Polarisation measurement using leptonic τ decays.

A. Kounine [a]

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A precision measurement of τ polarisation at LEP tests neutral couplings of τ leptons providing therefore a stringent test of the Standard Model predictions. The polarisation measurement using leptonic τ decays which constitute an important part of this analysis is discussed with a special emphasis on the analysis of the systematic errors. This measurement is based on approximately 600 000 $Z \to \tau^+\tau^-$ decays collected by the four LEP experiments during the 1990-1994 running periods.

1. INTRODUCTION

The dynamics of lepton pair production at LEP is fully determined by the vector ($g_{V\ell}$) and axial-vector ($g_{A\ell}$) couplings of leptons to the Z. In experiments with unpolarised beams at LEP, τ polarisation and its asymmetry in the reaction

$$e^+e^- \to Z \to \tau^+\tau^-(\gamma)\,. \qquad (1)$$

constitute one of the most precise determinations of neutral couplings of τ leptons to the Z [1].

The polarisation of outgoing τ^- leptons is defined as an asymmetry in the number of right- and left-handed τ^-,

$$\mathcal{P}_\tau = \frac{N(h>0) - N(h<0)}{N_{total}}\,. \qquad (2)$$

The spin content of the selected sample of τ pairs is analysed using decay products of five dominant τ decay modes: $\tau^- \to e^-\overline{\nu}_e\nu_\tau$; $\tau^- \to \mu^-\overline{\nu}_\mu\nu_\tau$; $\tau^- \to \pi^-(K^-)\nu_\tau$; $\tau^- \to \rho^-\nu_\tau$; $\tau^- \to a_1^-\nu_\tau$. For the leptonic decays all spin information is contained in the energy spectrum of the detected leptons. In the lowest order

$$\frac{1}{\sigma}\frac{d\sigma}{dx_\ell} = \frac{1}{3}[(5 - 9x_\ell^2 + 4x_\ell^3) + \mathcal{P}_\tau(1 - 9x_\ell^2 + 8x_\ell^3)]\,, \qquad (3)$$

where $x_\ell = E_\ell/E_{beam}$.

A determination of τ polarisation from this spectrum is conceptually straightforward. Nevertheless, the analysis of the associated errors becomes an essential part of this precision measurement. Moreover, when combining the results of the four LEP experiments, it is necessary to examine possible common systematic errors. A study of these errors is particularly important given the data sample accumulated at LEP.

2. DATA SAMPLE

The signature of $Z \to \tau^+\tau^-$ events at LEP is very clean: they consist of two well collimated

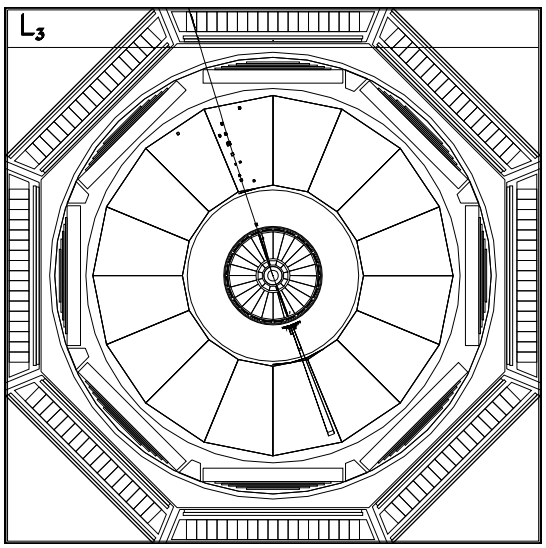

Figure 1. An example of a $Z \to \tau^+\tau^-$ event seen by the L3 detector. The event is reconstructed as a $\tau^- \to e^-\overline{\nu}_e\nu_\tau$ decay in the lower hemisphere and as a $\tau^+ \to \mu^+\nu_\mu\overline{\nu_\tau}$ decay in the upper hemisphere.

almost back to back jets with large missing energy due to undetected neutrinos from τ decays. These distinctive features greatly facilitate the selection of τ pairs. Figure 1 shows a $Z \to \tau^+\tau^-$ event, selected by L3 experiment, where both τ's decay into leptons.

The details of the selection of leptonic τ decays were discussed at the III Tau Workshop [10]. The most important subdetectors for electron identification are the tracking device and the electromagnetic calorimeter. The fine granularity of the electromagnetic calorimeter is an advantage as it helps differentiate between electrons in the final states and hadronic τ decays involving π^0. Identification of muons is based on the hit pattern in the muon chambers and the hadron calorimeter. Typical efficiencies for the selection of these decay modes range from 60% to 85%, depending on the channel and the experiment. A significant fraction of the background for the selection of exclusive leptonic decays comes from misclassified τ decays. Background from other processes consists of $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ decays and $e^+e^- \to e^+e^-f\bar{f}$ events.

The LEP machine was running in the vicinity of Z pole in 1990-1995. During this period each experiment recorded about 200K τ pairs. So far only ~70% of the full sample has been analysed. The results presented in this review are based on the preliminary analysis of 1990-1994 data by the DELPHI, L3 and OPAL experiments and on the final results from 1990-1992 data by the ALEPH experiment [2–5].

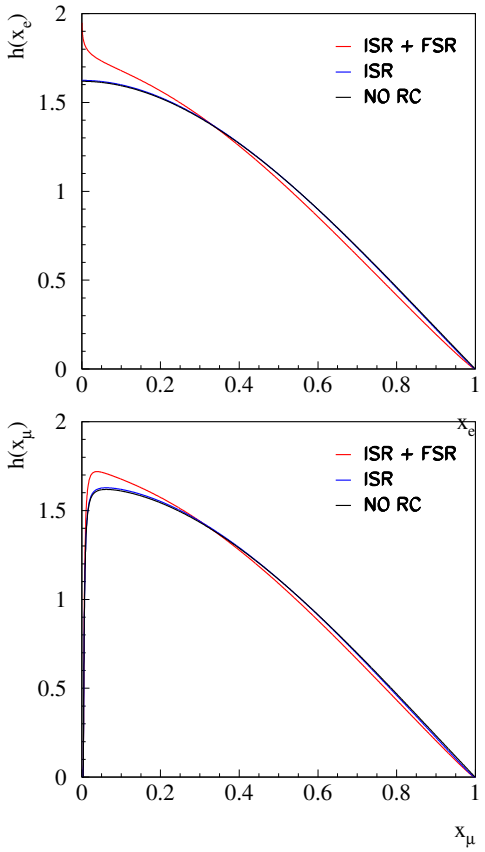

Figure 2. Energy spectra of electrons (a) and muons (b). The solid lines correspond to Eqn. 3. The dotted lines include the effects of initial and final state radiation.

3. SENSITIVITY TO POLARISATION

The measurement of the polarisation using $\tau^- \to e^-\overline{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\overline{\nu}_\mu\nu_\tau$ decays is based on the analysis of the energy spectrum of detected leptons (Eqn. 3). In this analysis the explicit assumption is made of the V-A structure of the τ decay vertex, as supported by other measurements [6].

The sensitivity of the energy spectrum to polarisation depends on its shape and the number of events in it. The part depending on the shape can be defined [7] as

$$S = \frac{1}{\sigma\sqrt{n}} = \sqrt{\frac{1}{n}\frac{d^2\mathcal{L}}{dp^2}}, \qquad (4)$$

where the likelihood function $\mathcal{L}$ is defined using Equation 3. There are a few factors reducing the sensitivity of leptonic spectra to the polarisation which must be taken into account in the polarisation fit.

Firstly, Equation 3 does not account for effects of initial and final state radiation which modify the energy spectra (Fig. 2), thus leading to the decrease of the sensitivity. The net effect is not negligible; it leads to a reduction of sensitivity of

5% for electrons and 3% for muons.

Secondly, in practice, the likelihood function $\mathcal{L}$ is defined using Monte Carlo predictions and not an analytical expression for the energy spectrum. This approach allows one to account for the dependence of the selection efficiency on energy which could induce polarisation dependent biases. On the other hand it leads to a further reduction of the sensitivity due to limited statistics in the Monte Carlo sample and introduces a systematic error in the polarisation due to the uncertainty in the selection efficiency. All experiments assume Poisson statistics for data when constructing the likelihood function $\mathcal{L}$. The Poisson probability to observe n_d^i events in the i-th bin of the data energy spectrum is

$$\mathcal{L} = P(n_d^i|\mu_d^i) = e^{-\mu_d^i}\,\frac{(\mu_d^i)^{n_d^i}}{n_d^i!}, \qquad (5)$$

where μ_d^i is unknown underlying mean which can be expressed in terms of Monte Carlo underlying means for τ helicity states $h = +1/2$ (μ_+^i), $h = -1/2$ (μ_-^i), and background (μ_b^i), with corresponding overall normalisations factors c_+, c_- and c_b: $\mu_d^i = c_+\mu_+^i + c_-\mu_-^i + c_b\mu_b^i$. The following step depends on the experiment. ALEPH assumes that the Monte Carlo prediction is the exact one (i.e. the Monte Carlo underlying means are given by the number of events observed in the Monte Carlo), while L3 and OPAL collaborations use Poisson statistics for the number of events observed in the Monte Carlo (see for example [8]) and perform an integration over unknown μ_+^i, μ_-^i and μ_b^i:

$$\mathcal{L} = P(n_d^i|n_+^i, n_-^i, n_b^i) \quad = \qquad (6)$$
$$\int\int\int P(n_d^i|\mu_d^i)\cdot P(\mu_d^i|n_+^i, n_-^i, n_b^i)\, d\mu_+^i\, d\mu_-^i\, d\mu_b^i$$

It should be noted that the first approach underestimates slightly the statistical errors and leads to a larger bias than the second one. (A discussion of biases due to the finite Monte Carlo statistics can be found, for instance, in reference [9]).

The third source of sensitivity reduction is non-τ background. Despite the small (of the order few percent) contamination, the background populates predominantly the regions which are most sensitive to the polarisation. Two-photon interactions $e^+e^- \to e^+e^-f\bar{f}$ occupy the low energy range and fake right-handed τ^- while $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ decays populate the high energy range, thus faking leptonic decays of left-handed τ^-. These impurities in the sample, if not fitted along with the polarisation, lead to a reduction of the sensitivity due to statistical fluctuations even in case the expected fraction of these backgrounds is known to high precision. In addition, uncertainties in these fractions add further to the systematic error on polarisation.

4. ERROR ANALYSIS

The experimental aspects of the error analysis were discussed in [10]. In this section a comparative analysis of errors on $\mathcal{P}_\tau$ is made using a simple toy Monte Carlo simulation. This approach allows to perform detector-independent estimates of statistical and systematic errors, and to investigate in detail uncertainties arising from the fitting procedure.

The "toy" experiment consists of computer-simulated samples of "data" and "real MC" events which are generated in the same proportion as the corresponding samples in the genuine experiment. The only advantage of this method is that "toy" experiment can be repeated hundreds times with statistically independent samples. This allows one to compare the actual spread of the fitted polarisation values with the error estimates given by the likelihood functions (Eqns 5,6). It also permits the introduction of various biases in the data sample corresponding to possible systematic effects and the examination of the resulting effects.

Estimates of the ideal sensitivity of the lepton spectra to the polarisation

$$\mathrm{S}^{\mathrm{e}}_{\mathcal{P}_\tau} = 0.209, \qquad \mathrm{S}^{\mu}_{\mathcal{P}_\tau} = 0.213 \qquad (7)$$

are performed using both the analytical expression given by Equation 4 and the toy Monte Carlo. They are found to agree well. These estimates do not account for dilutions of the sensitivity to energy dependence of the efficiency and due to statistical fluctuations of the background. The

Table 1
Comparison of the sensitivity of LEP experiments. The first column represents the number of events in the data sample, the second shows the corresponding statistical error from the fit, and the last shows the sensitivity derived using Eqn. 4.

Experiment		Data	$\Delta\mathcal{P}_\tau$	$S_{\mathcal{P}_\tau}$
ALEPH	$e^-\overline{\nu}_e\nu_\tau$	6037	0.063	0.204
1992	$\mu^-\overline{\nu}_\mu\nu_\tau$	8695	0.052	0.206
DELPHI	$e^-\overline{\nu}_e\nu_\tau$	13561	0.052	0.165
1992-1993	$\mu^-\overline{\nu}_\mu\nu_\tau$	15082	0.044	0.185
L3	$e^-\overline{\nu}_e\nu_\tau$	25285	0.036	0.175
1991-1994	$\mu^-\overline{\nu}_\mu\nu_\tau$	24793	0.033	0.192
OPAL	$e^-\overline{\nu}_e\nu_\tau$	26614	0.033	0.186
1991-1994	$\mu^-\overline{\nu}_\mu\nu_\tau$	23450	0.033	0.198

actual sensitivities from the experiments which account for these effects are presented in Table 1. As can be seen from the table the sensitivity reduction due to these effects varies between 5 and 10%. ALEPH results, which are close to the ideal numbers, can be partially explained by better acceptance in the low energy region.

One of the most important sources of systematic error is calibration uncertainty. It does not decrease sensitivity, but may introduce significant bias in the measurement. Discrepancy between the energy scales in the data and Monte Carlo simulation (Δs) leads to a significant shift in the measured polarisation value:

$$\frac{\Delta\mathcal{P}_\tau}{\Delta\mathrm{s}} \quad = \quad 5.3 \pm 0.4 \qquad (8)$$

This means that 0.01 shift in the energy scale for the data changes the measured polarisation by 0.053. The estimates are made using toy Monte Carlo for an "average" LEP experiment and assuming no dependence of the scale shift on energy. The error represents uncertainties due to acceptance and background contamination. In Table 2 the quoted scale uncertainties are compared with the corresponding systematic errors for each experiment. Due to the statistics of the control samples, the scale uncertainties vary from the low energy region to the high energy region.

Table 2
Summary of systematic errors due to the uncertainties in the energy scale. The first column represents the scale uncertainty itself, the second shows the corresponding systematic error on polarisation, and the last shows the ratio of these two quantities.

Experiment		Δs (%)	$\Delta\mathcal{P}_\tau$	$\frac{\Delta\mathcal{P}_\tau}{\Delta\mathrm{s}}$
ALEPH	$e^-\overline{\nu}_e\nu_\tau$	0.1-0.3	0.010	5.0
1992	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.1-0.3	0.010	5.0
DELPHI	$e^-\overline{\nu}_e\nu_\tau$	0.6	0.050	8.0
1992-1993	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.2	0.017	8.5
L3	$e^-\overline{\nu}_e\nu_\tau$	0.1-0.4	0.014	5.6
1991-1994	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.2	0.016	8.0
OPAL	$e^-\overline{\nu}_e\nu_\tau$	0.3	0.020	6.7
1991-1994	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.2	0.014	7.0

Table 3
Summary of the systematic errors on polarisation due to uncertainties in the background contamination.

Experiment		$\Delta^{\mathrm{ff}}\mathcal{P}_\tau$	$\Delta^{\gamma\gamma}\mathcal{P}_\tau$
ALEPH	$e^-\overline{\nu}_e\nu_\tau$	0.014	0.014
1992	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.014	0.010
DELPHI	$e^-\overline{\nu}_e\nu_\tau$	0.021	0.021
1992-1993	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.012	0.005
L3	$e^-\overline{\nu}_e\nu_\tau$	0.009	0.015
1991-1994	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.020	0.010
OPAL	$e^-\overline{\nu}_e\nu_\tau$	0.010	0.004
1991-1994	$\mu^-\overline{\nu}_\mu\nu_\tau$	0.010	0.008

These changes are reflected in the uncertainty intervals quoted by the ALEPH and L3 experiments. There is fair agreement between the expectations (Eqn. 8) and the numbers from the experiments, especially if one takes into account the size of the errors and possible rounding of the quoted values.

The purely statistical reduction of sensitivity due to the background contamination in the data sample was already discussed in the previous section. The systematic effects result from the uncertainty in the background fraction. The net effect depends on several factors (background frac-

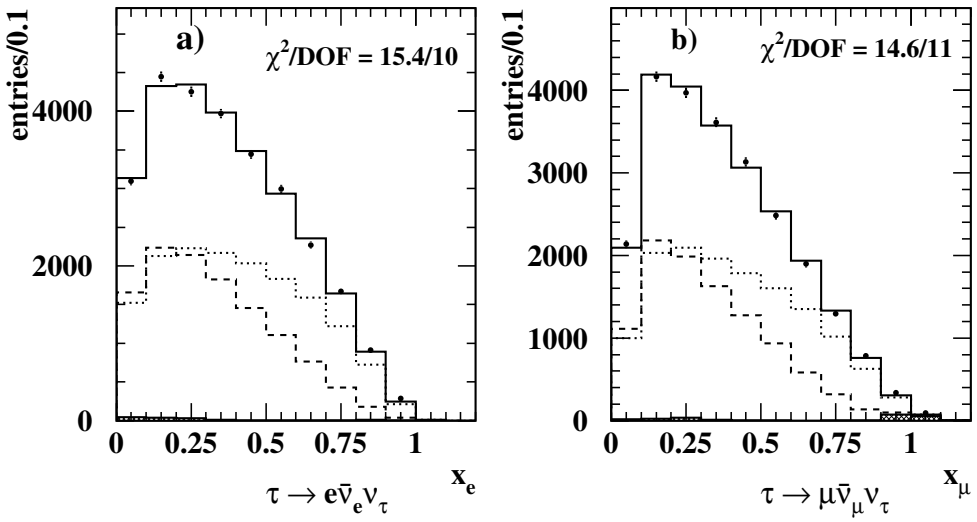

Figure 3. Energy spectra of electrons (a) and muons (b) as measured by the OPAL Collaboration. Data (dots) are shown in comparison with the Monte Carlo expectations (solid histogram). The contribution from the τ^- helicity state $+1/2$ and $-1/2$ are indicated by dashed and dotted histograms respectively.

tion, detector resolution and technicalities of the fitting procedure) and varies from experiment to experiment. A summary of this systematic effects is given in Table 3. It is crucial to keep the background contamination which is the most important deteriorating factor at the lowest possible level. For instance, OPAL numbers for the systematic error due to two-photon interactions $e^+e^- \to e^+e^-f\overline{f}$ are largely due to a very small contamination from this background source (see Fig. 3).

Possible bias in the fitted value of the polarisation due to the fitting procedure is studied using the toy Monte Carlo and repeating the experiments many times. A systematic shift in the fitted polarisation value is observed which is proportional to the difference of the polarisations in the data and MC samples. It is estimated to be of the order 0.1-0.2%, depending on the likelihood function, for typical data samples of 10000 events in the data, 100000 events in the Monte Carlo, and a difference in the polarisations of the simulated "data" and "MC" samples of 5%. The bias increases if smaller data samples are used. Although it could be potentially a source of systematics common between the experiments, in practice the fitted average polarisation values scatter around common mean, thus reducing the effect and making it negligible.

So far, the errors on the average tau polarisation have been discussed. Let us consider now errors relevant to the polarisation asymmetry measurement. In the Standard Model, the angular dependence of $\mathcal{P}_\tau$ on the polar angle is given by the equation [11]

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau + \mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}}{1 + \mathcal{A}_\tau\mathcal{A}_e \frac{2\cos\theta}{1+\cos^2\theta}}\,. \qquad (9)$$

This dependence of the polarisation on polar angle, as measured by the L3 Collaboration using $\tau^- \to \mu^-\overline{\nu}_\mu\nu_\tau$ decays, is shown in Fig 4.

Most of the systematic errors relevant to the measurement of the average polarisation do not exhibit significant left-right dependence and therefore cancel in the measurement of the polarisation asymmetry. The only exception is the background from $e^+e^- \to e^+e^-$ events which have significant charge asymmetry due to the t-channel contribution (for instance the fraction of this background changes by a factor 2 when the polar angle runs from 40^0 to 140^o). This may lead to ~ 0.01 uncertainty in the $\mathcal{A}_e$ measurement for

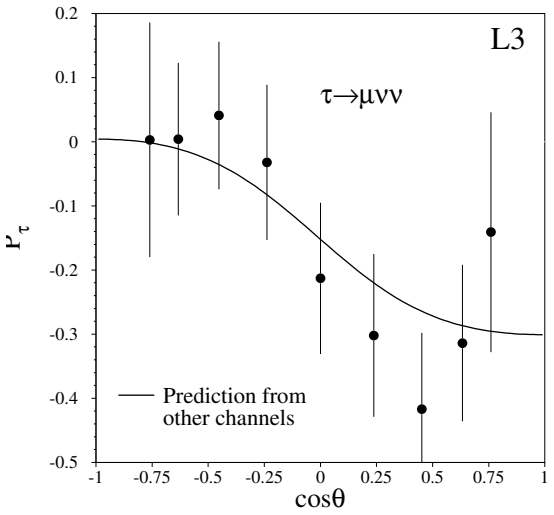

Figure 4. Angular dependence of the τ polarisation using $\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau$ decays. The solid curve corresponds to the Standard Model predictions with the parameters taken from the polarisation measurement using other τ decays.

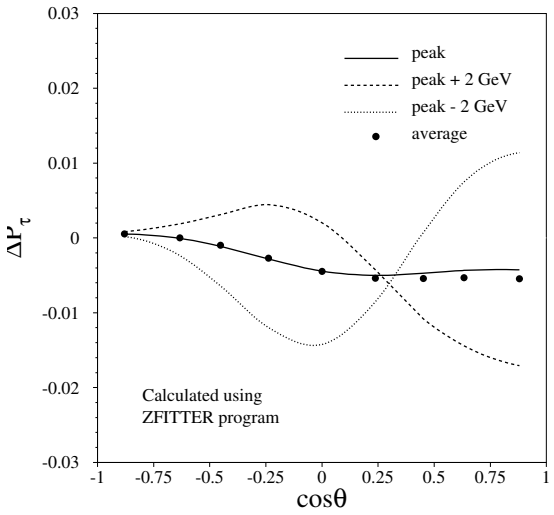

Figure 5. Corrections to the polarisation due to the effects of initial and final state radiation, γ-exchange and γZ-interference.

$\tau^- \to e^- \overline{\nu}_e \nu_\tau$ decays using the numbers quoted in Table 3. Other decay modes are affected to a lesser extent.

Another source of possible systematic error is the bias from the fitting procedure. This bias largely depends on fit technicalities. It reaches a maximum if the contents of two mirror $\cos\theta$ bins are summed up in the Monte Carlo sample while fitting individual $\cos\theta$ bins in the data. In this case the polarisation difference between data and Monte Carlo samples reaches $\sim 10\%$ decreasing therefore an $\mathcal{A}_e$ value by 0.002-004 depending on the choice of the likelihood function. On the other hand this technique has some advantages as it decreases the effects of MC statistics on the $\mathcal{A}_e$ error. In any event this effects could be responsible for a substantial part of common systematic error in the $\mathcal{A}_e$ measurement and certainly should be addressed when averaging all experiments and all channels.

In order to extract $\mathcal{A}_\tau$ and $\mathcal{A}_e$ from the dependence using Eqn.(9), the corrections due to initial and final state radiation, γ-exchange and γZ-interference must be applied. These corrections are calculated using the ZFITTER program [12] for every $\sqrt{s}$ point and averaged, weighted by the integrated luminosity at these points (Fig. 5). The absolute value of the correction to $\mathcal{A}_\ell$ is calculated to be 0.003-0.004 depending slightly on the breakdown of the data between the $\sqrt{s}$ points. The theoretical uncertainty in this correction is believed to be better than 0.001. It is common to all LEP experiments and affects both the $\mathcal{A}_\tau$ and $\mathcal{A}_e$ measurements.

5. DISCUSSION

The summary of the measurements of $\mathcal{A}_\tau$ and $\mathcal{A}_e$ is presented in Fig. 6-7. There is good agreement between the results of the four LEP experiments. The $\mathcal{A}_\tau$ measurement agrees well with the measurements using all τ decays. No significant systematic errors common between the experiments have been detected; therefore this measurement can be improved when the full data

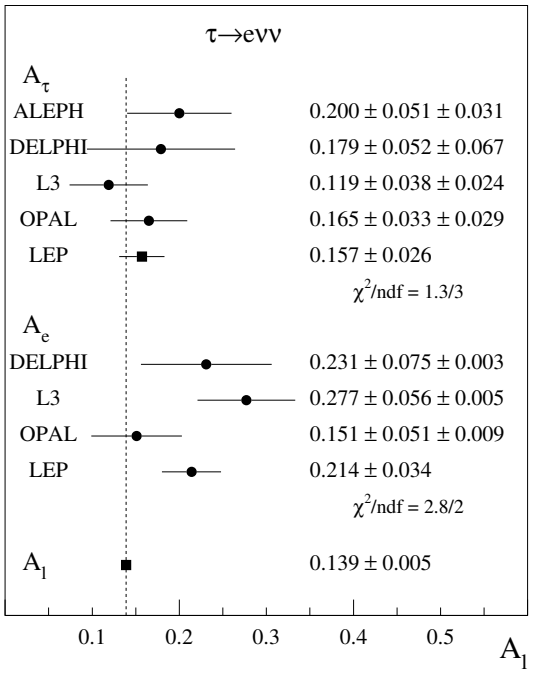

Figure 6. Summary of the polarisation measurements using $\tau^- \to e^- \overline{\nu}_e \nu_\tau$ decays. The dashed vertical line represents the average $\mathcal{A}_\ell$ value as measured at LEP.

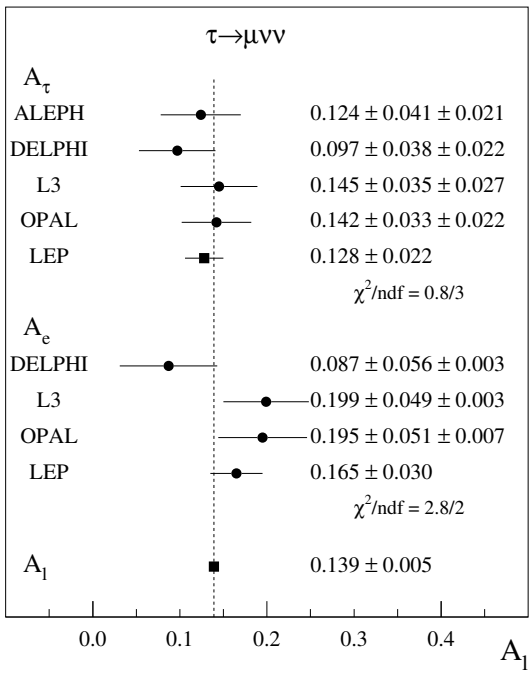

Figure 7. Summary of the polarisation measurements using $\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau$ decays. The dashed vertical line represents the average $\mathcal{A}_\ell$ value as measured at LEP.

sample is analysed.

The $\mathcal{A}_e$ average value from the $\tau^- \to e^- \overline{\nu}_e \nu_\tau$ channel shows 2σ deviation from the average $\mathcal{A}_\ell$ value measured at LEP. Although this deviation could be purely statistical in nature, the influence of possible systematic effects should be investigated further. Special attention should be paid to the background from $Z \to e^+e^-$ decays which may be responsible for part of this deviation. It is desirable also to evaluate in detail the influence of the fitting procedure on $\mathcal{A}_e$ results using all channels and not only leptonic τ decays. Nevertheless, the net effect is not expected to be large compared to the statistical errors as indicated by the preliminary studies presented in the previous section. Clearly this measurement will also profit from the increase in the statistics.

6. CONCLUSIONS

The LEP-I program was completed in 1995 resulting in about 200K τ pairs delivered to each of four LEP experiments. No increase of this data sample is foreseen for the future.

The progress in the precision electroweak measurements at LEP, including τ polarisation and its asymmetry, is impressive. Ultimately it leads to a precision test of the Standard Model at the level of one-loop electroweak corrections [13].

The measurements of $\mathcal{A}_\tau$ and $\mathcal{A}_e$ performed using leptonic τ decays and $\sim 70\%$ of the full data sample are in fair agreement with other measurements. There is a good understanding of the dominant systematic errors accompanying this measurement, although a few more checks are needed before the final LEP results can be released. However, the systematic errors common between the experiments are estimated to be small; therefore

one can double the precision of the measurement by averaging the results of the four LEP experiments.

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