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Oscillating H_q, Event Shapes, and QCD

W. J. Metzger
University of Nijmegen, Toernooiveld 1, 6525 ED Nijmegen, The Netherlands
E-mail: W.Metzger@cern.ch

The ratio of cumulant factorial to factorial moments of the charged particle multiplicity distribution shows oscillatory behavior similar to that predicted by NNLLA. However, an analysis of the jet multiplicity distribution finds these oscillations only at non-perturbative scales. Comparisons are made with Monte Carlo models, and the roles of energy conservation and the LPHD assumption are investigated.

1 Introduction

The shape of the charged particle multiplicity distribution is often studied using normalized factorial moments.[1] The normalised factorial moments, F_q, and their cumulants, K_q, of order q are defined in terms of the multiplicity distribution, P_n, by

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)....(n-q+1)P_n}{\left(\sum_{n=1}^{\infty} nP_n\right)^q}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m}F_m$$

The cumulant factorial moment, K_q, expresses the phase space integral over the genuine q-particle correlation function, *i.e.*, the function which describes the correlations between q particles. Their ratio, $H_q = K_q/F_q$, has been calculated[2] for the soft gluon multiplicity distribution in different approximations of perturbative QCD (pQCD). The expected behavior of H_q with respect to q depends strongly on the approximation: In the Double Leading Logarithm Approximation (DLA), H_q decreases to 0 as q^{-2}. In the Modified Leading Log Approximation (MLLA), H_q decreases to a negative minimum at $q \approx 5$ and then rises to approach 0 asymptotically. In the Next to Leading Log Approximation (NLLA), H_q decreases to a positive minimum at $q \approx 5$ and then increases to a positive constant value for large moment rank, q. In the Next to Next to Leading Log Approximation (NNLLA), H_q decreases to a negative first minimum at $q \approx 5$, and then shows quasi-oscillations about 0.

Validity of the Local Parton Hadron Duality hypothesis (LPHD) would imply similar behavior for the charged particle multiplicity distribution.

Previous measurements of the H_q for the charged particle multiplicity distribution in e^+e^- collisions[3,4] at 91.2 GeV and also in hadron collisions[5] have shown, qualitatively, the behavior predicted in NNLLA.

While it is tempting to conclude that the observed behavior is a confirmation of the NNLLA prediction, it must be realized that such a conclusion rests strongly on the validity of LPHD. Although the LPHD hypothesis works well for single-particle distributions and mean

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multiplicity, difficulties have been detected when resonances are involved.[6,7] Further, heavy quark decays cause a modification of the multiplicity distribution which is unlikely to be properly accounted for by LPHD.[8] For these reasons, we consider the assumption of LPHD as highly questionable.

Therefore, in this paper we present preliminary measurements[9] of the H_q not only for the charged particle multiplicity distribution but also for the jet multiplicity distributions. The latter should enable us to extend the analysis to the energy scales of partons, in particular to scales where perturbative QCD should be reliable ($\gtrsim 1-2$ GeV), thus reducing the dependence on the assumption of LPHD.

2 Charged particle multiplicity analysis

This analysis is based on data collected by the L3 detector[10] in 1994 at the energy of the Z and uses about 900000 selected hadronic events.[9]

An iterative Bayesian method[11] is used to unfold the charged particle multiplicity distribution from that of the raw data. The method makes use of a detector response matrix determined from a sample of events generated by JETSET 7.4 PS[12] or HERWIG 5.9,[13] which have been tuned using L3 data.[14,15] The events have been passed through a simulation of the L3 detector,[16] reconstructed, and subjected to the same selection procedure as the data. The unfolded distributions obtained using JETSET and HERWIG are consistent. The multiplicity distribution is further corrected to remove the decay products of K^0_S and Λ and for initial state radiation. The resulting multiplicity distribution, shown in Fig. 1, is well reproduced by JETSET, but HERWIG gives too broad a distribution. That consistent unfolded distributions are found using JETSET and HERWIG, even though their generator level distributions are different, gives us confidence in the unfolding method. The H_q moments are calculated from the multiplicity distribution. They are found[9] to agree well[†] with the SLD results,[3] showing the minimum plus oscillatory behavior expected by NNLLA.

The H_q are very sensitive to statistical fluctuations, which are large for high multiplicities due to the limited statistics of the sample. Therefore, the multiplicity distribution is truncated at 53. The H_q, shown in Fig. 2, have a negative first minimum at $q \approx 5.5$ and quasi-oscillations around 0 for greater q. JETSET is found to reproduce this behavior quite well, while HERWIG shows the same features but with much larger amplitudes.

Since the behavior of the H_q agrees qualitatively with that predicted in NNLLA, it is natural to interpret this agreement as a confirmation of the NNLLA prediction. However, JETSET (and ARIADNE,[18] not shown) agree with the data, and HERWIG shows a qualitatively similar behavior, even though their parton showers are closer to MLLA than to NNLLA. Hence their H_q should not be expected to oscillate. That they do, together with doubt about the LPHD assumption, suggests that we view the NNLLA interpretation with scepticism.

One of the improvements in going from DLA to NNLLA is a better treatment of energy conservation.[19] Further, the parton shower of ARIADNE is very similar to that of the DLA calculations, but with the addition of exact energy and momentum conservation at each branching.[20] In order to investigate the contribution of energy conservation as well as the LPHD assumption we have run ARIADNE allowing the parton shower to proceed to a very small scale and compared the resulting parton distributions with the data. In this approach the hadronization model is replaced by a more extensive parton shower and strict LPHD. It has been found to successfully mean multiplicity, and low order factorial moments.[20]

Run in this way ARIADNE has just two parameters, Λ and the shower cut-off, Q_0. To compare the parton multiplicity distribution with that of charged particles, we use 2/3 of the

[†]For this comparison we have not corrected for K^0_S and Λ, since SLD did not.

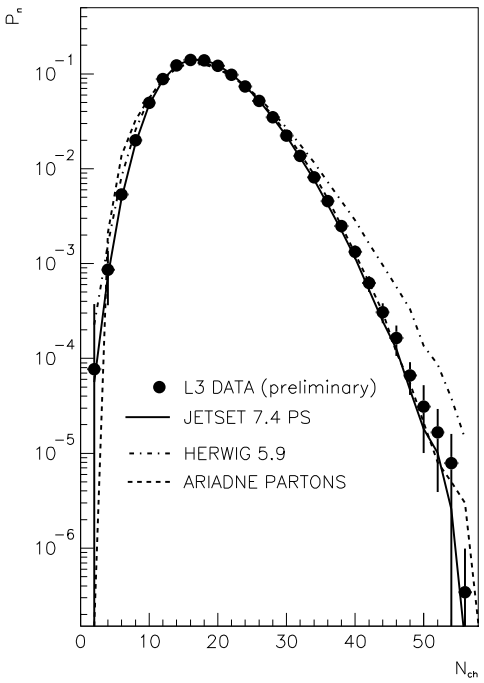

Figure 1: Charged particle multiplicity distribution.

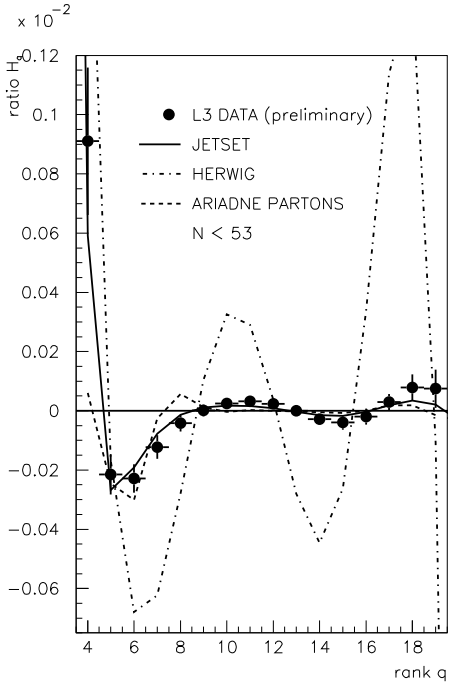

Figure 2: H_q *vs.* q.

number of partons rounded to an even number. Best agreement is found for $\Lambda = 388\,\mathrm{MeV}$ and $Q_0 = 388.01\,\mathrm{MeV}$. The ARIADNE multiplicity and H_q distributions are compared with the data in Figs. 1 and 2. While the multiplicity distribution agrees well, except at small multiplicities where the rounding of the number of gluons introduces the largest error and where fluctuations of the neutral/charged particle ratio are largest, the H_q can only be said to show the qualitative features of the data. A minimum is observed at the right place, but the oscillations do not agree quantitatively with those observed.

Further, we found that agreement is also only qualitative for shape variables such as thrust, C-parameter, scaled heavy jet mass, and 3-jet resolution parameter, as is shown in Fig. 3. The combined χ^2 for the distributions shown is 350, whereas for JETSET (not shown) agreement is also quantitatively good, $\chi^2 = 43$ for 56 degrees of freedom.

3 Jet multiplicity analysis

This analysis assumes that the jet multiplicity distribution is related to the parton multiplicity distribution at the energy scale corresponding to the jet resolution. By choosing a scale where perturbative QCD should be applicable ($\gtrsim$ 1 GeV), we should be able to test the pQCD predictions for the behavior of the H_q. We define jets using the Cambridge[21] jet algorithm. Since this study starts with the charged track multiplicity distribution, we have also constructed jets using only charged tracks. The jet multiplicity distributions have been unfolded using the same procedure as for the charged particle multiplicity distribution, again truncating the distribution at the highest multiplicities. These multiplicity distributions and the H_q calculated from them are shown in Figs. 4, 5 and 6 for three different values of the jet

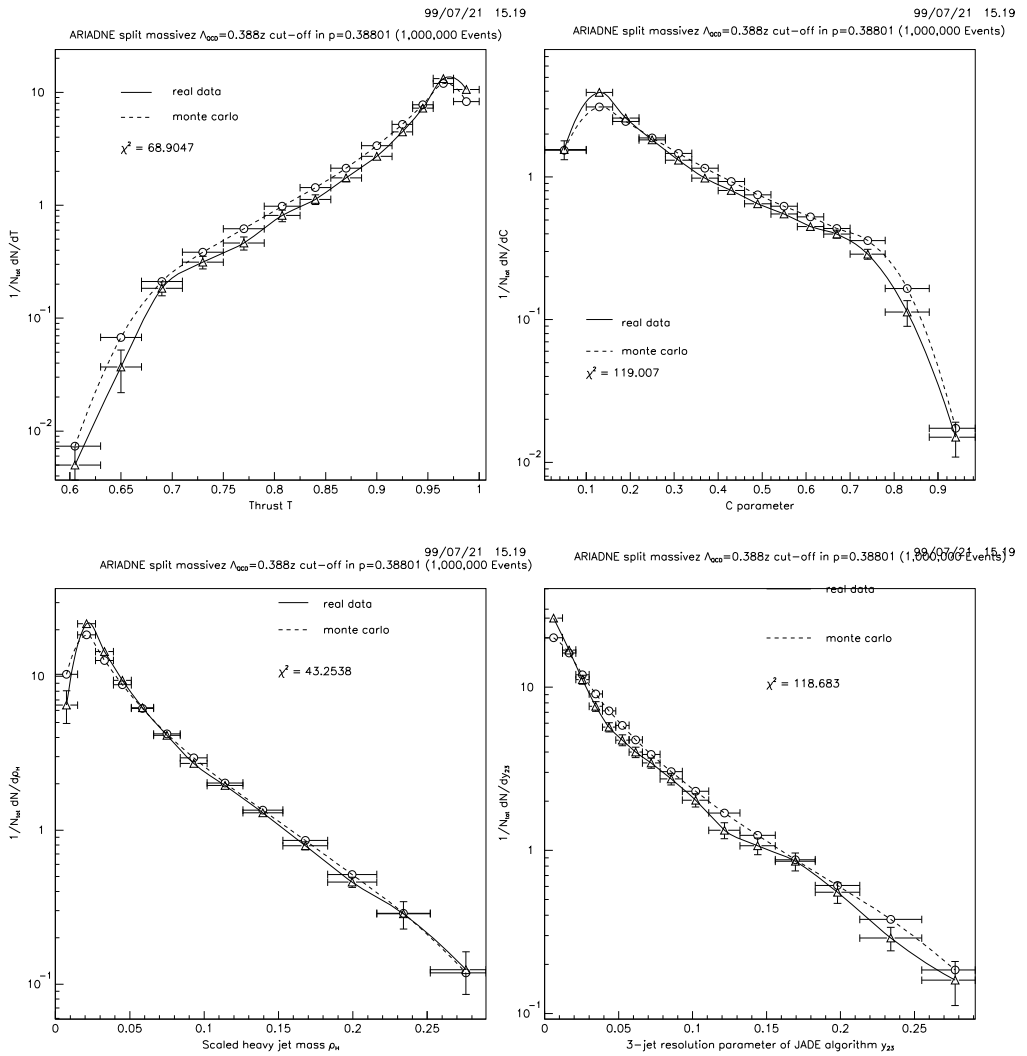

Figure 3: Distributions, measured by L3,[15] of the shape variables thrust, C-parameter, scaled heavy jet mass, and 3-jet resolution parameter for the JADE algorithm, compared to those obtained from ARIADNE with $\Lambda = 388\,\text{MeV}$, $Q_0 = 388.01\,\text{MeV}$ and no hadronization (see text).

resolution parameter, y_{cut}, representing non-perturbative (240 MeV) and perturbative (1.0 and 2.7 GeV) energy scales (scale $= E_{\text{cms}}\sqrt{y_{\text{cut}}}$).

The jet multiplicity distributions are well reproduced by JETSET, while HERWIG is somewhat too broad, especially at small y_{cut}. The ARIADNE parton model's distribution is too broad at small y_{cut} and too narrow at large y_{cut}.

The H_q calculated from JETSET agree well with the data for each y_{cut}, while the H_q calculated from HERWIG disagree in magnitude with the data for low values of y_{cut}, where the jet multiplicity distributions are closer to the charged particle multiplicity distribution.

For $y_{\text{cut}} = 6.8 \times 10^{-6}$ (energy scale $\approx$ 240 MeV) a first negative minimum is seen at $q \approx 4.5$ followed by oscillations about 0. This is qualitatively similar to that for the charged particle multiplicity distribution, but the position of the first minimum is shifted to a lower value of q.

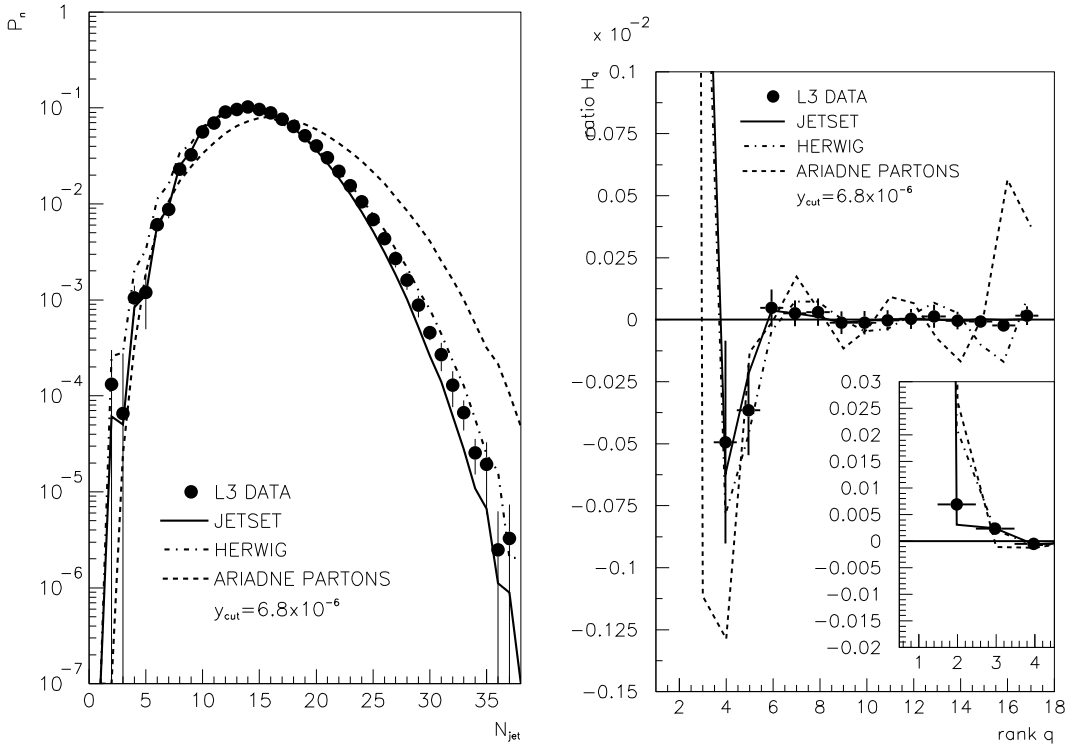

Figure 4: Jet multiplicity distribution and resulting H_q at a non-perturbative (240 MeV, $y_{\rm cut} = 6.8 \times 10^{-6}$) energy scale.

As $y_{\rm cut}$ is increased, the behavior of the H_q changes further. The negative first minimum near 4.5 disappears. For $q \leq 5$ each H_q alternates between positive and negative values. For greater q, the oscillations begin to disappear.

Before the perturbative region ($y_{\rm cut} \gtrsim 10^{-4}$) is reached, the oscillations and the negative first minimum have completely disappeared and the H_q are found to alternate between positive and negative values for each q. None of the predictions made at different approximations of pQCD for the H_q is observed at perturbative scales. Even though pQCD describes the average jet multiplicity well,[22–26] it seems to be unable to describe the shape of these distributions.

4 Conclusions

The H_q have been measured for both the charged particle and jet multiplicity distributions obtained with Cambridge algorithm at various scales. The NNLLA prediction of a first minimum at $q \approx 5$ and quasi-oscillation around 0 for higher q is observed for the charged particle multiplicity distribution, confirming previous results. However, our analysis of the H_q of the jet multiplicity distributions reveals that this behavior appears only for very small $y_{\rm cut}$, corresponding to energy scales $\lesssim 300$ MeV, which is far from the perturbative region. JETSET reproduces well the multiplicity distributions and H_q for both particles and jets. HERWIG produces a multiplicity distribution which is too broad; its H_q show a minimum and oscillations with amplitudes far larger than the data. ARIADNE, using JETSET for hadronization,

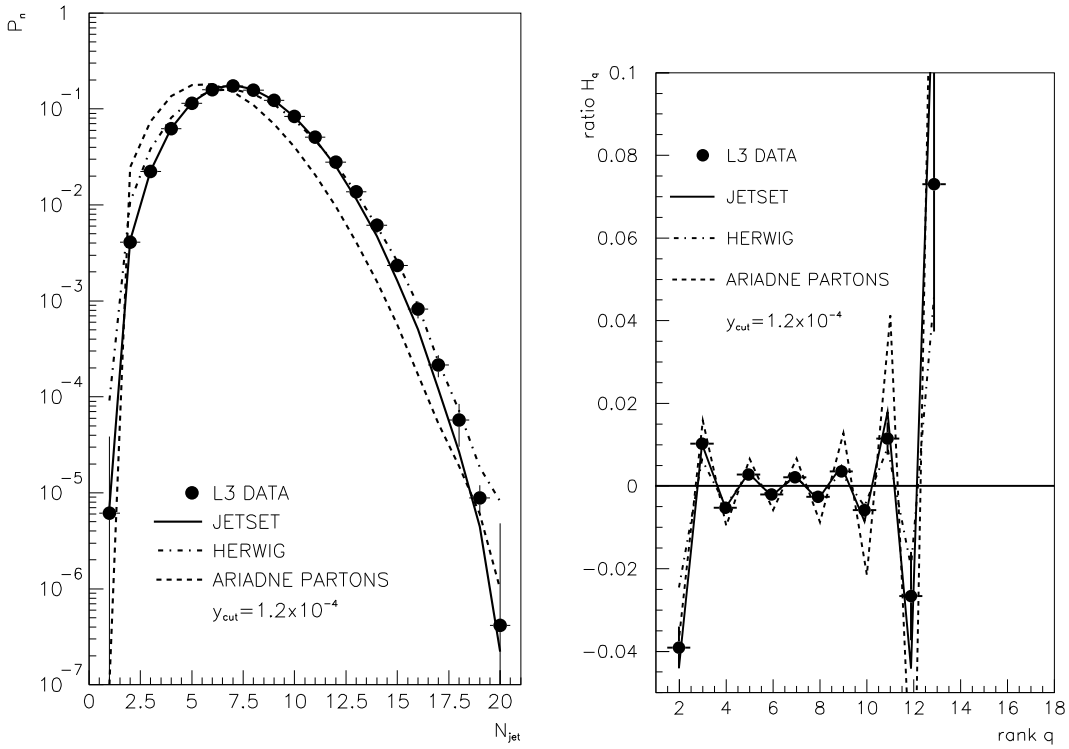

Figure 5: Jet multiplicity distribution and resulting H_q at a non-perturbative (1 GeV, $y_{\rm cut} = 1.2 \times 10^{-4}$) energy scale.

gives multiplicities and H_q very similar to JETSET.[27] ARIADNE, with its parton shower extending to a cut-off at 388 MeV and with LPHD rather than a hadronization model, is less successful at describing the H_q, jet multiplicities and event shapes. Nevertheless, its qualitative behavior is similar to the data. The above observations lead us to the conclusion that the minimum plus oscillatory behavior of the H_q is not only due to perturbative QCD, but that non-perturbative hadronization also plays a strong role. Further, since the Monte Carlo models are not full NNLLA, the qualitative success of ARIADNE+LPHD suggests that energy-momentum conservation is responsible for much of the effect.

Acknowledgments

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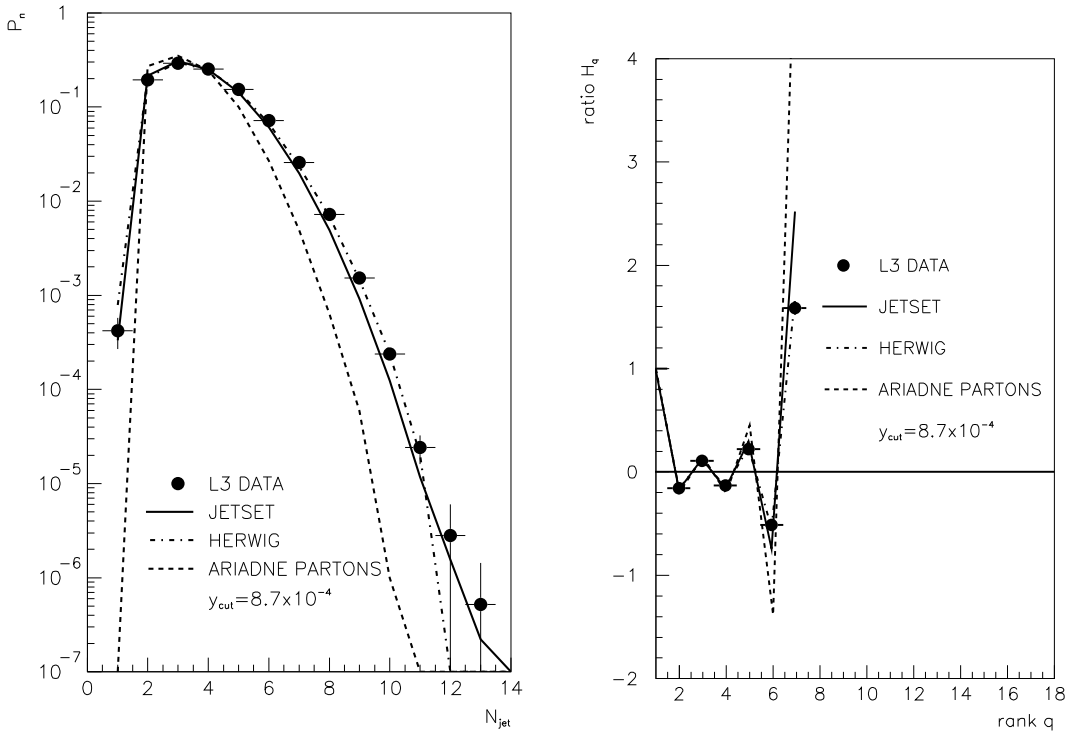

Figure 6: Jet multiplicity distribution and resulting H_q at a non-perturbative (1 GeV, $y_{\rm cut} = 1.2 \times 10^{-4}$) energy scale.

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