# QCD at LEP II

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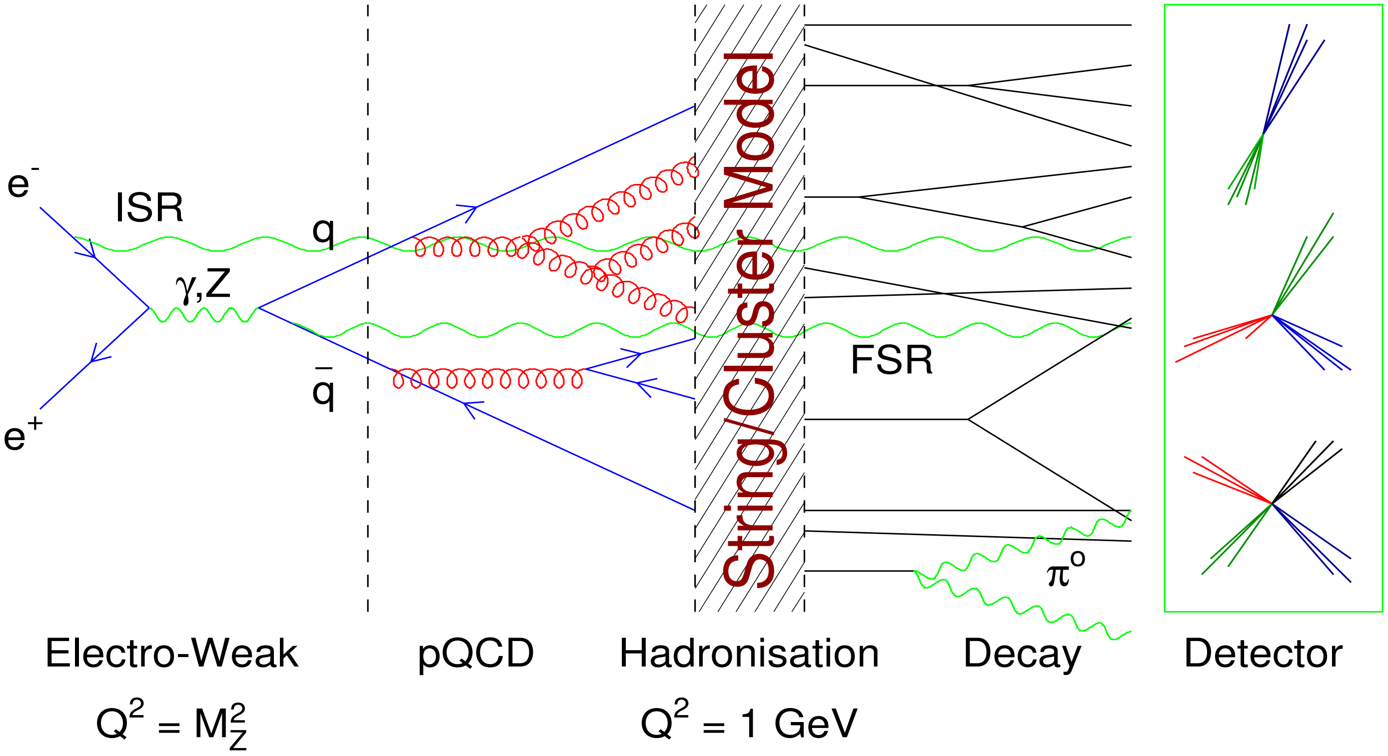
# Outline

- Introduction
- Event shape variables
- QCD at LEP
- Hadronic event selection at $\sqrt{\mathrm{s}} > \mathrm{m_Z}$
- Comparison with QCD models
  - Event shapes
  - Jet rates
  - Charged particle multiplicity, inclusive momentum spectra
- Power law description of fragmentation
- Determination of the strong coupling constant ($\alpha_\mathrm{s}$)
- Summary

# Introduction

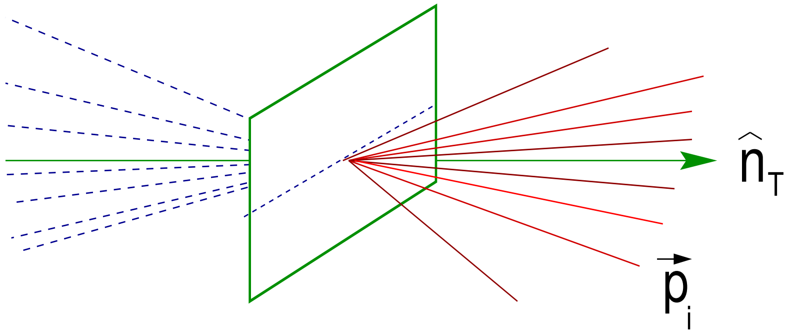



- Production process well defined, hadronic activity only in final state
- Effect of hadronisation decreases with increasing CM energy
  - ➜ Jets retain initial parton direction at high energy

$\boxed{\text{Allows clean test of QCD with minimum assumptions}}$

# Event Shape Variables

➢ Event shape variables, sensitive to hard gluon emission, describe multijet topology




➢ **Thrust**

$$T = \max \frac{\sum_a |\vec{p_a} \cdot \vec{n_T}|}{\sum_a |\vec{p_a}|}$$

➢ **Scaled Heavy Jet Mass**

$$M^2_{\pm} = \left( \sum_{a \in S_{\pm}} p_a \right)^2$$

$$M_H = \max[M_+(\vec{n_T}), M_-(\vec{n_T})]$$

$$\rho_H = M^2_H / s$$

➜ describe longitudinal development

➢ **Jet Broadening Variables**

$$B_{\pm} = \frac{\sum_{a \in S_{\pm}} |\vec{p_a} \times \vec{n_T}|}{2 \sum_a |\vec{p_a}|}$$

- Total Jet Broadening $B_T = B_+ + B_-$
- Wide Jet Broadening $B_W = \max(B_+, B_-)$

➢ **C Parameter**

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$$

$\lambda_1, \lambda_2, \lambda_3$ eigenvalues of spherocity tensor

$$\theta^{ij} = \frac{\sum_a p^i_a p^j_a / |\vec{p_a}|}{\sum_a |\vec{p_a}|} \quad i, j = 1, 2, 3$$

➜ describe transverse development

| Topology | |
|---|---|
| 2-Jet | $T = 1$; $\rho_H$, C, $B_{T,W} = 0$ |
| 3-Jet | $T_{max} = \frac{2}{3}$, $\rho_H = 1 - T$ |
| | $C_{max} = \frac{3}{4}$, $B^{max}_T = \frac{1}{2\sqrt{3}}$ |

# QCD at LEP

- At LEP
  - Data collected over a large range of CM energy
  - Detectors cover 99% of $4\pi$
  - Energy and direction of jets precisely measured

- Study QCD with small experimental and theoretical uncertainties

- Test QCD prediction of energy evolution of
  - Moments of event shape distributions
  - Strong coupling constant

  in a single experiment,
    - experimental errors at different energy points correlated

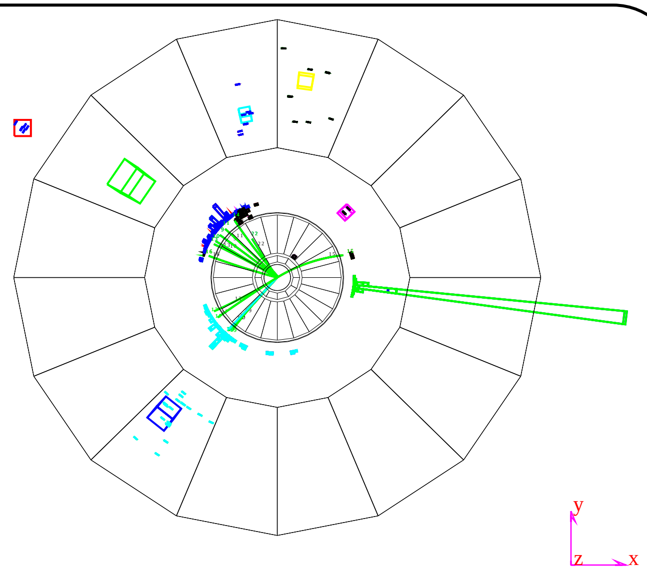
- New particle searches, W pair studies require a thorough understanding of QCD background

# LEP Data

➜ Reduced $\sqrt{s'}$ from Z decay with hard isolated photon (FSR, ISR)

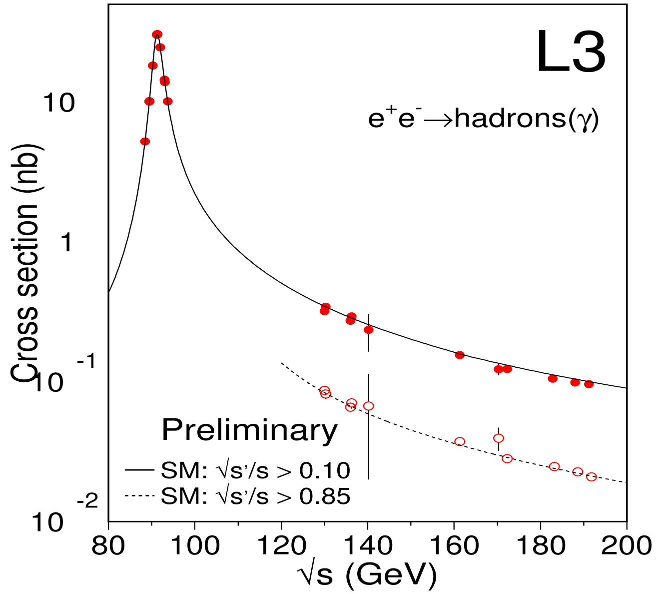
$$\sqrt{s'} \;=\; \sqrt{s\left(1-\frac{2\,E_\gamma}{\sqrt{s}}\right)}$$

L3 : 6 points from 30 to 86 GeV
DELPHI : 3 points from 32 to 80 GeV




➜ Hadronic Z decays at LEP I, $\sqrt{s} = m_Z$

➜ LEP II Data at $\sqrt{s} = 130, 136, 161, 172, 183, 189, 192, 196$ GeV.
Background contribution from
(a) "Radiative Return to Z"
(b) $W^+W^- \rightarrow 4\,\mathrm{Jets}$



# LEP Data

| LEP Run | Energy (GeV) | $\int \mathcal{L}\,(\mathrm{pb}^{-1})$/Expt | $\sim$ Events/Expt |
|---|---|---|---|
| LEP I | Z peak | 140 | 4000K |
| LEP 1.5 | 130, 136 | 6+6 | 600 |
| 1996 | 161 | 10 | 400 |
| | 172 | 10 | 300 |
| 1997 | 181–184 | 55 | 1500 |
| 1998 | 189 | 176 | 3500 |
| 1999 | 192 | 25 | 500 |
| 1999 | 196 | 25 | 500 |

➜ Hadronic events characterized by

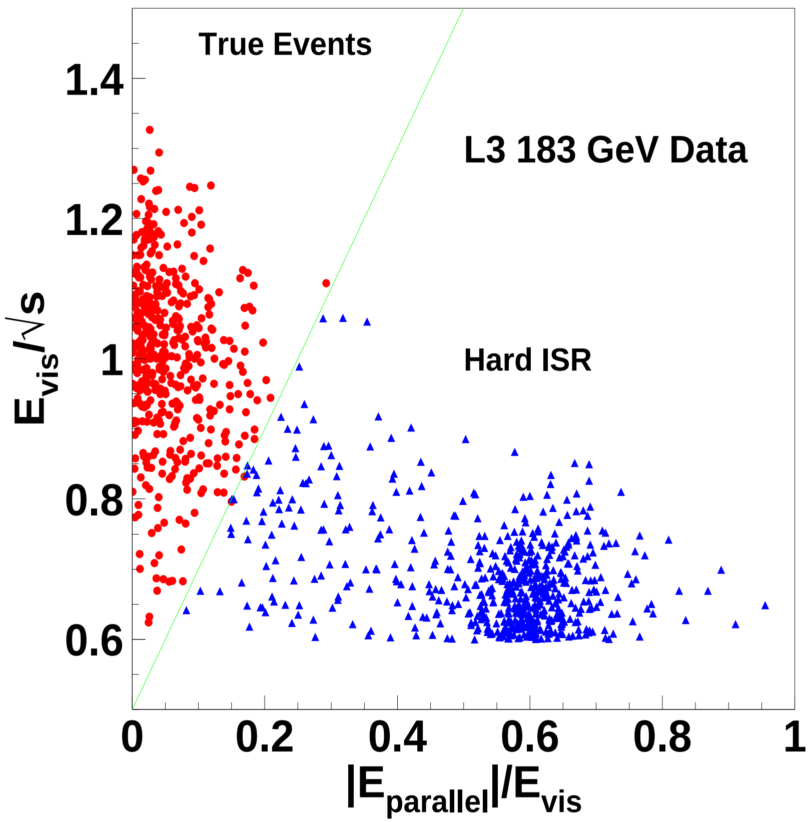
- ➢ Large visible energy
- ➢ Large number of particles in the final state

## Event Selection

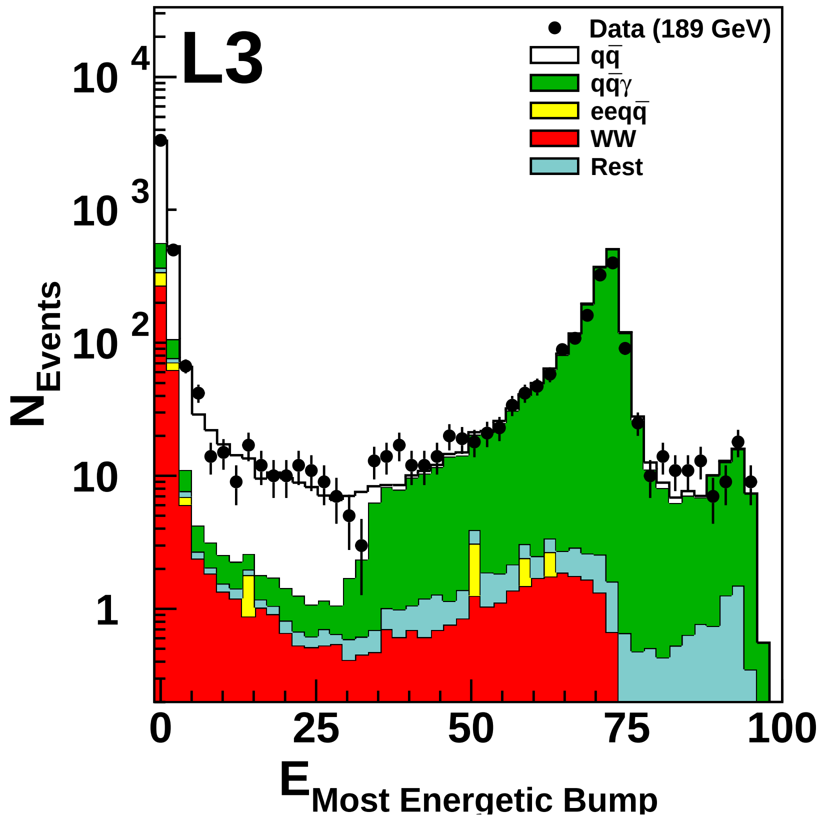
L3

Large contribution from radiative return to Z, characterized by

(a) Large $E_{||}$ due to missing $\gamma$




(b) Energetic high $p_T$ $\gamma$ in the detector




➜ Rejection requires

$E_{vis}/\sqrt{s} > 2.0\ (|\ E_{||}\ |/E_{vis}) + 0.5$

$E_\gamma < 30$ GeV

## Event Selection

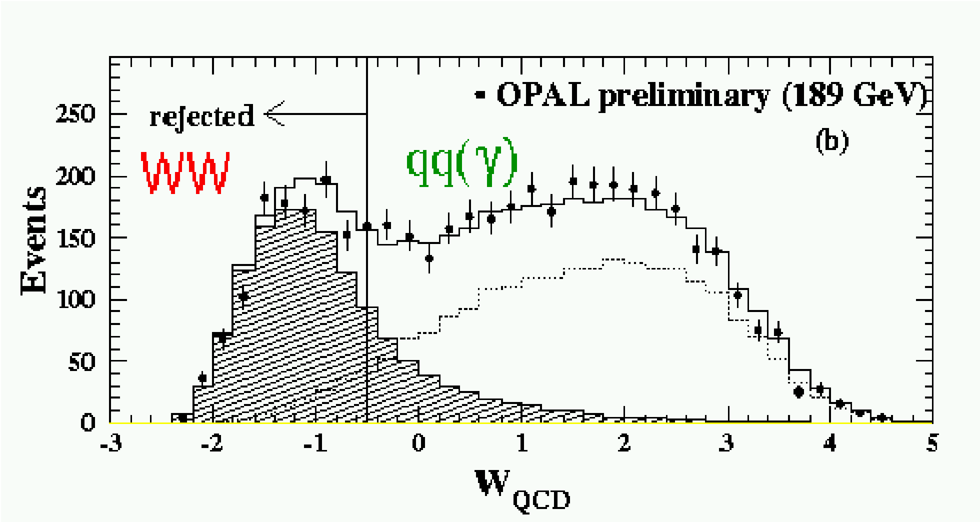
Large contribution from $W^+W^- \to$ 4 Jets beyond threshold

Test compatibility of events with QCD like production process.

OPAL

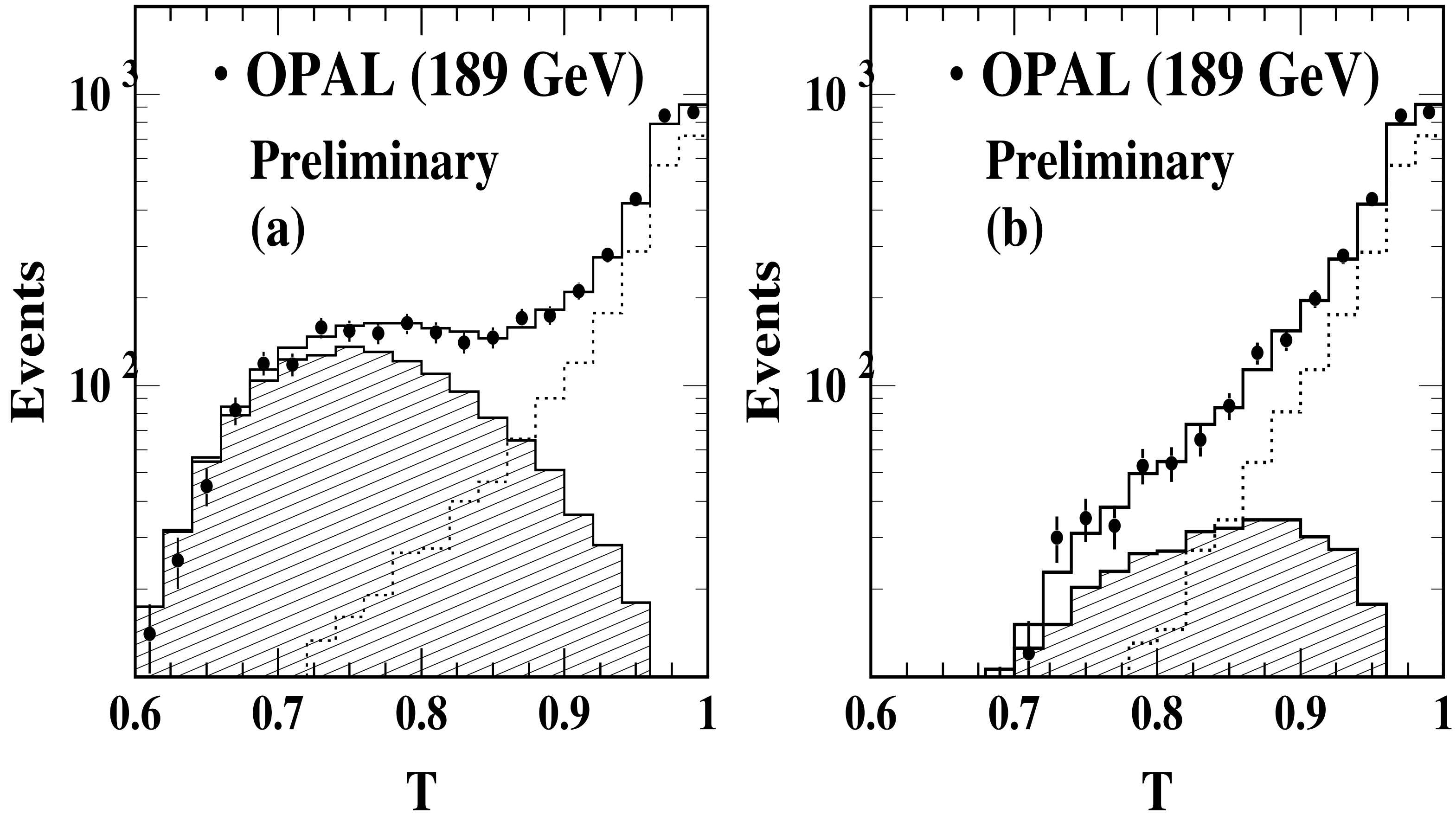
- Force events into 4-jet configuration using Durham algorithm
- Use reconstructed 4-momenta of the four jets to compute $\mathcal{O}(\alpha_s{}^2)$ matrix elements for processes $e^+e^- \to q\bar{q}q\bar{q}, q\bar{q}gg$
- No jet identification, calculate for each permutation of the jet momenta
- Define QCD event weight, the maximum of the permutations as

$$W_{QCD} = \max_{\{p_1,p_2,p_3,p_4\}} \log\left(|\,\mathcal{M}(p_1,p_2,p_3,p_4)\,|^2\right)$$




$W_{QCD} \geq -0.5$

## Correction of Event Shape Distributions




➜ Thrust (a) before and (b) after $W^+W^-$ rejection

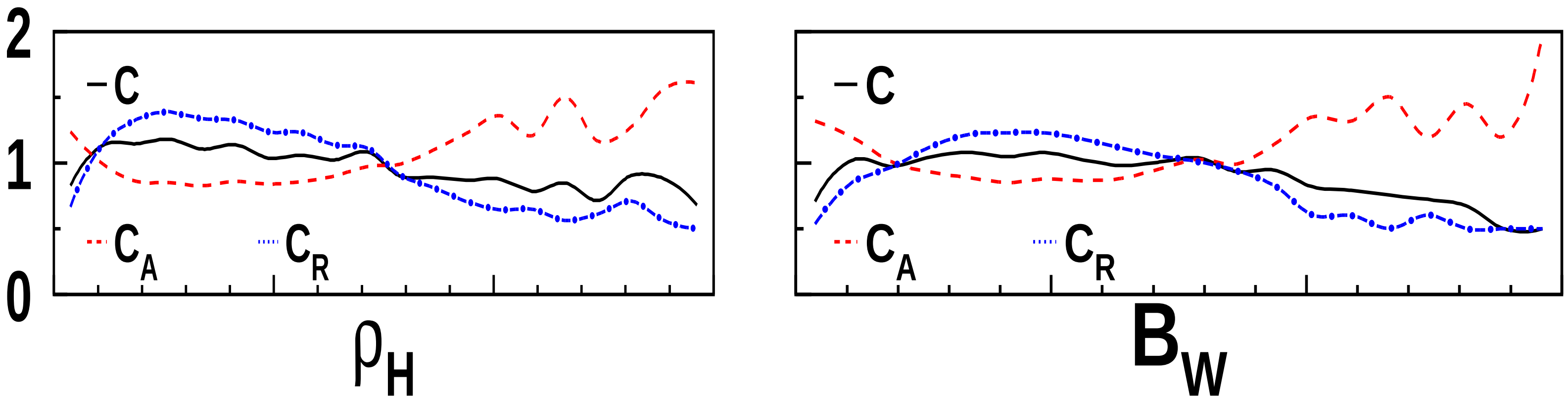
## Correction of Event Shape Distributions

Detector level distributions corrected bin-by-bin for

- ➢ Remaining background
- ➢ Finite detector resolution
- ➢ Acceptance
- ➢ Initial and final state radiation

➜ Charge multiplicity corrected using matrix unfolding method

L3




- ➢ Corrections smooth and small
- ➢ Corrected data compared to theory with hadronisation effects folded in

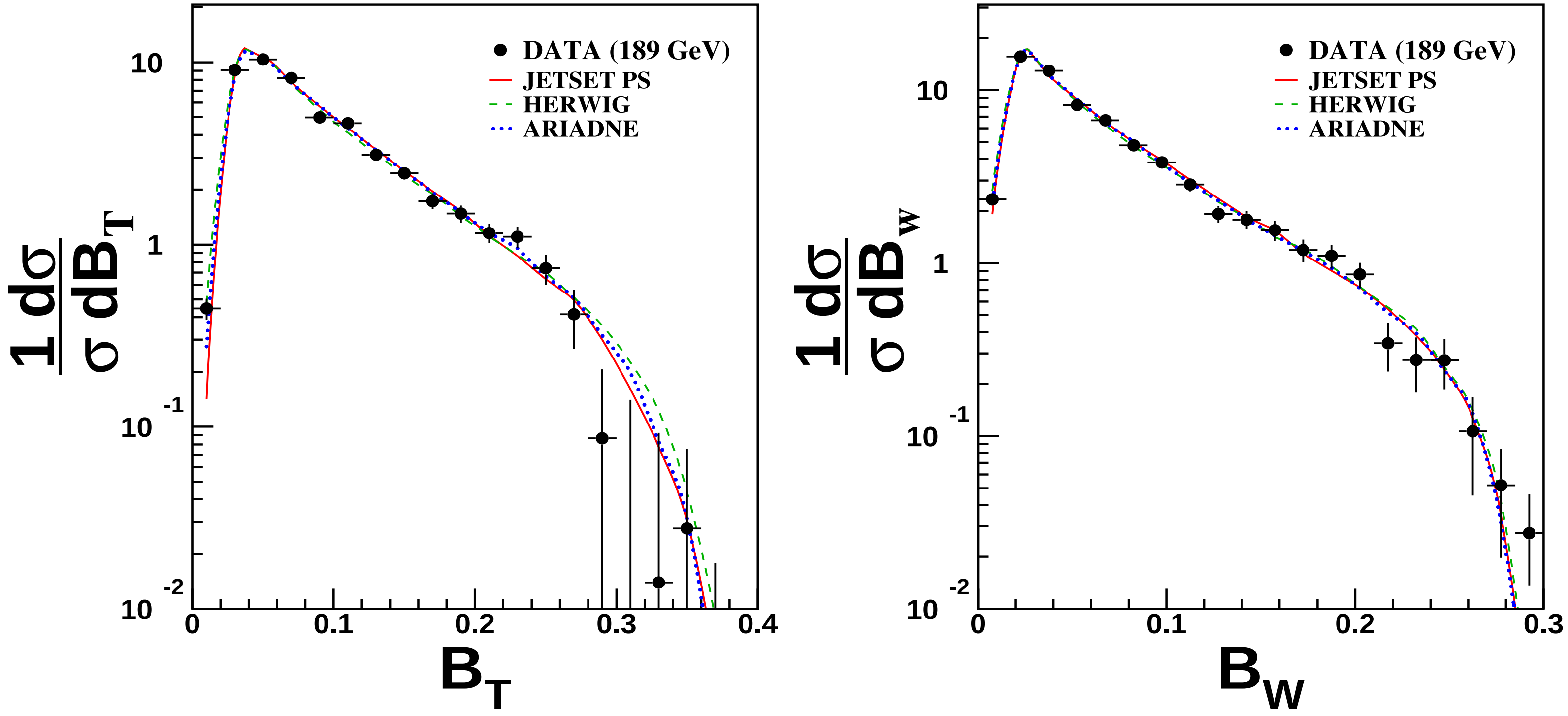
# QCD Models

| QCD Model | Shower Development | Fragmentation |
|---|---|---|
| JETSET | Coherent Parton Shower | String |
| HERWIG | Coherent Parton Shower | Cluster |
| ARIADNE | Color Dipole | String |
| COJETS | Incoherent Parton Shower | Independent |

All models tuned using LEP I data

L3

Comparison with QCD Predictions

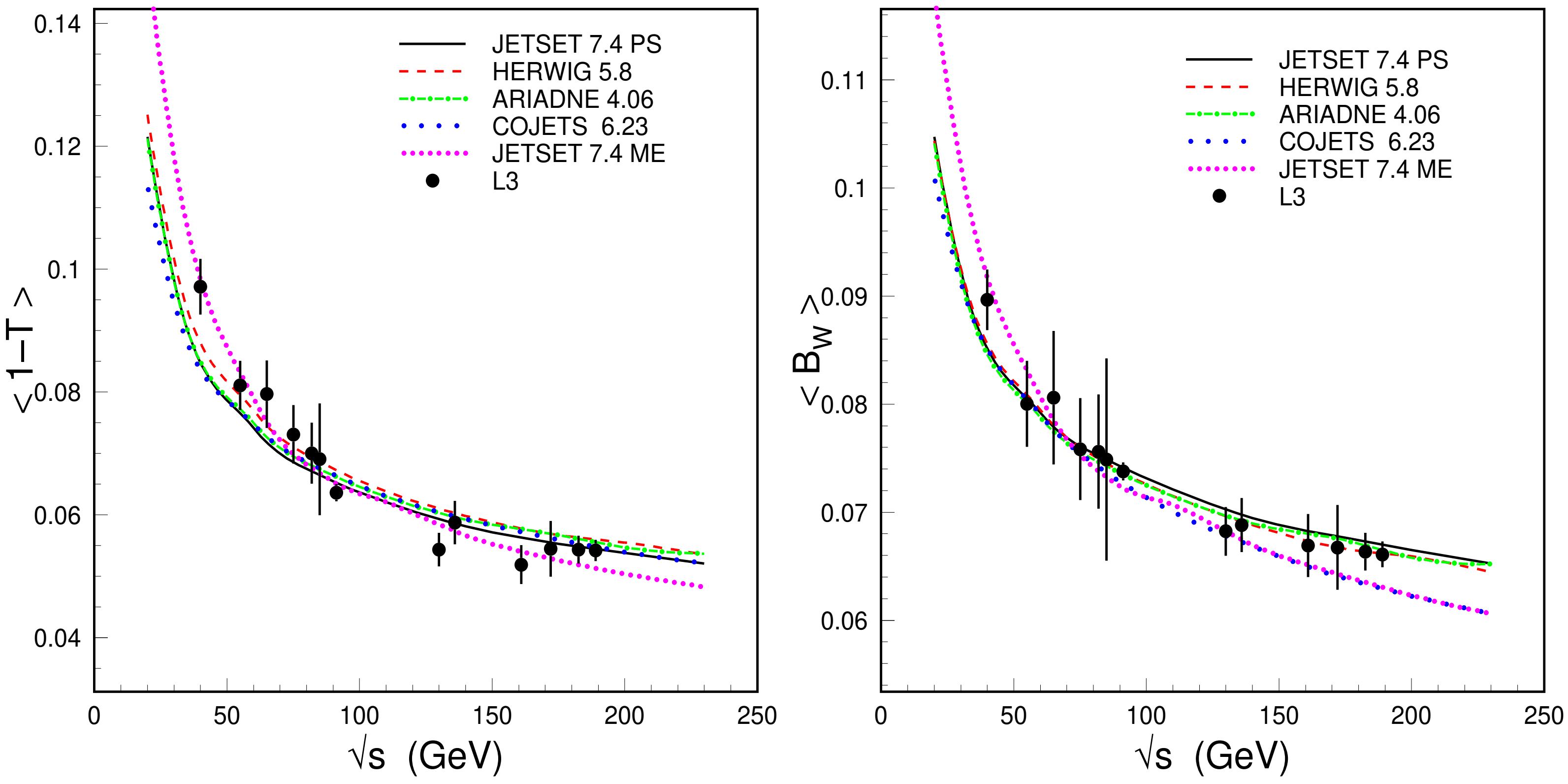


# Energy Evolution of Mean

Dependence of event shape variables on CM energy

- Logarithmic dependence of $\alpha_s$ on energy
- Fragmentation $\propto 1/Q^n$

L3

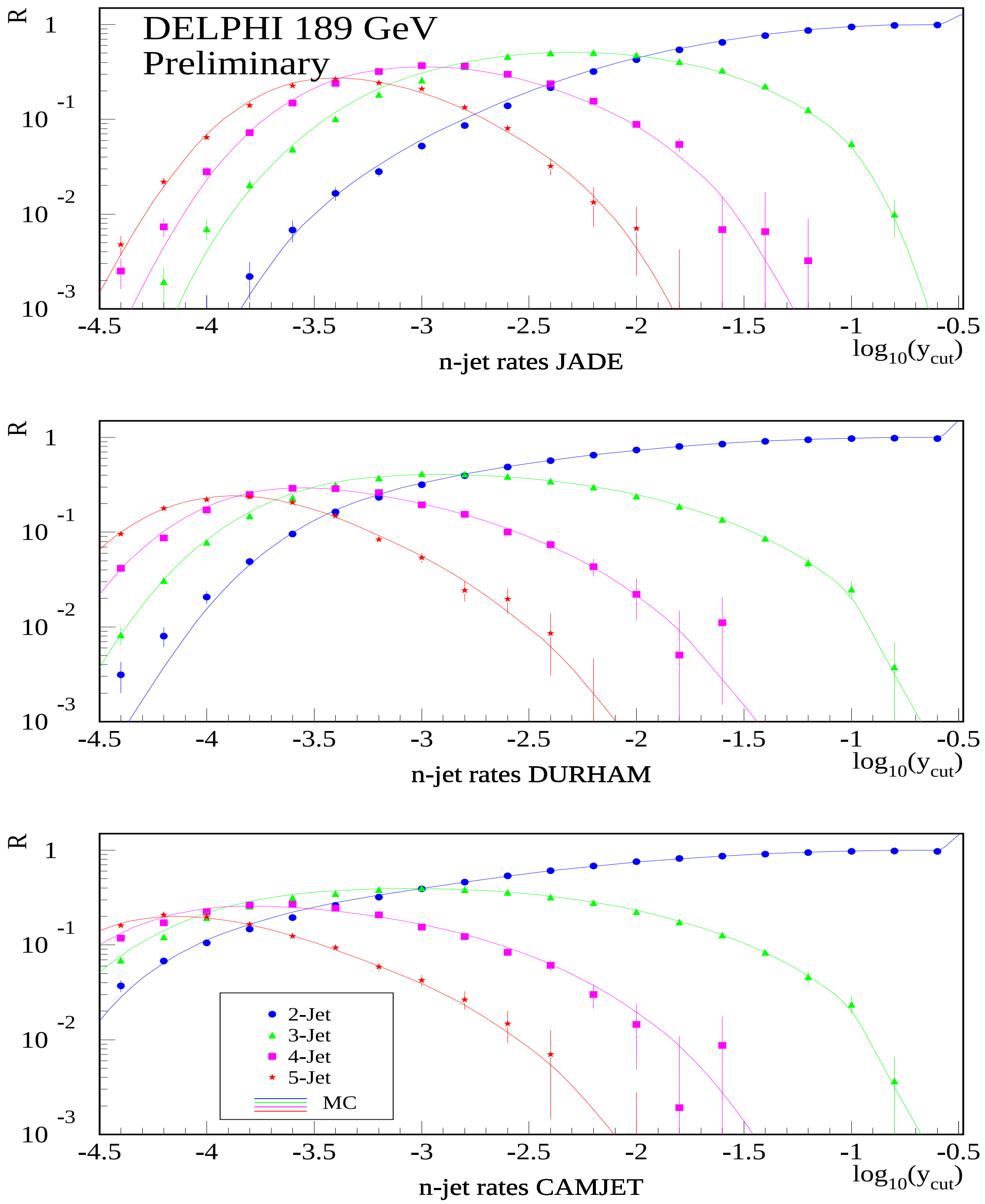



Models with coherent parton shower and string/cluster fragmentation describe data well

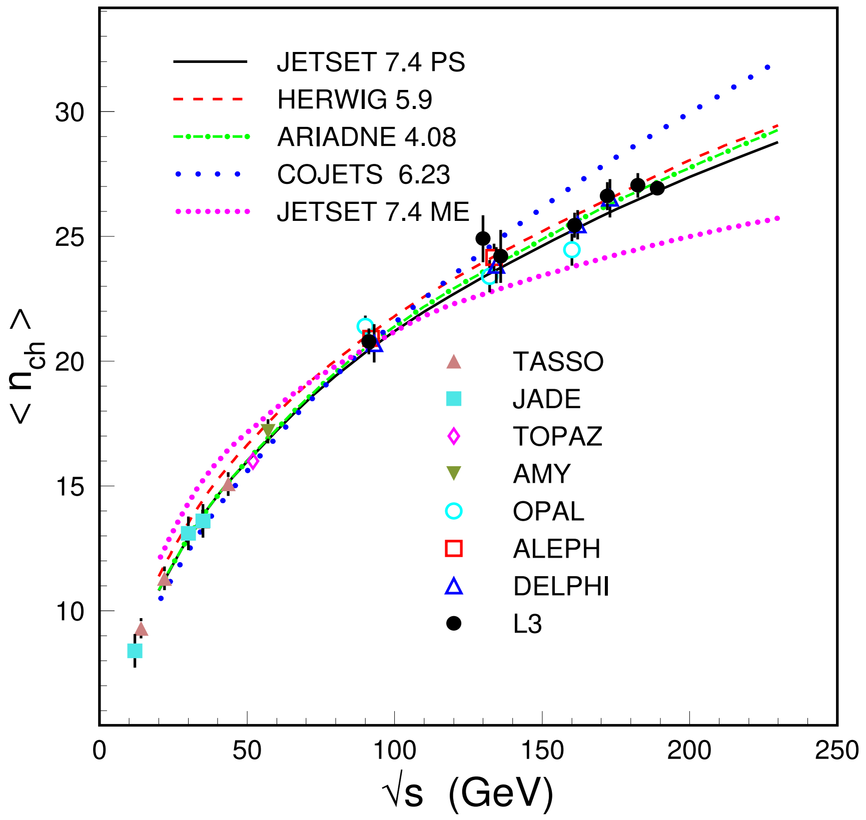
# Jet Rate

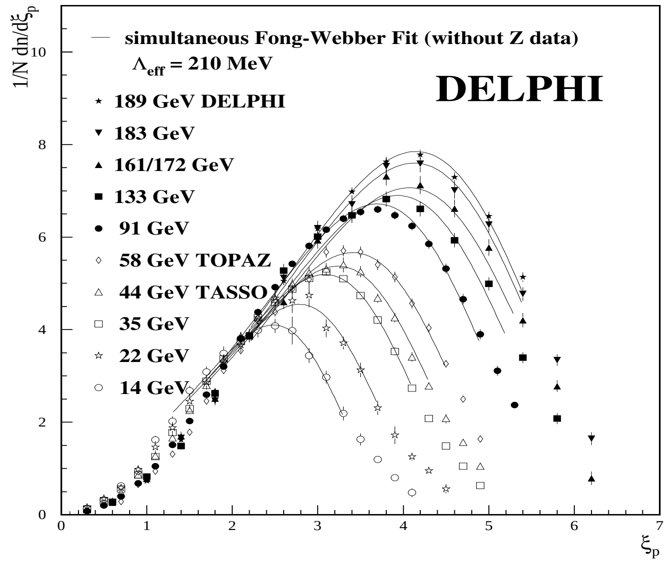
- ➢ Reconstruct jets : JADE, Durham, Cambridge algorithms
- ➢ Average jet multiplicity

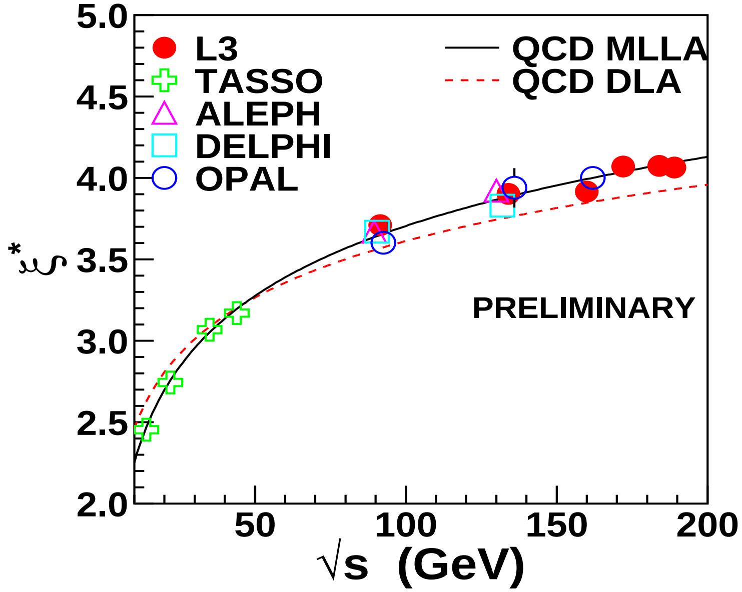
$$\langle n_{jet} \rangle \;=\; \frac{1}{N} \sum_{i=1}^{N} n^{i}_{jet}$$



# Charge Multiplicity




Inclusive Momentum Spectra

$$x_p = \frac{2\,p}{\sqrt{s}},\ \xi_p = \ln\left(\frac{1}{x_p}\right)$$

$$\xi^* \rightarrow \text{Peak position of } \xi_p$$

## Power Law Description of Fragmentation

First Moments:

Expressed as a sum of perturbative and power law contributions:

$$\langle f \rangle = \frac{1}{\sigma_{tot}} \int f \frac{d\sigma}{df} df = \underbrace{\langle f_{pert} \rangle}_{\mathcal{O}(\alpha_s{}^2)} + \underbrace{\langle f_{power} \rangle}_{c_f * \mathcal{P}}$$

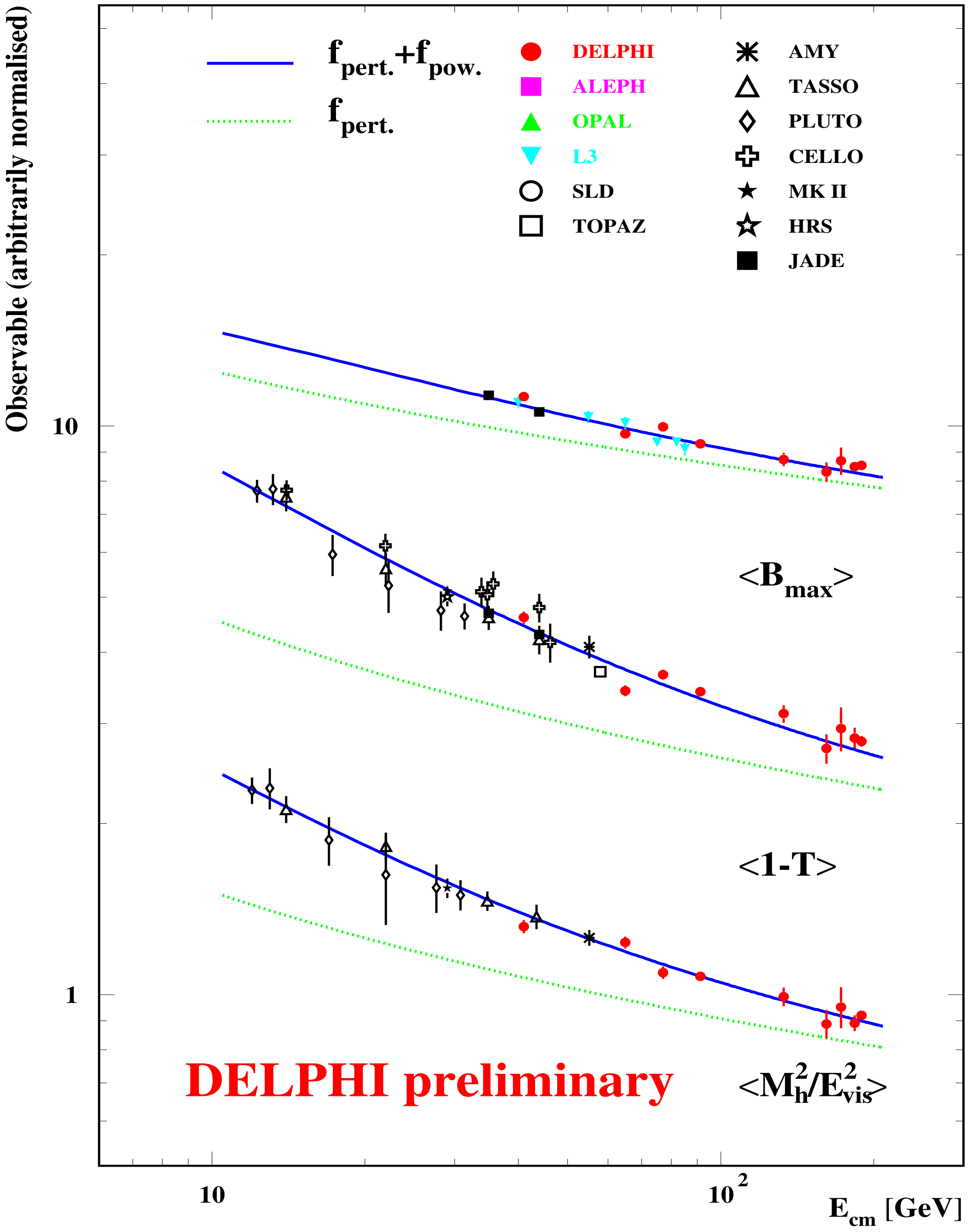
$$\langle f_{pert} \rangle = A \frac{\alpha_s(Q)}{2\pi} + (B + A \cdot (\beta_\circ \ln\mu - 2)) \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 \quad A,\ B \text{ known}$$

$$\langle f_{power} \rangle = c_f \cdot \mathcal{P}(\sqrt{s}, \alpha_s(\sqrt{s}), \mu_I, \alpha_\circ(\mu_I))$$

- $\mathcal{P}$ is universal
  - ➜ 'a shift $\propto 1/Q$', for 1-T, C, $\rho_H$
  - ➜ 'a shift $\propto 1/Q$' + 'logarithmic-skew', for $B_T$, $B_W$
- $c_f = 2, 2, 1, \frac{1}{2}$ and $3\pi$ for 1-T, $\rho$, $B_T$, $B_W$ and C resp.
- $\alpha_\circ$ accounts for contributions below matching scale $\mu_I$ ($= 2$ GeV)

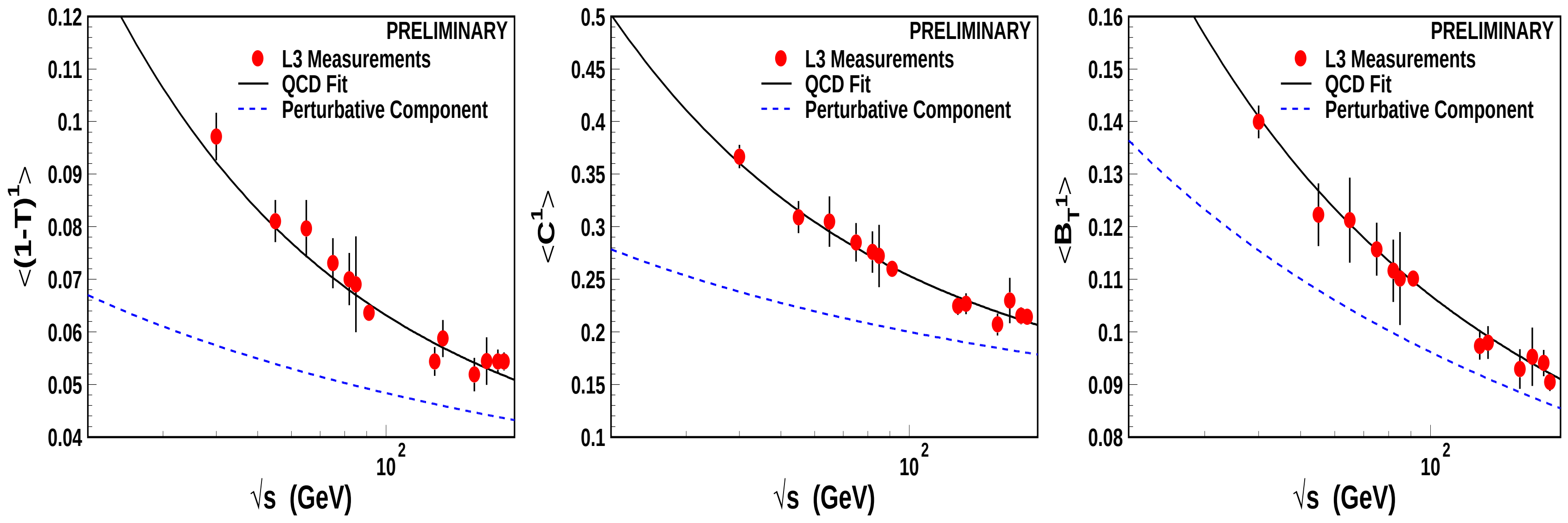
| Free parameters: |
|---|
| ❶ $\alpha_s(m_Z)$ |
| ❷ $\alpha_\circ(\mu_I) = \frac{1}{\mu_I} \int_0^{\mu_I} dq\, \alpha_{eff}(q)$ |

# Power Law




- ➢ Perturbative component alone cannot describe energy evolution of mean
- ➢ At high enough energies power correction term becomes less significant

# Power Law : $\alpha_s$




| Observable | $\alpha_\circ$ (2 GeV) | $\alpha_s(m_Z)$ | $\chi^2$/d.o.f. |
|---|---|---|---|
| $< 1 - T >$ | $0.567 \pm 0.054$ | $0.1097 \pm 0.0047$ | $10.4/11$ |
| $< \rho_H >$ | $0.469 \pm 0.041$ | $0.1032 \pm 0.0039$ | $5.6/11$ |
| $< B_T >$ | $0.480 \pm 0.034$ | $0.1148 \pm 0.0028$ | $4.5/11$ |
| $< B_W >$ | $0.450 \pm 0.069$ | $0.1131 \pm 0.0035$ | $2.4/11$ |
| $< C >$ | $0.482 \pm 0.031$ | $0.1120 \pm 0.0033$ | $6.2/11$ |
| Average: $\alpha_s(m_Z) = 0.1106 \pm 0.0036 \pm 0.0040$ | | | |

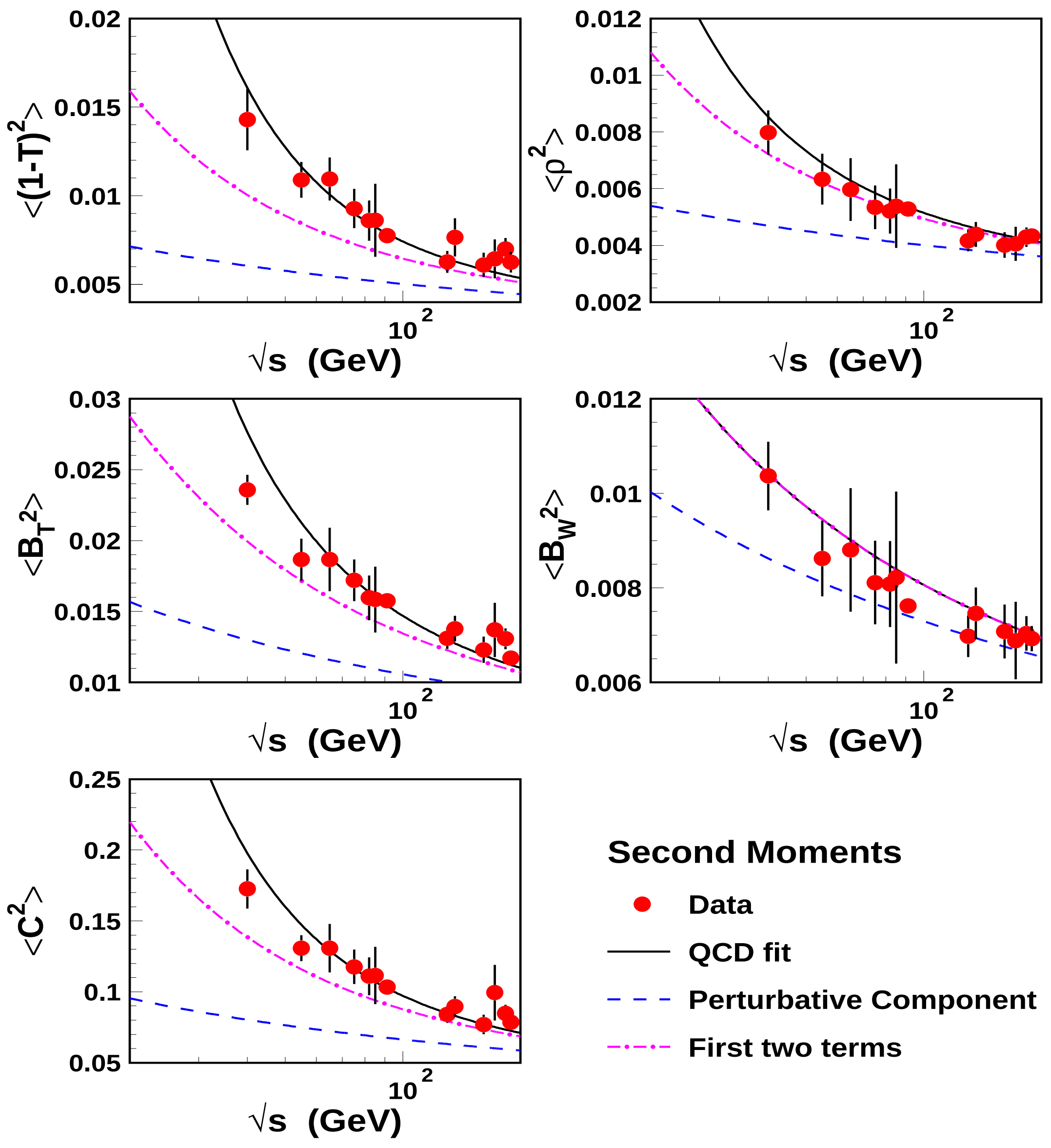
L3

Evidence of universality of the power law correction

# Second Moments

$$\langle \mathrm{f}^2 \rangle \quad = \quad \langle {\mathrm{f}_{\mathrm{pert}}}^2 \rangle + 2\, \langle \mathrm{f}_{\mathrm{pert}} \rangle \cdot \mathrm{c_f}\, \mathcal{P}(\sqrt{\mathrm{s}}) + \mathcal{O}\left(\frac{1}{\mathrm{Q}^2}\right)$$

- First two terms from fit results for first moments
- The remaining $\mathcal{O}\left(\frac{1}{Q^2}\right)$ term is fit for each variable
- The above formula is reliable for $1 - \mathrm{T}$, $\rho$ and C.



## Analytical Calculations

- Complete Calculations to $\mathcal{O}(\alpha_s{}^2)$

$$
\begin{aligned}
\mathrm{R}(\alpha_s, \mathrm{y}) &\equiv \int_0^{\mathrm{y}} \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{dy}} \\
&= \overline{\alpha}_s \mathrm{A}(\mathrm{y}) + \overline{\alpha}_s{}^2 \left[\mathrm{B}(\mathrm{y}) + 2\pi\beta_\circ \ln(\mu^2/\mathrm{s})\mathrm{A}(\mathrm{y})\right]
\end{aligned}
$$

→ Describe data well in the multi-jet region

→ But fail in the two jet region (small y) where multiple soft/collinear gluon emissions become dominant

- Resum

| | | |
|---|---|---|
| Leading-Log | $\sum(\alpha_s \mathrm{L}^2)^n$ | (L = ln(1/y)) |
| Next-to-Leading-Log | $\frac{1}{\mathrm{L}}\sum(\alpha_s \mathrm{L}^2)^n$ | terms |

- Combine the two

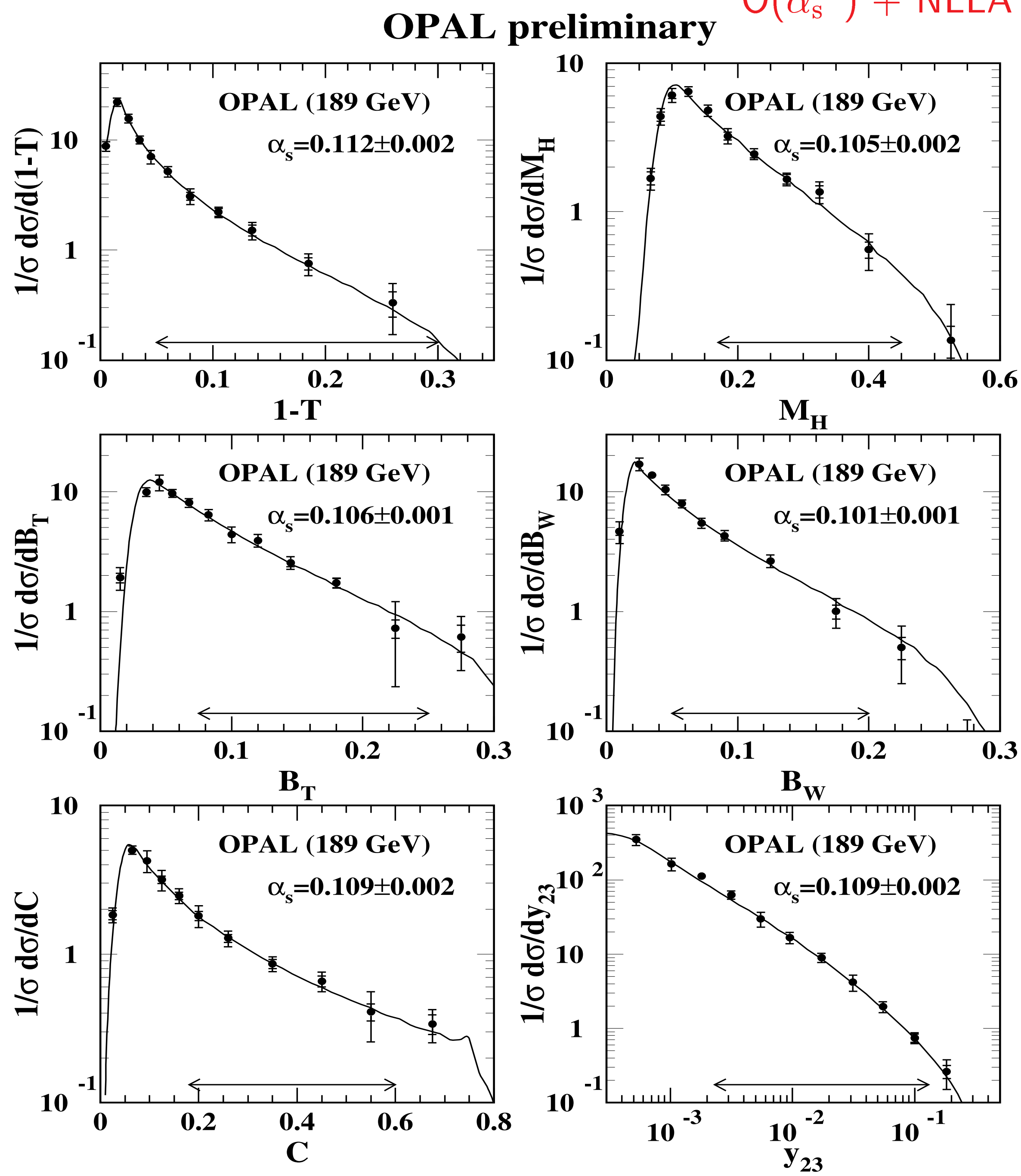
Describe data over wide range

- Improved analytical calculations exist for T, $\rho_{\mathrm{H}}$, $\mathrm{B_T}$, $\mathrm{B_W}$, C, $\mathrm{y_3}$, $\mathrm{D_2}$

- Hadronisation effect folded in using PS Monte Carlo

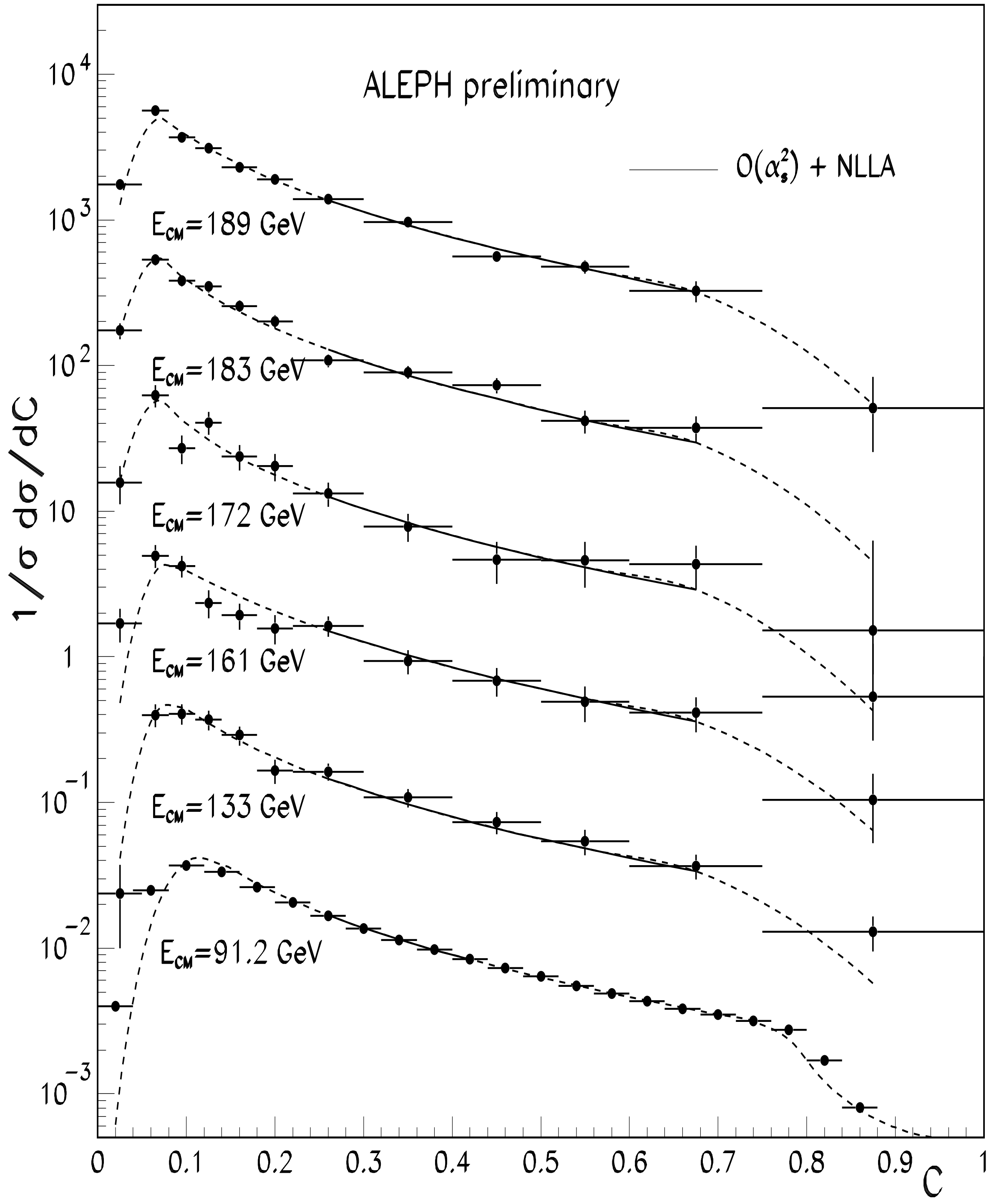
## Fit to QCD

$O(\alpha_s{}^2)$ + NLLA



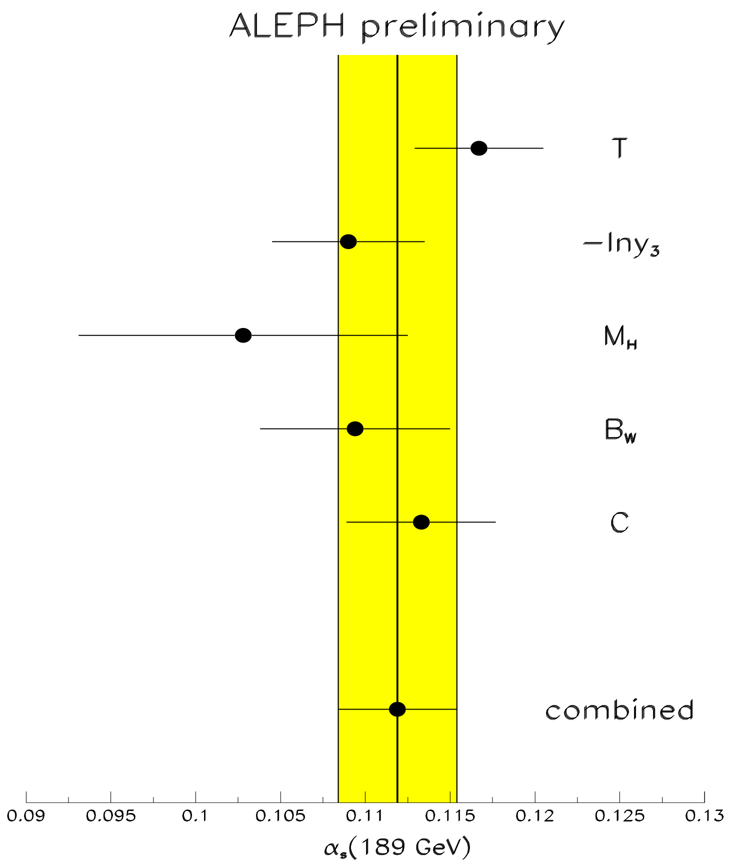

- ➢ Fit range narrower than region of agreement
- ➢ L3 uses similar variables, with wider range

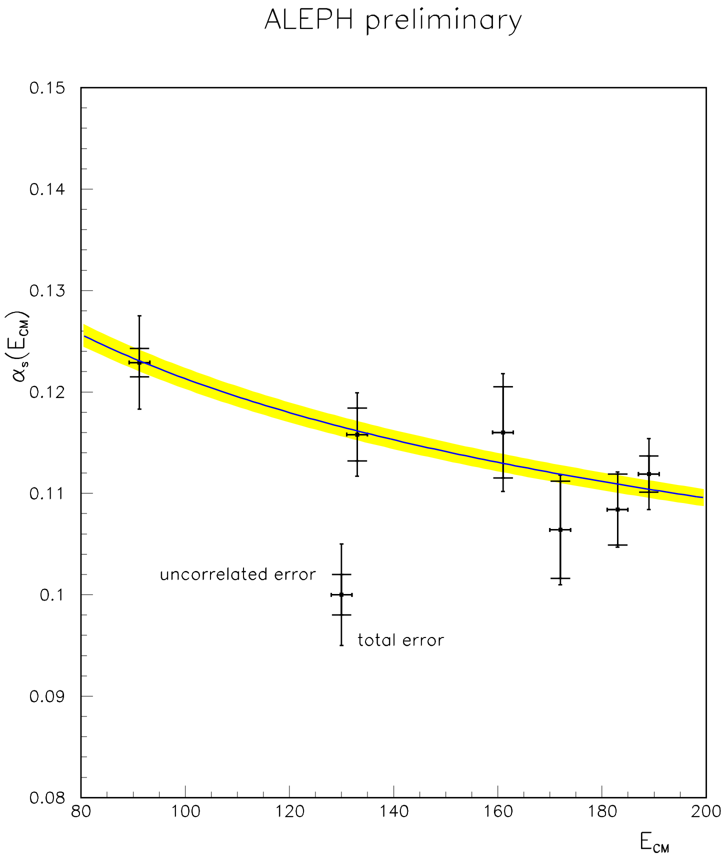
## Fit to QCD




➢ Good fit is obtained over a wide range of CM energies

## Strong Coupling Constant



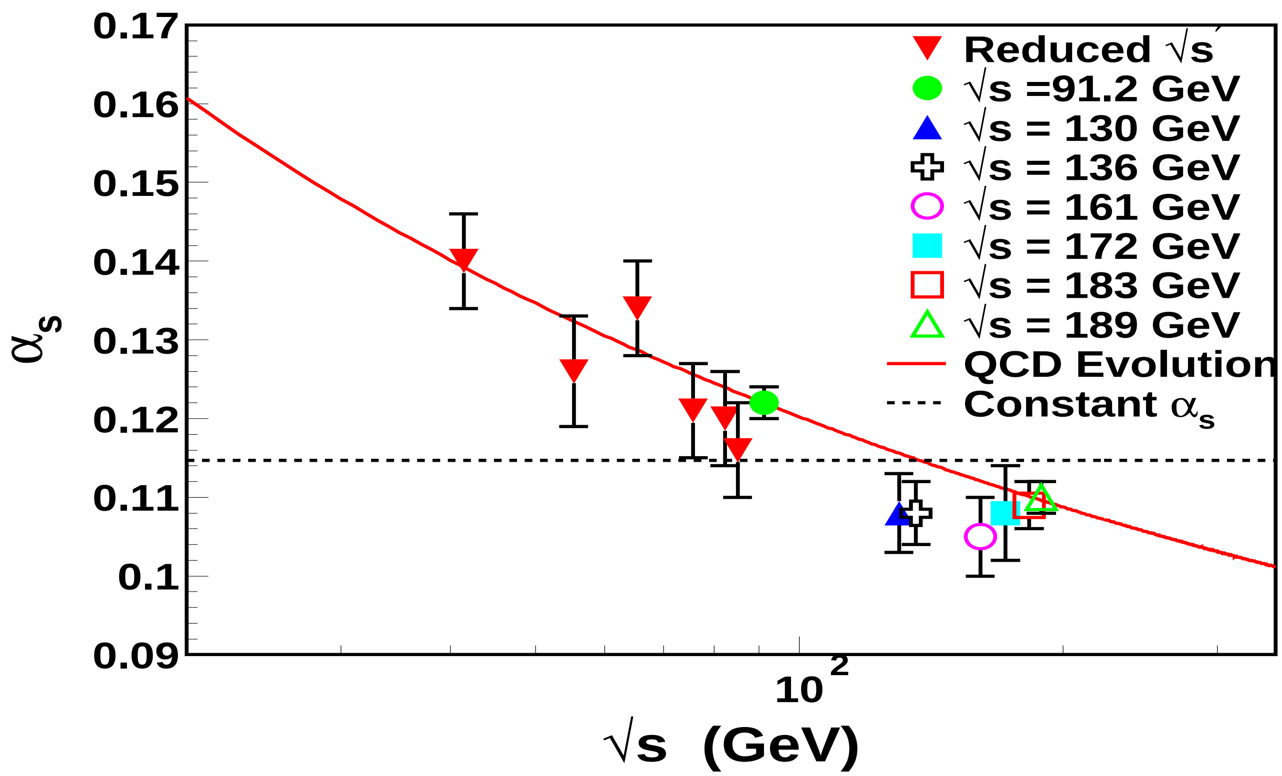




$$\alpha_s\,(189\,\mathrm{GeV}) = 0.1119 \pm 0.0015\,(\mathrm{stat}) \pm 0.0011\,(\mathrm{exp}) \pm 0.0030\,(\mathrm{theo})$$

$$\alpha_s\,(m_Z) = 0.1216 \pm 0.0039$$

# Strong Coupling Constant

L3




| | $\alpha_s(m_Z)$ | $\chi^2$/dof | Probability |
|---|---|---|---|
| QCD Evolution | $0.1220 \pm 0.0015 \pm 0.0060$ | $15.4/12$ | 0.22 |
| Constant | $0.1144 \pm 0.0015$ | $115/12$ | $1.5 \times 10^{-5}$ |

Supports QCD evolution

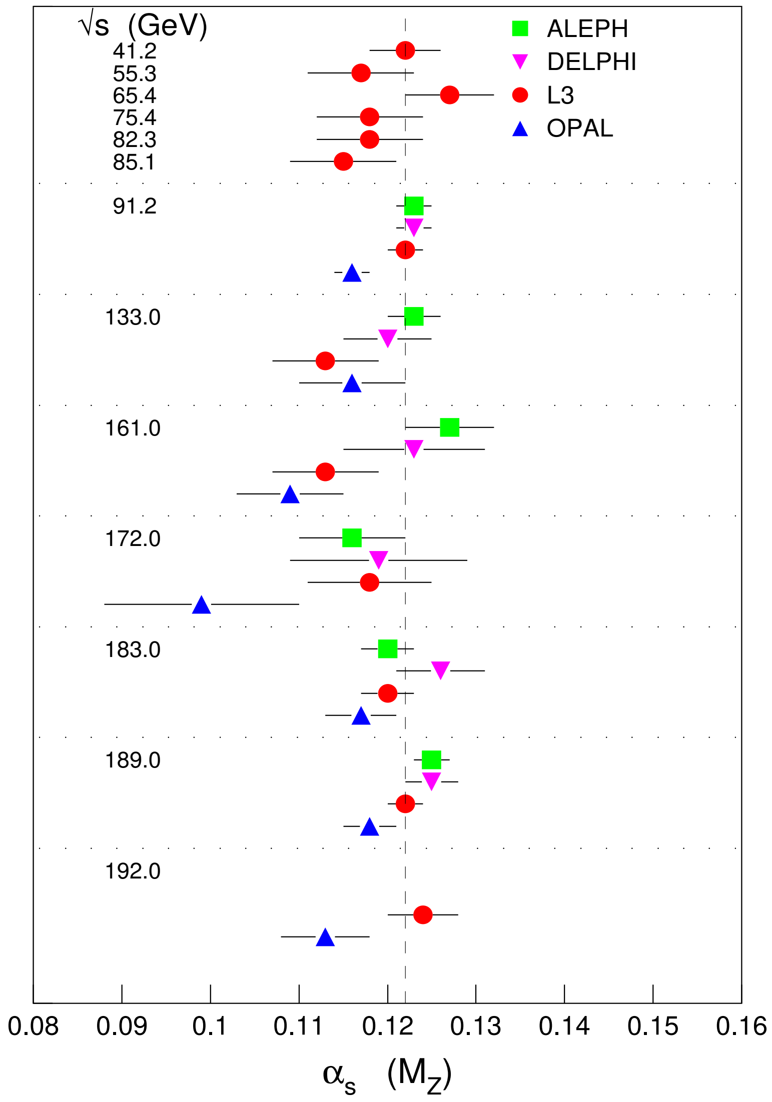
Number of active flavours: $N_f = 4.4 \pm 1.0\,(\mathrm{exp}) \pm 2.0\,(\mathrm{theo})$

## $\alpha_s$ at LEP

➢ $O(\alpha_s{}^2)$ + NLLA

| Experiment | Variables |
|---|---|
| ALEPH | T, $-\ln y_3$, $M_H$, $B_W$, C |
| DELPHI | T, $\rho_H$ |
| L3 | T, $\rho_H$, $B_T$, $B_W$, C |
| OPAL | T, $y_{23}$, $M_H$, $B_T$, $B_W$, C |

➢ For the same variable, different fit ranges used by different experiments



## Summary

- QCD studies : 30 GeV $\leq \sqrt{s} \leq$ 196 GeV at LEP
- Data and QCD models including coherence effects agree well
- Recent theoretical developments, e.g
  - Resummed $O(\alpha_s{}^2)$ calculation for C parameter
  - Power law description of fragmentation
  - Improved jet clustering algorithm (Cambridge)

  provide a better understanding of systematic errors
- Power correction investigated by all the 4 experiments
- Combination of results from individual experiments in progress.
  Useful for energies where the statistics is limited
- The running of $\alpha_s$ is demonstrated at LEP