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Updated Parameters of the Z^0 Resonance from Combined Preliminary Data of the LEP Experiments

The LEP Electroweak Working Group*

Internal Note

summarizing the combination of preliminary LEP data for the LaThuile and Moriond conferences, March 1994

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1 Introduction

We evaluate updated parameters of the Z^0 resonance derived from published data and new preliminary results. Most of these preliminary results are contributions to the 1994 winter conferences at LaThuile and Moriond. The emphasis of this note is a documentation of the combination of electroweak parameters of the four LEP experiments, taking account of errors which are correlated among the experiments. Several aspects of such a combination have already been studied in [1, 2].

2 Z^0 Lineshape and Lepton Forward-Backward Asymmetries

The results presented here are based on the results from energy scans in 1990 and 1991 with centre-of-mass energies in a range $\sqrt{s} = M_Z \pm 3$ GeV, the high statistics data collected at the Z^0 peak in 1992, and a preliminary analysis of the high precision scan in 1993. During this scan more than 18 pb^{-1} were recorded by each experiment at two centre-of-mass energy points roughly 1.8 GeV above and below the Z^0 mass while the remainder was within 200 MeV of M_Z.

The total statistics and the systematic errors of the individual LEP collaborations are given in Tables 1 and 2. Details of the individual analyses can be found in [3, 4, 5, 6]. An important aspect of the lineshape analysis is a precise knowledge of the LEP centre-of-mass energies. The treatment of the LEP centre-of-mass energies by the four LEP experiments is based on [7, 8]. For the combination with data recorded before 1993 only the earlier Γ_Z measurements contribute to the combined result. As recommended [8], for these preliminary results we do not use the information from the earlier M_Z measurement as the correlation of systematic errors between the 1991 and 1993 M_Z measurements has not yet been studied.

For the averaging of results the LEP experiments provide a standard set of 9 parameters describing the information contained in hadronic and leptonic cross-sections and leptonic forward-backward asymmetries [1]. The four sets of 9 parameters provided by the LEP experiments from fits to the data taken until the end of 1992 and preliminary data for 1993 are presented in Table 3. The full covariance matrix of the input parameters is given in Tables 4, 5 and 6. The combined parameter set and its correlation matrix are given in Tables 7 and 8.

Lepton universality can be used to reduce the set of 9 parameters given above to a set of 5 parameters. Due to mass corrections to $\Gamma_{\tau\tau}$ we expect, however, a small (0.2%) difference between the values for R_e and R_μ and the value for R_τ. We define $R_\ell \equiv \Gamma_{had}/\Gamma_{\ell\ell}$, where $\Gamma_{\ell\ell}$ refers to the partial Z^0 width for the decay into a pair of massless charged leptons.

The data of each of the four LEP experiments and their combination are consistent with lepton universality. Based on this assumption Table 9 provides the five parameters M_Z, Γ_Z, σ^0_h, R_ℓ and $A^{0,\ell}_{FB}$ for the individual LEP experiments. The four experiments now all use the above definition of $\Gamma_{\ell\ell}$ (for other choices c.f. [1]). Therefore we have two choices to derive a combined 5-parameter result: as a transformation of the combined 9-parameter result or as a combination of the separate 5-parameter results of the 4 experiments. The latter procedure allows to incorporate from the 1993 data of DELPHI not only the data of the the hadronic channel but also a flavour independent lepton pair analysis. The full covariance matrix for the combination of the 5-parameter results of the 4 experiments can be constructed from Tables 4, 10 and 11.

Tables 12 and 13 provide the five parameters M_Z, Γ_Z, σ^0_h, R_ℓ and $A^{0,\ell}_{FB}$ and the corresponding correlation matrix for the combined result of the 4 LEP experiments. Figure 1 shows, for each

leptonic species and for the combination assuming lepton universality, the resulting one standard deviation contours in the R_ℓ-$A_{FB}^{0,\ell}$ plane.

The parameter R_ℓ can be used to determine the strong coupling constant α_s. For $M_Z = 91.1895$ GeV, $M_t = 150$ GeV and $M_H = 300$ GeV we obtain from $R_\ell = 20.789 \pm 0.040$ alone:

$$\alpha_s(M_Z^2) = 0.124 \pm 0.006 \,.$$

The error quoted accounts for experimental uncertainties only. A discussion of theoretical uncertainties can be found in [9]. A more precise value of $\alpha_s(M_Z^2)$ will be given in the context of a global fit to all electroweak precision measurements in section 7.

The parameters M_Z, Γ_Z, σ_h^0, R_ℓ and $A_{FB}^{0,\ell}$ are convenient for fitting and averaging since they have minimal correlations amongst them. However a number of other parameters, which can be derived from the previous set, are of physical importance. Table 14 gives a number of commonly used parameters derived from the results of the 9-parameter (Tables 7 and 8) and the 5-parameter fit (Tables 12 and 13). Among these parameters we quote $\sin^2\theta_{eff}^{lept}$ which is defined as:

$$\sin^2\theta_{eff}^{lept} \equiv \frac{1}{4}(1 - g_{V_\ell}/g_{A_\ell}) \,. \qquad (1)$$

As an indication of the power of these new data we use them to derive the number of light neutrino species. Using the results of Tables 12 and 13 we find:

$$\Gamma_{inv}/\Gamma_{\ell\ell} \;=\; 5.947 \pm 0.046 \,.$$

Taking the Standard Model value for the ratio of the partial widths to neutrinos and charged leptons:

$$\Gamma_\nu/\Gamma_{\ell\ell} \;=\; 1.992 \pm 0.003 \,,$$

where the central value is evaluated for $M_Z = 91.1895$ GeV, $M_t = 150$ GeV, $M_H = 300$ GeV and $\alpha_s(M_Z^2) = 0.123$ and the error quoted accounts for a variation of M_t in the range $100 < M_t(\mathrm{GeV}) < 200$ and a variation of M_H in the range $60 < M_H(\mathrm{GeV}) < 1000$, we find:

$$N_\nu \;=\; 2.985 \pm 0.023 \,.$$

The parameters discussed above, however, do not describe the properties of the Z^0 completely, as they do not contain information about the interference of the Z^0 exchange with the photon exchange. In the case of charged leptons, assuming no modifications in the Born diagrams, the γZ^0 interference term could be expressed in terms of the leptonic effective vector and axial-vector couplings, g_{V_ℓ} and and g_{A_ℓ}, which can be determined by the measurements discussed above. However, this is not the case for the inclusive hadronic final state, which is summed over all colours and open quark flavours. Thus, the hadronic γZ^0 interference term remains as an additional undetermined parameter unless it is fixed to its Standard Model expectation value.

Because of the asymmetric nature of the γZ^0 interference cross section as a function of the centre-of-mass energy when crossing the Z^0 pole, the hadronic γZ^0 interference term is strongly anti-correlated with M_Z. Therefore, its uncertainty substantially increases the total error on M_Z (by ≈ 8 MeV for a single LEP experiment) compared to the situation where it is taken from the Standard Model. The errors on the total and partial widths of the Z^0 increase only marginally.

Work is in progress to implement the γZ^0 interference term as an input parameter in the various fitting programs in order to include its effects in more detail in future analyses. For the results presented below, the hadronic γZ^0 interference term is fixed to its Standard Model value.

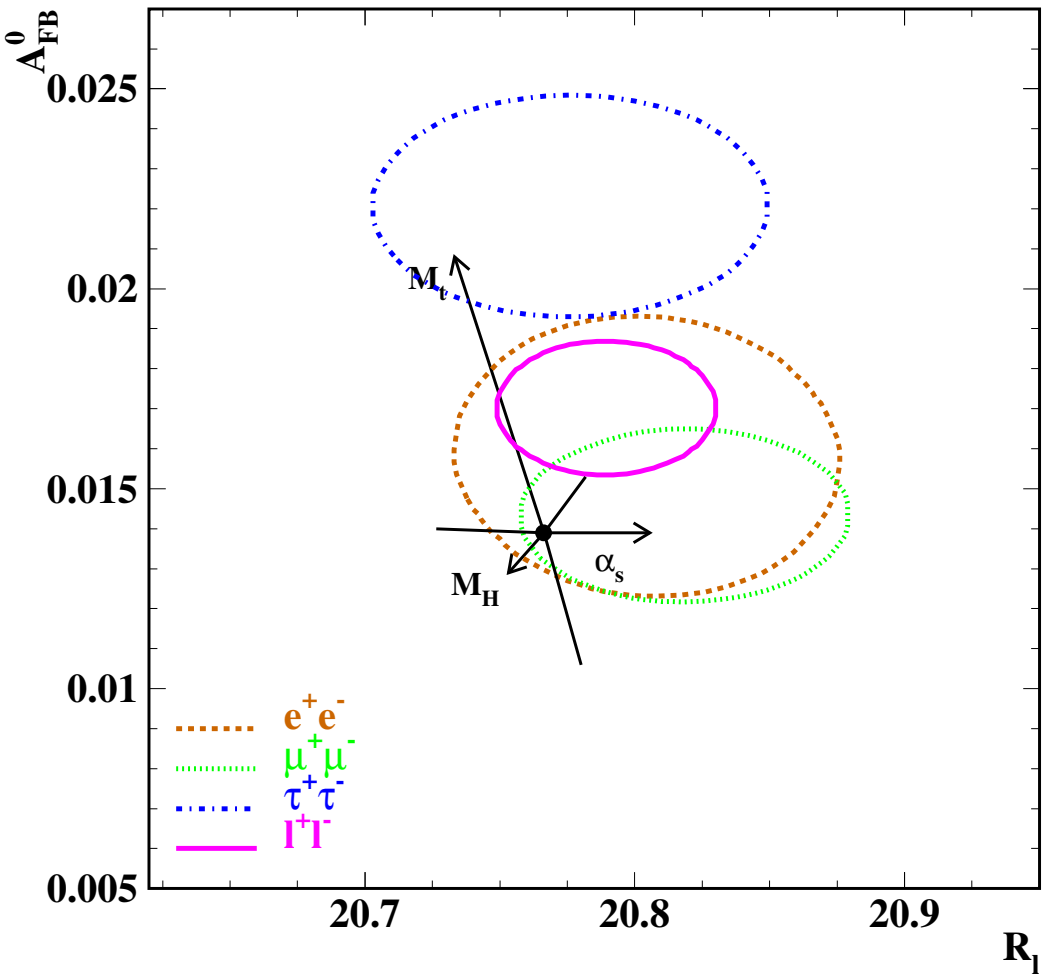

Figure 1: One standard deviation contours (39% probability) in the R_ℓ-$A_{FB}^{0,\ell}$ plane. Also shown as dotted symbol is the Standard Model prediction for $M_Z = 91.1895$ GeV, $M_t = 150$ GeV, $M_H = 300$ GeV, $\alpha_s = 0.123$. The lines with arrows correspond to the variation of the Standard Model prediction when M_t, M_H or α_s are varied in the intervals $50 < M_t(\mathrm{GeV}) < 250$, $60 < M_H(\mathrm{GeV}) < 1000$ and $\alpha_s(M_Z^2) = 0.123 \pm 0.006$, respectively. The arrows point in the direction of increasing values for M_t, M_H and α_s.

		ALEPH	DELPHI	L3	OPAL	LEP
$q\bar{q}$	'90-'91	451	356	416	454	1677
	'92	680	697	678	733	2788
	'93 prel.	653	677	671	656	2657
	total	1784	1730	1765	1843	7122
$\ell^+\ell^-$	'90-'91	55	37	39	58	189
	'92	82	69	59	88	298
	'93 prel.	79	67	$-^{(a)}$	83	229
	total	216	173	98	229	716

Table 1: The LEP statistics in units of 10^3 events used for the analysis of the Z^0 line shape and lepton forward-backward asymmetries.
$^{(a)}$No preliminary result quoted yet.

	ALEPH		DELPHI		L3		OPAL	
	'92	'93 prel.	'92	'93 prel.	'92	'93 prel.	'92	'93 prel.
$\mathcal{L}^{\text{exp.}\ (b)}$	0.15%	0.09%	0.38%	0.28%	0.5 %	0.16%$^{(c)}$	0.41%	$-^{(a)}$
σ_{had}	0.14%	0.14%	0.13%	0.13%	0.15%	0.25%	0.20%	0.24%
σ_{e}	0.4 %	0.4%	0.59%	$-^{(a)}$	0.3 %	$-^{(a)}$	0.22%	0.35%
σ_{μ}	0.5 %	0.5%	0.37%	0.5%	0.5%	$-^{(a)}$	0.19%	0.22%
σ_{τ}	0.3%	0.3%	0.63%	$-^{(a)}$	0.7 %	$-^{(a)}$	0.44%	0.51%
$A_{\text{FB e}}$	0.0029	0.0029	0.003	$-^{(a)}$	0.002	$-^{(a)}$	0.002	0.002
$A_{\text{FB }\mu}$	0.001	0.001	0.001	0.002	0.002	$-^{(a)}$	0.001	0.001
$A_{\text{FB }\tau}$	0.0005	0.0005	0.0017	$-^{(a)}$	0.003	$-^{(a)}$	0.002	0.002

Table 2: The experimental systematic errors for the analysis of the Z^0 line shape and lepton forward-backward asymmetries. The errors quoted do not include the common uncertainty due to the LEP energy calibration. For the treatment of correlations between the errors for different years we refer to [3, 4, 5, 6].
$^{(a)}$No preliminary result quoted yet.
$^{(b)}$Only the experimental error including the statistics of small angle Bhabha events is quoted. In addition, there is a theoretical error for the calculation of the small angle Bhabha cross section. The experiments have a different acceptance for small angle Bhabha scattering and quote a different value for this theoretical error: 0.25% for ALEPH, DELPHI and L3; 0.3% for OPAL.
$^{(c)}$The L3 Collaboration has given a preliminary update of M_{Z} and Γ_{Z} which includes the 1993 data. Their preliminary 1993 absolute luminosity measurement is not yet used for the results quoted in the following Tables.

	ALEPH	DELPHI	L3	OPAL
$M_{\rm Z}$(GeV)	91.1915±0.0052	91.1883±0.0052	91.1951±0.0054	91.1860±0.0054
$\Gamma_{\rm Z}$(GeV)	2.4959±0.0061	2.4953±0.0059	2.5028±0.0057	2.4947±0.0063
$\sigma^0_{\rm h}$(nb)	41.59±0.13	41.24±0.17	41.39±0.26	41.69±0.23
R_e	20.67±0.13	20.75±0.18	20.96±0.15	20.88±0.13
R_μ	20.91±0.14	20.59±0.12	21.02±0.16	20.871±0.098
R_τ	20.69±0.12	20.69±0.18	20.80±0.20	20.94±0.13
$A^{0,\,e}_{\rm FB}$	0.0212±0.0054	0.0247±0.0088	0.0104±0.0092	0.0075±0.0063
$A^{0,\,\mu}_{\rm FB}$	0.0189±0.0039	0.0128±0.0037	0.0179±0.0061	0.0110±0.0036
$A^{0,\,\tau}_{\rm FB}$	0.0253±0.0043	0.0215±0.0069	0.0265±0.0088	0.0180±0.0044
$\chi^2/(d.o.f.)$	172/178	124/116	84/103[a]	–[b]

Table 3: Line shape and asymmetry parameters from 9-parameter fits to the data of the four LEP experiments.
[a]The $\chi^2/(d.o.f.)$ is for the 1992 data only.
[b]This parameter set has been obtained from a parameter transformation applied to the 15 parameters of the OPAL model independent fit, which treats the $\gamma{\rm Z}^0$ interference terms for leptons as free parameters. The extra parameters for the $\gamma{\rm Z}^0$ interference terms have not been fixed in the transformation. The $\chi^2/(d.o.f.)$ for the direct fit to the data is 91.8/133.

$\mathcal{V}$:	ALEPH	DELPHI	L3	OPAL
ALEPH	$\sigma_A A \sigma_A^T$	$(C_E + C_L)^T$	$(C_E + C_L)^T$	$(C_E + C_L)^T$
DELPHI	$C_E + C_L$	$\sigma_D D \sigma_D^T$	$(C_E + C_L)^T$	$(C_E + C_L)^T$
L3	$C_E + C_L$	$C_E + C_L$	$\sigma_L L \sigma_L^T$	$(C_E + C_L)^T$
OPAL	$C_E + C_L$	$C_E + C_L$	$C_E + C_L$	$\sigma_O O \sigma_O^T$

Table 4: The covariance matrix $\mathcal{V}$ used for the LEP average of parameters. The parameter errors of the individual experiments (σ_A, σ_D, σ_L, σ_O) and the correlation matrices of the individual experiments, A, D, L and O, (see Table 5 for the combination of 9 parameter results and Table 10 for the combination of 5 parameter results, respectively) include the effect of experiment specific and common uncertainties. The submatrix C_E (see Table 6 for the combination of 9 parameter results and Table 11 for the combination of 5 parameter results, respectively) accounts for common systematic uncertainties due to the LEP energy calibration. The submatrix C_L accounts for the common systematic error of the absolute normalization due to the theoretical uncertainty in the small angle Bhabha cross section. For simplicity we use for these preliminary results a submatrix C_L which is zero everywhere except for the diagonal element for $\sigma^0_{\rm h}$, which is $C_L(\sigma^0_{\rm h}, \sigma^0_{\rm h}) = 0.0025 \cdot 0.0025 \cdot {\sigma^0_{\rm h}}^2$.

A:	M_{Z}	Γ_{Z}	σ^0_{h}	R_{e}	R_μ	R_τ	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\,\mu}$	$A_{\mathrm{FB}}^{0,\,\tau}$
M_{Z}	1.000	0.053	0.015	0.002	0.002	−0.002	0.047	0.065	0.058
Γ_{Z}	0.053	1.000	−0.226	−0.004	0.006	−0.012	0.000	0.000	0.000
σ^0_{h}	0.015	−0.226	1.000	0.111	0.102	0.123	0.003	0.003	0.003
R_{e}	0.002	−0.004	0.111	1.000	0.064	0.071	0.006	0.000	0.000
R_μ	0.002	0.006	0.102	0.064	1.000	0.067	0.000	0.011	0.000
R_τ	−0.002	−0.012	0.123	0.071	0.067	1.000	0.000	0.000	0.007
$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	0.047	0.000	0.003	0.006	0.000	0.000	1.000	0.031	0.035
$A_{\mathrm{FB}}^{0,\,\mu}$	0.065	0.000	0.003	0.000	0.011	0.000	0.031	1.000	0.027
$A_{\mathrm{FB}}^{0,\,\tau}$	0.058	0.000	0.003	0.000	0.000	0.007	0.035	0.027	1.000

D:	M_{Z}	Γ_{Z}	σ^0_{h}	R_{e}	R_μ	R_τ	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\,\mu}$	$A_{\mathrm{FB}}^{0,\,\tau}$
M_{Z}	1.000	0.052	0.007	0.001	−0.003	0.013	0.007	0.031	0.009
Γ_{Z}	0.052	1.000	−0.175	−0.006	−0.002	−0.004	0.000	−0.006	−0.001
σ^0_{h}	0.007	−0.175	1.000	0.059	0.098	0.060	−0.002	−0.006	−0.004
R_{e}	0.001	−0.006	0.059	1.000	0.046	0.038	0.006	0.001	0.001
R_μ	−0.003	−0.002	0.098	0.046	1.000	0.046	0.000	0.017	0.000
R_τ	0.013	−0.004	0.060	0.038	0.046	1.000	−0.008	−0.011	0.006
$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	0.007	0.000	−0.002	0.006	0.000	−0.008	1.000	0.026	0.019
$A_{\mathrm{FB}}^{0,\,\mu}$	0.031	−0.006	−0.006	0.001	0.017	−0.011	0.026	1.000	0.028
$A_{\mathrm{FB}}^{0,\,\tau}$	0.009	−0.001	−0.004	0.001	0.000	0.006	0.019	0.028	1.000

L:	M_{Z}	Γ_{Z}	σ^0_{h}	R_{e}	R_μ	R_τ	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\,\mu}$	$A_{\mathrm{FB}}^{0,\,\tau}$
M_{Z}	1.000	0.079	−0.004	0.008	0.001	0.000	0.033	0.055	0.038
Γ_{Z}	0.079	1.000	−0.106	−0.011	−0.006	−0.002	0.004	0.000	0.001
σ^0_{h}	−0.004	−0.106	1.000	−0.045	0.069	0.055	0.088	−0.009	−0.006
R_{e}	0.008	−0.011	−0.045	1.000	0.067	0.052	−0.102	0.002	0.001
R_μ	0.001	−0.006	0.069	0.067	1.000	0.050	−0.001	0.012	0.000
R_τ	0.000	−0.002	0.055	0.052	0.050	1.000	−0.001	0.000	0.010
$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	0.033	0.004	0.088	−0.102	−0.001	−0.001	1.000	0.016	0.011
$A_{\mathrm{FB}}^{0,\,\mu}$	0.055	0.000	−0.009	0.002	0.012	0.000	0.016	1.000	0.018
$A_{\mathrm{FB}}^{0,\,\tau}$	0.038	0.001	−0.006	0.001	0.000	0.010	0.011	0.018	1.000

O:	M_{Z}	Γ_{Z}	σ^0_{h}	R_{e}	R_μ	R_τ	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\,\mu}$	$A_{\mathrm{FB}}^{0,\,\tau}$
M_{Z}	1.000	0.063	−0.032	0.138	−0.002	0.001	−0.048	0.073	0.065
Γ_{Z}	0.063	1.000	−0.032	0.008	0.035	0.026	0.010	0.001	0.017
σ^0_{h}	−0.032	−0.032	1.000	0.049	0.233	0.170	0.095	−0.017	−0.013
R_{e}	0.138	0.008	0.049	1.000	0.184	0.098	−0.134	0.049	0.042
R_μ	−0.002	0.035	0.233	0.184	1.000	0.162	0.004	0.008	0.000
R_τ	0.001	0.026	0.170	0.098	0.162	1.000	0.006	0.000	0.008
$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	−0.048	0.010	0.095	−0.134	0.004	0.006	1.000	−0.025	−0.020
$A_{\mathrm{FB}}^{0,\,\mu}$	0.073	0.001	−0.017	0.049	0.008	0.000	−0.025	1.000	0.023
$A_{\mathrm{FB}}^{0,\,\tau}$	0.065	0.017	−0.013	0.042	0.000	0.008	−0.020	0.023	1.000

Table 5: The matrices A, D, L, and O used in the covariance matrix $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 9 parameter fits from the individual LEP experiments.

C_E:	σ_E	C								
		$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	$R_{\rm e}$	R_μ	R_τ	$A^{0,\,e}_{FB}$	$A^{0,\,\mu}_{FB}$	$A^{0,\,\tau}_{FB}$
$M_{\rm Z}$(GeV)	4 MeV	1.	0.	0.	0.	0.	0.	0.	0.	0.
$\Gamma_{\rm Z}$(GeV)	2.7 MeV	0.	1.	0.	0.	0.	0.	0.	0.	0.
$\sigma^0_{\rm h}$(nb)	0	0.	0.	1.	0.	0.	0.	0.	0.	0.
$R_{\rm e}$	0	0.	0.	0.	1.	0.	0.	0.	0.	0.
R_μ	0	0.	0.	0.	0.	1.	0.	0.	0.	0.
R_τ	0	0.	0.	0.	0.	0.	1.	0.	0.	0.
$A^{0,\,e}_{FB}$	0.0008	0.	0.	0.	0.	0.	0.	1.	$-1.$	$-1.$
$A^{0,\,\mu}_{FB}$	0.0008	0.	0.	0.	0.	0.	0.	$-1.$	1.	1.
$A^{0,\,\tau}_{FB}$	0.0008	0.	0.	0.	0.	0.	0.	$-1.$	1.	1.

Table 6: The matrix $C_E = \sigma_E C \sigma_E^T$ used in the covariance matrix, $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 9 parameter fits from the individual LEP experiments. Due to the t-channel contribution $A^{0,\,e}_{FB}$ has a slope with centre-of-mass energy which is of opposite sign to that of $A^{0,\,\mu}_{FB}$ and $A^{0,\,\tau}_{FB}$; hence the anticorrelation.

Parameter	Average Value
$M_{\rm Z}$(GeV)	91.1899±0.0044
$\Gamma_{\rm Z}$(GeV)	2.4971±0.0038
$\sigma^0_{\rm h}$(nb)	41.51±0.12
$R_{\rm e}$	20.800±0.071
R_μ	20.815±0.061
R_τ	20.773±0.073
$A^{0,\,e}_{FB}$	0.0158±0.0035
$A^{0,\,\mu}_{FB}$	0.0144±0.0021
$A^{0,\,\tau}_{FB}$	0.0221±0.0027

Table 7: Average line shape and asymmetry parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The $\chi^2/(d.o.f.)$ of the average is 28.7/27.

	$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	$R_{\rm e}$	R_μ	R_τ	$A^{0,\,e}_{FB}$	$A^{0,\,\mu}_{FB}$	$A^{0,\,\tau}_{FB}$
$M_{\rm Z}$	1.000	0.030	0.003	0.023	-0.001	0.001	0.009	0.031	0.026
$\Gamma_{\rm Z}$	0.030	1.000	-0.092	-0.002	0.002	-0.001	0.001	-0.001	0.004
$\sigma^0_{\rm h}$	0.003	-0.092	1.000	0.055	0.078	0.078	0.010	-0.001	-0.001
$R_{\rm e}$	0.023	-0.002	0.055	1.000	0.106	0.068	-0.049	0.012	0.010
R_μ	-0.001	0.002	0.078	0.106	1.000	0.086	-0.003	0.011	-0.001
R_τ	0.001	-0.001	0.078	0.068	0.086	1.000	-0.002	-0.002	0.008
$A^{0,\,e}_{FB}$	0.009	0.001	0.010	-0.049	-0.003	-0.002	1.000	-0.051	-0.034
$A^{0,\,\mu}_{FB}$	0.031	-0.001	-0.001	0.012	0.011	-0.002	-0.051	1.000	0.099
$A^{0,\,\tau}_{FB}$	0.026	0.004	-0.001	0.010	-0.001	0.008	-0.034	0.099	1.000

Table 8: The correlation matrix for the set of parameters given in Table 7.

	ALEPH	DELPHI	L3	OPAL
$M_{\rm Z}$(GeV)	91.1915±0.0052	91.1870±0.0052	91.1951±0.0054	91.1858±0.0054
$\Gamma_{\rm Z}$(GeV)	2.4959±0.0061	2.4952±0.0059	2.5028±0.0057	2.4938±0.0063
$\sigma^0_{\rm h}$(nb)	41.59±0.13	41.24±0.17	41.42±0.26	41.66±0.23
R_ℓ	20.730±0.078	20.682±0.076	20.93±0.10	20.882±0.077
$A^{0,\ell}_{\rm FB}$	0.0216±0.0026	0.0159±0.0031	0.0184±0.0045	0.0127±0.0026
$\chi^2/(d.o.f.)$	175/182	124/120	87/107[a]	7.2/8[b]

Table 9: Line shape and asymmetry parameters from 5-parameter fits to the data of the four LEP experiments, assuming lepton universality.
[a]The $\chi^2/(d.o.f.)$ is for the 1992 data only.
[b]This parameter set has been obtained from a parameter transformation to the 15 parameters of the OPAL model independent fit, which treats the $\gamma{\rm Z}^0$ interference terms for leptons as free parameters.

A:	$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$M_{\rm Z}$(GeV)	1.000	0.053	0.016	0.001	0.096
$\Gamma_{\rm Z}$(GeV)	0.053	1.000	−0.226	−0.006	0.000
$\sigma^0_{\rm h}$(nb)	0.016	−0.226	1.000	0.183	0.005
R_ℓ	0.001	−0.006	0.183	1.000	0.008
$A^{0,\ell}_{\rm FB}$	0.096	0.000	0.005	0.008	1.000
D:	$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$M_{\rm Z}$(GeV)	1.000	0.058	0.005	−0.003	0.031
$\Gamma_{\rm Z}$(GeV)	0.058	1.000	−0.172	−0.004	−0.004
$\sigma^0_{\rm h}$(nb)	0.005	−0.172	1.000	0.144	−0.008
R_ℓ	−0.003	−0.004	0.144	1.000	0.011
$A^{0,\ell}_{\rm FB}$	0.031	−0.004	−0.008	0.011	1.000
L:	$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$M_{\rm Z}$(GeV)	1.000	0.079	−0.003	0.006	0.074
$\Gamma_{\rm Z}$(GeV)	0.079	1.000	−0.108	−0.010	0.002
$\sigma^0_{\rm h}$(nb)	−0.003	−0.108	1.000	0.043	0.028
R_ℓ	0.006	−0.010	0.043	1.000	−0.020
$A^{0,\ell}_{\rm FB}$	0.074	0.002	0.028	−0.020	1.000
O:	$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$M_{\rm Z}$(GeV)	1.000	0.067	−0.013	0.055	0.071
$\Gamma_{\rm Z}$(GeV)	0.067	1.000	−0.033	0.040	0.015
$\sigma^0_{\rm h}$(nb)	−0.013	−0.033	1.000	0.249	0.019
R_ℓ	0.055	0.040	0.249	1.000	0.009
$A^{0,\ell}_{\rm FB}$	0.071	0.015	0.019	0.009	1.000

Table 10: The matrices A, D, L, and O used in the covariance matrix $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 5 parameter fits from the individual LEP experiments.

C_E:	σ_E	C				
		$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$M_{\rm Z}$(GeV)	4 MeV	1.	0.	0.	0.	0.
$\Gamma_{\rm Z}$(GeV)	2.7 MeV	0.	1.	0.	0.	0.
$\sigma^0_{\rm h}$(nb)	0	0.	0.	1.	0.	0.
R_ℓ	0	0.	0.	0.	1.	0.
$A^{0,\ell}_{\rm FB}$	0.0008	0.	0.	0.	0.	1.

Table 11: The matrix $C_E = \sigma_E C \sigma_E^T$ used in the covariance matrix, $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 5 parameter fits from the individual LEP experiments.

Parameter	Average Value
$M_{\rm Z}$(GeV)	91.1895±0.0044
$\Gamma_{\rm Z}$(GeV)	2.4969±0.0038
$\sigma^0_{\rm h}$(nb)	41.51±0.12
R_ℓ	20.789±0.040
$A^{0,\ell}_{\rm FB}$	0.0170±0.0016

Table 12: Average line shape and asymmetry parameters from the results of the four LEP experiments given in Table 3, assuming lepton universality. The $\chi^2/(d.o.f.)$ of the average is 23.2/15.

	$M_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$M_{\rm Z}$	1.000	0.031	0.003	0.010	0.038
$\Gamma_{\rm Z}$	0.031	1.000	−0.091	0.001	0.002
$\sigma^0_{\rm h}$	0.003	−0.091	1.000	0.114	0.002
R_ℓ	0.010	0.001	0.114	1.000	0.004
$A^{0,\ell}_{\rm FB}$	0.038	0.002	0.002	0.004	1.000

Table 13: The correlation matrix for the set of parameters given in Table 12.

Without Lepton Universality:	
$\Gamma_{\rm ee}$(MeV)	83.96±0.22
$\Gamma_{\mu\mu}$(MeV)	83.90±0.31
$\Gamma_{\tau\tau}$(MeV)	84.07±0.36
With Lepton Universality:	
Γ_l(MeV)	83.98±0.18
$\Gamma_{\rm had}$(MeV)	1745.8±4.0
$\Gamma_{\rm inv}$(MeV)	499.4±3.5
$g^2_{V_l}$	0.00144±0.00014
$g^2_{A_l}$	0.25123±0.00056
$\sin^2\theta^{\rm lept}_{eff}$	0.23107±0.00092

Table 14: Average values of some derived parameters.

3 The τ Polarization

3.1 Introduction

In the combination of the τ polarization results by the LEP Electroweak Working Group for the Marseille and Cornell Conferences in the summer of 1993 [1], it was stated that "Common systematic uncertainties among the experiments are expected to be much smaller than the present error of the average and have been neglected". More precise data sets are now available and a further significant reduction on the uncertainties can be anticipated in the future. It is thus desirable to study in more detail the magnitude of common systematic uncertainties between the four experiments. Such an estimate is presented in [10] and in this note. The results of the $\mathcal{P}_\tau$ measurements produced by the four LEP experiments for the La Thuile and Moriond Conferences in March 1994 are combined.

3.2 Relation between couplings and measured quantities

The τ polarization $\mathcal{P}_\tau$ is determined by indirect measurement of the longitudinal polarization of τ pairs produced in Z^0 decays. It is defined as

$$\mathcal{P}_\tau = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \tag{2}$$

where σ_R and σ_L are the cross-sections for the production of a right-handed and left-handed τ^-, respectively.

The quantity which can be extracted from P_τ is the ratio of the effective vector (g_{V_ℓ}) and axial-vector (g_{A_ℓ}) couplings of the leptons to the Z^0. The combination of g_{V_ℓ} and g_{A_ℓ} which enters the polarization is

$$\mathcal{A}_\ell = \frac{2 g_{V_\ell} g_{A_\ell}}{g_{V_\ell}^2 + g_{A_\ell}^2} \,. \tag{3}$$

Neglecting corrections (which are actually small) the angular distribution of $\mathcal{P}_\tau$ as a function of the angle θ between the e^- and the τ^-, for $\sqrt{s} = M_Z$ and ignoring the effects of γ exchange, is given by:

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau + \mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}}{1 + \mathcal{A}_\tau \mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}} \,, \tag{4}$$

with $\mathcal{A}_e$ and $\mathcal{A}_\tau$ defined in eqn. (3). When averaged over all production angles $\mathcal{P}_\tau$ is a measurement of $\mathcal{A}_\tau$, while as a function of $\cos\theta$, $\mathcal{P}_\tau(\cos\theta)$ provides nearly independent determinations of both $\mathcal{A}_\tau$ and $\mathcal{A}_e$, allowing thus a test of the universality of the couplings of the Z^0 to e and τ.

3.3 Comparison of $\mathcal{P}_\tau$ measurements

Each experiment has made separate $\mathcal{P}_\tau$ measurements using up to five tau decay modes ($e\nu\overline{\nu}$, $\mu\nu\overline{\nu}$, $\pi\nu$, $\rho\nu$ and $a_1\nu$). Some experiments have measured significant correlations in the systematic uncertainties between decay modes; for other experiments the correlations are small. It is not foreseen to combine

the results of a given decay mode for the four experiments, but to combine only the results already averaged by each experiment over decay modes[1].

There are systematic effects in the measurements which are common to the four experiments. The first class affect all decay modes in the same way. In general these have been studied previously and the uncertainties in the corrections applied are small. Other effects are different for each tau decay mode. In general these are not as well studied and may be more serious than the first class of systematics.

3.4 Possible common systematics

In order to extract $\sin^2\theta^{\rm lept}_{eff}$, or equivalently g_{V_ℓ}/g_{A_ℓ}, from the measured values of $\mathcal{P}_\tau$ and its variation as a function of polar-angle, the following effects have been taken into account in previous publications. These affect all tau decay modes equally:

- $\sqrt{s}$ dependence, due to Born level direct γ and $\gamma - {\rm Z}^0$ interference contributions. The results are corrected to correspond to $\mathcal{P}_\tau(\sqrt{s} = M_{\rm Z})$.
- Electromagnetic radiative corrections for initial state radiation from the e^+ and e^- and final state radiation from the τ^+ and τ^-.
- Born level mass terms leading to helicity flip configurations.

These three effects are theoretically well defined and have been calculated to adequate precision for present needs [11], [12], [13]. Their combined effect is to reduce the strength of the measured polarization by a small amount. The experiments correct for this by adding about 0.003 to the measured values of $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$. The uncertainty in this correction is about ± 0.001, or smaller, and is under study.

The τ polarization is not measured directly; properties of the particular τ decay modes are used. This introduces new theoretical uncertainties, in particular from the radiative corrections and model dependencies for the hadronic τ decays.

A survey of the systematic errors given by the four experiments for the decay modes analysed has shown that three possible systematic errors should be considered when combining the results. Other sources of systematic errors examined remain negligible. The three common errors are:

- "Theory error" for the a_1. Model dependent uncertainties in a_1 decay have been evaluated to give the following systematic errors in $\mathcal{P}_\tau$ for the a_1 decay mode :

  ALEPH $\pm$ 0.012, DELPHI $\pm$ 0.015, L3 $\pm$ 0.020.

  OPAL have not analysed this channel, but this term effects the error on the ρ channel. Although the meaning of the theory error differs somewhat from experiment to experiment, in principle this error should be taken to be fully correlated between experiments. A more detailed investigation

[1] The L3 results are included with the reservation, by the L3 Collaboration, that there could be an anomaly in the ρ channel, which is included in the L3 average. L3 measures a possible dependence of the polarization on rho mass. The ALEPH and DELPHI Collaborations have studied their data in this regard and see no anomaly in the ρ channel and have no reservation about including that channel in the average. The ρ channel is included without reservation in the combined results of the OPAL Collaboration.

of this error in the future is probably desirable and this would probably reduce the common error.

In practice the a_1 decay mode does not carry a large weight in the average polarization. The "theory error" induces an uncertainty ± 0.001 on the final polarization value (i.e. after combining the decay channels). This is essentially negligible at the present level of precision. This same applies for the effect on the forward-backward polarization asymmetry.

- Radiative corrections for τ hadronic final states Unlike radiation from leptons, there is no precise formalism for handling these corrections. The impact of these corrections on the extracted polarization has been estimated and a systematic error has been included by the OPAL collaboration, but not yet by the other collaborations. The effect is quantified by analysing a KORALZ (version 3.8) event sample after full simulation as though it were data. An analysis is performed on the complete event sample and for a sample which excludes events with radiated photons in the tau decay process. The difference in the results is an estimate of the magnitude of the correction. Following reference [12], the correction is assumed to have a systematic error of the order of $1/\ln(m_\tau/m_h)$ of the correction itself where m_h is the mass of the hadron in the decay. Using this procedure, the uncertainty is approximately ± 0.005 for the ρ decay mode and ± 0.004 for the π(K) mode. Further, there is an additional uncertainty in that version 3.8 of KORALZ did not include a complete description of the radiative corrections for the leptonic decay modes of the tau. As the relative sign of the uncertainties for the various modes is unknown, the most conservative approach is to take this error to be fully correlated between modes. Doing so, OPAL estimates an uncertainty of ± 0.006 on its measurement of $\mathcal{A}_\tau$ and ± 0.001 on $\mathcal{A}_e$. In the future this uncertainty can be reduced as can be seen from reference [14] where a comparison with an analytical calculation for the $\tau \to \pi\nu$ mode is reported. Some experimental input from low energy interactions can reduce this uncertainty further.

- Uncertainty in the V-A theory The V-A theory for the τ decay is normally assumed implicitly in the experimental analysis. Of course, this theory, which is part of the Standard Model, has been tested to good accuracy in μ decays. However, if the V-A theory in τ decay is not imposed in the analysis, then the extracted precision of $\mathcal{P}_\tau$ is much reduced. Analyses of the V-A structure of τ decays have been carried out by the ALEPH [15] and ARGUS [16] Collaborations. These publications show that if the V-A assumption in tau decays is dropped the additional uncertainty which should be included in the average tau polarization is about ± 0.015

  It is proposed that this V-A uncertainty is not treated as a common systematic error in the same way as the other two. However, it is important to stress that the Standard Model has been assumed for tau decays.

3.5 Results

In this section a combination is made of the results produced for the La Thuile and Moriond Conferences in March 1994.

Tables 15 and 16 show the results for $\mathcal{A}_\tau$ and $\mathcal{A}_e$ respectively obtained by the four experiments and their combination.

No common systematics have, in fact, been included in these averages. If the largest potentially common uncertainty, ± 0.006 coming from uncertainties in radiation from hadronic decay products of the tau, is included the final error on $\mathcal{A}_\tau$ becomes ± 0.012 instead of the quoted ± 0.010. The additional uncertainty in $\mathcal{A}_e$ from the same source is much smaller.

The correlation between the extracted values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$ is small (≤ 4 %) and can be neglected. The values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$ are compatible with lepton universality.

For the future, further studies of the uncertainties from the effects of radiative corrections for the τ decay products is recommended.

ALEPH	('90 + '91 + '92), prel.	0.137 ± 0.014
DELPHI	('90 + '91 + '92), prel.	0.154 ± 0.023
L3	('90 + '91 + '92), prel.	0.174 ± 0.021
OPAL	('90 + '91 + '92), prel.	0.150 ± 0.025
LEP Average ($\chi^2/(d.o.f.)$=2.2/3)		0.150 ± 0.010

Table 15: LEP results for $\mathcal{A}_\tau$.

ALEPH	('90 + '91 + '92), prel.	0.127 ± 0.017
DELPHI	('90 + '91 + '92), prel.	0.081 ± 0.033
L3	('90 + '91 + '92), prel.	0.130 ± 0.025
OPAL	('90 + '91 + '92), prel.	0.117 ± 0.033
LEP Average ($\chi^2/(d.o.f.)$=1.7/3)		0.120 ± 0.012

Table 16: LEP results for $\mathcal{A}_e$.

3.6 Determination of the effective Z^0 couplings for charged leptons

The partial widths of the Z^0 into leptons, the lepton forward-backward asymmetries, the τ polarization and the τ polarization asymmetry can all be combined to determine the vector and axial vector couplings for e, μ and τ. The asymmetries determine the ratio g_{V_ℓ}/g_{A_ℓ} while the axial vector coupling squared is derived from the leptonic partial width. The corresponding results for the effective lepton couplings are given in Table 17. Figure 2 shows the one standard deviation contours in the g_{A_ℓ}-g_{V_ℓ} plane. The measured ratios of the e, μ and τ couplings provide a test of universality:

$$g_{A_\mu}/g_{A_e}=0.9997 \pm 0.0023 \ , \ g_{A_\tau}/g_{A_e}=1.0011 \pm 0.0024 \ ,$$

$$g_{V_\mu}/g_{V_e} =0.99 \pm 0.20 \qquad , \ g_{V_\tau}/g_{V_e} =1.15 \pm 0.11 \ .$$

g_{V_e}	-0.0347 $\pm$ 0.0023
g_{V_μ}	-0.0341 $\pm$ 0.0057
g_{V_τ}	-0.0399 $\pm$ 0.0024
g_{A_e}	-0.50141 $\pm$ 0.00066
g_{A_μ}	-0.5013 $\pm$ 0.0010
g_{A_τ}	-0.5020 $\pm$ 0.0011
g_{V_ℓ}	-0.0364 $\pm$ 0.0014
g_{A_ℓ}	-0.50135 $\pm$ 0.00055

Table 17: Results for the leptonic vector and axial vector couplings without and with the assumption of lepton universality.

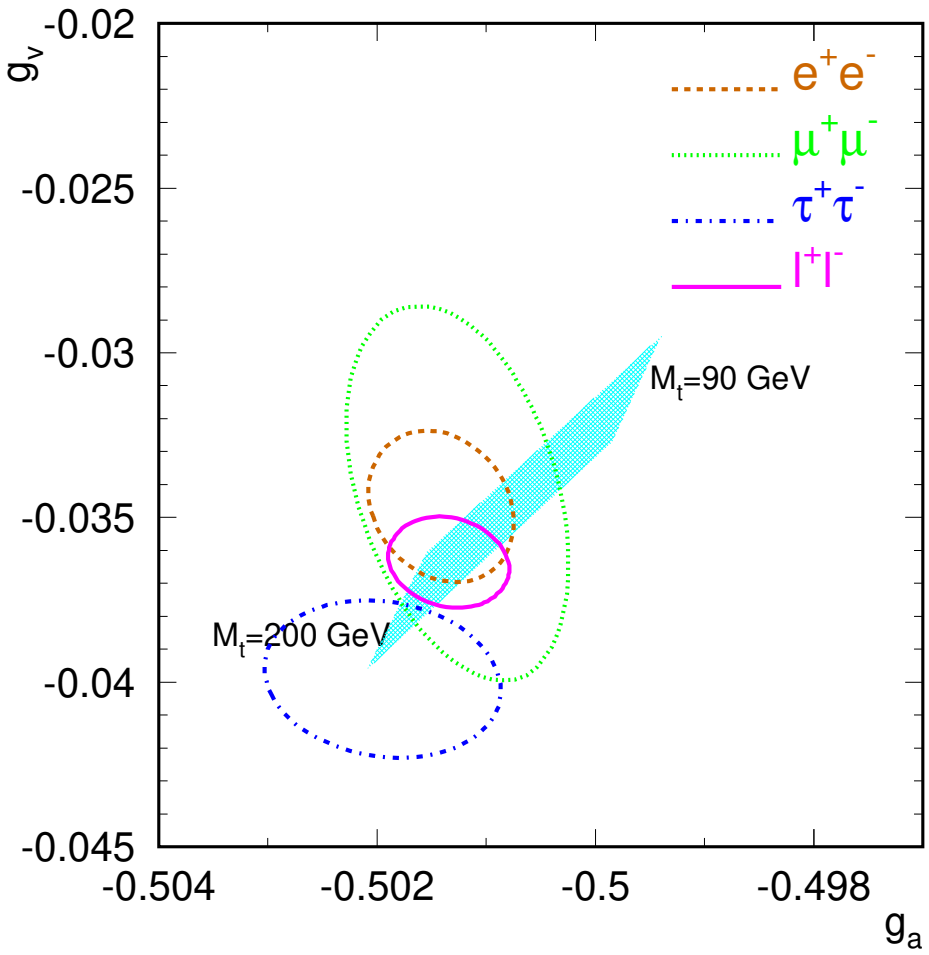

Figure 2: One standard deviation contours (39% probability) in the g_{V_t}-g_{A_t} plane. The solid contour results from a fit assuming lepton universality. The shaded band represents the Standard Model prediction.

4 Electroweak Results with b and c Quarks

4.1 Introduction

An ongoing effort is made by the heavy flavour sub-group of the LEP combined electroweak working group to establish a framework in which the abundant results on electroweak measurements involving heavy quarks from the different LEP experiments can be combined into a consistent set of LEP averages. Recently this effort has resulted in a note describing the treatment and presentation of common systematic errors [17].

Although many of the new results that have been prepared for the 1994 winter/spring conferences follow (more or less) the conventions laid down in [17], the older measurements are still providing a significant input to the LEP averages. An attempt has been made to adapt wherever possible the treatment of systematic errors in these older measurements to comply with [17], but unavoidably some assumptions and approximations had to be made.

In the following a framework for the combination of heavy flavour electroweak measurements is described. Inside the framework approximations have been made, that can be refined in future averages.

4.2 Combination strategy

The strategy for the combined heavy flavour electroweak fit as presented here was inspired by the availability of lepton fit results from the different LEP collaborations. A common set of parameters measured in such fits has been chosen to consist of: the heavy quark partial widths $R_{\rm b} \equiv \Gamma_{\rm b\overline{b}}/\Gamma_{\rm had}$ and $R_{\rm c} \equiv \Gamma_{\rm c\overline{c}}/\Gamma_{\rm had}$, the lepton average B mixing parameter $\overline{\chi_{\rm B}}$, the semi-leptonic branching ratios $Br(\rm b \to \ell)$ and $Br(\rm b \to c \to \ell)$, and the heavy quark asymmetries $A_{\rm FB}^{\rm b}$ and $A_{\rm FB}^{\rm c}$. The lepton fit results for these parameters, including their covariance matrices, are available now from ALEPH, DELPHI and OPAL. For L3 the available double lepton tag measurements have been taken from two different internal notes and a covariance matrix has been constructed from the statistical and systematic error information given in the available documentation.

Most of the seven parameters listed above have also been measured by other methods (including some inclusive single lepton measurements, which are more conveniently combined with these non-lepton fit results.) For the partial widths, especially $R_{\rm b}$, these non-lepton-fit measurements yield more precise results than the lepton fits. The average values of the chosen parameter set from these non-lepton ("OTHER") measurements have been found as follows. Firstly, where several measurements of the same quantity by one collaboration exist, then these are combined to form an average value. Then these averages from each collaboration are combined to give a LEP average for that quantity from non-lepton measurements.

The average per experiment, and the LEP average for each quantity were each formed by deriving weights (represented by the vector α) for each of the measurements participating in the average according to [18]:

$$\alpha = \frac{C^{-1}U}{U^T C^{-1} U} \ ,$$

where C is the covariance matrix for the measurements and U is the vector of which all components are unity. The covariance matrix C is the sum of the covariance matrices for the statistical and

systematic errors, but is constructed to exclude the error contributions from the other parameters that take part in the full fit. The statistical and systematic covariance matrices and the dependence on the other parameters in the full fit for the average results have been computed using the weights α at each of the two steps of the averaging procedure. The statistical and systematic errors needed to construct the covariance matrices were taken from the information in the available documentation for each measurement.

In a final step, the overall LEP averages from the lepton fits and other measurements were formed. The combined covariance matrices (statistical and systematic errors are treated independently in this case) for this full fit have the form:

$$\begin{pmatrix} \text{ALEPH} \\ \text{lepton fit} & \cdots & \cdots & \cdots & \cdots \\ & \text{DELPHI} & & & \\ \cdots & \text{lepton fit} & \cdots & \cdots & \cdots \\ & & \text{L3} & & \\ \cdots & \cdots & \text{lepton fit} & \cdots & \cdots \\ & & & \text{OPAL} & \\ \cdots & \cdots & \cdots & \text{lepton fit} & \cdots \\ & & & & \text{OTHER} \\ \cdots & \cdots & \cdots & \cdots & \text{measurements} \end{pmatrix} ,$$

where the blocks along the diagonal are 7×7 matrices that are obtained as described above and the off-diagonal part indicated by the " $\cdots$ " is derived from the information on the uncertainties as obtained from the available documentation. The fit is performed by minimising the χ^2 defined by:

$$\chi^2 = D^T \; C^{-1} \; D \; ,$$

where C is the 35×35 covariance matrix that is the sum of the covariance matrices for the statistical and systematic errors and D is the 35 component vector (which repeats itself every 7 elements) defined by the differences between the measured values and the fitted values:

$$D = \begin{pmatrix} (R_{\mathrm{b}})_{\mathrm{ALEPH\,lepton\,fit}} & - & R_{\mathrm{b}} \\ (R_{\mathrm{c}})_{\mathrm{ALEPH\,lepton\,fit}} & - & R_{\mathrm{c}} \\ (\overline{\chi_{\mathrm{B}}})_{\mathrm{ALEPH\,lepton\,fit}} & - & \overline{\chi_{\mathrm{B}}} \\ (Br(\mathrm{b} \to \ell))_{\mathrm{ALEPH\,lepton\,fit}} & - & Br(\mathrm{b} \to \ell) \\ (Br(\mathrm{b} \to \mathrm{c} \to \ell))_{\mathrm{ALEPH\,lepton\,fit}} & - & Br(\mathrm{b} \to \mathrm{c} \to \ell) \\ (A_{\mathrm{FB}}^{\mathrm{b}})_{\mathrm{ALEPH\,lepton\,fit}} & - & A_{\mathrm{FB}}^{\mathrm{b}} \\ (A_{\mathrm{FB}}^{\mathrm{c}})_{\mathrm{ALEPH\,lepton\,fit}} & - & A_{\mathrm{FB}}^{\mathrm{c}} \\ (R_{\mathrm{b}})_{\mathrm{DELPHI\,lepton\,fit}} & - & R_{\mathrm{b}} \\ \vdots & & \\ (A_{\mathrm{FB}}^{\mathrm{c}})_{\mathrm{DELPHI\,lepton\,fit}} & - & A_{\mathrm{FB}}^{\mathrm{c}} \\ (R_{\mathrm{b}})_{\mathrm{L3\,lepton\,fit}} & - & R_{\mathrm{b}} \\ \vdots & & \\ (A_{\mathrm{FB}}^{\mathrm{c}})_{\mathrm{L3\,lepton\,fit}} & - & A_{\mathrm{FB}}^{\mathrm{c}} \\ (R_{\mathrm{b}})_{\mathrm{OPAL\,lepton\,fit}} & - & R_{\mathrm{b}} \\ \vdots & & \\ (A_{\mathrm{FB}}^{\mathrm{c}})_{\mathrm{OPAL\,lepton\,fit}} & - & A_{\mathrm{FB}}^{\mathrm{c}} \\ (R_{\mathrm{b}})_{\mathrm{OTHER\,measurements}} & - & R_{\mathrm{b}} \\ \vdots & & \\ (A_{\mathrm{FB}}^{\mathrm{c}})_{\mathrm{OTHER\,measurements}} & - & A_{\mathrm{FB}}^{\mathrm{c}} \end{pmatrix} .$$

4.3 The lepton fit results

The values of the quantities R_b, R_c, $\overline{\chi_B}$, $Br(b \to \ell)$, $Br(b \to c \to \ell)$, A_{FB}^b and A_{FB}^c measured with a lepton fit for the different experiments are listed in Table 18. In the last column of the same table the non-lepton fit measurements are listed.

The asymmetries have been corrected by +3% for A_{FB}^b and +2% for A_{FB}^c for QCD effects that smear the thrust axis direction with respect to the initial parton direction before gluon radiation. The values have all been shifted to those measured with the ACCMM model.[2] For ALEPH and OPAL the modelling error has been increased to be the full difference between the ACCMM model and ISGW** model.

The correlation matrices for the measurements within the experiments is given per experiment in the Tables 19, 20, 21 and 22, for ALEPH, DELPHI, L3 and OPAL respectively. The systematic error correlation between the L3 R_b and $Br(b \to \ell)$ measurements has been estimated to be -0.3 [49].

4.4 Combination of results other than from lepton fits

The strategy to combine the results that are derived from measurements other than lepton fits, is to first combine the different measurements for each experiment separately and then combine the averages per experiment into a global LEP average. The errors are presented in blocks that are separated by horizontal lines in the tables. In the first block are the statistical errors (also from the Monte Carlo) that are taken to be uncorrelated, unless specifically commented on. The following blocks contain the correlated errors, that are taken to be 100% correlated for each individual entry in the error tables. The first of the correlated error blocks contain uncertainties due to various experimental and theoretical sources, excluding the uncertainties from explicit dependences on other parameters that take part in the global heavy quark electroweak fit, which are listed in the bottom blocks of the tables, when appropriate.

4.4.1 R_b

The ALEPH, DELPHI, L3 and OPAL measurements included in the non-lepton fit R_b average and their uncertainties are listed in Tables 23, 24, 25 and 26 respectively.

Combining the measurements per experiment results in the averages presented in Table 27. In this table the first section with uncorrelated uncertainties also contains entries that are uncorrelated among the experiments, although they may be correlated among measurements in the same experiment. The χ^2/d.o.f. values for the averages per experiment are 1.65/2, 0.90/3, 0.42/1 and 0.47/4 for ALEPH, DELPHI, L3 and OPAL respectively.

The average value found from Table 27 for $R_c = 0.171$ is:

$$R_b = 0.2208 \pm 0.0013(\text{stat}) \pm 0.0021(\text{syst}) ,$$

and the explicit experimental dependence of R_b on R_c is given by:

$$\Delta R_b = -0.096 \Delta R_c .$$

[2] See Ref. [17] and the relevant publications for each experiment for a discussion of the heavy quark semi-leptonic decay models and the specific parameters in these models that were used.

The χ^2/d.o.f.$= 0.30/3$ for the average.

Many of these R_b results use a double tag technique, in which possible correlations between the tagging efficiencies in the two hemispheres of the event must be taken into account. These hemisphere correlations lead to systematic errors, and the degree to which these systematic errors are correlated among the experiments is somewhat uncertain. In the average quoted above, this source of systematic error was taken to be fully correlated. If instead it is taken to be uncdrrelated then the central value is unchanged, and the systematic error goes down from 0.0021 to 0.0020, which has a negligible effect on the final result of the full fit.

4.4.2 R_c

The DELPHI and OPAL measurements included in the non-lepton fit R_c average and their uncertainties are listed in Tables 28 and 29 respectively. ALEPH has a single measurement, that is listed in Table 30.

Combining the measurements per experiment result in the averages presented in Table 30. In this table the first section with uncorrelated uncertainties also contains entries that are specific to the experiments and have been determined more or less independently per experiment. The tiny dependence of R_c on R_b due to one of the OPAL measurements has been neglected. The χ^2/d.o.f. values for the averages per experiment are $0.36/2$ and $0.00/1$ for DELPHI and OPAL respectively.

The average value found from Table 30 is:

$$R_c = 0.172 \pm 0.013(\text{stat}) \pm 0.018(\text{syst}) .$$

The χ^2/d.o.f.$= 0.36/2$ for the average.

4.4.3 A_{FB}^b

The ALEPH, DELPHI and OPAL collaborations each have one measurement of A_{FB}^b other than from lepton fits. They are listed in Table 31. The measurements have been shifted to a centre-of-mass energy of $\sqrt{s} = 91.3$ GeV and if necessary corrected for QCD effects (as described for the lepton fit asymmetry measurements).

The average value found from Table 31, assuming $A_{FB}^c = 0.06$, $R_b = 0.217$, $R_c = 0.171$ and $\overline{\chi_B} = 0.132$ is:

$$A_{FB}^b = 0.1016 \pm 0.0063(\text{stat}) \pm 0.0030(\text{syst}) ,$$

where the explicit dependences on the assumed quantities are given by:

$$\begin{aligned} \Delta A_{FB}^b &= 0.135 \Delta A_{FB}^c , \\ \Delta A_{FB}^b &= -0.378 \Delta R_b , \\ \Delta A_{FB}^b &= 0.004 \Delta R_c , \\ \Delta A_{FB}^b &= 0.008 \Delta \overline{\chi_B} . \end{aligned}$$

The χ^2/d.o.f.$= 0.88/2$ for the average.

4.4.4 A_{FB}^{c}

The ALEPH, DELPHI and OPAL collaborations each have one measurement of A_{FB}^{c} other than from lepton fits. They are listed in Table 31. The measurements have been shifted to a centre-of-mass energy of $\sqrt{s} = 91.3$ GeV and if necessary corrected for QCD effects (as described above for lepton fit results). The D^* measurement by DELPHI uses decay length information to separate $b\overline{b}$ from $c\overline{c}$ events and is therefore sensitive to the time dependence of the $B^0\overline{B^0}$ mixing. The time dependence of the B_d^0 mixing is treated independently from the average mixing for B^0.

The average value found from Table 32, assuming $A_{FB}^{b} = 0.11$, and $\overline{\chi_B} = 0.132$ is:

$$A_{FB}^{c} = 0.081 \pm 0.018(\text{stat}) \pm 0.007(\text{syst}) ,$$

where the explicit dependences on the assumed quantities are given by:

$$\begin{aligned} \Delta A_{FB}^{c} &= 0.302 \Delta A_{FB}^{b} , \\ \Delta A_{FB}^{c} &= 0.042 \Delta \overline{\chi_B} . \end{aligned}$$

The χ^2/d.o.f.$= 0.56/2$ for the average.

4.4.5 $\overline{\chi_B}$

The ALEPH and DELPHI collaborations each have one measurement of $\overline{\chi_B}$ other than from lepton fits. They are listed in Table 33. The measured values have been corrected to the lepton average mixing assuming $f_s = 0.117$ and $\chi_s = 0.25 \pm 0.25$. The additional error due to this Ansatz is small compared to both the statistical error and the total systematic error from other sources.

The average value found from Table 33 is:

$$\overline{\chi_B} = 0.142 \pm 0.012(\text{stat}) \pm 0.014(\text{syst}) .$$

The χ^2/d.o.f.$= 0.62/1$ for the average.

4.4.6 Results and correlation matrix for non-lepton fit measurements

The statistical and systematic correlation matrix for non-lepton fit results have been derived from the results in the previous section and are listed in Table 34. A summary of the non-lepton fit results is given in the last column of Table 18.

4.5 Inter experimental correlated errors

The correlation coefficients between the measurements from different experiments and between results from lepton fits and the "other" measurements have been calculated from the uncertainties listed in Table 35.

The correlation coefficients between measurements of different quantities by different experiments were taken to be zero. It was found that at present the information on the systematic errors, especially the sign, was not sufficient to reconstruct these correlations with sufficient confidence. The correlation

between measurements of the same quantity by different experiments have been taken into account exactly according to the correlated systematic errors as quoted in the relevant papers.

The difference between taking the correlations between the same quantity but different experiments into account and ignoring these correlations is:

$$\begin{aligned}
\Delta R_{\mathrm{b}} &= -0.0005\,, \\
\Delta R_{\mathrm{c}} &= +0.0002\,, \\
\Delta \overline{\chi_{\mathrm{B}}} &= +0.0020\,, \\
\Delta Br(\mathrm{b} \to \ell) &= -0.0013\,, \\
\Delta Br(\mathrm{b} \to \mathrm{c} \to \ell) &= +0.0044\,, \\
\Delta A_{\mathrm{FB}}^{\mathrm{b}} &= +0.0002\,, \\
\Delta A_{\mathrm{FB}}^{\mathrm{c}} &= +0.0003\,.
\end{aligned}$$

The total error of the results in this case stay similar for R_{b}, $\overline{\chi_{\mathrm{B}}}$, $Br(\mathrm{b} \to \mathrm{c} \to \ell)$, $A_{\mathrm{FB}}^{\mathrm{b}}$ and $A_{\mathrm{FB}}^{\mathrm{c}}$, but decrease significantly for R_{c} and $Br(\mathrm{b} \to \ell)$. The relatively large shifts in $\overline{\chi_{\mathrm{B}}}$, $Br(\mathrm{b} \to \ell)$ and $Br(\mathrm{b} \to \mathrm{c} \to \ell)$ are not relevant to the remainder of this note.

4.6 The full heavy flavour electroweak fit results

The results for the full fit are listed in Table 36. The fit resulted in $\chi^2/\mathrm{d.o.f.} = 20.3/(30-7)$. The uncertainties have been split artificially into the statistical and systematic contributions respectively by deriving the total errors in the correct way and the statistical error separately by setting the systematic errors to zero in the fit. This splitting is only approximate, but the total uncertainty is mathematically correct. The total correlation matrix for the results is also given in Table 36.

The overall average values for the partial widths are:

$$\begin{aligned}
R_{\mathrm{b}} &= 0.2208 \pm 0.0013(\mathrm{stat}) \pm 0.0020(\mathrm{syst}) \\
R_{\mathrm{c}} &= 0.1697 \pm 0.0035(\mathrm{stat}) \pm 0.0134(\mathrm{syst})\ ,
\end{aligned}$$

with a correlation coefficient of -0.384 between these two values. If the value of R_{c} is fixed to 0.171, then $R_{\mathrm{b}} = 0.2207 \pm 0.0022$.

The asymmetries $A_{\mathrm{FB}}^{\mathrm{b}}$ and $A_{\mathrm{FB}}^{\mathrm{c}}$ are quoted at $\sqrt{s} = 91.3$GeV and have been corrected for QCD effects. The relative uncertainty due to the QCD correction is estimated to be 1% for both asymmetries. This estimate is such that the QCD correction that follows from the JETSET parton shower Monte Carlo (a little over 2% for $A_{\mathrm{FB}}^{\mathrm{b}}$) and the extreme correction presented in [50] (about 4% for $A_{\mathrm{FB}}^{\mathrm{b}}$) are included within one standard deviation. This can be regarded as a conservative estimate.

After applying a correction due to the energy shift from $\sqrt{s} = 91.3$ GeV to $\sqrt{s} = 91.19$ GeV (–0.0020 for $A_{\mathrm{FB}}^{\mathrm{b}}$ and –0.0051 for $A_{\mathrm{FB}}^{\mathrm{c}}$), due to initial and final state photon radiation (0.0041 for $A_{\mathrm{FB}}^{\mathrm{b}}$ and 0.0104 for $A_{\mathrm{FB}}^{\mathrm{c}}$), and due to photon exchange and the $\gamma\mathrm{Z}^0$-interference (–0.0003 for $A_{\mathrm{FB}}^{\mathrm{b}}$ and –0.0008 for $A_{\mathrm{FB}}^{\mathrm{c}}$), the pole asymmetries are found to be:

$$\begin{aligned}
A_{\mathrm{FB}}^{0,\,\mathrm{b}} &= 0.0960 \pm 0.0038(\mathrm{stat}) \pm 0.0021(\mathrm{syst}) \\
A_{\mathrm{FB}}^{0,\,\mathrm{c}} &= 0.0700 \pm 0.0080(\mathrm{stat}) \pm 0.0072(\mathrm{syst})\ ,
\end{aligned}$$

where the systematic errors now also include the uncertainty due to the QCD correction.

4.7 Energy dependence of $A_{\rm FB}^{\rm b}$

The value of $A_{\rm FB}^{\rm b}$ has been calculated as a function of centre-of-mass energy for three different energy points in Table 37. The information from references [45, 47, 48] has been used. The centre-of-mass energy of the two off-peak points has been calculated as a weighted mean, which is a good approximation given the nearly linear behaviour of the asymmetries around $\sqrt{s} = M_{\rm Z^0}$. The average for the on-peak point was given in the previous subsection. The values in Table 37 are corrected for QCD effects, but not for initial and final state photon radiation.

	ALEPH[45]	DELPHI[46]	L3[47]	OPAL[48]	non-lepton
R_b	0.216±0.006 ±0.005	0.216±0.009 ±0.005	0.218±0.008 ±0.008	0.224±0.011 ±0.007	0.2208±0.0018 ±0.0027
R_c	0.167±0.005 ±0.020	0.176±0.008 ±0.019	–	–	0.172 ±0.013 ±0.018
$\overline{\chi_B}$	0.106±0.007 ±0.010	0.137±0.020 ±0.018	0.118±0.012 ±0.010	0.144±0.022 ±0.007	0.142 ±0.012 ±0.014
$Br(b \to \ell)$	0.112±0.003 ±0.005	0.112±0.004 ±0.006	0.117±0.005 ±0.005	0.106±0.006 ±0.006	–
$Br(b \to c \to \ell)$	0.088±0.003 ±0.012	0.079±0.005 ±0.010	–	0.083±0.004 ±0.007	–
A_{FB}^b	0.087±0.007 ±0.002	0.118±0.014 ±0.006	0.095±0.012 ±0.004	0.090±0.010 ±0.004	0.1016±0.0063 ±0.0030
A_{FB}^c	0.094±0.020 ±0.017	0.079±0.025 ±0.022	0.063±0.022 ±0.028	0.043±0.013 ±0.009	0.081 ±0.018 ±0.007

Table 18: Results from lepton fits and in the last column the combined result for non-lepton fit measurements. The first error is statistical and the second systematic.

	statistical							systematic						
	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c
R_b	1.00	-0.48	0.00	-0.94	-0.38	0.00	0.05	1.00	0.30	0.06	-0.20	-0.24	0.00	0.12
R_c	-0.48	1.00	0.00	0.47	-0.31	-0.02	-0.07	0.30	1.00	-0.22	-0.05	0.16	-0.31	-0.29
$\overline{\chi_B}$	0.00	0.00	1.00	0.00	0.00	0.22	0.00	0.06	-0.22	1.00	0.13	0.12	-0.30	0.37
$b\to\ell$	-0.94	0.47	0.00	1.00	0.25	0.00	-0.04	-0.20	-0.05	0.13	1.00	0.02	0.16	0.23
$b\to c\to\ell$	-0.38	-0.31	0.00	0.25	1.00	0.00	-0.01	-0.24	0.16	0.12	0.02	1.00	-0.45	-0.45
A_{FB}^b	0.00	-0.02	0.22	0.00	0.00	1.00	0.11	0.00	-0.31	-0.30	0.16	-0.45	1.00	0.33
A_{FB}^c	0.05	-0.07	0.00	-0.04	-0.01	0.11	1.00	0.12	-0.29	0.37	0.23	-0.45	0.33	1.00

Table 19: ALEPH lepton fit correlation matrices.

	statistical							systematic						
	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c
R_b	1.00	-0.41	-0.08	-0.94	-0.44	0.00	0.00	1.00	-0.15	-0.50	-0.54	0.26	0.35	0.11
R_c	-0.41	1.00	0.14	0.36	-0.39	0.00	0.00	-0.15	1.00	0.13	0.17	0.23	0.13	0.38
$\overline{\chi_B}$	-0.08	0.14	1.00	0.11	-0.13	0.29	0.00	-0.50	0.13	1.00	0.77	-0.78	0.43	0.00
$b\to\ell$	-0.94	0.36	0.11	1.00	0.32	0.00	0.00	-0.54	0.17	0.77	1.00	-0.56	0.07	-0.14
$b\to c\to\ell$	-0.44	-0.39	-0.13	0.32	1.00	0.00	0.00	0.26	0.23	-0.78	-0.56	1.00	0.33	0.43
A_{FB}^b	0.00	0.00	0.29	0.00	0.00	1.00	0.28	0.35	0.13	0.43	0.07	0.33	1.00	0.49
A_{FB}^c	0.00	0.00	0.00	0.00	0.00	0.28	1.00	0.11	0.38	0.00	-0.14	0.43	0.49	1.00

Table 20: DELPHI lepton fit correlation matrices.

	statistical							systematic						
	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c
R_b	1.00	0.00	0.00	-0.98	0.00	0.00	0.00	1.00	0.00	0.00	-0.30	0.00	0.00	0.00
R_c	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
$\overline{\chi_B}$	0.00	0.00	1.00	0.00	0.00	0.27	0.00	0.00	0.00	1.00	0.00	0.00	0.14	0.01
$b\to\ell$	-0.98	0.00	0.00	1.00	0.00	0.00	0.00	-0.30	0.00	0.00	1.00	0.00	0.00	0.00
$b\to c\to\ell$	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
A_{FB}^b	0.00	0.00	0.27	0.00	0.00	1.00	0.20	0.00	0.00	0.14	0.00	0.00	1.00	-0.01
A_{FB}^c	0.00	0.00	0.00	0.00	0.00	0.20	1.00	0.00	0.00	0.01	0.00	0.00	-0.01	1.00

Table 21: L3 lepton fit correlation matrices.

	statistical							systematic						
	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c
R_b	1.00	0.00	-0.12	-0.93	0.07	-0.12	0.09	1.00	0.00	-0.30	0.16	0.02	-0.35	0.01
R_c	0.00	1.00	0.00	0.00	0.00	0.11	-0.17	0.00	1.00	0.00	0.00	0.00	0.00	0.00
$\overline{\chi_B}$	-0.12	0.00	1.00	0.14	-0.14	0.53	-0.01	-0.30	0.00	1.00	0.39	-0.34	0.35	-0.22
$b\to\ell$	-0.93	0.00	0.14	1.00	-0.15	0.12	-0.08	0.16	0.00	0.39	1.00	-0.02	0.04	-0.33
$b\to c\to\ell$	0.07	0.00	-0.14	-0.15	1.00	-0.08	0.01	0.02	0.00	-0.34	-0.02	1.00	-0.20	-0.15
A_{FB}^b	-0.12	0.11	0.53	0.12	-0.08	1.00	0.23	-0.35	0.00	0.35	0.04	-0.20	1.00	0.12
A_{FB}^c	0.09	-0.17	-0.01	-0.08	0.01	0.23	1.00	0.01	0.00	-0.22	-0.33	-0.15	0.12	1.00

Table 22: OPAL lepton fit correlation matrices.

Reference	[19]	[20]	[21]
R_b	0.228	0.2187	0.220
experimental statistical error	2.2%	1.0%	7.3%
Monte Carlo statistical error	–	0.6%	–
hemisphere correlations NN and VTX	0.8%	0.3%	–
hemisphere correlations leptons	0.6%	–	–
charm efficiency for NN	0.4%	–	–
lepton fit b purity	1.6%	–	–
lepton fit c purity	0.3%	–	–
background	0.2%	–	–
acceptance	0.1%	0.1%	–
$f(D^\pm) : f(D^0) : f(D_s) : f(\Lambda_c^\pm)$	–	0.5%	–
K^0 and hyperon production	–	0.1%	–
charmed hadron lifetime	–	0.2%	–
detector effects	0.3%	0.8%	–
Br(b→l) (0.102±0.010)	–	–	9.8%
c fragm. (ϵ_c±0.02)	–	0.1%	–
b/c from fragmentation	–	0.2%	–
inclusive lepton 1989 systematic	–	–	4.6%
R_c (±0.014)	0.2%	0.5%	0.9%
Total systematic error (excl. R_c)	2.0%	1.2%	10.8%

Table 23: ALEPH non-lepton fit R_b results. The uncertainties are *relative*. The measurement from [21] is from 1989 data alone and has a weight of only 1.2% in the ALEPH average. The abbreviations NN and VTX indicate Neural Network and lifetime tagging analysis respectively.

Reference	[22]	[23]	[24]	[25]
R_b	0.2257	0.2201	0.232	0.218
experimental statistical error	1.5%	1.8%	2.2%	6.4%
Monte Carlo statistical error	–	–	–	4.6%
efficiency correlation	0.9%	–	–	–
event selection bias	–	0.5%	–	–
tracking resolution	1.2%	1.2%	2.0%	1.8%
tracking resolution	–	–	4.3%	–
average beam spot	–	0.7%	–	–
$Br(b \to \ell)$	–	–	0.9%	–
b semi-leptonic decay model	2.1%	–	–	–
b fragmentation	–	–	1.7%	4.1%
b-flavoured hadron lifetime	–	0.5%	–	–
b decay multiplicity	–	–	–	–
c fragmentation	–	0.2%	1.7%	1.4%
$Br(c \to \ell)$	–	–	0.9%	–
c decay model	0.1%	–	–	–
$f(D^{\pm}) : f(D^0) : f(D_s) : f(\Lambda_c^{\pm})$	0.2%	0.5%	–	–
charmed hadron lifetime	0.1%	0.5%	–	–
c decay multiplicity	–	0.4%	–	–
light flavour multiplicity	0.1%	–	–	–
K^0 and hyperon production	–	0.8%	–	–
JETSET parameters	–	–	4.0%	–
event generator	–	–	3.4%	4.1%
b/c from fragmentation	–	0.5%	–	–
B hadron boost estimate	–	–	–	3.2%
jet clustering dependence	–	–	–	1.8%
R_c (± 0.014)	0.4%	0.9%	–	0.7%
Total systematic error (excl. R_c)	2.6%	2.0%	7.6%	8.6%

Table 24: DELPHI non-lepton fit R_b results. The uncertainties are *relative*. The measurements from [24] and [25] only get weights of 2.9% and 1.5% in the DELPHI average respectively.

Reference	[26]	[27]
R_b	0.222	0.204
experimental statistical error	1.4%	6.9%
$Br(b \to \ell)$ (L3 measurement stat)	0.1%	9.0%
b fragm. ($\epsilon_b \pm 0.004$)	1.4%	–
c fragm. ($\epsilon_c \pm 0.02$)	0.8%	–
JETSET PS parameters	2.2%	–
$m_b \pm 0.5$ GeV	0.3%	–
calorimeter $\sigma(E)$	0.9%	–
1989 systematic	–	6.9%
R_c (± 0.014)	0.8%	1.2%
Total systematic error (excl. R_c)	2.9%	11.3%

Table 25: L3 non-lepton fit R_b results. The uncertainties are *relative*. The measurement from [27] gets a weight of 5.1% in the average L3 result.

Reference	[28]	[28]	[29]	[30]	[31]
R_b	0.216	0.224	0.222	0.218	0.2197
experimental statistical error	1.4%	1.4%	3.2%	2.9%	1.41%
Monte Carlo statistical error	1.1%	1.2%	–	0.9%	0.28%
efficiency correlation	–	–	–	0.9%	–
efficiency correlation 1992	–	–	–	–	0.61%
event selection efficiency	0.4%	0.4%	–	–	–
bias due to kinematic cuts	–	–	1.8%	–	–
event selection bias	0.11%	0.11%	0.11%	0.11%	0.11%
photon conversions	0.9%	–	–	0.8%	0.15%
electron hadronic background	0.2%	–	–	1.3%	0.16%
muon hadronic background	–	1.7%	–	1.3%	0.21%
electron id. eff. for b decays	2.9%	–	–	–	0.07%
muon id. eff. for b decays	–	3.7%	–	–	0.02%
lepton id. eff. for background	–	–	–	0.7%	–
electron radiation loss	0.8%	–	–	–	–
θ determination	–	0.4%	–	–	–
tracking resolution	–	–	0.5%	0.2%	0.15%
tracking non-Gaussian tail	–	–	0.5%	–	–
average beam spot	–	–	0.2%	0.2%	–
$Br(\mathrm{b} \to \ell)$	4.0%	4.2%	–	–	–
$Br(\mathrm{b} \to \mathrm{c} \to \ell)$	1.9%	1.0%	–	–	–
$Br(\mathrm{b} \to \mathrm{J}/\psi \to \ell)$	0.5%	0.4%	–	–	–
$Br(\mathrm{b} \to \tau \to \ell)$	0.6%	0.4%	–	–	–
b semi-leptonic decay model	0.5%	0.3%	–	–	–
b fragmentation	1.4%	1.2%	0.5%	0.4%	–
b-flavoured hadron lifetime	–	–	0.5%	0.2%	–
b decay multiplicity	–	–	1.8%	–	–
c fragmentation	0.8%	0.3%	0.5%	0.9%	0.37%
$Br(\mathrm{c} \to \ell)$	0.8%	0.5%	–	1.2%	0.27%
c decay model	0.9%	0.5%	–	1.4%	0.12%
$f(\mathrm{D}^\pm) : f(\mathrm{D}^0) : f(\mathrm{D_s}) : f(\Lambda_c^\pm)$	0.5%	0.3%	0.9%	0.9%	0.50%
charmed hadron lifetime	–	–	–	–	0.19%
c decay multiplicity	–	–	–	–	0.61%
light flavour multiplicity	–	–	0.5%	–	–
K^0 and hyperon production	–	–	–	0.2%	0.19%
event generator	–	–	1.8%	–	–
b/c from fragmentation	0.5%	0.4%	–	–	0.54%
R_c ($\pm$ 0.014)	0.7%	0.4%	1.1%	1.2%	0.74%
Total systematic error (excl. R_c)	6.0%	6.3%	3.5%	3.4%	1.3%

Table 26: OPAL non-lepton fit R_b results. The uncertainties are *relative*.

Experiment	ALEPH	DELPHI	L3	OPAL	Combined
R_{b}	0.2205	0.2228	0.2211	0.2199	0.2208
experimental statistical error	0.91%	1.16%	1.37%	1.16%	0.58%
Monte Carlo statistical error	0.48%	–	–	0.23%	0.24%
charm efficiency for NN	0.08%	–	–	–	0.04%
lepton fit b purity	0.30%	–	–	–	0.14%
lepton fit c purity	0.06%	–	–	–	0.03%
background	0.04%	–	–	–	0.02%
acceptance	0.10%	–	–	–	0.05%
detector effects	0.70%	1.22%	0.85%	0.28%	0.41%
Aleph inclusive lepton 1989 systematic	0.06%	–	–	–	0.03%
L3 1989 systematic	–	–	0.35%	–	0.03%
$Br(\mathrm{b} \to \ell)$ (L3 measurement stat)	–	–	0.55%	–	0.05%
lepton id. eff. and background	–	–	–	0.45%	0.11%
average beam spot	–	0.38%	–	0.03%	0.07%
event selection bias	–	0.27%	–	0.11%	0.07%
hemisphere correlations	0.41%	0.37%	–	0.47%	0.38%
$Br(\mathrm{b} \to \ell)$ (0.102 ± 0.010)	0.12%	0.03%	–	0.23%	0.12%
$Br(\mathrm{b} \to \mathrm{c} \to \ell)$	–	–	–	0.07%	0.02%
$Br(\mathrm{b} \to \mathrm{J}/\psi \to \ell)$	–	–	–	0.02%	0.00%
$Br(\mathrm{b} \to \tau \to \ell)$	–	–	–	0.03%	0.01%
b semi-leptonic decay model	–	0.87%	–	0.02%	0.15%
c decay model	–	0.04%	–	0.21%	0.06%
b fragm. ($\epsilon_b \pm 0.004$)	–	0.11%	1.33%	0.15%	0.19%
c fragm. ($\epsilon_c \pm 0.02$)	0.08%	0.18%	0.76%	0.42%	0.25%
$Br(\mathrm{c} \to \ell)$	–	0.03%	–	0.32%	0.09%
b-flavoured hadron lifetime	–	–	–	0.07%	0.02%
charmed hadron lifetime	0.16%	0.31%	–	0.15%	0.17%
JETSET PS parameters	–	0.12%	2.09%	–	0.23%
event generator	–	0.10%	–	0.20%	0.07%
$f(\mathrm{D}^{\pm}) : f(\mathrm{D}^0) : f(\mathrm{D_s}) : f(\Lambda_{\mathrm{c}}^{\pm})$	0.40%	0.35%	–	0.56%	0.39%
$m_b \pm 0.5$ GeV	–	–	0.28%	–	0.03%
b decay multiplicity	–	–	–	0.20%	0.05%
c decay multiplicity	0.24%	0.22%	–	0.47%	0.27%
light flavour multiplicity	–	0.04%	–	0.06%	0.02%
K^0 and hyperon production	0.08%	0.43%	–	0.16%	0.15%
b/c from fragmentation	0.06%	0.27%	–	0.44%	0.19%
R_{c} ($\pm$ 0.014)	0.45%	0.66%	0.82%	0.80%	0.61%
Total systematic error (excl. R_{c})	1.1%	1.8%	2.8%	1.4%	0.97%

Table 27: LEP non-lepton fit R_{b} results per experiment. The uncertainties are *relative*. The last column gives the break down for the errors of the combined result.

Reference	[32]	[24]	[33]
R_c	0.187	0.151	0.162
experimental statistical error	16.6%	5.3%	18.5%
Monte Carlo statistical error	3.0%	–	–
CLEO D^0 and D^+ error	10.5%	–	–
CLEO D^* error	–	–	16.0%
background estimation	10.0%	–	17.9%
reconstruction bias	5.6%	–	7.4%
b-hadron lifetime (1.3±0.1 ps)	1.4%	0.7%	–
b fragm. ($\epsilon_b = 0.0085 \pm 0.0045$)	1.4%	1.8%	–
D's in uds events	1.4%	–	–
$Br(D^0 \to K^- \pi^+)$	5.8%	–	–
$Br(D^+ \to K^- \pi^+ \pi^+)$	10.0%	–	–
$Br(D^{*\pm} \to D^0 \pi^\pm)$	–	1.7%	–
$Br(c \to \ell)$ (9.6±0.6%)	–	1.7%	–
JETSET PS parameters	–	15.5%	–
event generator	–	21.9%	–
detector resolution	–	6.0%	–
electron hadronic background	–	–	1.2%
Fitting procedure	–	–	17.8%
Total systematic error	19.8%	27.0%	30.8%

Table 28: DELPHI non-lepton fit R_c results. The uncertainties are *relative*.

Reference	[34]	[35]
R_c	0.186	0.183
experimental statistical error	18.8%	14.2%
$Br(c \to D^*)$ (0.0073±0.0006)	8.2%	–
$Br(b \to D^*)$	6.3%	–
b fragm. ($\epsilon_b = 0.008 \pm 0.002$)	3.5%	–
background estimation	8.1%	–
$Br(c \to \ell)$ (9.6±0.6%)	–	6.3%
detector resolution	–	4.5%
lepton background shape	–	6.8%
muon efficiency	–	1.7%
cascade contribution	–	6.3%
fitting bias	–	21.0%
fragmentation function	–	4.5%
R_b (0.215±0.018)	1.5%	–
Total systematic error	13.7%	24.7%

Table 29: OPAL non-lepton fit R_c results. The uncertainties are *relative*.

Experiment/Reference	ALEPH[36]	DELPHI	OPAL	Combined
R_c	0.148	0.165	0.185	0.172
experimental statistical error	29.7%	6.9%	12.6%	7.4%
Monte Carlo statistical error	–	1.3%	–	0.6%
lepton tagging eff. and background	9.0%	0.2%	2.8%	1.6%
detector resolution	–	2.4%	1.8%	1.3%
background estimation	–	7.4%	4.9%	4.0%
Fitting procedure	–	3.1%	8.4%	3.6%
reconstruction bias	–	3.7%	–	1.8%
$Br(\mathrm{b}\to \ell)$ (0.102±0.010)	14.0%	–	–	1.9%
$Br(\mathrm{b}\to \tau, \psi, \bar{\mathrm{c}} \to \ell)$	17.0%	–	–	2.3%
cascade contribution	–	–	2.5%	1.0%
CLEO D^0 and D^+ error	–	4.4%	–	2.1%
CLEO D^* error	–	2.8%	6.2%	3.7%
b-hadron lifetime (1.3±0.1 ps)	–	0.9%	–	0.4%
b fragm. ($\epsilon_b = 0.0085\pm0.0015$)	–	1.3%	1.6%	1.2%
D's in uds events	–	0.6%	–	0.3%
$Br(\mathrm{D}^0 \to \mathrm{K}^-\pi^+)$	–	2.4%	–	1.2%
$Br(\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+)$	–	4.2%	–	2.0%
$Br(\mathrm{D}^{*\pm} \to \mathrm{D}^0\pi^\pm)$	–	0.7%	–	0.3%
$Br(\mathrm{c} \to \ell)$ (9.6±0.6%)	6.6%	0.7%	2.5%	2.2%
JETSET PS parameters	–	6.2%	–	3.0%
event generator	–	8.8%	–	4.2%
Total systematic error	25%	16%	13%	10.2%

Table 30: LEP non-lepton fit R_c results per experiment. The uncertainties are *relative*. The last column gives the break down for the errors of the combined result.

Experiment/reference	ALEPH[37]	DELPHI[38]	OPAL[39]	Combined
$A_{\rm FB}^{\rm b}$	0.1019	0.117	0.096	0.1016
experimental statistical error	8.2%	14.9%	11.7%	6.2%
Monte Carlo statistical error	–	–	0.4%	0.1%
detector effects	1.7%	–	1.1%	1.0%
δ_b determination	3.8%	–	–	2.0%
lifetime tag purity	1.7%	4.4%	0.4%	1.1%
tag acceptance	0.8%	1.8%	0.4%	0.5%
thrust direction resolution	0.4%	–	1.2%	0.5%
$Br(\mathrm{b} \to \ell)$ (11$\pm$0.5%)	–	0.2%	–	0.0%
$Br(\mathrm{b} \to \mathrm{c} \to \ell)$ (8.7$\pm$0.9%)	–	1.4%	–	0.2%
$Br(\mathrm{c} \to \ell)$ (9$\pm$1%)	–	0.1%	–	0.0%
b fragm. ($\epsilon_b = 0.006 \pm 0.002$)	–	0.2%	–	0.0%
c fragm. ($\epsilon_c = 0.049 \pm 0.015$)	–	2.6%	2.2%	1.1%
muon tagging eff. and background	–	1.2%	–	0.2%
b jet charge tag probability	–	4.8%	–	0.6%
JETSET PS parameters	–	0.9%	–	0.1%
MC generator	–	0.9%	–	0.1%
hemisphere correlation	–	–	0.6%	0.2%
background asymmetry	0.9%	–	0.4%	0.6%
$A_{\rm FB}^{\rm c}$ ($\pm$10%)	–	3.5%	1.1%	0.8%
$R_{\rm b}$ (0.217$\pm$0.0043)	2.1%	–	1.3%	1.6%
$R_{\rm c}$ (0.171$\pm$0.0014)	0.1%	0.1%	0.1%	0.1%
$\mathrm{B^0\overline{B^0}}$ mixing($\overline{\chi_{\rm B}}$=0.132$\pm$0.027)	–	1.8%	–	0.2%
Total systematic error (excl. $A_{\rm FB}^{\rm c}$, $R_{\rm b}$, $R_{\rm c}$ and $\mathrm{B^0\overline{B^0}}$ mixing)	4.7%	8.5%	3.4%	2.9%

Table 31: LEP non-lepton fit $A_{\rm FB}^{\rm b}$ results per experiment. The uncertainties are *relative*. These results are all based on a combination of vertex tagging and jet-charge measurements. The $A_{\rm FB}^{\rm b}$ values have been normalised to $\sqrt{s}$ = 91.3GeV and a QCD correction has been applied. QED corrections have not been applied to these numbers.

Experiment/reference	ALEPH[42]	DELPHI[40]	OPAL[41]	Combined
$A_{\rm FB}^{\rm c}$	0.075	0.067	0.095	0.081
experimental statistical error	61.8%	39.4%	26.1%	21.8%
detector effects	–	6.1%	2.2%	2.7%
thrust axis resolution	–	1.5%	1.1%	0.8%
binning effects in fit / fitting method	–	7.6%	–	3.1%
Aleph syst from 90-91 data	13.2%	–	–	2.0%
b decay modelling	–	–	4.4%	2.0%
c decay modelling	–	–	1.1%	0.5%
b fragm. ($\epsilon_b = 0.006 \pm 0.002$)	–	2.1%	1.2%	1.4%
c fragm. ($\epsilon_b = 0.049 \pm 0.015$)	–	2.1%	0.6%	1.1%
b-flavoured hadron lifetime	–	1.5%	2.2%	1.6%
D^* rate prediction	–	3.4%	–	1.4%
D^* background ($\pm 15\%$)	–	4.5%	9.1%	5.9%
slow pion rate	–	6.1%	–	2.5%
$A_{\rm FB}^{\rm b}$ ($\pm 10\%$)	–	4.5%	4.3%	3.7%
time dependent $B_d^0\overline{B_d^0}$ mixing($x_d = 0.69 \pm 0.07$)	–	3.0%	–	1.2%
$B^0\overline{B^0}$ mixing($\overline{\chi_B} = 0.132 \pm 0.027$)	–	–	3.2%	1.4%
Total systematic error (excl. $A_{\rm FB}^{\rm b}$ and $B^0\overline{B^0}$ mixing)	13%	13%	11%	8.6%

Table 32: LEP non-lepton fit $A_{\rm FB}^{\rm c}$ results per experiment. The uncertainties are *relative*. The measurements are from analyses involving tagged D^* mesons.

Experiment/reference	ALEPH[43]	DELPHI[44]	Combined
$\overline{\chi_B}$	0.121	0.147	0.142
experimental statistical error	18.8%	9.7%	8.6%
Monte Carlo statistical error	2.4%	–	0.5%
lepton background	1.2%	2.1%	1.9%
flavour composition of sample	3.5%	–	0.7%
$Br(b \to \ell)$	2.6%	2.7%	2.7%
$Br(b \to c \to \ell)$	2.7%	3.5%	3.4%
b fragmentation	17.1%	4.2%	6.7%
c fragmentation	–	1.4%	1.1%
$Br(c \to \ell)$	0.8%	1.4%	1.3%
light flavour fragmentation	–	6.2%	5.0%
momentum weighting dependence	1.8%	–	0.3%
correction to lepton average mixing	6.6%	2.0%	2.9%
Total systematic error	19%	9.4%	10%

Table 33: LEP non-lepton fit $\overline{\chi_B}$ results per experiment. The uncertainties are *relative*. The measured values have been corrected to the lepton average mixing assuming $f_s = 0.117$ and $\chi_s = 0.25 \pm 0.25$.

	statistical							systematic						
	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$b\to\ell$	$b\to c\to\ell$	A_{FB}^b	A_{FB}^c
R_b	1.00	-0.69	0.00	0.00	0.00	-0.08	0.00	1.00	-0.64	-0.04	0.00	0.00	-0.26	0.00
R_c	-0.69	1.00	0.00	0.00	0.00	0.01	0.00	-0.64	1.00	0.36	0.00	0.00	0.02	0.00
$\overline{\chi_B}$	0.00	0.00	1.00	0.00	0.00	0.02	-0.03	-0.04	0.36	1.00	0.00	0.00	0.08	-0.04
$b\to\ell$	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
$b\to c\to\ell$	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
A_{FB}^b	-0.08	0.01	0.02	0.00	0.00	1.00	0.38	-0.26	0.02	0.08	0.00	0.00	1.00	0.33
A_{FB}^c	0.00	0.00	-0.03	0.00	0.00	0.38	1.00	0.00	0.00	-0.04	0.00	0.00	0.33	1.00

Table 34: Non-lepton fit correlation matrices. The two leptonic branching ratios $Br(b \to \ell)$ and $Br(b \to c \to \ell)$ are dummy.

Table 35: Systematic errors specified per source. The systematic uncertainty from experimental sources that are not in common between the experiments are combined into the entry *experimental systematic*. This entry contains also systematic errors that are correlated between the different observables within one experiment.

	ALEPH							DELPHI						
	R_b	R_c	$\overline{\chi_B}$	$Br(b \to \ell)$	$Br(b \to c \to \ell)$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$Br(b \to \ell)$	$Br(b \to c \to \ell)$	A_{FB}^b	A_{FB}^c
experimental systematic	0.0033	0.0149	0.0029	0.34%	0.41%	0.10%	1.54%	0.0047	0.0079	0.007	0.33%	0.55%	0.37%	1.53%
R_b	0.	0.	0.	0. %	0. %	0. %	0. %	0.	0.	0.	0. %	0. %	0.05%	0.05%
R_c	0.	0.	0.	0. %	0. %	0. %	0. %	0.	0.	0.	0. %	0. %	0.11%	-0.48%
$Br(b \to \ell)$	0.	0.	0.	0. %	0. %	0. %	0. %	0.	0.	0.	0. %	0. %	0.18%	0.22%
$Br(b \to c \to \ell)$	0.	0.	-0.040	0. %	0. %	0.04%	0.61%	0.	0.	0.	0. %	0. %	0. %	0. %
$b \to \ell$ decay model	0.0026	-0.0025	0.0025	0.40%	-0.62%	-0.05%	-0.72%	-0.0002	-0.0002	0.007	0.39%	-0.57%	-0.22%	-0.90%
$c \to \ell$ decay model	0.0009	0.0040	0.0044	0.09%	0.03%	-0.06%	-0.11%	0.0007	0.0049	-0.006	-0.16%	-0.03%	-0.12%	-0.02%
$Br(c \to \ell)$	-0.0010	-0.0093	0.	-0.05%	0.02%	0.05%	-0.04%	0.0022	-0.0157	-0.003	-0.09%	-0.07%	0.09%	-0.70%
$Br(b \to \overline{c} \to \ell)$	0.0006	0.0050	0.0030	-0.03%	-0.20%	0. %	0.47%	0.0004	0.0017	0.004	0.01%	-0.43%	0. %	0. %
$Br(b \to \tau \to \ell)$	0.0003	0.0006	0.	-0.03%	-0.09%	0. %	0.11%	-0.0002	-0.0002	0.001	-0.03%	-0.13%	0. %	0. %
$Br(b \to \psi \to \ell)$	0.0003	0.0006	0.	0.03%	0.06%	0. %	0.02%	0.	0.	0.	0. %	0. %	0. %	0. %
$b \to u$	0.0022	0.0006	0.0016	0.04%	0.41%	0.03%	0.15%	0.	0.	0.	0. %	0. %	0. %	0. %
b fragmentation	0.	0.0005	0.0001	0. %	0. %	0.01%	0. %	0.	0.	0.	0. %	0. %	-0.03%	-0.14%
c fragmentation	0.	0.	0.	0. %	0. %	0.07%	0. %	-0.0015	0.0056	0.003	0.17%	-0.28%	0.21%	-0.08%
b baryon polarization	0.	0.	0.	0. %	0. %	0. %	0. %	0.	0.	0.	0. %	0. %	0. %	0. %
$B \to D$	-0.0004	0.0054	0.0020	0.14%	-0.79%	0.01%	0.14%	0.	0.	0.	0. %	0. %	0. %	0. %
light flavour fragm.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

	L3							OPAL						
	R_b	R_c	$\overline{\chi_B}$	$Br(b \to \ell)$	$Br(b \to c \to \ell)$	A_{FB}^b	A_{FB}^c	R_b	R_c	$\overline{\chi_B}$	$Br(b \to \ell)$	$Br(b \to c \to \ell)$	A_{FB}^b	A_{FB}^c
experimental systematic	0.0047	–	0.010	0.28%	– %	0.49%	2.28%	0.0044	–	0.0036	0.39%	0.28%	0.21%	0.51%
R_b	0.	0.	0.	0. %	– %	0.04%	0.10%	0.	0.	0.	0. %	0. %	0. %	0. %
R_c	-0.017	0.	0.	0.02%	– %	0.03%	-0.20%	-0.003	0.	0.003	0.05%	0.07%	0.09%	-0.21%
$Br(b \to \ell)$	0.	0.	0.002	0. %	– %	0.08%	0.20%	0.	0.	0.	0. %	0. %	0. %	0. %
$Br(b \to c \to \ell)$	0.	0.	0.	0. %	– %	0. %	0. %	0.	0.	0.	0. %	0. %	0. %	0. %
$b \to \ell$ decay model	0.	–	0.	0.40%	– %	0. %	0. %	0.0030	–	0.0020	0.40%	-0.21%	-0.02%	-0.50%
$c \to \ell$ decay model	0.	–	0.	0.13%	– %	0. %	0. %	0.0010	–	0.0010	0.05%	0.34%	-0.12%	-0.02%
$Br(c \to \ell)$	0.	–	0.001	0. %	– %	0.17%	-1.30%	-0.0010	–	0.0020	-0.01%	0.04%	0.20%	-0.33%
$Br(b \to \overline{c} \to \ell)$	0.	–	0.	0. %	– %	0. %	0. %	0.0010	–	0.	-0.06%	-0.07%	0.07%	0.36%
$Br(b \to \tau \to \ell)$	0.	–	0.	0. %	– %	0. %	0. %	0.0010	–	0.	-0.07%	-0.04%	0.02%	0.15%
$Br(b \to \psi \to \ell)$	0.	–	0.	0. %	– %	0. %	0. %	0.	–	0.	0. %	0. %	-0.01%	0.06%
$b \to u$	0.	–	0.	0. %	– %	0. %	0. %	0.	–	0.	0. %	0. %	0. %	0. %
b fragmentation	-0.0010	–	0.001	-0.18%	– %	-0.03%	-0.20%	-0.0010	–	0.0010	-0.16%	-0.07%	-0.03%	-0.09%
c fragmentation	-0.0001	–	0.	0.04%	– %	0.08%	-0.40%	-0.0020	–	0.0010	0.04%	0.05%	0.16%	-0.14%
b baryon polarization	0.	–	0.	0. %	– %	0. %	0. %	0.	–	0.	-0.01%	0.08%	0.01%	-0.05%
$B \to D$	-0.0031	–	0.	0.21%	– %	0. %	0. %	-0.0014	–	0.0042	0.07%	-0.49%	0.04%	0.19%
light flavour fragm.	0.	–	0.	0. %	– %	0. %	0. %	0.	–	0.	0. %	0. %	0. %	0. %

	OTHER MEASUREMENTS						
	R_b	R_c	$\overline{\chi_B}$	$Br(b \to \ell)$	$Br(b \to c \to \ell)$	A_{FB}^b	A_{FB}^c
experimental systematic	0.0034	0.014	0.0070	–	–	0.28%	0.66%
R_b	0.	–	–	–	–	–	0. %
R_c	0.	–	–	–	–	–	0. %
$Br(b \to \ell)$	0.0002	0.0033	0.0036	–	–	0. %	0. %
$Br(b \to c \to \ell)$	0.0000	0.0040	0.0045	–	–	0.02%	0. %
$b \to \ell$ decay model	0.0003	0.	0.	–	–	0. %	-0.16%
$c \to \ell$ decay model	0.0001	0.	0.	–	–	0. %	-0.04%
$Br(c \to \ell)$	-0.0002	0.0040	0.0018	–	–	0. %	0. %
$Br(b \to \overline{c} \to \ell)$	0.	–	–	–	–	0. %	0. %
$Br(b \to \tau \to \ell)$	0.	0.0020	0.	–	–	0. %	0. %
$Br(b \to \psi \to \ell)$	0.	0.0020	0.	–	–	0. %	0. %
$b \to u$	0.	0.	0.	–	–	0. %	0. %
b fragmentation	-0.0004	0.0020	0.0095	–	–	0. %	-0.11%
c fragmentation	-0.0005	0.	0.0016	–	–	0.11%	-0.09%
b baryon polarization	0.	0.	0.	–	–	0. %	0. %
$B \to D$	-0.0008	0.0040	0.	–	–	0. %	0.11%
light flavour fragm.	0.0004	0.0090	0.0071	–	–	0. %	0.

	result	Correlation coefficients						
		R_b	R_c	$\overline{\chi_B}$	b$\to \ell$	b$\to$c$\to \ell$	A_{FB}^b	A_{FB}^c
R_b	0.2208±0.0013±0.0020	1.000	-0.384	0.081	-0.313	-0.084	-0.057	0.061
R_c	0.1698±0.0035±0.0134	-0.384	1.000	0.006	0.166	0.035	-0.023	-0.025
$\overline{\chi_B}$	0.1182±0.0050±0.0045	0.081	0.006	1.000	0.036	-0.242	0.141	0.048
b$\to \ell$	0.1115±0.0009±0.0040	-0.313	0.166	0.036	1.000	-0.246	0.057	0.069
b$\to$c$\to \ell$	0.0796±0.0019±0.0055	-0.084	0.035	-0.242	-0.246	1.000	-0.024	0.022
A_{FB}^b	0.0942±0.0038±0.0019	-0.057	-0.023	0.141	0.057	-0.024	1.000	0.209
A_{FB}^c	0.0655±0.0080±0.0071	0.061	-0.025	0.048	0.069	0.022	0.209	1.000

Table 36: Results for the combined LEP heavy flavour electroweak fit.

$\sqrt{s}$	A_{FB}^b
89.51 GeV	0.039 ± 0.019
91.19 GeV	0.0942 ± 0.0042
93.31 GeV	0.100 ± 0.015

Table 37: Values of A_{FB}^b as a function of centre-of-mass energy.

5 The Hadronic Charge Asymmetry

The status of LEP results related to the hadronic charge asymmetry is unchanged with respect to [1].

6 The Effective Electroweak Mixing Angle $\sin^2\theta^{\rm lept}_{eff}$

Table 38 shows the results of various determinations of $\sin^2\theta^{\rm lept}_{eff}$ and their combination. Also the comparison with the measurement of the left-right asymmetry, $A_{\rm LR}$, at SLC is given [51].

	$\sin^2\theta^{\rm lept}_{eff}$	average by group of observations	$\chi^2/(d.o.f.)$	cumulative average	$\chi^2/(d.o.f.)$
$A_{\rm FB}^{0,\ell}$	0.23107 ± 0.00092				
$\mathcal{A}_\tau$	0.2311 ± 0.0013				
$\mathcal{A}_{\rm e}$	0.2350 ± 0.0015	0.2319 ± 0.0007	5.5/2	0.2319 ± 0.0007	5.5/2
$A_{\rm FB}^{0,\rm b}$	0.23279 ± 0.00077				
$A_{\rm FB}^{0,\rm c}$	0.2324 ± 0.0026	0.2327 ± 0.0007	0.0/1	0.2323 ± 0.0005	6.2/4
$\langle Q_{\rm FB}\rangle$	0.2320 ± 0.0016	0.2320 ± 0.0016	–	0.2322 ± 0.0005	6.3/5
$A_{\rm LR}$ (SLC)	0.2294 ± 0.0010	0.2294 ± 0.0010	–	0.2317 ± 0.0004	12.8/6

Table 38: Comparison of several determinations of $\sin^2\theta^{\rm lept}_{eff}$ from asymmetries. Averages are obtained as weighted averages assuming no correlations.

7 Standard Model Constraints

The precision measurements collected in the preceeding sections can be used to check the validity of the Standard Model and to infer valuable information about its basic parameters. Their accuracy makes them sensitive to the top quark mass, $M_{\rm t}$, and the mass of the Higgs boson, $M_{\rm H}$, through the loop corrections. The leading top quark dependence is quadratic and allows a determination of $M_{\rm t}$. The main dependence on $M_{\rm H}$ is logarithmic and, with the present data accuracy, it is not expected to allow a meaningful determination of $M_{\rm H}$.

The measurements collected above, are summarized in Table 39 and presented in Figure 3 together with their Standard Model prediction as a function of $M_{\rm t}$. The bands in the Standard Model predictions reflect the linear sum of the expected variations of each quantity due to a variation of the strong coupling constant $\alpha_s(M_{\rm Z}^2) = 0.123 \pm 0.006$ [53] and $M_{\rm H}$ in the interval $60 \leq M_{\rm H}$ [GeV] ≤ 1000 for $M_{\rm Z} = 91.1895$ GeV.

Table 40 shows the constraints obtained on $M_{\rm t}$ and $\alpha_s(M_{\rm Z}^2)$ when fitting all the above measurements in Table 39 to the most up to date Standard Model calculations [54]. Since the data are not expected to have a sensitivity with respect to $M_{\rm H}$, the fits have been repeated for $M_{\rm H} = 60$, 300 and 1000 GeV and the difference in the fitted parameters is quoted as an uncertainty. We present the

	measurement	Standard Model fit	pull
a) LEP			
line-shape and lepton asymmetries:			
$M_{\rm Z}$ [GeV]	91.1895 ± 0.0044	91.192	0.6
$\Gamma_{\rm Z}$ [GeV]	2.4969 ± 0.0038	2.4967	0.1
$\sigma^0_{\rm h}$ [nb]	41.51 ± 0.12	41.44	0.6
R_ℓ	20.789 ± 0.040	20.781	0.2
$A^{0,\ell}_{\rm FB}$	0.0170 ± 0.0016	0.0152	1.1
+ correlation matrix (Table 12)			
τ polarization:			
$\mathcal{A}_\tau$	0.150 ± 0.010	0.142	0.8
$\mathcal{A}_{\rm e}$	0.120 ± 0.012	0.142	1.8
b and c quark results:			
$R_{\rm b} = \Gamma_{\rm b\overline{b}}/\Gamma_{\rm had}$	0.2208 ± 0.0024	0.2158	2.0
$R_{\rm c} = \Gamma_{\rm c\overline{c}}/\Gamma_{\rm had}$	0.170 ± 0.014	0.172	0.1
$A^{0,b}_{\rm FB}$	0.0960 ± 0.0043	0.0997	0.8
$A^{0,c}_{\rm FB}$	0.070 ± 0.011	0.071	0.1
+ correlation matrix (Table 36)			
$\rm q\overline{q}$ charge asymmetry:			
$\sin^2\theta^{\rm lept}_{eff}$ from $\langle Q_{\rm FB}\rangle$	0.2320 ± 0.0016	0.2321	0.1
b) $\rm p\overline{p}$ and νN			
M_W [GeV] (CDF, CDF prel., D0 prel., UA2; [52])	80.23 ± 0.18	80.31	0.4
$1 - M^2_{\rm W}/M^2_{\rm Z}(\nu N)$	0.2256 ± 0.0047	0.2246	0.2
c) SLC			
$\sin^2\theta^{\rm lept}_{eff}$ from $\mathcal{A}_{\rm e}$	0.2294 ± 0.0010	0.2321	2.7

Table 39: Summary of measurements included in the combined analysis of Standard Model parameters. Section a) summarizes LEP averages, section b) electroweak precision tests from hadron colliders and νN-scattering, section c) gives the result for $\sin^2\theta^{\rm lept}_{eff}$ from the measurement of the left-right polarization asymmetry at SLC. The Standard Model fit result in column 3 and the pulls in column 4 are derived from the fit including all data (Table 40, column 4) for a fixed value of $M_{\rm H} = 300$ GeV.

results obtained using only the LEP data, the results obtained using in addition the measurements of $M_{\rm W}$ from UA2 [55], CDF [56, 57] and D0 [57], and the measurements of the neutrino neutral to charged current ratios from CDHS [58], CHARM [59] and CCFR [60] and finally we also include the SLC result for the left-right asymmetry, $A_{\rm LR}$ [51]. The $\chi^2/(d.o.f.)$ for all these fits is acceptable. The value of $\alpha_s(M^2_{\rm Z})$ resulting from these fits is in very good agreement with the value obtained from event shape measurements at LEP ($\alpha_s(M^2_{\rm Z}) = 0.123 \pm 0.006$ [53]) and of similar precision.

The theoretical uncertainties are dominated by the uncertainty in the value of $\alpha(M^2_{\rm Z})$ due to the contribution of light quarks to the vacuum-polarization of the photon. An uncertainty $\delta\Delta\alpha = 0.0009$

has been propagated in the fits.

	LEP	LEP + Collider and ν data	LEP + Collider and ν data + $A_{\rm LR}$ from SLC
$M_{\rm t}$ (GeV)	$172^{+13\ +18}_{-14\ -20}$	$170^{+12\ +18}_{-12\ -19}$	$177^{+11\ +18}_{-11\ -19}$
$\alpha_s(M_{\rm Z}^2)$	$0.125 \pm 0.005 \pm 0.002$	$0.125 \pm 0.005 \pm 0.002$	$0.124 \pm 0.005 \pm 0.002$
$\chi^2/(d.o.f.)$	11.4/9	11.5/11	19.1/12
$\sin^2\theta^{\rm lept}_{eff}$	$0.2323 \pm 0.0002\ ^{+0.0001}_{-0.0002}$	$0.2324 \pm 0.0002\ ^{+0.0001}_{-0.0002}$	$0.2320 \pm 0.0003\ ^{+0.0001}_{-0.0002}$
$1 - M_{\rm W}^2/M_{\rm Z}^2$	$0.2251 \pm 0.0015\ ^{+0.0003}_{-0.0003}$	$0.2253 \pm 0.0013\ ^{+0.0003}_{-0.0002}$	$0.2243 \pm 0.0012\ ^{+0.0003}_{-0.0002}$
$M_{\rm W}$ (GeV)	$80.28 \pm 0.08\ ^{+0.01}_{-0.02}$	$80.26 \pm 0.07\ ^{+0.01}_{-0.01}$	$80.31 \pm 0.06\ ^{+0.01}_{-0.01}$

Table 40: Results of fits to LEP and other data for $M_{\rm t}$ and $\alpha_s(M_{\rm Z}^2)$. No external constraint on $\alpha_s(M_{\rm Z}^2)$ has been imposed. In the third column also the combined data from the $\rm p\overline{p}$ experiments UA2 [55], CDF [56, 57] and D0 [57]: $M_{\rm W} = 80.22 \pm 0.16$ GeV and from the neutrino experiments, CDHS [58], CHARM [59] and CCFR [60]: $1 - M_{\rm W}^2/M_{\rm Z}^2 = 0.2256 \pm 0.0047$ are included. The fourth column gives the result when also the SLD measurement of the left-right asymmetry at SLC [51], $\sin^2\theta^{\rm lept}_{eff} = 0.2294 \pm 0.0010$, is added. The central values and the first errors quoted refer to $M_{\rm H} = 300$ GeV. The second errors correspond to the variation of the central value when varying $M_{\rm H}$ in the interval $60 \leq M_{\rm H}\ [\rm GeV] \leq 1000$.

The fact that the $R_{\rm b}$ measurement has an accuracy of the order of one per cent, allows the estimation of the top mass entering the $\rm Z^0 b\overline{b}$ vertex correction in a way which is largely free from assumptions on the Higgs sector structure. Figure 4 shows the χ^2 curves as a function of $M_{\rm t}$ obtained by fitting this measurement alone.

It should be emphasized that the averaging of $R_{\rm b}$ values obtained from different methods with sometimes very different corrections or assumptions should be considered very preliminary and will require further study.

Nevertheless, if taken at face value, $R_{\rm b}$ differs from the Standard Model prediction by about 2 standard deviations (see Table 39). Since the b partial width is a component of the total hadronic width, a possible effect could be revealed in the ratio R_ℓ.

In Figure 5 the measured values of $\sin^2\theta^{\rm lept}_{eff}$ and $R_{\rm b}$ are plotted against each other as a vertical and a horizontal band respectively. If one assumes the Standard model dependence on $\sin^2\theta^{\rm lept}_{eff}$ for the light quark widths, and assuming for $\alpha_s(M_{\rm Z}^2)$ the value 0.123 ± 0.006, R_ℓ imposes a constraint on these two variables, which is represented by the inclined band. While the intersect of the $R_{\rm b}$ and R_ℓ bands agree marginally with each other, R_ℓ agrees with the Standard Model, while $R_{\rm b}$ does not. This graph also shows that both $R_{\rm b}$ and R_ℓ pull the best Standard Model fit towards low Higgs mass values. Confidence contours of a fit which determines $R_{\rm b}$ and $\sin^2\theta^{\rm lept}_{eff}$ with the assumptions above from the measured values of $R_{\rm b}$, $R_{\rm c}$, R_ℓ, $\sigma^0_{\rm h}$ and $\sin^2\theta^{\rm lept}_{eff}$ of LEP and SLC are shown in in Figure 6.

Figure 7 shows, for the fit in Table 40 column 4, the χ^2 value as a function of $M_{\rm t}$ for the three values of $M_{\rm H}$ considered in Table 40.

At present, the increase in χ^2 when $M_{\rm H}$ is changed between the two extreme values is about 2.2 for the LEP data alone and about 3.7 when including collider, νN and SLC data, which does not

allow the derivation of meaningful constraints on $M_{\rm H}$. For the fit to the LEP data alone, most of this increase is still traced back to the contribution of the $R_{\rm b}$ measurement, as described above. In the fit to all data, the trend to low Higgs mass values is enhanced by the SLC measurement of $A_{\rm LR}$. Our conclusion in [1] that the present increase in χ^2 with $M_{\rm H}$ is 'artificial', and hence its interpretation has to be taken with care, still holds.

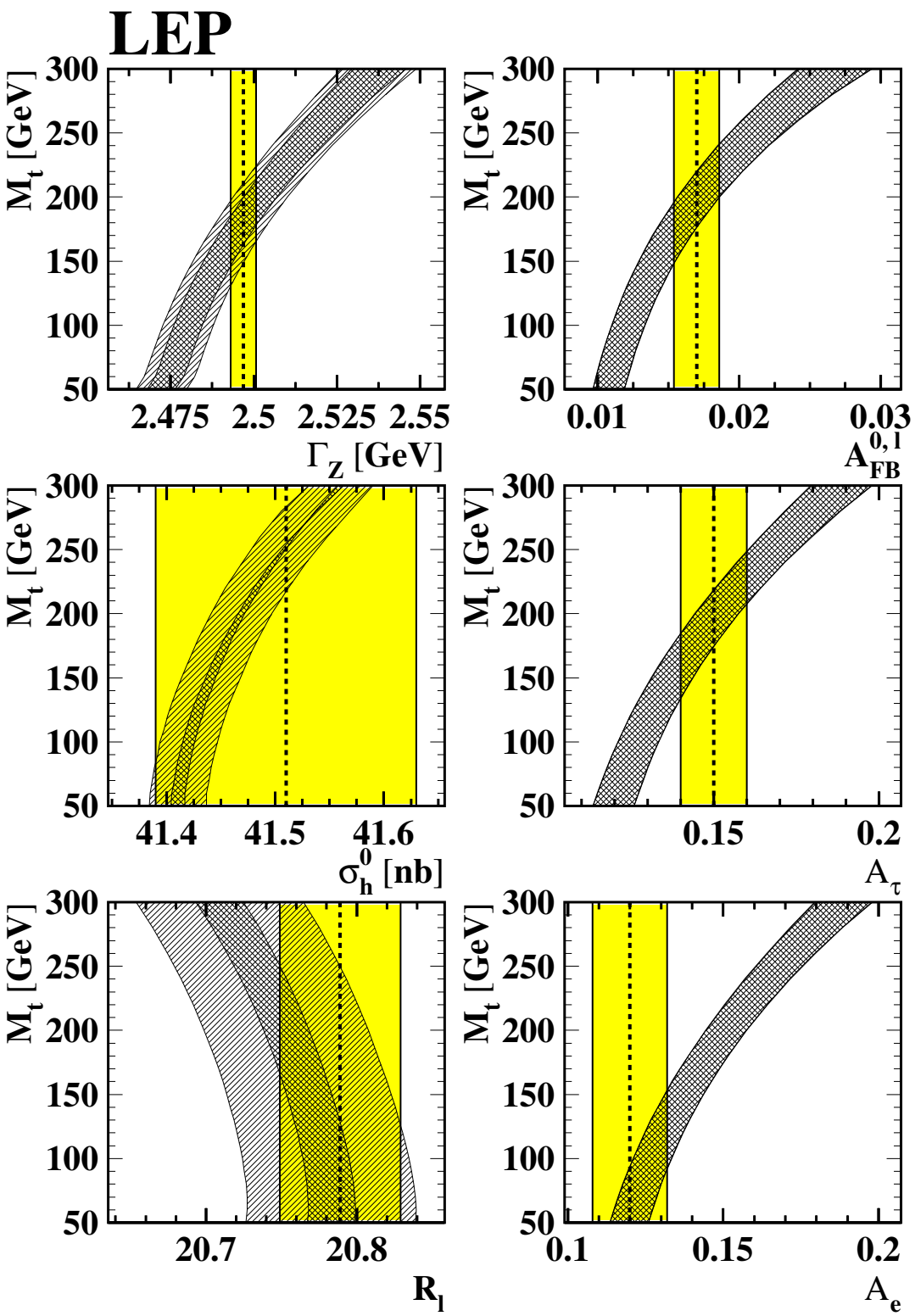

Figure 3: a) Comparison of LEP measurements with the Standard Model prediction as a function of M_t. The cross-hatched area shows the variation of the Standard Model prediction with M_H spanning the interval $60 < M_H\,(\mathrm{GeV}) < 1000$ and the singly-hatched area corresponds to a variation of $\alpha_s(M_Z^2)$ within the interval $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. The total width of the band corresponds to the linear sum of both uncertainties. The experimental errors on the parameters are indicated as vertical bands.

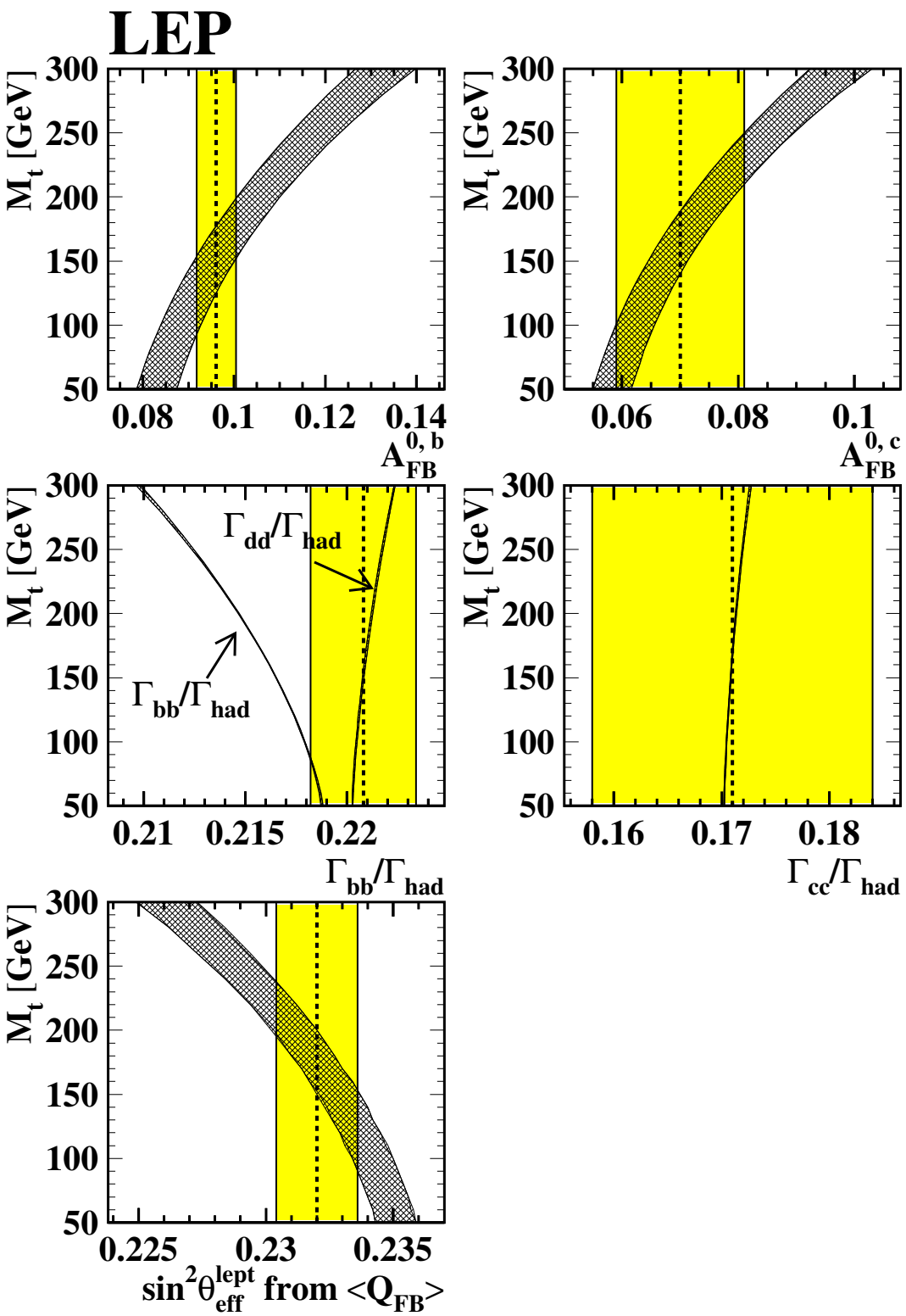

Figure 3: b) Comparison of LEP measurements with the Standard Model prediction as a function of M_{t}. The cross-hatched area shows the variation of the Standard Model prediction with M_{H} spanning the interval $60 < M_{\mathrm{H}}\,(\mathrm{GeV}) < 1000$ and the singly-hatched area corresponds to a variation of $\alpha_s(M_{\mathrm{Z}}^2)$ within the interval $\alpha_s(M_{\mathrm{Z}}^2) = 0.123 \pm 0.006$. The total width of the band corresponds to the linear sum of both uncertainties. For the ratios of partial widths, $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$ and $\Gamma_{\mathrm{d\bar{d}}}/\Gamma_{\mathrm{had}}$, this variation nearly cancels. The experimental errors on the parameters are indicated as vertical bands.

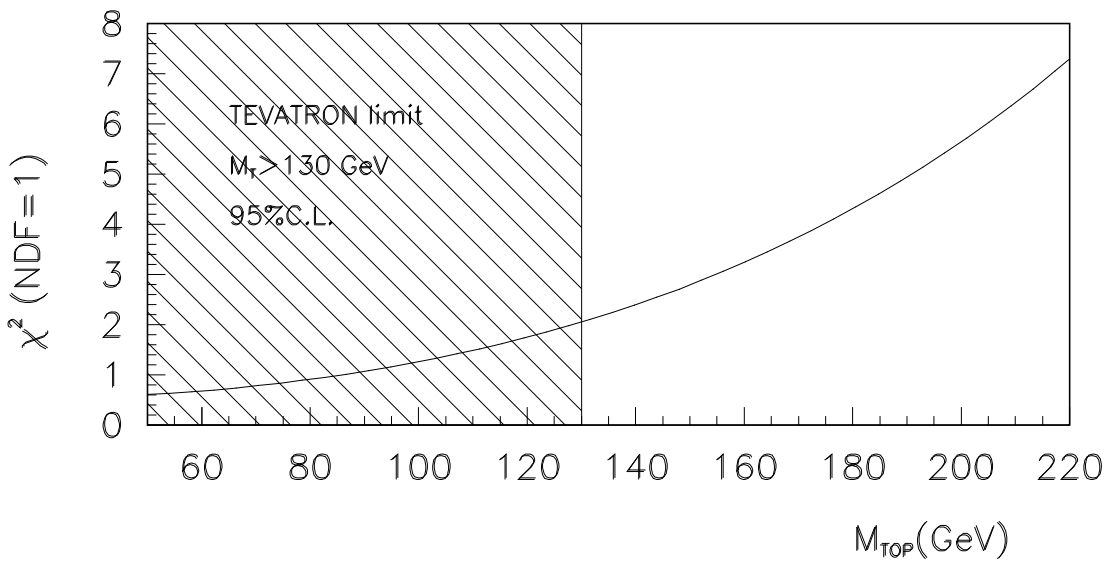

Figure 4: The χ^2 curve for the Standard Model fit to R_{b} alone. M_{Z} and α_s have been varied as free parameters constrained by their input values given in Table 39. The χ^2 curve is not sensitive to the value of M_{H} when varied in the interval $60 < M_{\mathrm{H}}\,(\mathrm{GeV}) < 1000$.

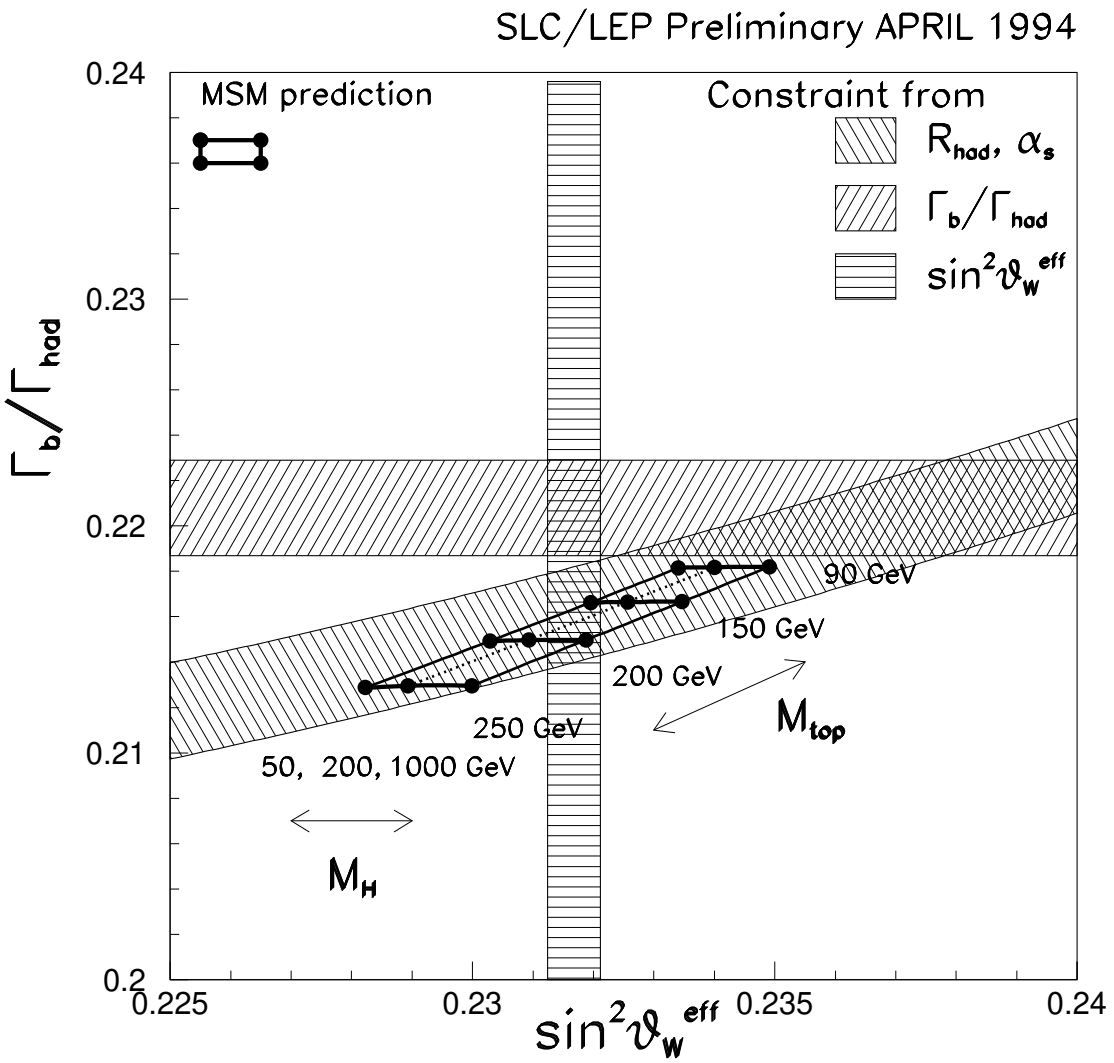

Figure 5: The LEP/SLC measurements of $\sin^2\theta^{\rm lept}_{eff}$ and $R_{\rm b}$ and the Standard Model prediction. Also shown is the constraint resulting from the measurement of R_ℓ on these variables, assuming the world average $\alpha_s(M_Z^2) = 0.123 \pm 0.006$, as well as the Standard model dependence of light quark partial widths on $\sin^2\theta^{\rm lept}_{eff}$.

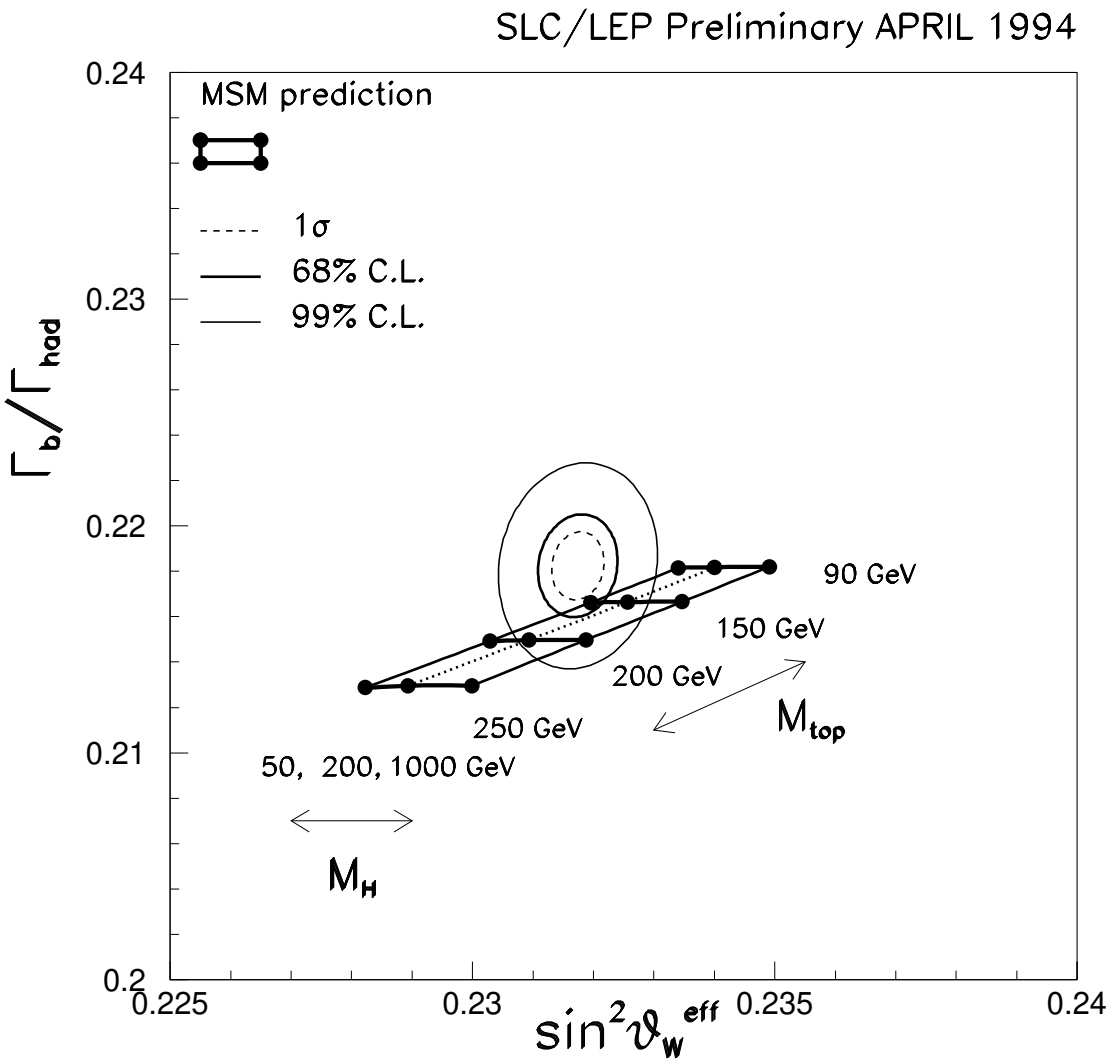

Figure 6: Confidence contours of a fit to the measured values of $R_{\rm b}$, $R_{\rm c}$, R_ℓ, $\sigma^0_{\rm h}$ and $\sin^2\theta^{\rm lept}_{eff}$ of LEP and SLC, assuming the world average $\alpha_s(M_Z^2) = 0.123 \pm 0.006$, as well as the Standard model dependence of light quark partial widths on $\sin^2\theta^{\rm lept}_{eff}$. The $\chi^2/(d.o.f.)$ is 3.0/3, dominated by the contribution of $R_{\rm b}$.

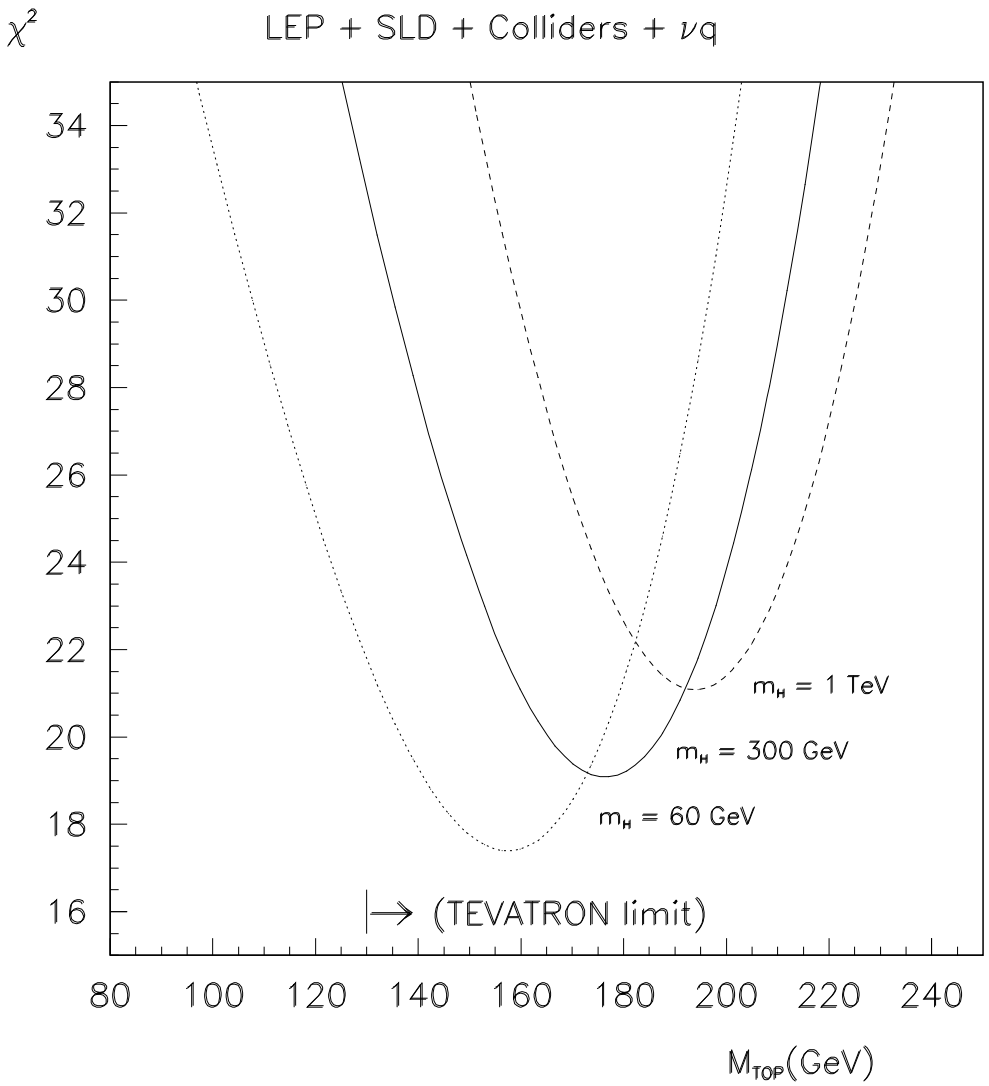

Figure 7: The χ^2 curves for the Standard Model fit in Table 40, column 4 to the electroweak precision measurements listed in Table 39 as a function of $M_{\rm t}$ for three different Higgs mass values spanning the interval $60 \leq M_{\rm H}$ [GeV] ≤ 1000.

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