Multiplicity Fluctuations and Multiparticle Correlations

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Abstract

Mini-review (revue ?) of papers

542	L3	Local Multiplicity Fluctuations
297	OPAL	Intermittency and Correlations in 1, 2, 3 dim.
543	L3	Angular Multiplicity Fluctuations and QCD
802	ZEUS	Angular Correlations
149	DELPHI	Angular Correlations and QCD
541	L3	Oscillations in H_q moments

6	papers
104	pages
43	figures
15	minutes

Common theme of these papers

1. Are there correlations/fluctuations?

2. Why?

   Compare with

   - MC models (JETSET, HERWIG, ...)

   - analytic calculations

     – pQCD calculations for partons
     (LLA, DLLA, MLLA, NLLA, NNLLA)

     – LPHD to transform
     parton predictions to particle predictions
     $\mathcal{O}(\text{particle}) \propto \mathcal{O}(\text{parton})$

Fluctuations

Usually studied using Factorial and Cumulant moments of the charged particle multiplicity distribution, P_n

$$F_q = \frac{\sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n}{\left(\sum_{n=1}^{\infty} n\, P_n\right)^q}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \binom{q-1}{m} K_{q-m}\, F_m$$

and their ratio

$$H_q = \frac{K_q}{F_q}$$

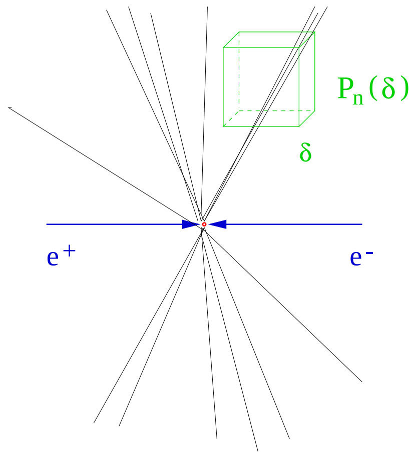
F_q measures q-particle correlations
K_q measures genuine q-particle correlations

The normalization $\left(\sum_{n=1}^{\infty} n\, P_n\right)^q$
can be done differently,
and various "corrections" can be used.

(If you are an expert, you know this.
If not, sorry, but there is no time to discuss it.
The conclusions will not greatly depend on it.)

Local Particle Fluctuations

To study dynamical fluctuations in particle density in small regions, δ, of phase space,



Usual method:

Normalized Factorial Moments, $F_q(\delta)$

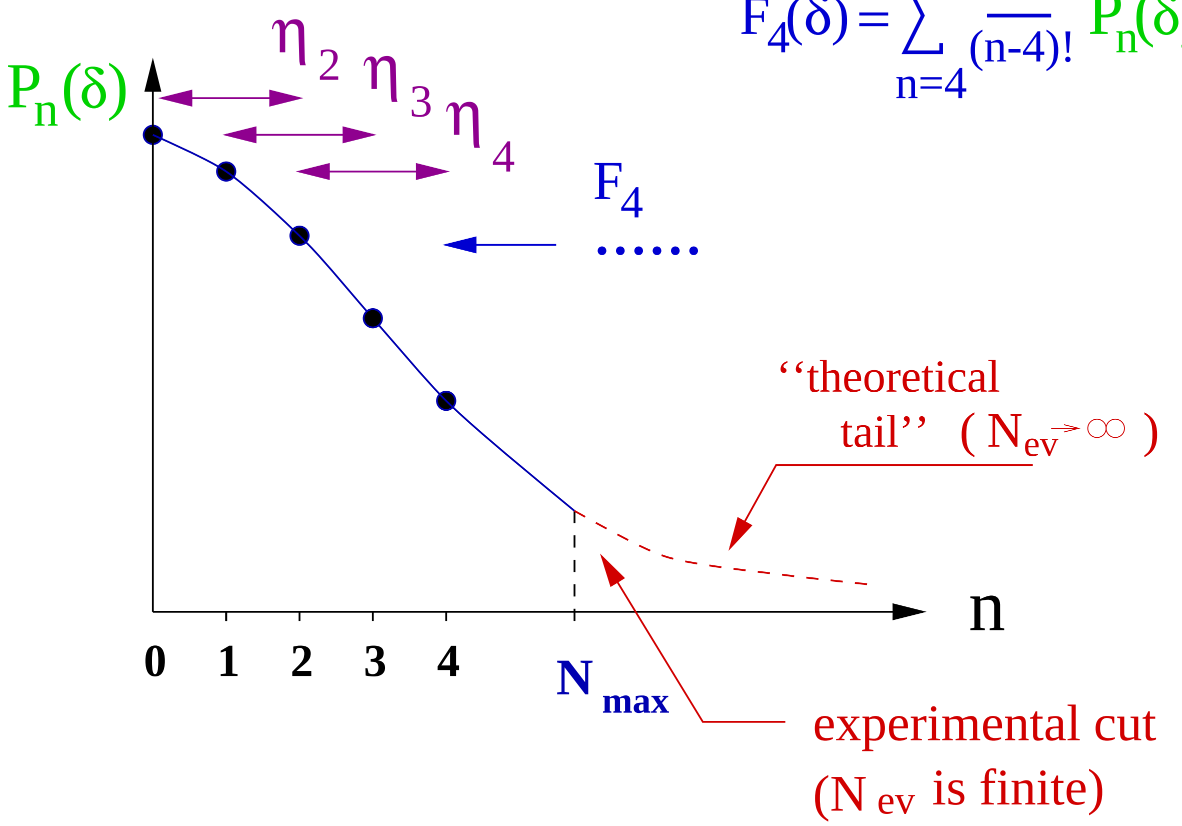
New method used by L3: (paper 542)

Bunching Parameters, $\eta_q(\delta)$

$$\eta_q(\delta) = \frac{q}{q-1} \frac{P_q(\delta) P_{q-2}(\delta)}{P_{q-1}^2(\delta)}$$

NFMs:

$$F_4(\delta) = \sum_{n=4}^{\infty} \frac{n!}{(n-4)!} P_n(\delta)$$



Local Particle Fluctuations

Disadvantages of Normalized Factorial Moments:

- very sensitive to tail of multiplicity dist.
  resulting in a bias to lower values
  Bias decreases with increased statistics

- require resolution of max. number of particles

- "insensitivity" to the underlying physics,
  e.g., NFM behavior for rapidity w.r.t. thrust axis
  very similar to rapidity w.r.t. beam

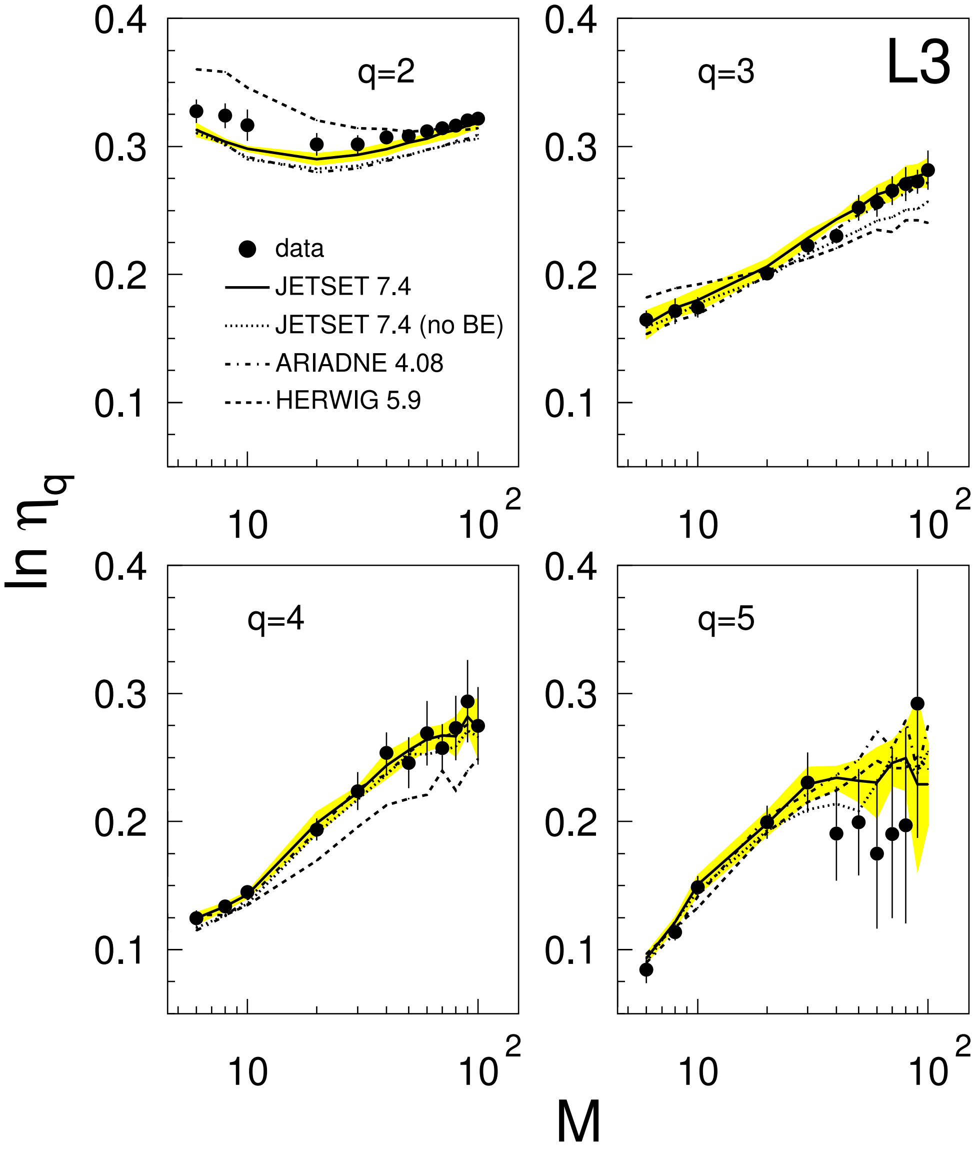
NFMs and BPs

As function of bin size δ:

"Statistical" fluctuations = Indep. production	
P_n is Poisson	$F_q(\delta) = \eta_q(\delta) = 1$
Dynamical fluctuations, scale-inv. correlations	
Intermittency:	$F_q(\delta) \propto \delta^{-\phi_q}$, $\phi_q > 0$
	$\eta_2(\delta) \propto \delta^{-\phi_2}$
anomalous fractal dimensions, $d_q = \frac{\phi_q}{q-1}$	
monofractal fluctuations	$\eta_2(\delta) \propto \delta^{-d_2}$
(d_q indep. of q)	$\eta_{q>2}(\delta) \approx$ const
multifractal fluct. (QCD)	$\eta_2(\delta) \propto \delta^{-d_2}$
(d_q depend on q)	$\eta_{q>2}(\delta) \approx$ not const

Local Particle Fluctuations – BPs in y

L3 (paper 542) ($|y| < 5$) w.r.t. thrust axis:



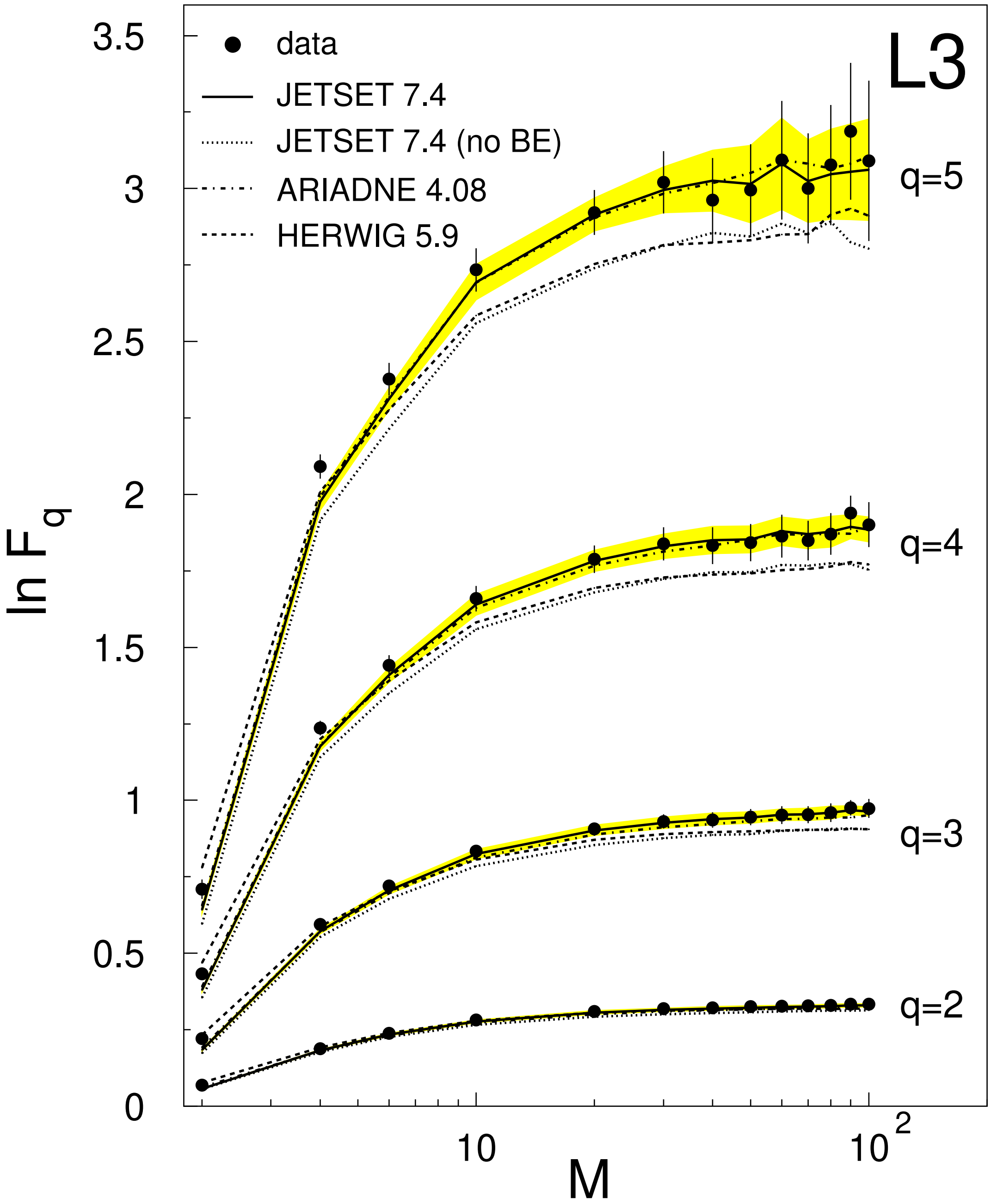
- η_2: - disagreement with all MC's for large bins (small M)
  - HERWIG overestimates data at small M
  $\Longrightarrow$ mult. dist. too broad
- $\eta_{q\geq 3}$: good agreement (HERWIG slopes slightly too low)
- $\eta_{q\geq 3} \propto \delta^{-d} \Longrightarrow$ local fluctuations are multifractal (as expected in QCD)

yellow indicates syst. err. on JETSET;

err. on ARIADNE ∼same; err. on HERWIG ∼50% larger

Local Particle Fluctuations – NFMs in y

L3 (paper 542) ($|y| < 5$) w.r.t. thrust axis:



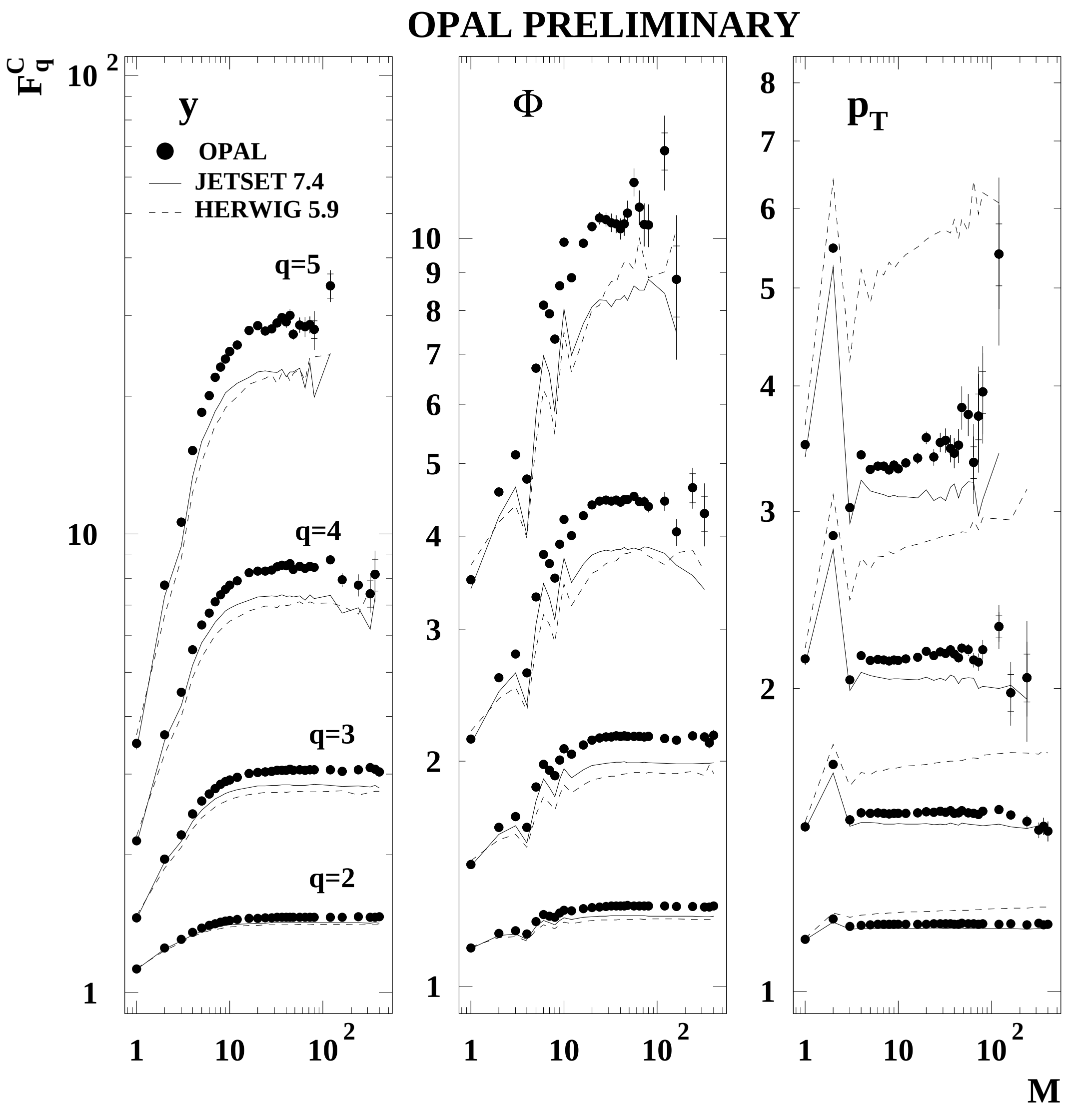
- JETSET, ARIADNE agree well with data
- HERWIG and JETSET(no BE) slightly below the data

yellow indicates syst. err. on JETSET;

err. on ARIADNE ∼same; err. on HERWIG ∼50% larger

Local Particle Fluctuations – 1-dim

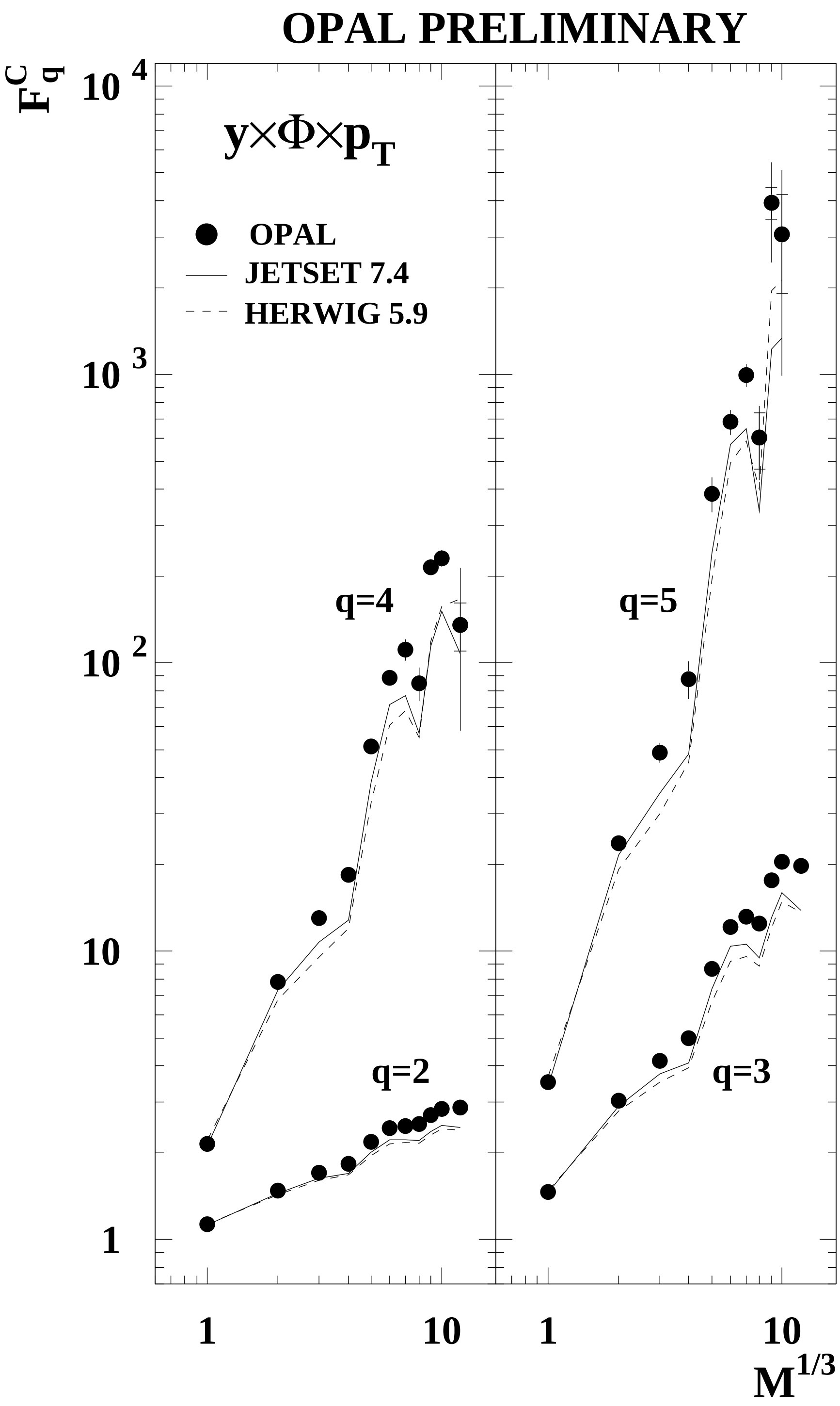
OPAL (paper 297) Factorial Moments in 1 dimension:
(y, p_{T}, ϕ w.r.t. sphericity axis) ($|y| < 2,\ -2.4 < \ln p_{\mathrm{T}} < 0.7$)



- $F_q(y)$ similar to L3
- MCs agree reasonably well with data
  but have slightly lower values
  except HERWIG: too high for p_{T}

Local Particle Fluctuations – 3-dim

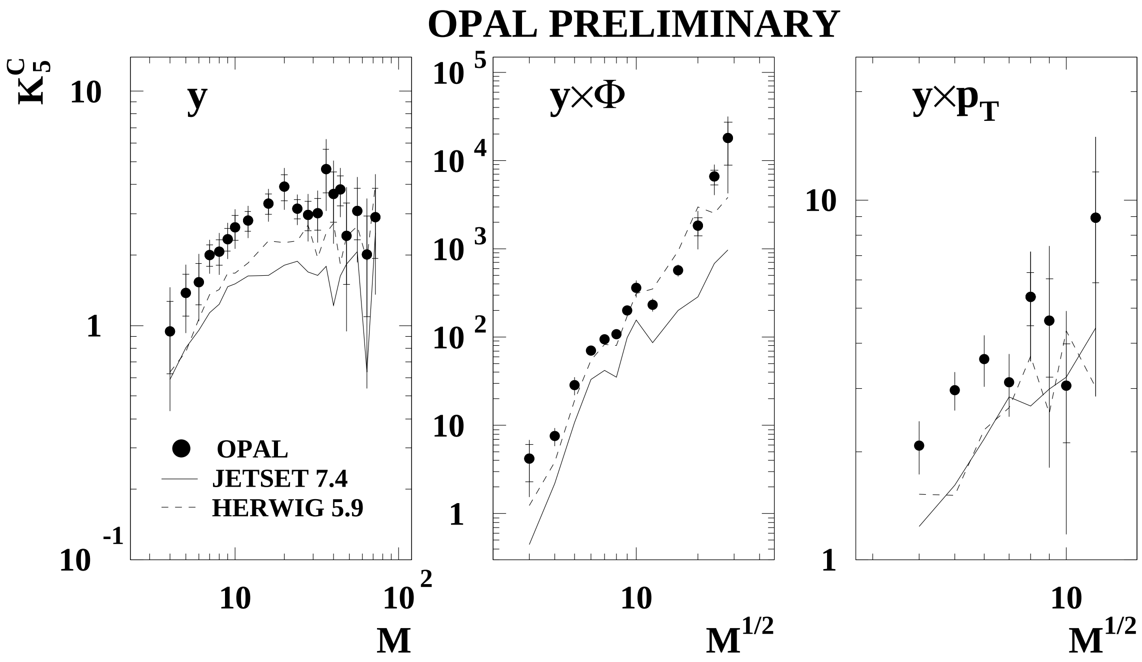
OPAL (paper 297) Factorial Moments in 3 dimensions:
(y, p_T, ϕ w.r.t. sphericity axis) ($|y| < 2,\ -2.4 < \ln p_\mathrm{T} < 0.7$)

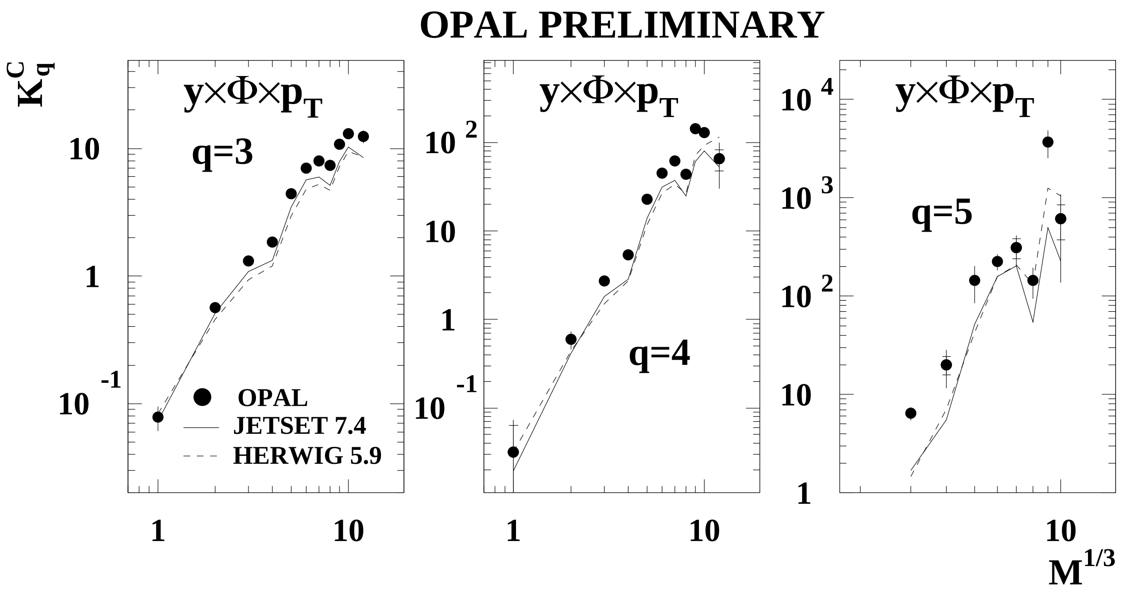


Good agreement with MCs (too good?)

Local Particle Fluctuations – 1, 2, 3 dim

Using cumulants to observe genuine q-particle correlations:





- Good agreement with MCs
- Saturation at large M only in 1 dim.

Local Particle Fluctuations – Conclusions

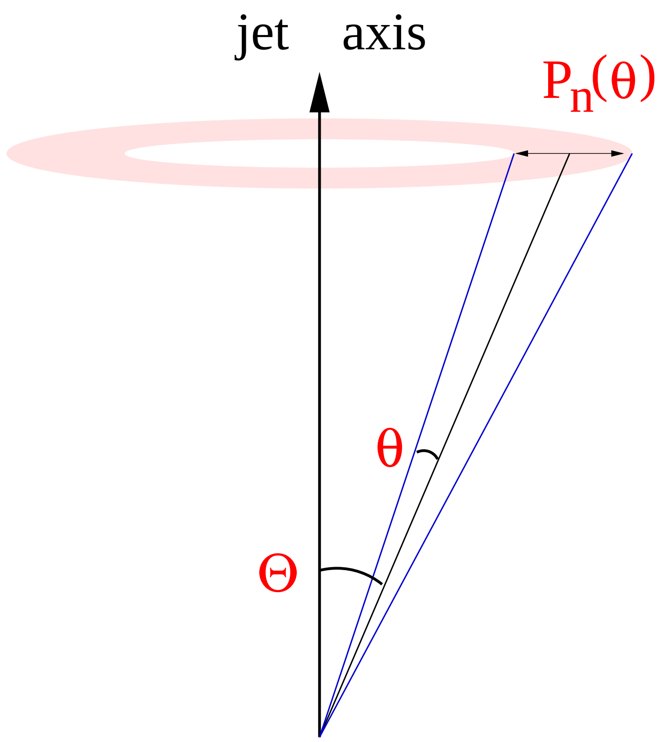
Local fluctuations, as measured by NFMs and BPs, are well described by MCs.

- trend is very well described
- magnitude sometimes somewhat underestimated by MCs

There is little need for additional physics in the MCs.

Angular Intermittency

L3 (paper 543) and
ZEUS (paper 802)
study Factorial moments in
polar-angular bins
w.r.t. jet axis



$P_n(\theta)$ - multiplicity dist. of charged particles within ring of half-width θ at Θ

- $\theta \quad \Rightarrow \quad \epsilon = z = \dfrac{\ln(\Theta/\theta)}{\ln(E_{\text{jet}}\Theta/\Lambda)}$

  $0 < \theta < \Theta \quad \Rightarrow \quad \infty > z > 0$

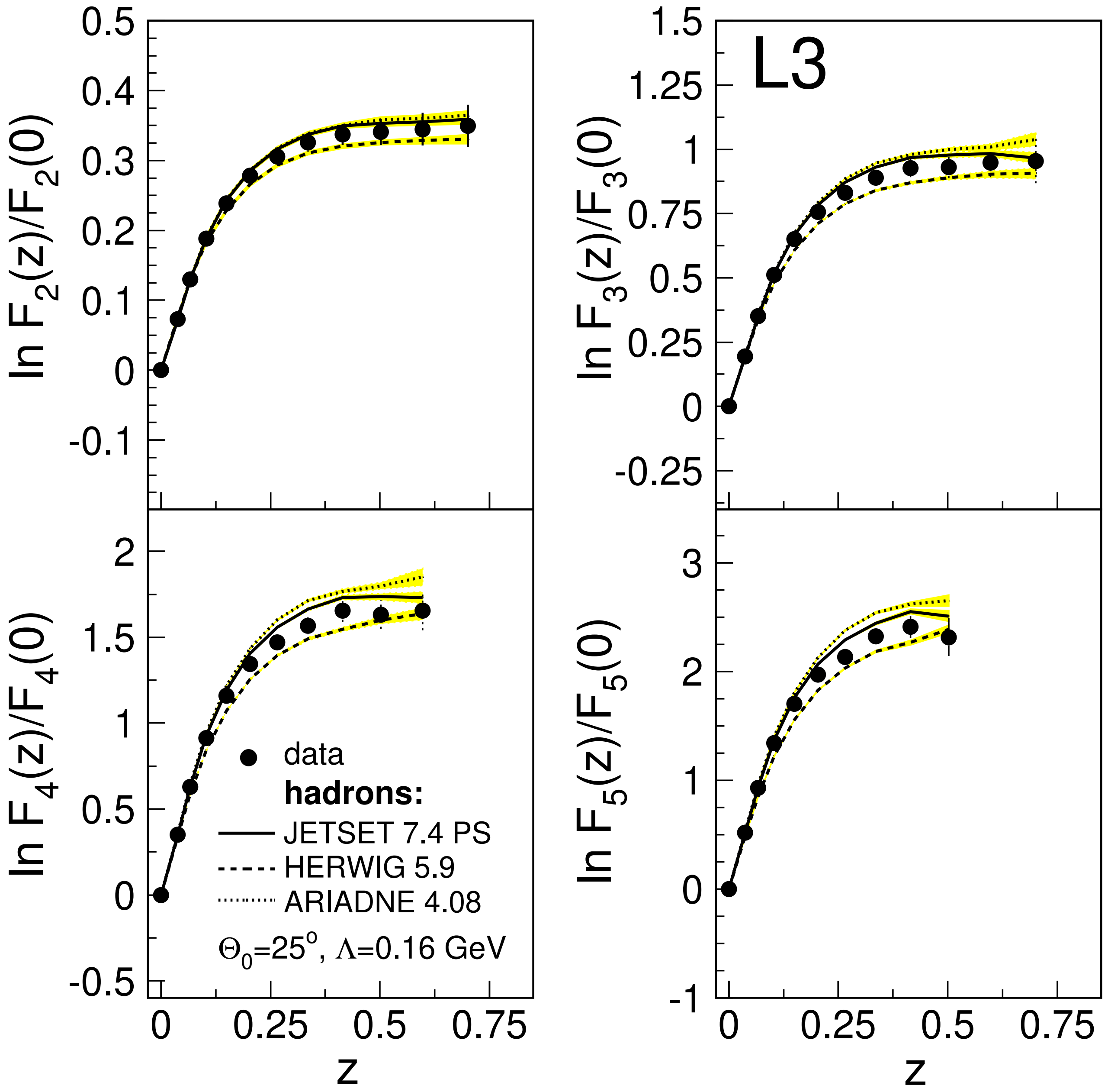
- $P_n(\theta) \quad \Rightarrow \quad P_n(z) \quad \rightarrow \quad F_q(z)$

  normalized factorial moments

expt.		jet axis	E_{jet}
L3		thrust	$\sqrt{s}/2 = 45.6$ GeV
ZEUS	Breit frame	exchanged γ	$Q/2 \approx 31$ GeV ($Q^2 > 2000$)

Angular Intermittency Test of QCD

use $\frac{F_q(z)}{F_q(0)}$ to minimize hadronization effects, syst. errors



- Good agreement with MCs

Angular Intermittency Test of QCD

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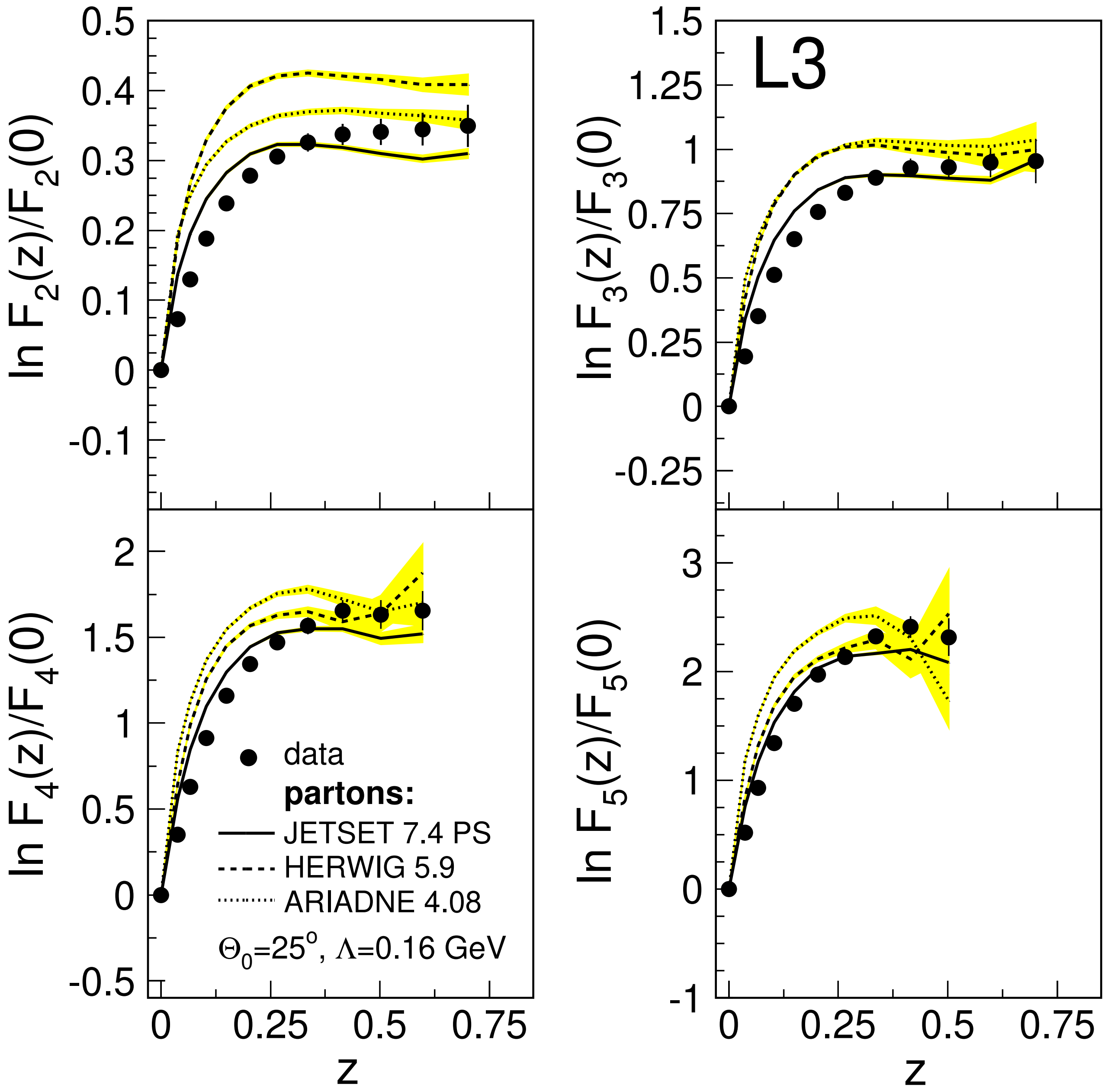


- agreement of MCs at parton level less good

Angular Intermittency Test of QCD

- Fixed coupling regime in DLLA:

$$D_q = \gamma_0 \frac{q+1}{q},$$

$\gamma_0 = \sqrt{2\, C_{\rm A} \alpha_s / \pi}.$

- Running coupling regime in DLLA:

Yu.Dokshitzer, I.M.Dremin, Nucl.Phys. B402 (1993) 139

$$D_q \simeq \gamma_0 \frac{q+1}{q} \left(1 + \frac{q^2+1}{4q^2} z \right) \qquad (a)$$

- Another DLLA estimation:

Ph.Brax, J.-L.Meunier, R.Peschanski, Z.Phys. C62 (1994) 649

$$D_q \simeq 2\, \gamma_0 \frac{q+1}{q} \left(\frac{1-\sqrt{1-z}}{z} \right) \qquad (b)$$

- Running coupling regime in MLLA:

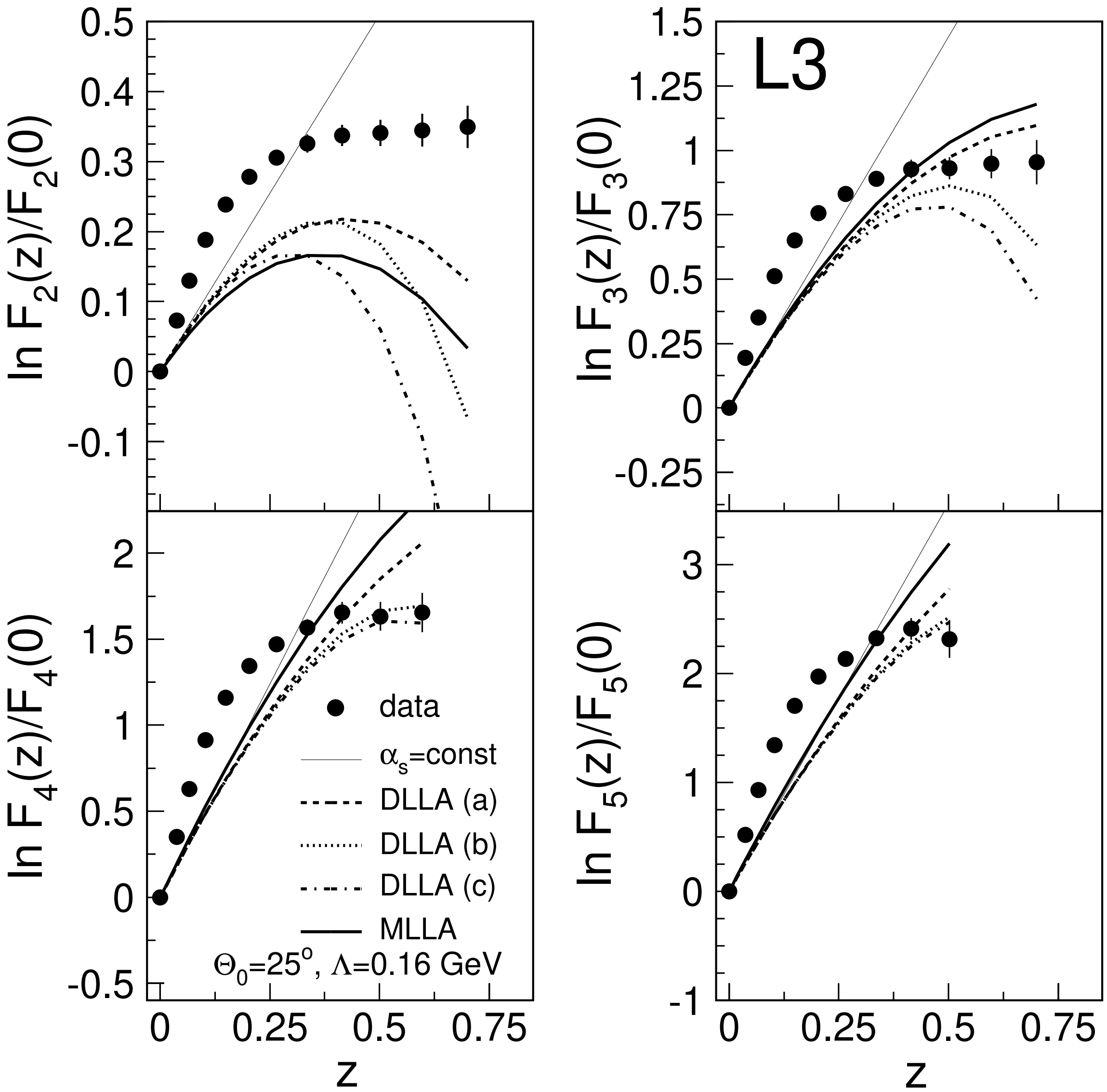
Yu.Dokshitzer, I.M.Dremin, Nucl.Phys. B402 (1993) 139

$$\gamma_0 \rightarrow \gamma_0^{\rm eff} = \gamma_0 + \gamma_0^2 \frac{b}{4C_{\rm A}} \left[-B \frac{q-1}{2(q+1)} + S_q \right]$$

$$S_q = \frac{q-1}{2(q+1)(q^2+1)} + \frac{1}{4}, \quad b = \frac{11C_{\rm A}}{3} - \frac{2n_f}{3}, \quad B = \frac{1}{b}\left[\frac{11C_{\rm A}}{3} + \frac{2n_f}{3C_{\rm A}^2} \right].$$

Angular Intermittency Test of QCD

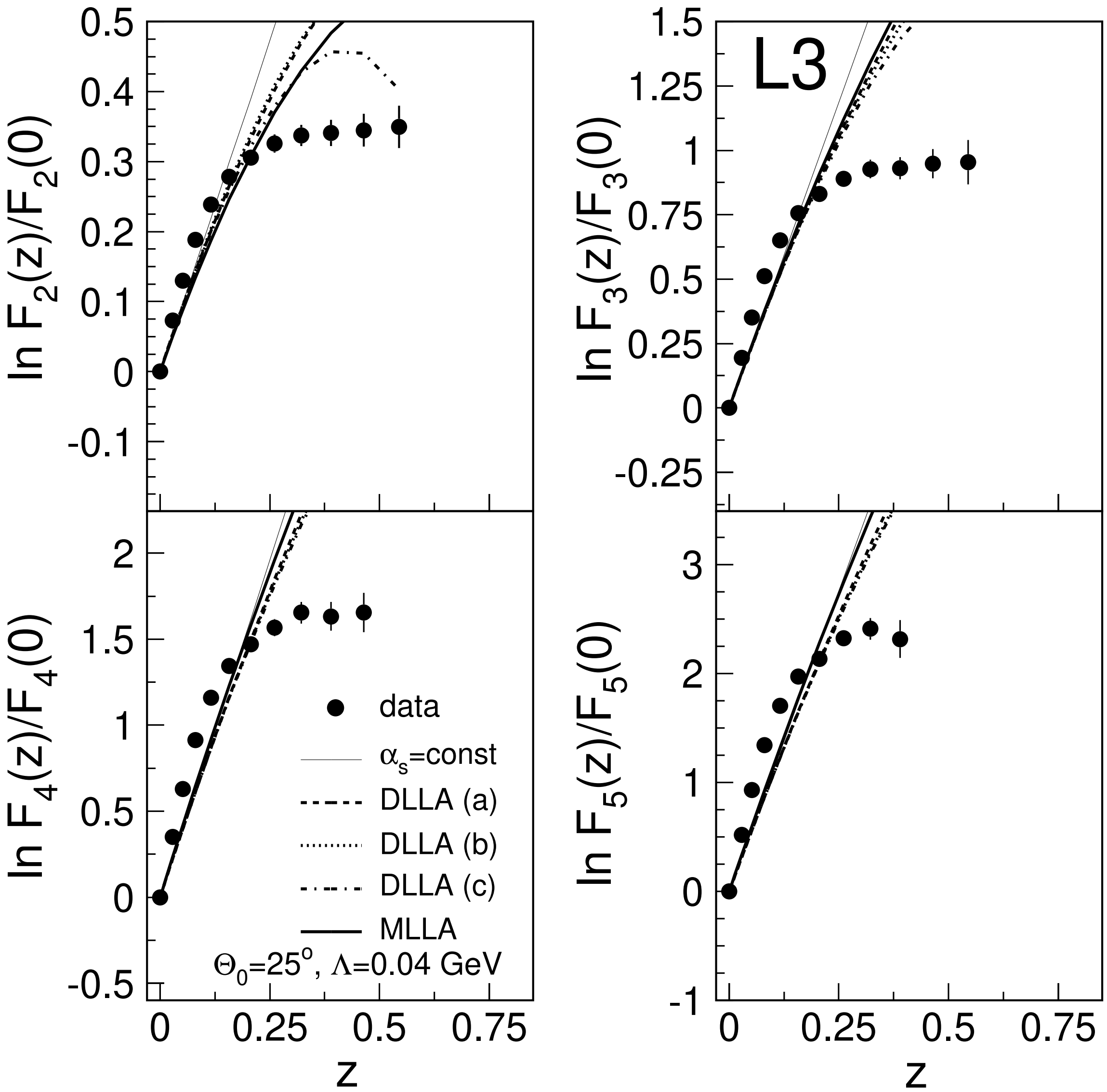
$\Theta_0 = 25°$ $\Lambda = 0.16\,\mathrm{GeV}$ $n_f = 5$



None of the pQCD predictions agree with data

Angular Intermittency Test of QCD

$\Theta_0 = 25°$ $\Lambda = 0.04\,\mathrm{GeV}$ $n_\mathrm{f} = 3$



Disagreement is less for smaller Λ, n_f
but still not good

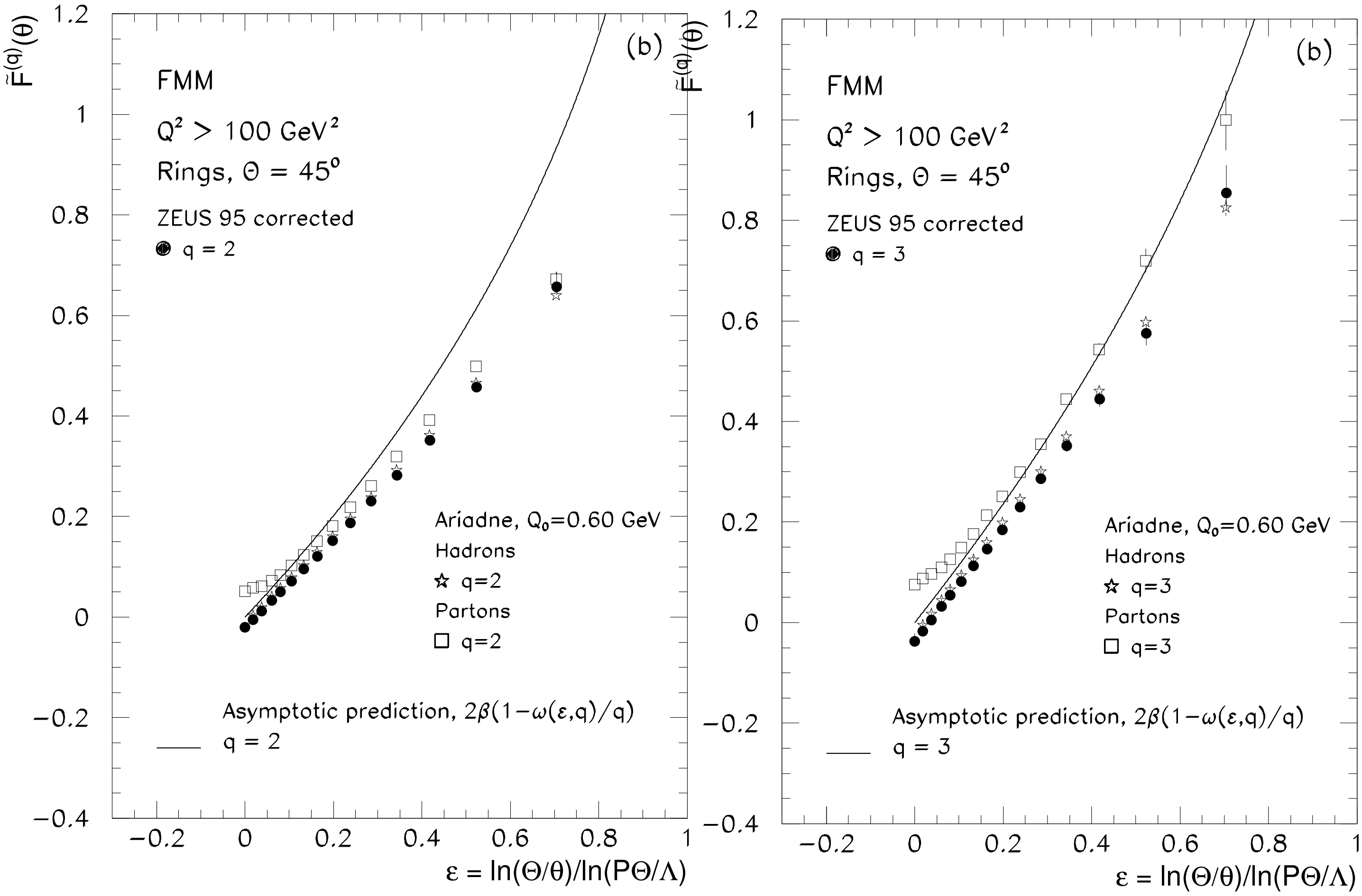
Possible reasons:

Need higher orders of pert. QCD
Non-pert. effects, *i.e.*, LPHD is invalid

Angular Intermittency Test of QCD

"QCD inspired normalization"
$\Longrightarrow$ less E dependence.
Compare to
asymptotic QCD prediction

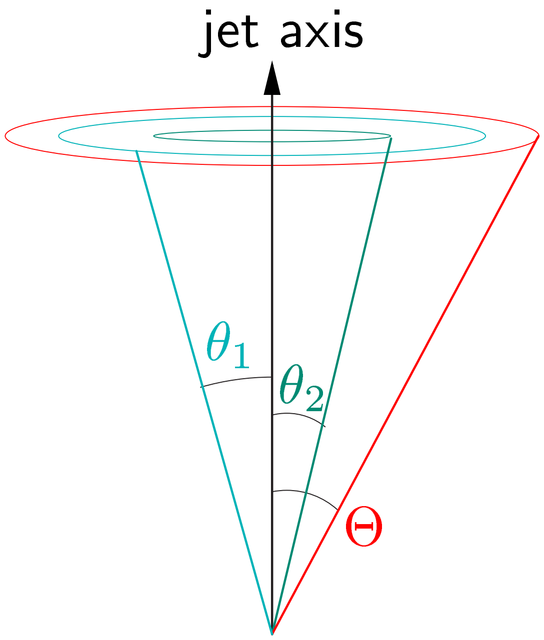
$$F_q \Longrightarrow \tilde{F}_q = -\frac{\log\left[\left(\frac{\theta}{\Theta}\right)^{q-1} F_q\right]}{q\sqrt{\log\left(\frac{E_{\rm jet}\Theta}{\Lambda}\right)}}$$



- QCD prediction disagrees
- ARIADNE agrees well with data at hadron level

Angular Correlations in Cone

ZEUS (paper 802) and DELPHI (paper 149) study polar-angular correlations between particles within a cone, Θ, about jet axis



N particles in cone Θ

$$\theta_{12} = \theta_1 - \theta_2$$

$$\hat{r}_2(\theta_{12}) = \frac{\rho_2(\theta_{12})}{\langle N(\Theta)\rangle^2}$$

$$r_2(\theta_{12}) = \frac{\rho_2(\theta_{12})}{\rho_1 \otimes \rho_1(\theta_{12})}$$

$$\rho_2(\theta_{12}) = \int_\Theta \mathrm{d}^3k_1 \mathrm{d}^3k_2\, \rho_2(k_1, k_2)\, \delta\left(\theta_{12} - \theta(k_1, k_2)\right)$$

$$\rho_1 \otimes \rho_1(\theta_{12}) = \int_\Theta \mathrm{d}^3k_1 \mathrm{d}^3k_2\, \rho_1(k_1)\rho_1(k_2)\, \delta\left(\theta_{12} - \theta(k_1, k_2)\right)$$

Can write

$$\rho_2(\theta_{12}) = \kappa(\theta_{12}) + \rho_1 \otimes \rho_1(\theta_{12})$$

correlations = genuine correlations + 'trivial' correlations

It turns out that

$\hat{r}_2$ – dominated by 'trivial' correlations
r_2 – measures genuine correlations

ρ_2 – measured from events
$\rho_1 \otimes \rho_1$ – measured from event mixing

“Trivial” Angular Correlations in Cone — $\hat{r}_2$

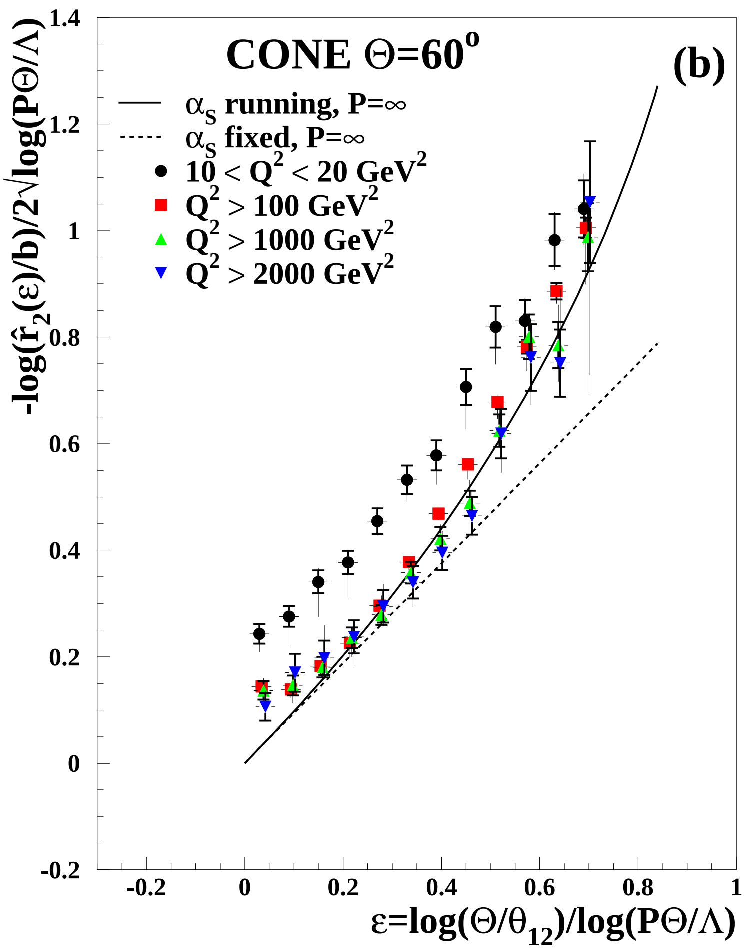
Transform to “scaling variable”

$\theta_{12} \Longrightarrow \epsilon$

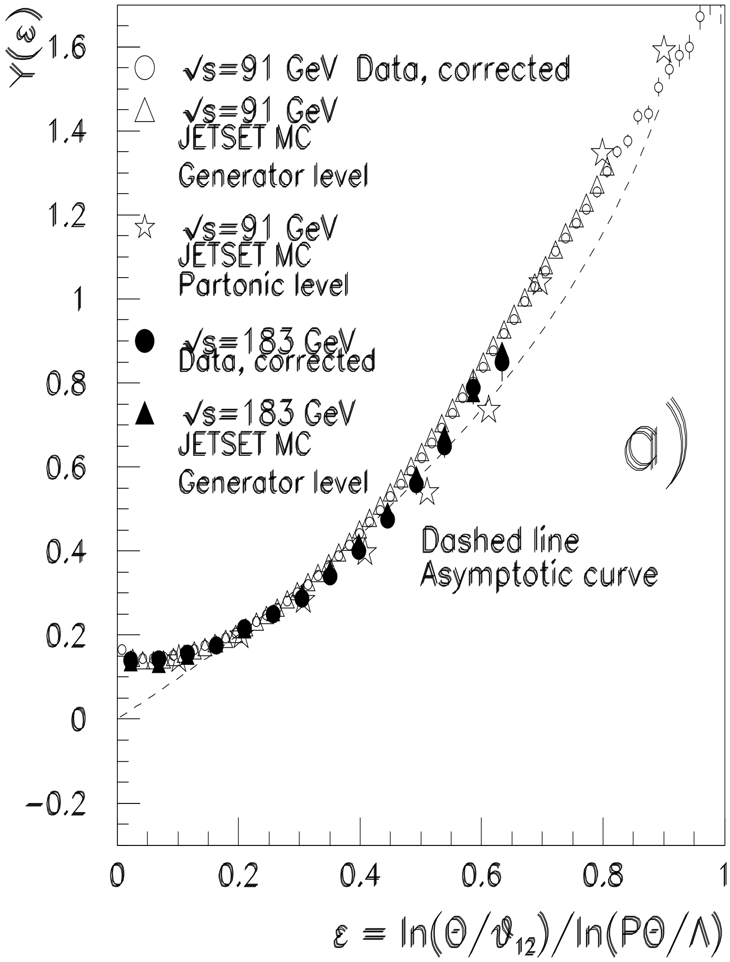
$$\epsilon = \frac{\log\left(\frac{\Theta}{\theta_{12}}\right)}{\log\left(\frac{P\Theta}{\Lambda}\right)}$$

Compare to asymptotic QCD prediction $\Lambda = 150$ MeV $\quad P = E_{\text{jet}}$

ZEUS



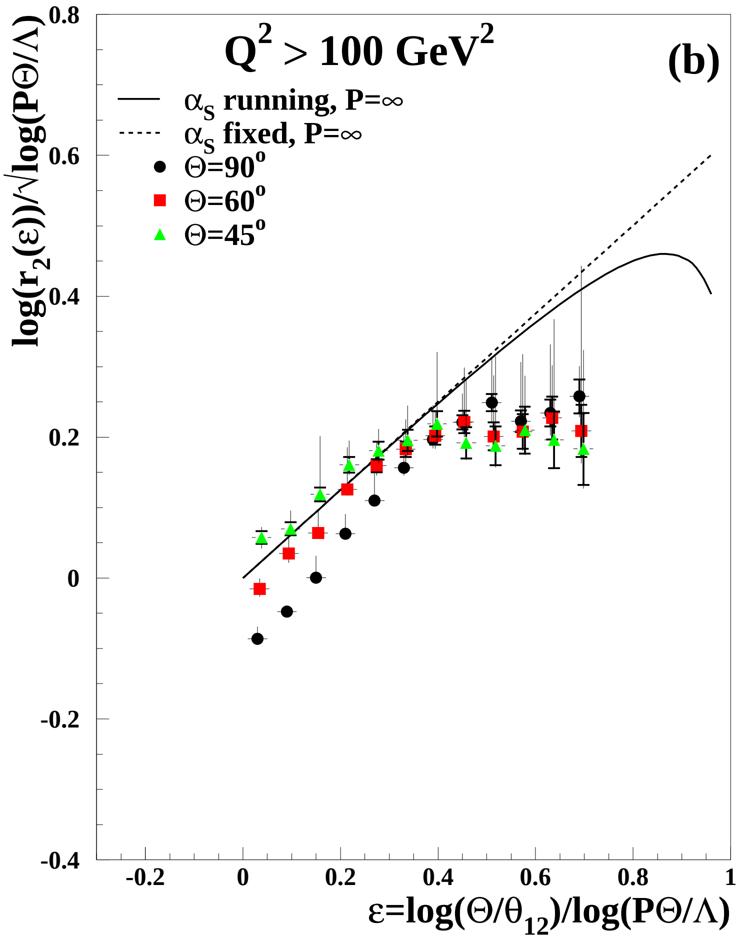
DELPHI



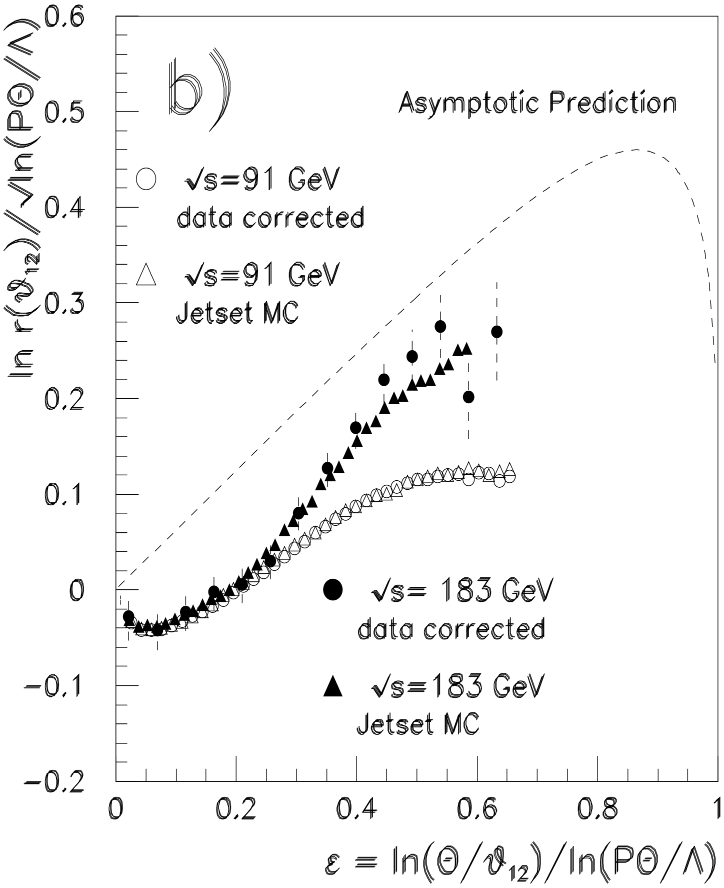
- ZEUS: as Q^2 (E_{jet}) increases, values of $\hat{r}_2$ decrease
- ZEUS data agree with asymptotic QCD for highest E_{jet} (~ 31 GeV) ▼
- But DELPHI ($E_{\text{jet}} = 45$ GeV) ○ is above the prediction
- DELPHI data agree with asymptotic QCD for $\Lambda = 300$
- JETSET △ works perfectly

Genuine Angular Correlations in Cone — r_2

ZEUS



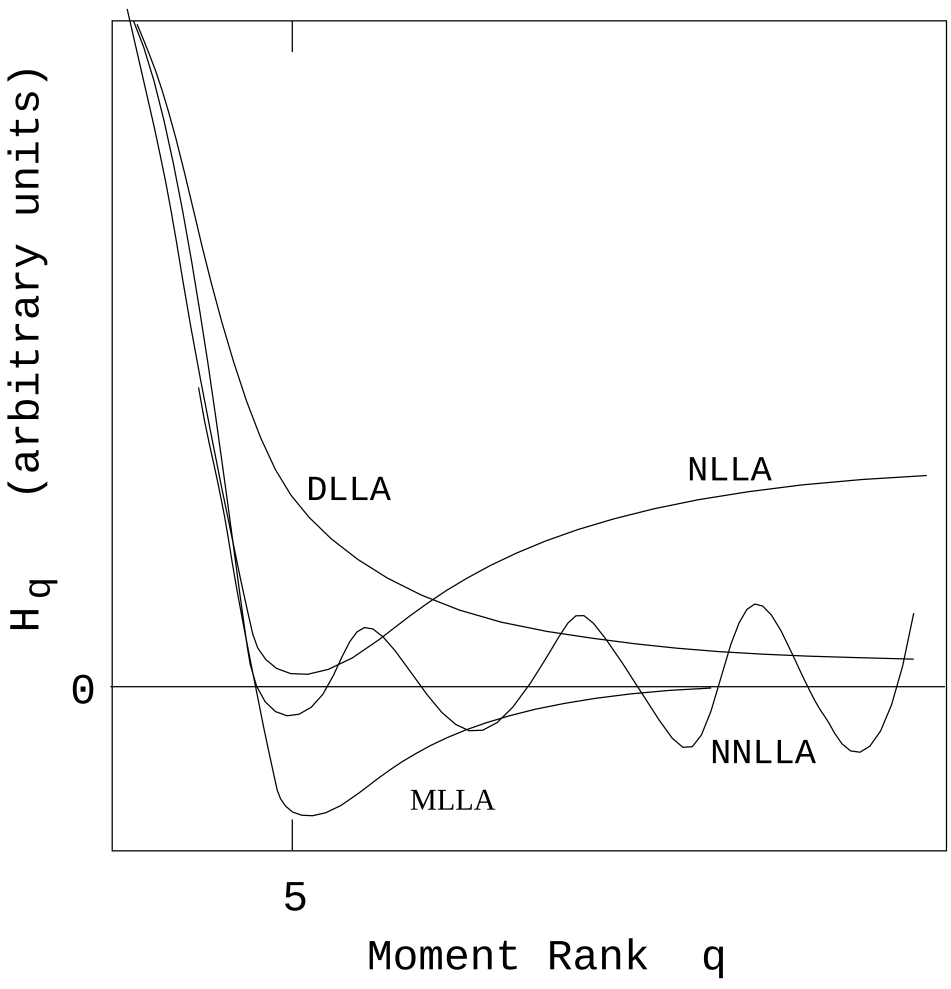
DELPHI 45° cone



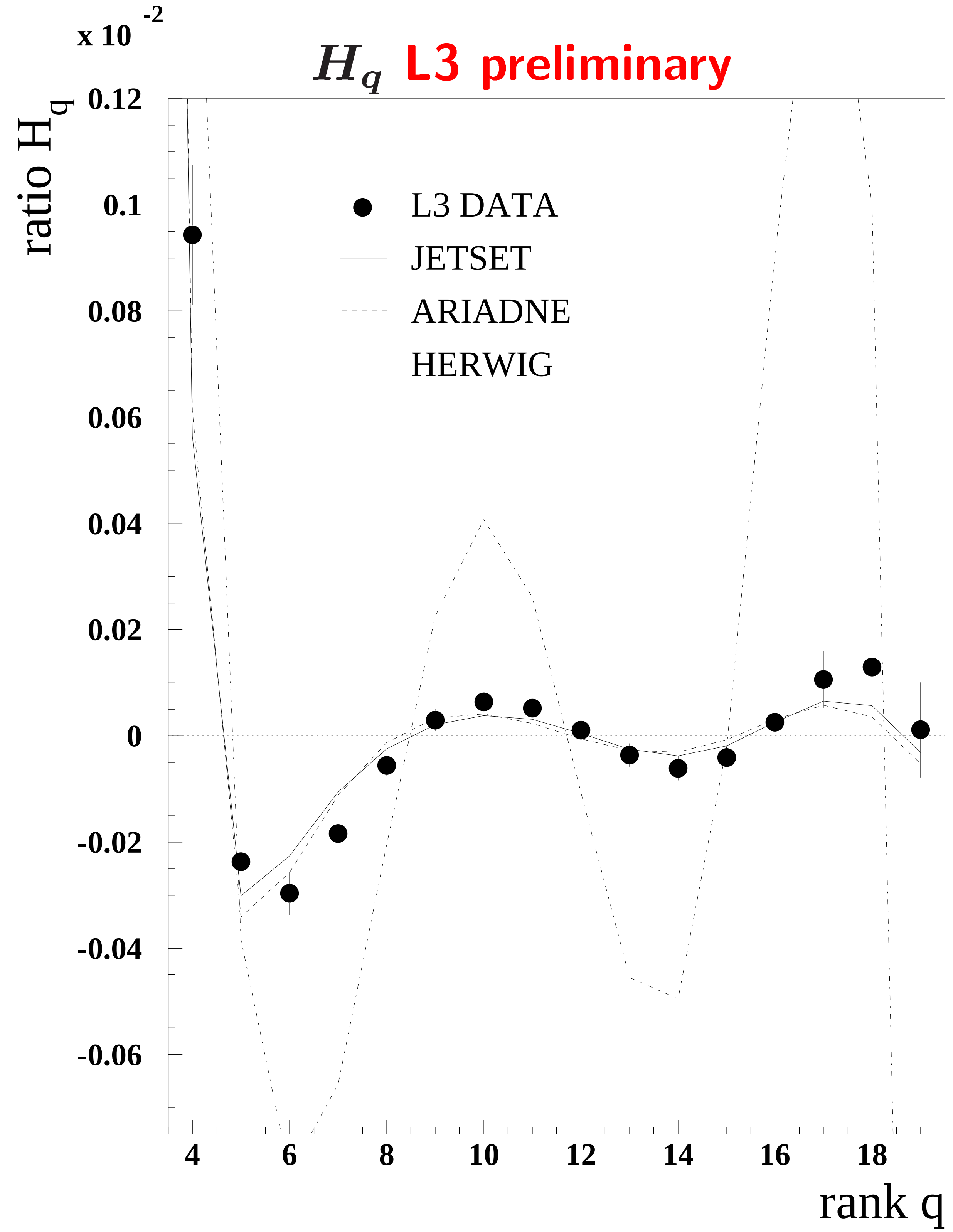
- Data are far from asymptotic prediction
  but getting closer with higher E_{jet} (DELPHI)
- ZEUS r_2 values ≈ DELPHI high-E_{jet} (91 GeV) values, ▲
  but ZEUS E_{jet} is lower — puzzling
- JETSET △ works perfectly

Charged Particle Mult. in full phase space

$H_q = \frac{K_q}{F_q}$ are calculable in pQCD



Assuming LPHD, same behaviour should occur for charged particle mult. dist.
SLD observed the behaviour predicted by NNLLA, *i.e.*, min. + oscillations (SLD, P.L. B371 (1996) 149)



- min. + oscillations seen – similar to SLD
- JETSET, ARIADNE reproduce data
- HERWIG has min. + oscillations

  But much larger amplitude of oscillations

Analytical QCD + LPHD or models

- QCD calculations are for partons, not particles
  – can not calculate at such small scales (pion mass)

- to compare with data, must rely on
  - assumptions like LPHD
  - models like string fragmentation in JETSET

- So you do not really know if you are testing QCD
  or hadronization model or assumptions

◇ Are the oscillations we see a
confirmation of NNLLA pQCD
or not ???

◇ Agreement with MCs is suspicious
– they are only ∼MLLA, not NNLLA

◇ Try to get away from LPHD assumption
by using jet multiplicity (at different scales)
instead of particle multiplicity.

Analytical QCD + LPHD or models

- Agreement with MCs is suspicious
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- Are the oscillations we see a
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  by using jet multiplicity (at different scales)
  instead of particle multiplicity.

Jets at various scales

A possible way out:

- do not use particles
- but use jets defined at a scale
  - smaller than that use to classify events as 2-, 3-, 4-jet
  - large enough that QCD calculations still valid
- Should get a sort of snapshot of the parton shower at various scales.

Using Durham ($k_\perp$) jet algorithm,

$$y_{ij} = \frac{2\ \min(E_i^2, E_j^2)}{E_{\text{vis}}^2} (1 - \cos\theta_{ij})$$

- Combine particles i,j with smallest y_{ij} into new "particle"
- Repeat until no pair left with $y_{ij} < y_{\text{cut}}$
- Result is jets at scale corresponding to y_{cut}
- $2\ \min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) \approx p_{\text{t}}^2$ of particle i w.r.t. j (to second order in θ_{ij})
- p_{t}^2 is a reasonable choice of QCD scale Q^2
- Thus the energy scale of a jet is $E_{\text{cms}}\sqrt{y_{\text{cut}}}$

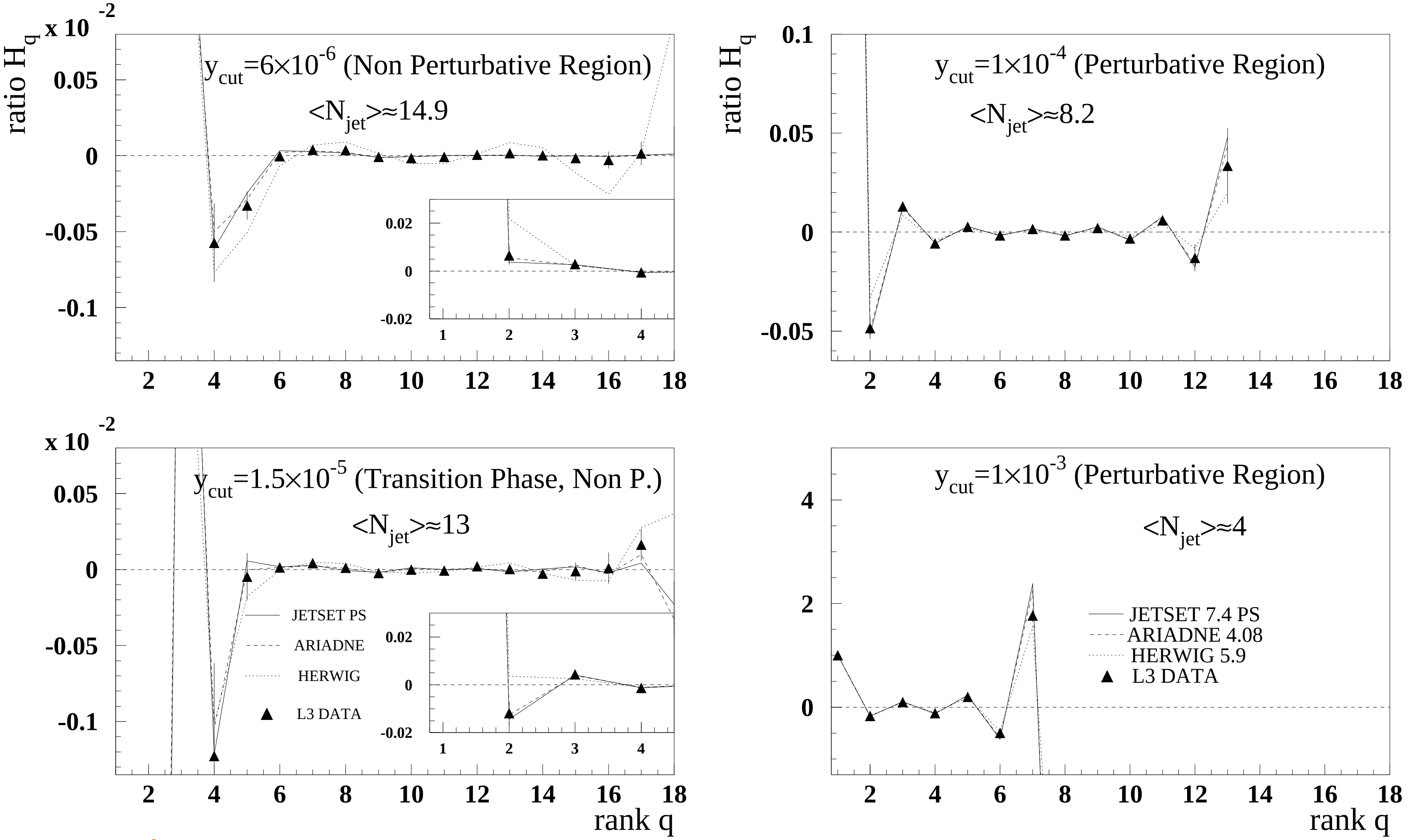
Mean Jet Multiplicity for 2- and 3-jet events agrees well with analytic QCD calculations of the mean gluon multiplicity

H_q from Jet (Durham) Multiplicity

$y_{\mathrm{cut}} = 6 \times 10^{-6}$ (220 MeV)   L3 preliminary

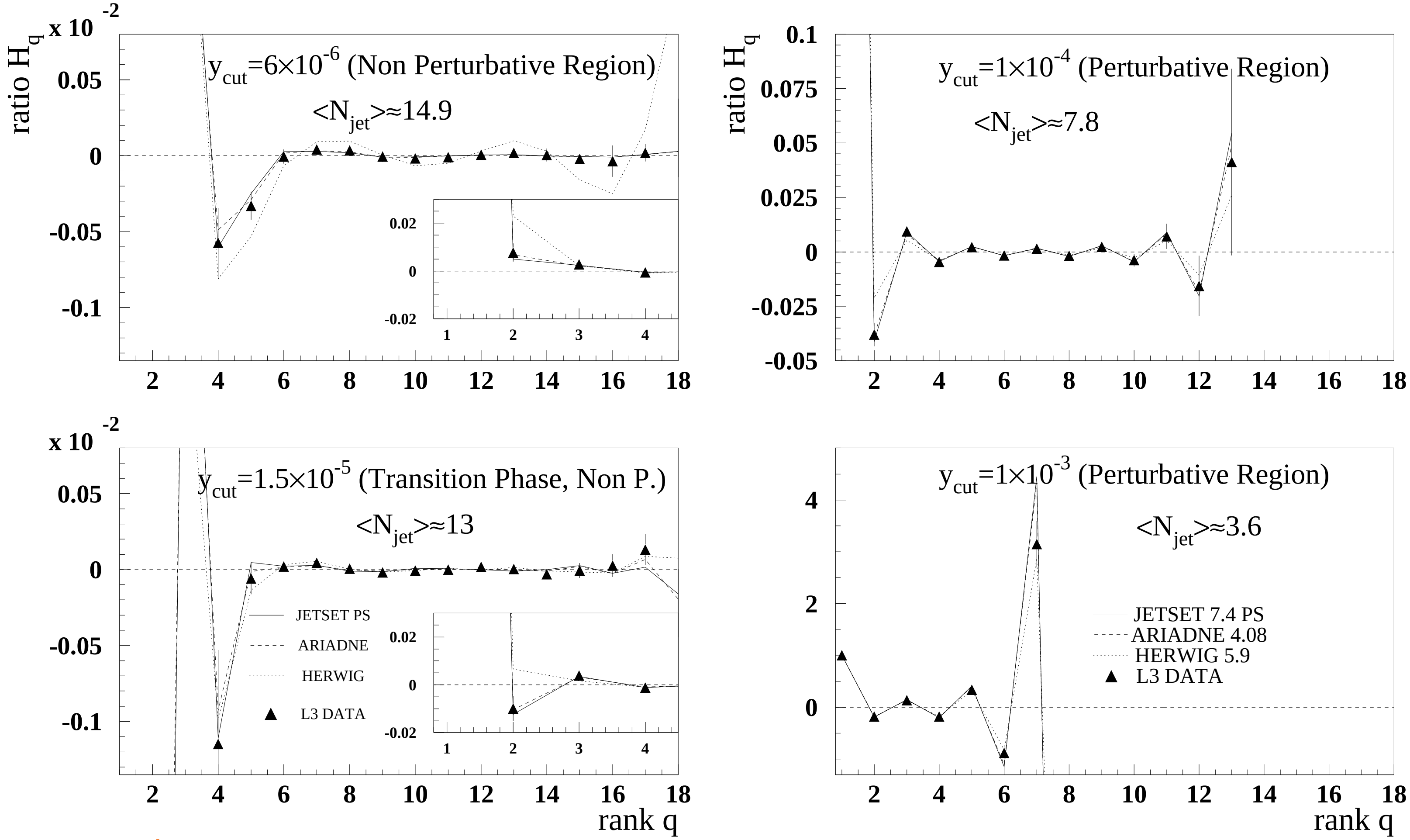
Minimum + oscillations for 'jets' at very small scale

H_q from Jet (Durham) Multiplicity L3 preliminary



As $y_{\rm cut}$ (E scale) increases, min. + oscillations disappear long before pert. region

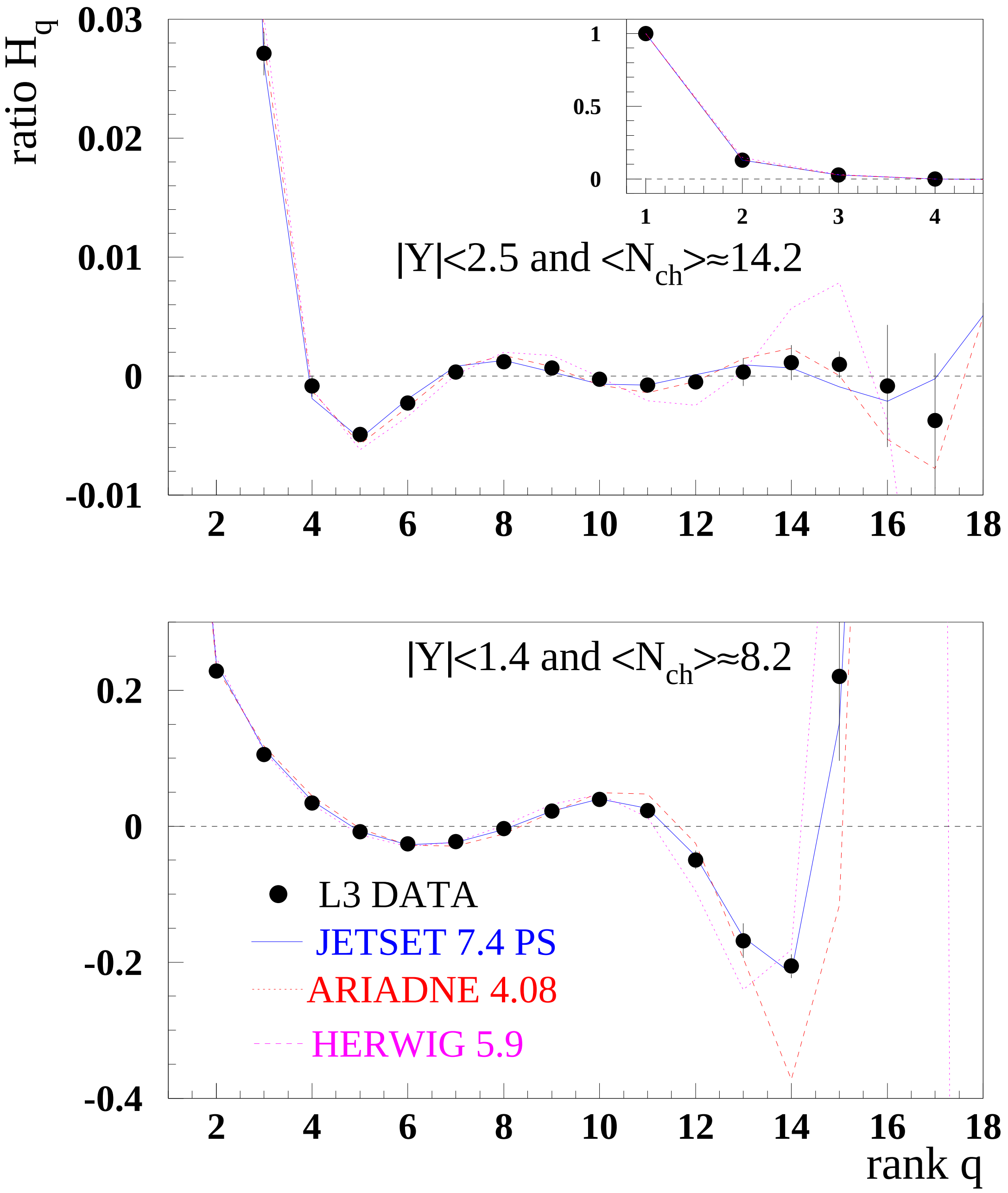
H_q from Jet (Cambridge) Multiplicity L3 preliminary



As y_{cut} (E scale) increases, min. + oscillations disappear long before pert. region

A worry

Is disappearance of osc. related to decrease in $< N >$?
Look at charged track mult. in rapidity intervals



Summary

- Charged Track Multiplicities:
  - L3 confirms H_q min. + oscillations observed by SLD
  - JETSET, ARIADNE agree with data
  - HERWIG shows min. + oscillations, but much larger amplitudes

- Agreement of MCs with data puzzling since MCs are only $\sim$MLLA, not NNLLA

- NNLLA $\Longrightarrow$ particles depends heavily on LPHD

- Jet Multiplicities:
  - H_q min. + oscillations disappear as $y_{\text{cut}} \to$ pert. scales

Conclusion

- Min. + oscillations are associated with non-pert. hadronization.

- Data do NOT confirm NNLLA prediction for H_q.

- Agreement of H_q for charged tracks with NNLLA is purely fortuitous

Conclusion on H_q oscillations

- Min. $+$ oscillations are associated with non-pert. hadronization.

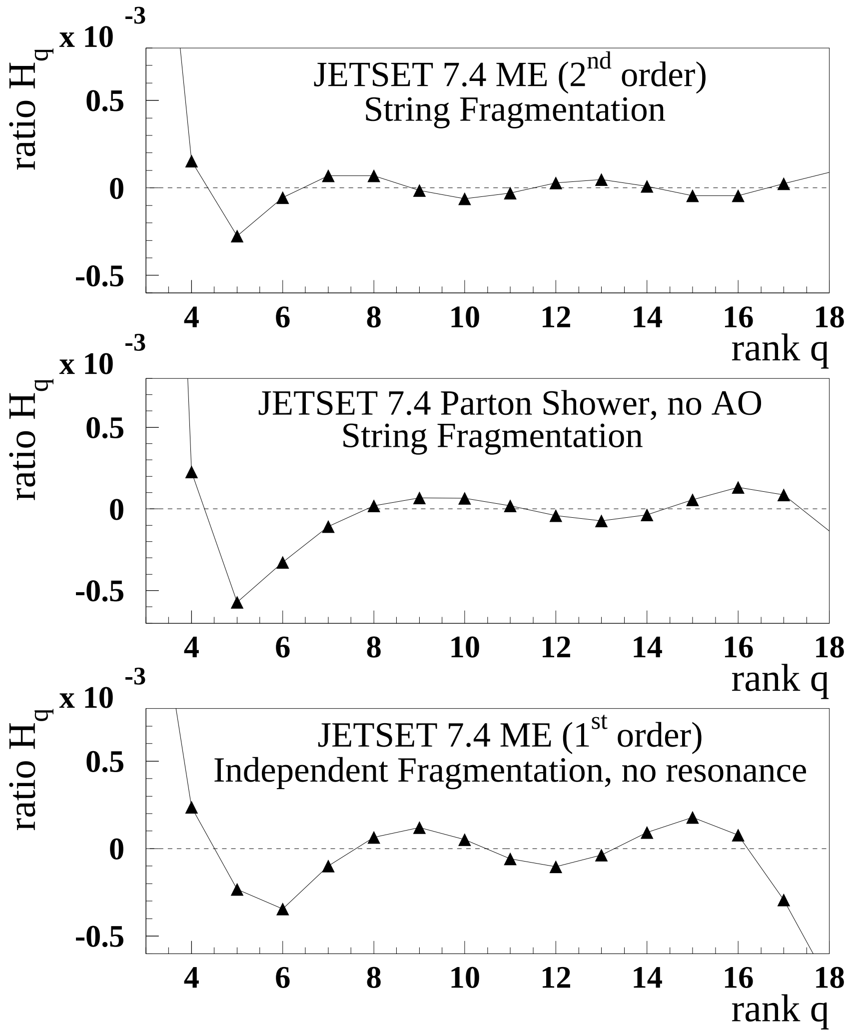
- Data do NOT confirm NNLLA prediction for H_q.

- Agreement of H_q for charged tracks with NNLLA is purely fortuitous

MC Study

So if not NNLLA, what the hell is it?
Using JETSET:

- No angular ordering parton shower($\sim$LLA)
- Matrix elements: $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s)$, $q\bar{q}$ only
- For both String and Independant Fragmentation



Conclusions

- Fluctuations and Correlations are well described by MCs.
- There is little need for additional physics in the MCs.
- pQCD + LPHD is less successful
  - Comparisons are close enough to give hope,
  - but far enough away to tell you that there is still lots to do.
  - Calculations are not accurate enough:
  - From success of MCs, pQCD calculations need
    * better approximations (many are only DLA)
    * finite E; quarks with mass
    * (better) treatment of E, p conservation
- In some cases parton, particle levels of MCs give similar results.
  However, this seems to depend on
  - which variables
  - which MC

for transparencies:

`http://wwwinfo.cern.ch/~metzger/talks.html`