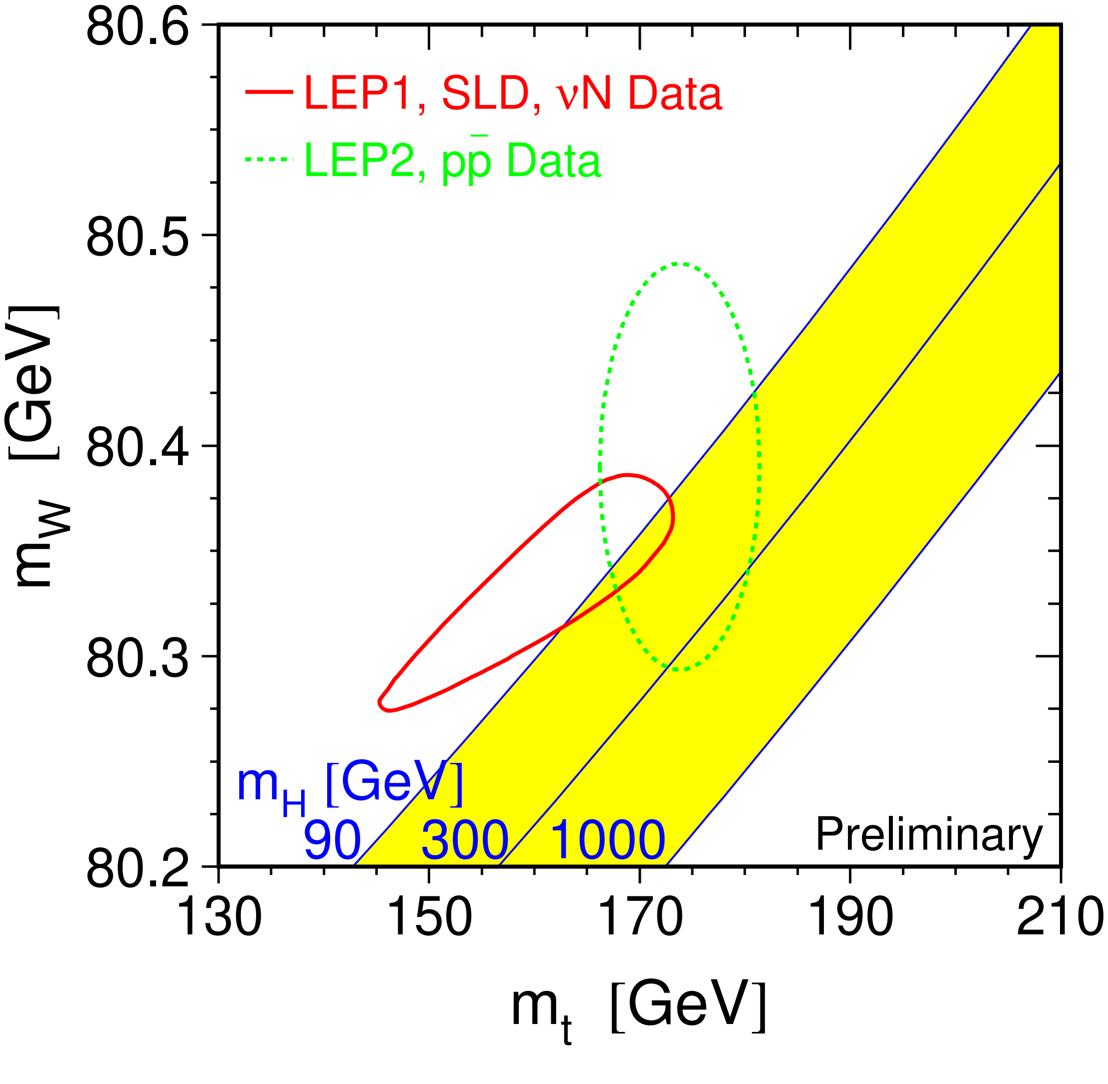
$\chi^2/\text{dof} = 14/12\ (30\%)$

$$M_t = 158^{+9}_{-8}\ \text{GeV}$$

$$\log_{10} M_H = 1.53^{+0.37}_{-0.28} \qquad (34^{+45}_{-16}\ \text{GeV})$$

$$\alpha_S(M_Z^2) = 0.120 \pm 0.003$$

c.f. TEVATRON: $M_t = 173.8 \pm 5.0$ GeV



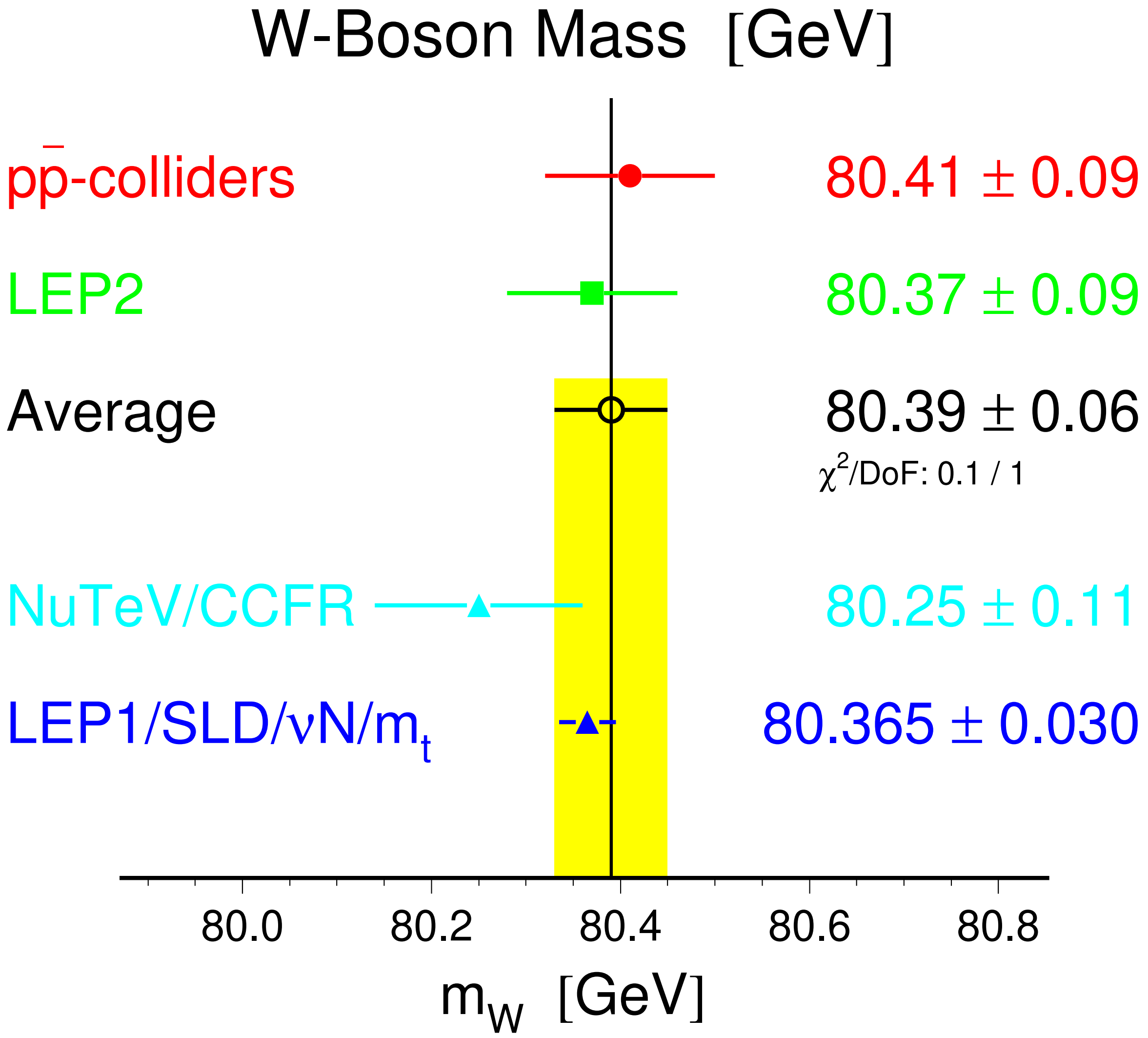
Tests of the Standard Model

Best prediction of M_W (includes direct M_t):

$$M_W = 80.365 \pm 0.030 \text{ GeV}$$

Direct measurement at TEVATRON and LEP2:

$$M_W = 80.39 \pm 0.06 \text{ GeV}$$



Aim for 30 MeV M_W error at LEP2

Constraints on the Standard Model

Standard-Model analysis of results:

Include direct measurements of M_W and M_t

$\chi^2/\mathrm{dof} = 16/15\ (35\%)$

$$M_t = 171.3 \pm 4.9\ \mathrm{GeV}$$

$$\log_{10} M_H = 1.92^{+0.32}_{-0.41} \qquad (84^{+91}_{-51}\ \mathrm{GeV})$$

$$\alpha_S(M_Z^2) = 0.119 \pm 0.003$$

75 % correlation between M_H and $\alpha(M_Z^2)$

Derived quantities:

$$\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = 0.23156 \pm 0.00019$$

$$1 - M_W^2/M_Z^2 = 0.22318 \pm 0.00052$$

$$M_W = 80.370 \pm 0.027\ \mathrm{GeV}$$

With $1/\alpha(M_Z^2) = 128.923 \pm 0.036$ instead:

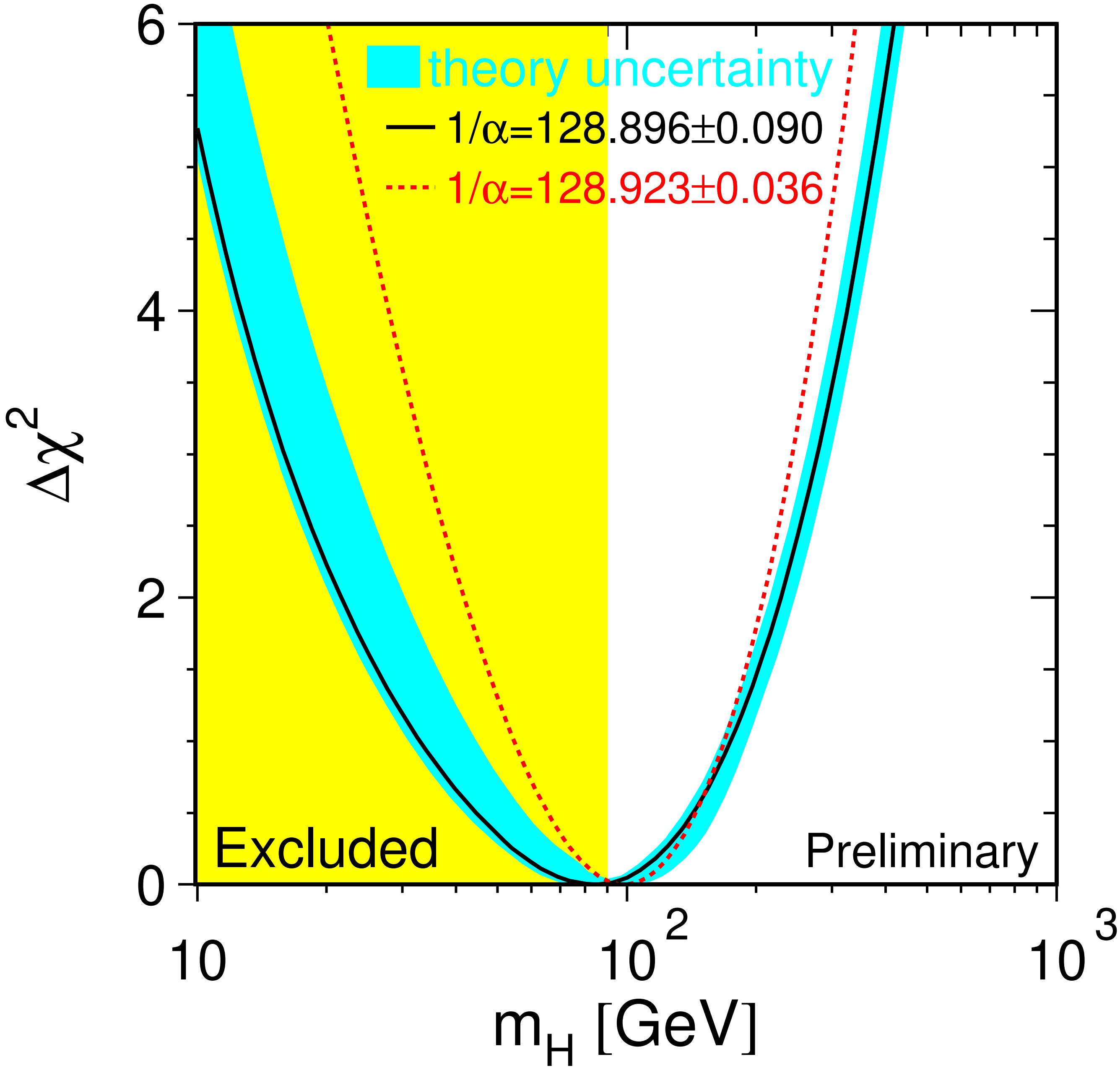
Davier and Höcker, PLB419 (1998) 419

$$\log_{10} M_H = 1.99^{+0.23}_{-0.25} \qquad (98^{+70}_{-44}\ \mathrm{GeV})$$

38 % correlation between M_H and $\alpha(M_Z^2)$

Missing higher-order ew corrections:

$\delta \log_{10} M_H = \pm 0.05$

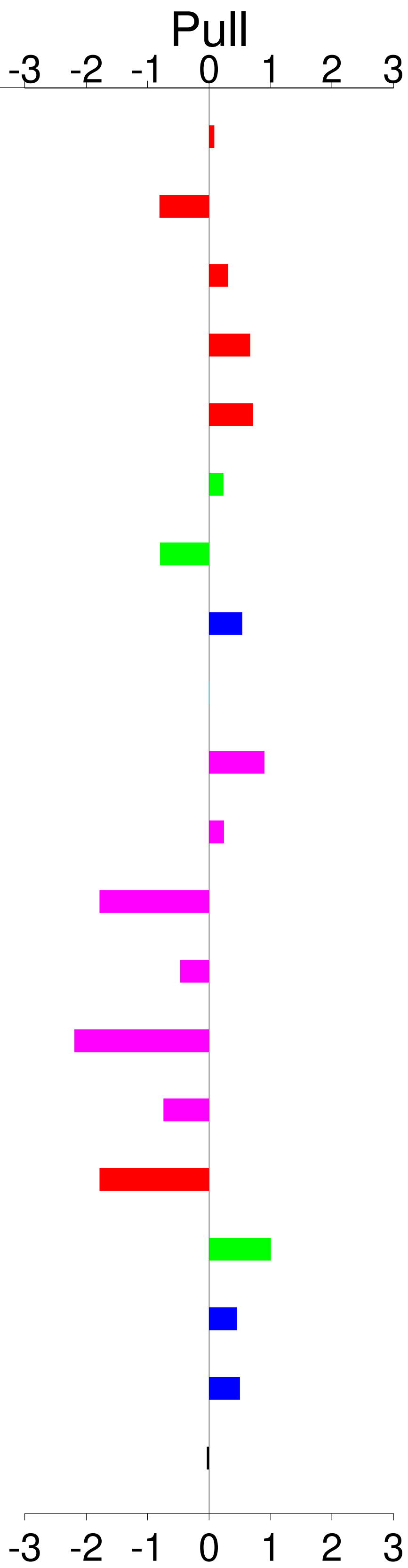


One-sided 95% CL upper limit:

$M_H < 280$ GeV

Tests of the Standard Model

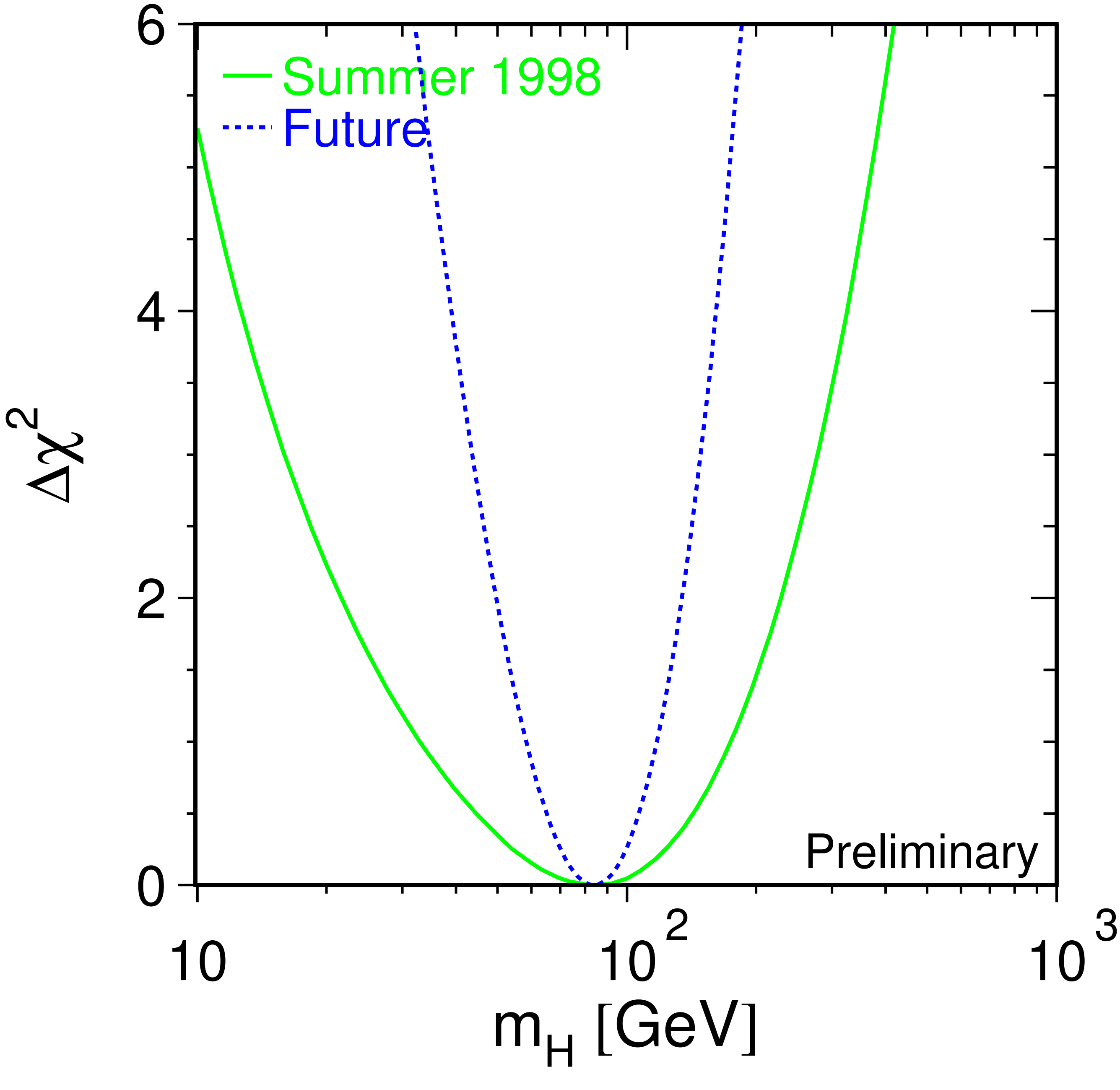
	Measurement	Pull
m_Z [GeV]	91.1867 ± 0.0021	.08
Γ_Z [GeV]	2.4939 ± 0.0024	-.80
σ^0_{hadr} [nb]	41.491 ± 0.058	.31
R_e	20.765 ± 0.026	.66
$A^{0,e}_{fb}$	0.01683 ± 0.00096	.72
A_e	0.1479 ± 0.0051	.24
A_τ	0.1431 ± 0.0045	-.80
$\sin^2\theta^{lept}_{eff}$	0.2321 ± 0.0010	.54
m_W [GeV]	80.370 ± 0.090	.01
R_b	0.21656 ± 0.00074	.90
R_c	0.1733 ± 0.0044	.24
$A^{0,b}_{fb}$	0.0991 ± 0.0021	-1.78
$A^{0,c}_{fb}$	0.0714 ± 0.0044	-.47
A_b	0.856 ± 0.036	-2.18
A_c	0.638 ± 0.040	-.74
$\sin^2\theta^{lept}_{eff}$	0.23101 ± 0.00031	-1.78
$\sin^2\theta_W$	0.2255 ± 0.0021	1.06
m_W [GeV]	80.410 ± 0.090	.45
m_t [GeV]	173.8 ± 5.0	.50
$1/\alpha$	128.896 ± 0.090	-.04



Future

Measurements more precise:

$$\delta(1/\alpha(M_Z^2)) = \pm 0.01$$
$$\delta M_t = \pm 2\ \text{GeV}$$
$$\delta M_W = \pm 30\ \text{MeV}$$



Factor 2.5 improvement in relative error

Conclusion

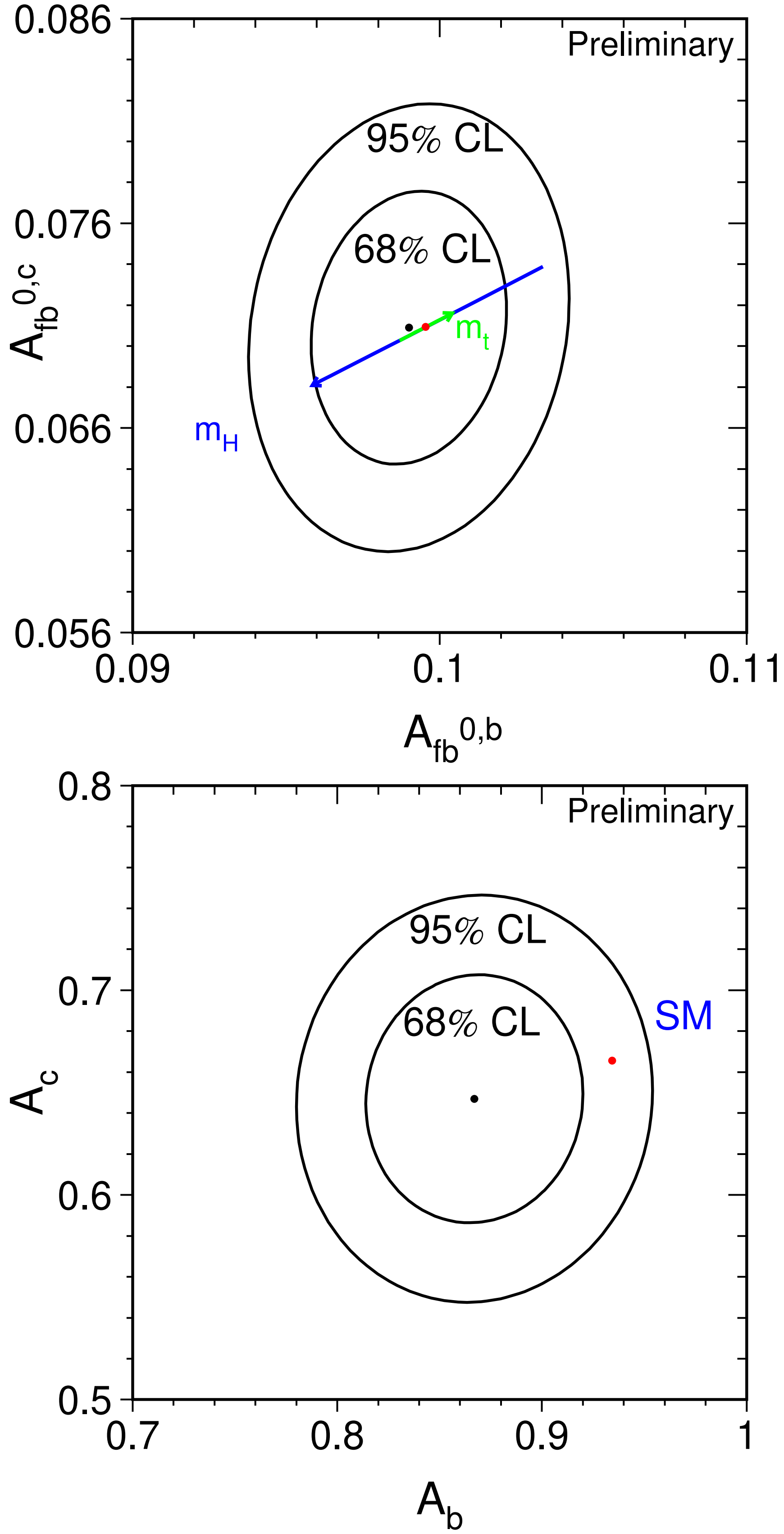
LEP, SLD, NuTeV, TEVATRON:
Many precision electroweak measurements

Status of the Standard Model
Describes successfully all measurements
Problems with b quarks?

Tests of the Standard Model:
Analysis of radiative corrections
Predictions of M_t and M_W agree well
with direct measurements
Prediction of M_H selfconsistent within SM
$M_H < 280$ GeV (95% CL)

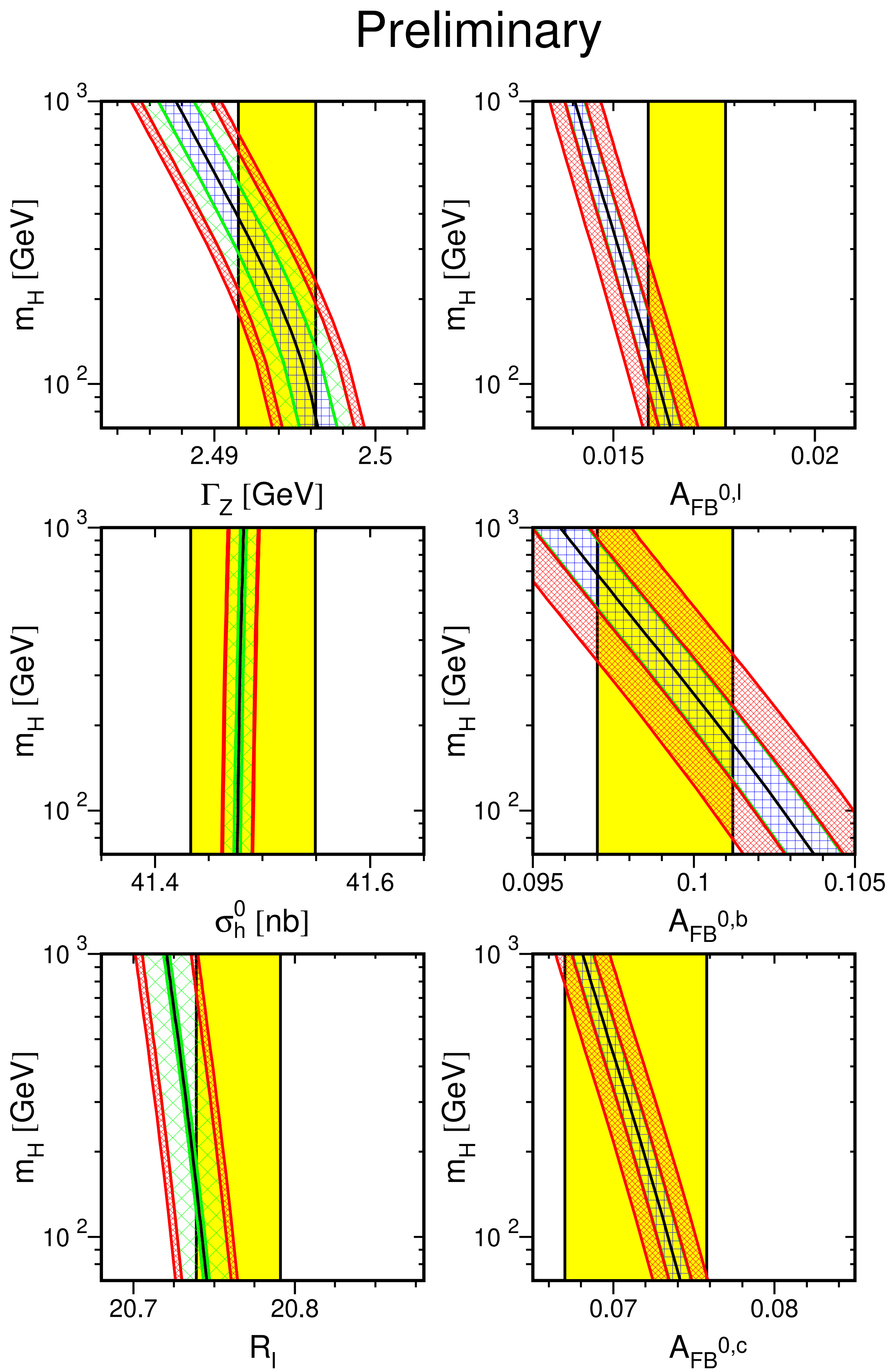
Future:
Soon final results from LEP1, SLD, NuTeV
Improvements in M_W, M_t (LEP2, TEVATRON)
Discovery of the Higgs Boson?

Heavy flavour results



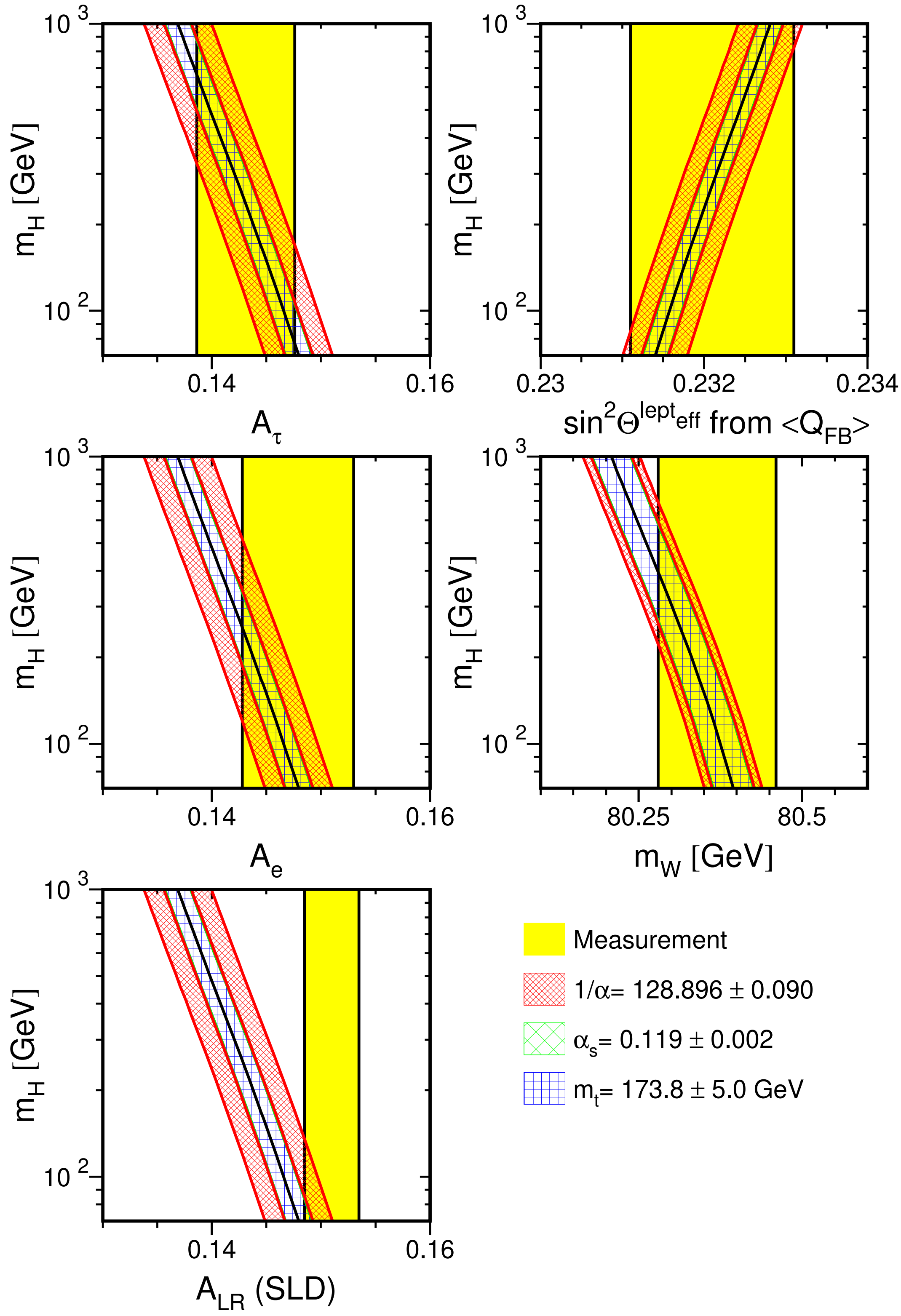
Including late SLD results

Higgs sensitivity



Higgs sensitivity

Preliminary



Top and Higgs