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Inclusion of Tau Anomalous Magnetic and Electric Dipole Moments in the KORALZ Monte Carlo

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Abstract

We describe modifications made to `KORALZ` version 4.03 in order to allow for anomalous magnetic and electric dipole moments of the τ. We discuss the verification of the method at LEP1 energies.

1 Introduction

The analysis of radiative τ pair production provides a means to determine the anomalous magnetic and electric dipole moments of the τ at $q^2 = 0$. An anomalous magnetic dipole moment at $q^2 = 0$, $F_2(0)$, or electric dipole moment, $F_3(0)$, affects the total cross section for the process $e^+e^- \to \tau^+\tau^-\gamma$ as well as the shape of energy and angular distributions of the three final state particles [1–3]. Previous experimental limits [3–5] on $F_2(0)$ and $F_3(0)$ have been based on approximate calculations of the $e^+e^- \to \tau^+\tau^-\gamma$ cross section and photon energy distribution. These calculations do not include the important effects of interference between anomalous and Standard Model amplitudes, and furthermore can not be used to properly account for detector acceptance and selection cuts.

In order to address these problems, a tree level calculation of the squared matrix element for the process $e^+e^- \to \tau^+\tau^-\gamma$ including the effects of non-zero $F_2(0)$ and $F_3(0)$ has been carried out. This matrix element calculation, which has been dubbed `TTG` [1], may be used in the generation of event samples with probabilistic weights, or may be applied in a Monte Carlo rejection method to produce events with weights of unity. Perhaps more practically, the matrix element may be used to compute weights for any desired values of $F_2(0)$ or $F_3(0)$ given a set of 4-vectors for the final state particles in $e^+e^- \to \tau^+\tau^-\gamma$. Thus it is straightforward to interface this calculation with `KORALZ` [6], thereby providing all the Monte Carlo tools necessary for a meaningful interpretation of the data.

This note describes the combination of `KORALZ` with `TTG`. First a brief description of the `TTG` and `KORALZ` programs is given. This is followed by description of how the two are interfaced and a description of the verification of the method for LEP1 energies. Finally, we provide technical information on how to use the program.

2 `TTG` program

One may parametrize the effects of anomalous electromagnetic couplings in $e^+e^- \to \tau^+\tau^-\gamma$ by replacing the usual γ^μ by more general Lorentz-invariant form of the coupling of a tau to a photon[1]:

$$\Gamma_\mu = F_1(q^2)\gamma_\mu + i\frac{F_2(q^2)}{2m_\tau}\sigma_{\mu\nu}q^\nu - F_3(q^2)\sigma_{\mu\nu}q^\nu\gamma_5, \tag{1}$$

where m_τ is the mass of the τ lepton, and $q = p' - p$ is the momentum transfer. As can be verified using the Gordon decomposition [7], the q^2-dependent form-factors, $F_i(q^2)$, have familiar interpretations for $q^2 = 0$ and with the τ on mass-shell: $F_1(0) \equiv q_\tau$ is the electric charge of the tau, $F_2(0) \equiv a_\tau = (g-2)/2$ is the static anomalous magnetic moment of the tau (where g is the gyromagnetic ratio), and $F_3(0) \equiv d_\tau/q_\tau$, where d_τ is the static electric dipole moment of the tau and q_τ is its charge. Using this parametrization, we consider all the Standard Model and anomalous amplitudes for the diagrams shown in Figure 1. The corresponding matrix element is then evaluated using the symbolic manipulation package FORM [8] without making any simplifying assumptions. In particular, no interference terms are neglected and no fermion masses are assumed to be zero. The matrix element is available from the authors in the form of a FORTRAN subroutine.

Following an initial presentation of these results [9], an analytical calculation for the process $e^+e^- \to \tau^+\tau^-\gamma$ was carried out [2]. This calculation makes some approximations, but importantly does not assume zero tau mass and does not neglect interference between Standard Model and anomalous final states. This provides with a means to crosscheck the results of `TTG`. The `TTG` program and the crosschecks are described in detail in reference [1].

[1] in general there are 5 independent lorentz invariant currents for spin 1/2 particle coupling to a photon.

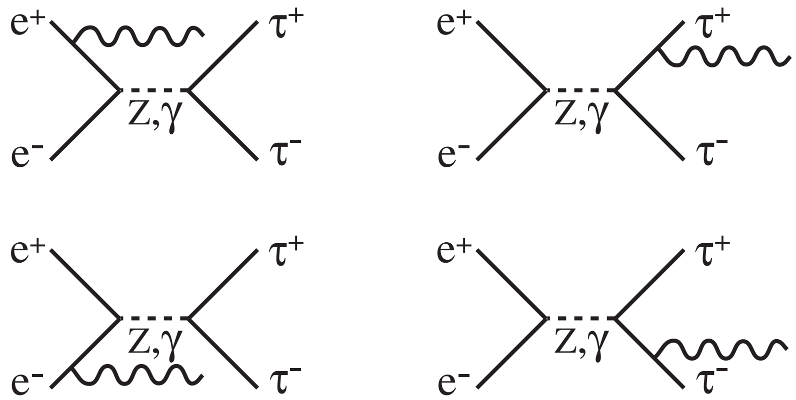

Figure 1: Diagrams contributing to $e^+e^- \to \tau^+\tau^-\gamma$

3 KORALZ Program

In this section we briefly review the properties of KORALZ which are relevant to the modifications we describe in section 4. The algorithm employed by KORALZ for generation of $e^+e^- \to \tau^+\tau^-(\gamma)$, including radiative corrections and τ decay, is described in detail in reference [6].

Real photon radiation in KORALZ is controlled by the KEYRAD flag. The program may be run at Born level (KEYRAD=0), may include order α QED corrections (KEYRAD=1), or order α^2 QED corrections including exclusive exponentiation (KEYRAD=12). The Born level differential distribution is used as a starting point in calculating the matrix element at $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$.

In the case of KEYRAD=12, the user may switch on and off contributions from initial state radiation (ISR) and final state radiation (FSR) using the NPAR(12) card. For testing purposes, three additional KEYRAD options have also been introduced which allow one to turn on (or off) ISR, FSR, and interference for the case of single bremsstrahlung, as summarized below.

KEYRAD=0	Born level
KEYRAD=1	$\mathcal{O}(\alpha)$, ISR, FSR, interference
KEYRAD=2	$\mathcal{O}(\alpha)$, ISR, FSR
KEYRAD=3	$\mathcal{O}(\alpha)$, ISR
KEYRAD=4	$\mathcal{O}(\alpha)$, FSR
KEYRAD=12, NPAR(12)=1000011	$\mathcal{O}(\alpha^2)$+ exponentiation, ISR, FSR
KEYRAD=12, NPAR(12)=1000001	$\mathcal{O}(\alpha^2)$+ exponentiation, ISR
KEYRAD=12, NPAR(12)=1000010	$\mathcal{O}(\alpha^2)$+ exponentiation, FSR

4 Merging TTG with KORALZ

KORALZ and TTG have been merged such that for each event generated by KORALZ, a weight is computed by TTG for a given $F_2(0)$ or $F_3(0)$. Details of the weight calculation and how the information may be accessed is given in section 7. Since we are only interested in events with photons, events without photons can be rejected by setting the KORALZ internal weight to zero [2]. In this case, the total cross section given at the end of the KORALZ run will include only contributions from configurations with a real hard photon

[2]For that purpose, the internal input parameter IRECSOFT in routine kzphynew(XPAR,NPAR) should be set to 1.

above the KORALZ internal parameters xk0 or vvmin [3].

Since TTG provides an $\mathcal{O}(\alpha)$ calculation, this procedure is straightforward and unambiguous for KEYRAD = 1,2,3,4, where there is at most a single bremsstrahlung photon in the event. In this case, the KORALZ/TTG program simply works as a single bremsstrahlung generator with anomalous contributions included.

For multiple photon events (KEYRAD=12), the situation is not as simple because the weight factor (see equation 2 of section 6) for the anomalous contribution can no longer be calculated in a direct way. In this case we need to rely on a reduction procedure in which all photons except for the one with largest p_T are incorporated into the 4-momenta of effective initial or final state leptons. This approach is founded on the basic factorization properties of QED. In the infrared limit, as well as for important regions of phase space which give leading log corrections, the matrix element can be written (up to non-leading terms) as a product of the Born level matrix element multiplied by factors S_i corresponding to photon(s) emission. The S_i are independent from the particular hard process under consideration. A similar property holds for phase space. See for example reference [10] for an introductory presentation and references. Here we will assume that anomalous couplings of the photon to the τ do not affect these properties and that their effect can be described as correction or perturbation to the individual factors S_i [4]. Thus if one assumes that non-leading corrections are small and can be neglected, then such a reduction procedure may be combined with the calculations of TTG in order to account for anomalous contributions in the case of multiple photon radiation.

We now present details of the reduction procedure just discussed. This procedure is performed in the routine WTANOM, which is called in the case of flag IFKALIN=2. For each generated event which contains more than one real photon, the following algorithm is applied:

- The invariant mass, m_i^k, for each pair of particles containing a photon, $i = 1...n$, and a lepton, $k = 1...4$ where $1, 2 = e^{\pm}$ and $3, 4 = \tau^{\pm}$, is calculated.
- The masses m_i^k are multiplied by the square of the sum of the photon and Z propagators. The energy transfer for $k = 3, 4$ is taken to be the center-of-mass energy, whereas for $k = 1, 2$ this transfer is reduced by $1 - E_i/E_{\text{beam}}$. This increases the value of m_i^k for the case of a hard photon paired with a beam electron, reflecting the fact that a narrow resonance cuts off contributions from hard ISR.
- The minimum m_i out of $m_i^{k=1..4}$ is selected.
- The maximum m of the m_i is selected. The corresponding photon is stored as the highest p_T photon; this is the photon that will be passed to TTG to compute the weights corresponding to anomalous moments. The four-momentum of each remaining photon, i, is added to one of the final state τ's or subtracted from one of the initial state e's, depending on which $m_i^{k=1..4}$ is the smallest. The resulting lepton-photon combinations are referred to as "effective" τ's or beams.
- The 4-momenta of the effective τ's and beams as well as the highest p_T photon are boosted into the rest frame of the effective beams. We call this the rest frame of the effective reaction.
- The 4-momenta of the boosted effective beams are modified such that they are back-to-back in the rest frame of the effective reaction, and are consistent with the electron mass.
- The 4-momenta of the effective τ's are boosted into the rest frame of the τ pair.

[3] Cross sections corresponding to realistic cuts on minimal photon energies will not be affected by the choice of these parameters.

[4] In fact, as it will be explained later, we will take this perturbation for only the photon of the highest p_T with respect to leptons. We will also assume that anomalous contributions are not of the infrared divergent or collinear divergent type.

- The 4-momenta of these boosted effective τ's are modified such that they are back-to-back and consistent with the τ mass.
- These modified τ 4-vectors are boosted back into the frame of the effective reaction.

At this point we have constructed the kinematical configuration of the reaction $e^+e^- \to \tau^+\tau^-\gamma$, ensuring all the leptons are on mass shell. These 4-vectors may then be used by `TTG` to calculate the standard model or anomalous matrix element for this process in an unambiguous way.

In the final step of the event generation, the τ decay is simulated using TAUOLA [11]. In the calculation of spin effects which occurs at this point, any effects of anomalous contributions are neglected.

A rigorous evaluation of the quality of the algorithm described above would require careful comparison of the generated distributions with those of the exact matrix element calculations including anomalous contributions to at least $\mathcal{O}(\alpha^2)$. This is impossible at the moment, but similar tests for the process $e+e- \to \nu\bar{\nu}\gamma$'s were performed in [12] and the approximation worked quite well.

The above procedure introduces systematic uncertainties only in the *anomalous contributions* to the distribution of the final state particles (except at the single photon level, where no such systematic is introduced). Moreover, these uncertainties do not affect corrections at the leading-log QED level or infrared/collinear regions of the distribution of radiated photons. Only the next-to-leading-log contributions of higher orders are affected.

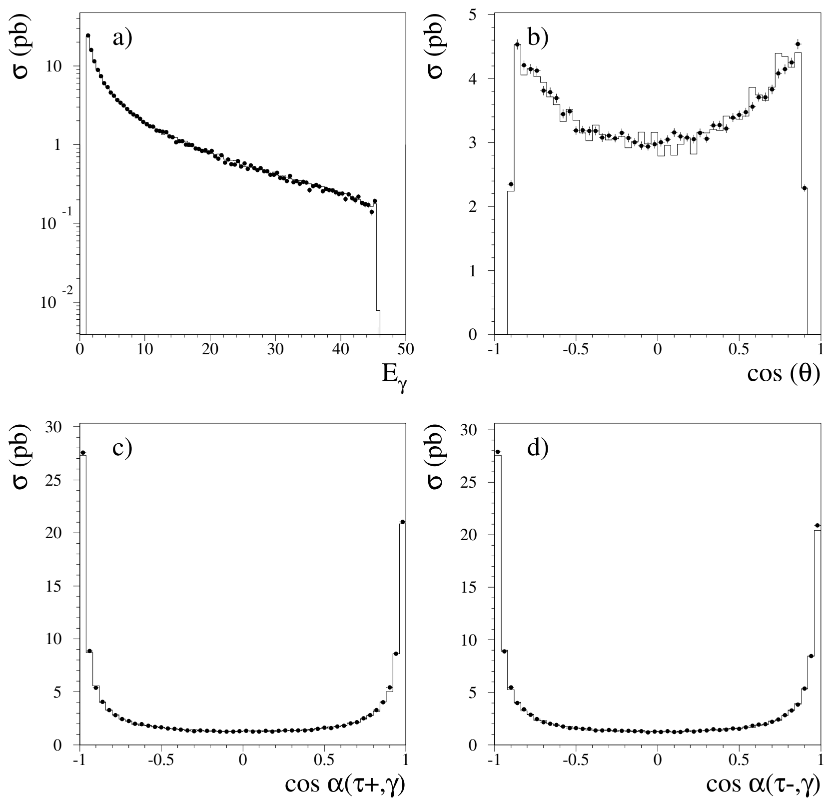

Figure 2: Comparison of predictions of `TTG` (histogram) with `KORALZ` (dots) for a) photon energy, b) angle between photon and beam electron, c) angle between photon and τ^+, d) angle between photon and τ^-. The cuts defined in section 5 have been applied.

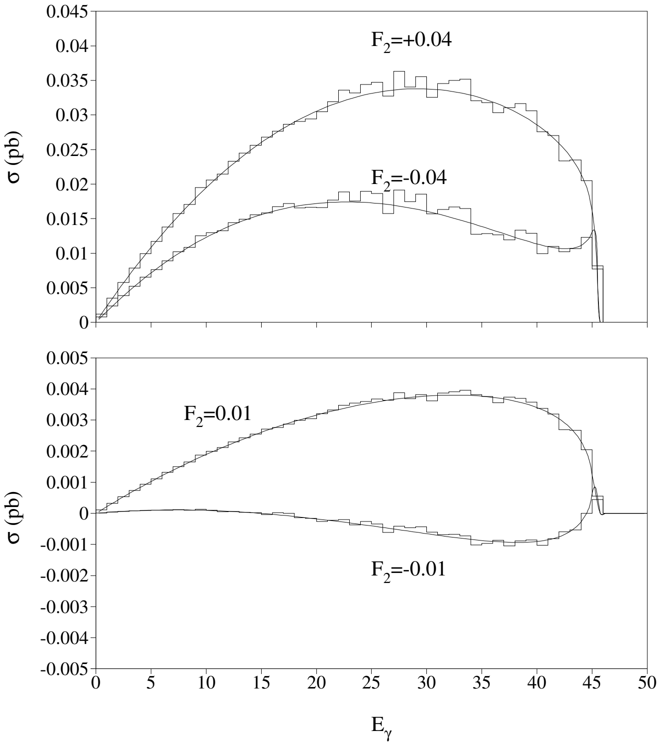

Figure 3: The anomalous contribution to the cross section as a function of energy for several values of $F_2(0)$. The histogram is the result of the KORALZ/TTG Monte Carlo and the curve is from the analytical calculation. No cuts have been applied.

5 Crosschecks at $\mathcal{O}(\alpha)$

As a technical crosscheck, we compare the predictions of TTG with settings $F_2(0) = F_3(0) = 0$ with those of KORALZ. Since TTG provides only an $\mathcal{O}(\alpha)$ calculation, KORALZ is run with KEYRAD=1 so that only single photon radiation is considered. In order to prevent infrared divergences in the TTG calculation, we impose cuts on the minimum photon energy ($E_{\rm min} > 1$ GeV), the angle between the photon and the beam electron ($|\cos\theta| < 0.9$), and the angle between the photon and closest tau ($\cos\alpha_{\rm min} < 0.995$). With these cuts, the total cross sections predicted by KORALZ and TTG agree to with about 0.1%. Figure 2 shows a comparison of the energy and angular distributions computed by the two programs.

Next we verify that the combined KORALZ/TTG program correctly calculates the anomalous contribution to the cross section for $F_2(0) \neq 0$ or $F_3(0) \neq 0$. For this check, we make use of the analytical calculation described in reference [2]. This calculation neglects anomalous contributions from initial-final state interference, from γZ interference, and from γ exchange, so from purposes of comparison we remove these terms from the TTG calculation and we run KORALZ with KEYRAD=4, in which case only FSR is considered. To remove any ambiguity concerning the validity of comparing the non-QED genuine weak corrections computed by KORALZ with the improved Born approximation approach used in the analytical calculation, we set KEYGSW=1 in KORALZ and use the Born approximation. The anomalous contribution to the cross section computed by KORALZ/TTG agrees with that of the analytical calculation to 1%. Figure 3 shows, for several values of $F_2(0)$, a comparison of the anomalous contribution to the photon energy spectrum computed by KORALZ/TTG with the predictions of the analytical calculation.

6 Results for Multiple Photon Radiation

We now present numerical results of the KORALZ/TTG program with multiple photon radiation included. The goal is to demonstrate that the reduction scheme described in section 4 gives results which are consistent with expectations from the $\mathcal{O}(\alpha)$ calculation, and to indicate how one might estimate the systematic errors associated KORALZ/TTG simulation including multiple photon radiation.

First, we check that the interference between initial and final state bremsstrahlung at $\mathcal{O}(\alpha)$ contributes negligibly to our observables. To this end, we compare our observables as they are computed by KORALZ/TTG using KEYRAD=1 with the results using KEYRAD=2. Figure 4 shows the ratios for the two calculations. It is necessary to check the effects of interference at $\mathcal{O}(\alpha)$, as interference is not included

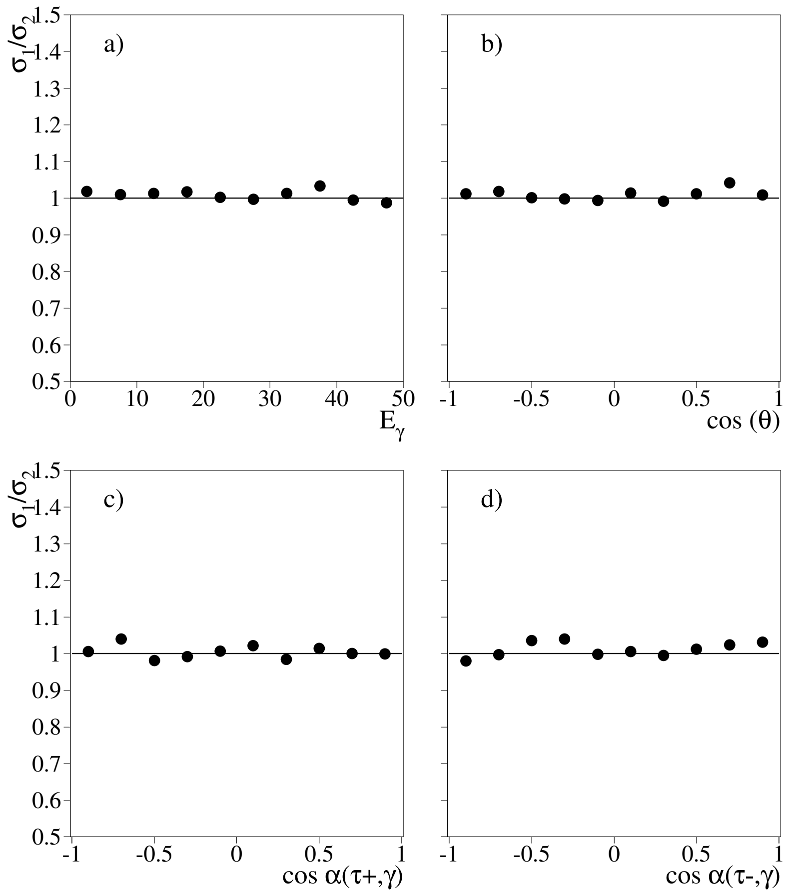

Figure 4: Ratios of differential cross sections predicted using KEYRAD=1 (ISR,FSR,interference included) to that predicted using KEYRAD=2 (ISR,FSR) for a) photon energy, b) angle between photon and beam electron c) angle between photon and τ^+, d) angle between photon and τ^-. The value $F_2(0) = +0.04$ has been used and the cuts specified in section 5 have been applied.

in the simulation of multiple photon radiation (KEYRAD=12). As the contribution from interference turns out to be very small for the single photon calculation, we may safely proceed with our KEYRAD=12 checks with further consideration of possible ISR/FSR interference effects.

From now on we will exploit the fact that, to a good approximation, the single (or highest p_T) photon distribution can be represented as a simple sum of the ISR and FSR contributions. For each event, the weight is calculated by TTG to be

$$w = \frac{|M_I + M_F + M_A|^2}{|M_I + M_F|^2} \tag{2}$$

Here M_I, M_F, and M_A denote respectively the matrix element for photon emission from initial, final states and from the final state tau through anomalous coupling. Let us denote the distributions which include anomalous contributions as:

$$\begin{aligned} d\sigma_I^A &= d\sigma_I \; w \\ d\sigma_F^A &= d\sigma_F \; w \\ d\sigma^A &= d\sigma \; w \end{aligned}$$

with

$$\begin{aligned} d\sigma_I &= |\mathcal{M}_I|^2 \, d\Omega \\ d\sigma_F &= |\mathcal{M}_F|^2 \, d\Omega \end{aligned}$$

where $\mathcal{M}$ is the matrix element and $d\Omega$ is the invariant phase space element. Thanks to the smallness of the ISR/FSR interference, we can say to good approximation that

$$d\sigma = d\sigma_I + d\sigma_F \tag{3}$$

and as a consequence

$$d\sigma^A = d\sigma_I^A + d\sigma_F^A. \tag{4}$$

In this sense we can separate total anomalous contributions into independent contribution from initial and final states.

In Figure 5, we compare the single bremsstrahlung calculations of the *anomalous* contribution to the differential cross section including ISR only (KEYRAD=3), FSR only (KEYRAD=4), and all contributions (KEYRAD=1). Note that the bulk of the anomalous contribution arises from final state radiation; this is expected, since the photons from phase space regions where ISR dominates produce rather small anomalous corrections to the amplitudes.

Next, we compare the anomalous contribution to the cross section calculated assuming single bremsstrahlung with that including multiple bremsstrahlung. Figure 6 shows this comparison for the case of ISR alone, and Figure 7 shows the same comparison for the case of FSR. In the case of multiple bremsstrahlung, the reduction procedure of section 4 has been employed, and the energies and angles plotted are those of effective reaction [5]. We can see that the difference between the two calculations is not dramatic [6], especially for the more sensitive regions of the distributions.

Finally, Figure 8 gives a comparison of the single and multiple bremsstrahlung calculations for the anomalous cross section including both ISR and FSR (and interference, in the single bremsstrahlung case). Again, the reduction scheme has been applied in the case of multiple bremsstrahlung. As expected, the overall anomalous cross section is suppressed by including multiple photon radiation in the initial state, but the overall shape is not strongly affected.

From these consistency checks, we conclude that the reduction algorithm gives sensible results. Aside from the expected overall scaling of the cross section due to ISR, the effects of higher order corrections on anomalous contributions to the differential cross section are small compared to the anomalous contributions themselves. This suggests that the related systematic errors on measurement of $F_2(0)$ and $F_3(0)$ should be small at LEP1 energies.

[5] As we will show in Figure 9, it makes essentially no difference whether we use the angles for the effective reaction or the angles in the real particles in the laboratory system.

[6] Keep in mind that the the *total* anomalous cross section shown in these plots (corresponding to $F_2 = 0.04$) produces only a 1% or so effect on the total $\tau\tau\gamma$ cross section.

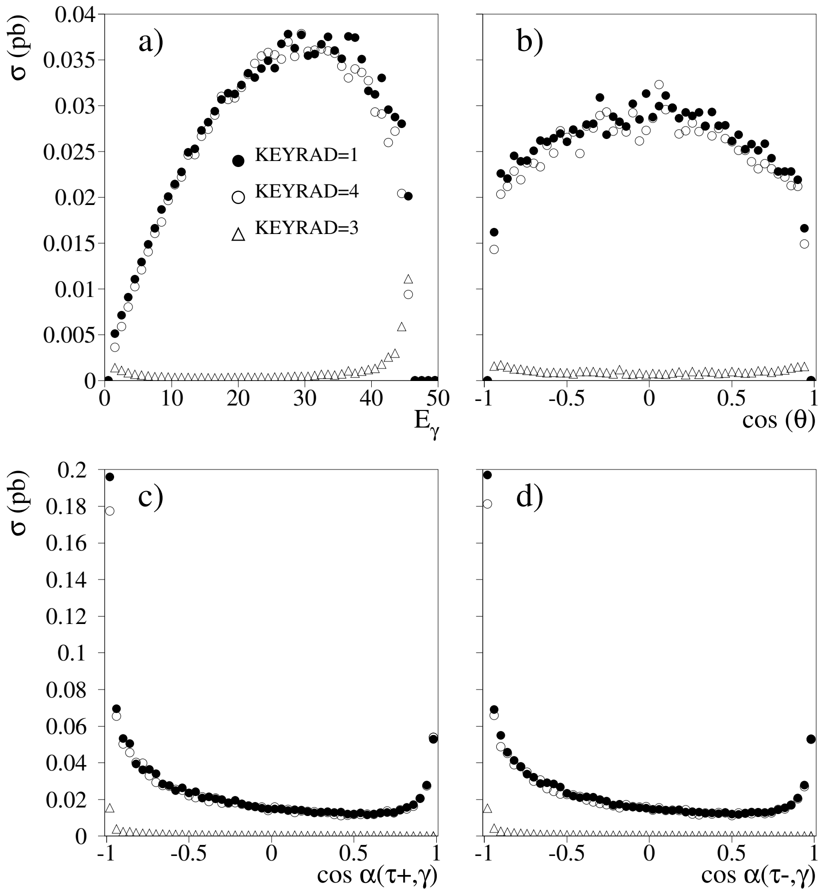

Figure 5: Anomalous contribution to differential cross section for the case of ISR (`KEYRAD=3`), FSR (`KEYRAD=4`), and ISR+FSR+interference (`KEYRAD=1`) shown as a function of a) photon energy, b) angle between photon and beam electron c) angle between photon and τ^+, d) angle between photon and τ^-. The cuts specified in section 5 have been applied, and the value $F_2(0) = +0.04$ has been used.

We may also estimate the size of systematic errors associated with higher order corrections to anomalous contributions by simulating events including multiple bremsstrahlung and comparing the anomalous distributions obtained using the reduction procedure to those obtained without using it. In order to do this, we select only events with *exactly* one photon satisfying certain energy and angle requirements. Two weights are then computed, the first using the 4-vectors of the selected photon and the two taus, and the second using the reduced 4-vectors which take into account any additional soft photons that may be present (but which do not pass the selection). The ratio of the anomalous photon energy distributions for these two approaches is shown in Figure 9a for two sets of selection cuts. In all cases, the photon energy stored in the histogram is that of the selected photon, not the photon energy seen in the effective frame. The discrepancy is significant at low energies, but essentially vanishes in the interesting high energy regions. Figure 9b is a similar comparison for a slightly different selection; in this case, we ask for one *or more* photons to pass the selection criteria, and compute one weight using only the highest energy selected photon and the taus and and the second weight using the reduction method. Again, there is only a small difference between the two approaches in the high energy regions.

Despite the fact that the effects of multiple bremsstrahlung appear small in the regions of interest,

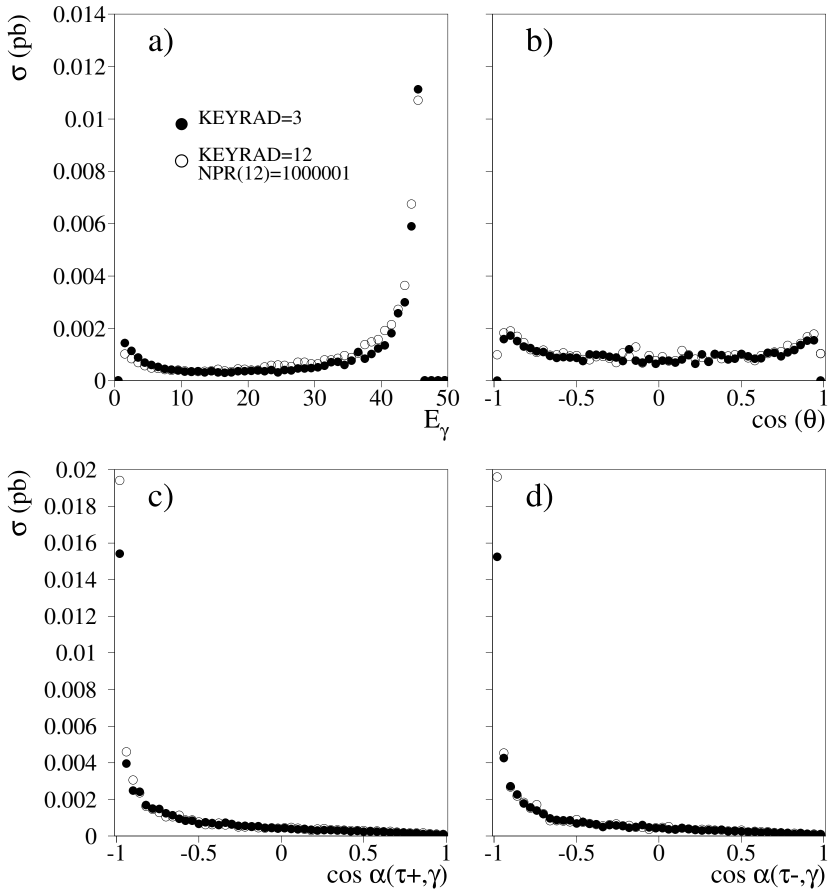

Figure 6: Comparison of the anomalous contribution to the differential cross section for single and multiple bremsstrahlung including only ISR (`KEYRAD=3` compared to KEYRAD=12,NPR(12)=1000001). Differential cross sections are shown as a function of a) photon energy, b) angle between photon and beam electron c) angle between photon and τ^+, d) angle between photon and τ^-. The cuts specified in section 5 have been applied, and the value $F_2(0) = +0.04$ has been used.

they are nonetheless included in the `KORALZ/TTG` Monte Carlo. This simplifies the selection and fitting. Finally, let us stress that our estimation of the systematic error is valid only for the observables, cuts, and center-of-mass energies defined here. For other choices, checks similar to those presented here should be performed.

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7 How to Use the Program

As discussed in sections 1 and 4, the strategy to account for anomalous magnetic and electric dipole moments involves using `KORALZ` to generate τ pairs with one or more radiated photons, applying a reduction procedure in the case of multiple photon radiation, and computing a weight, w, *for the event* as,

$$w = \frac{|\mathcal{M}_{\rm ano}|^2}{|\mathcal{M}_{\rm SM}|^2}, \tag{5}$$

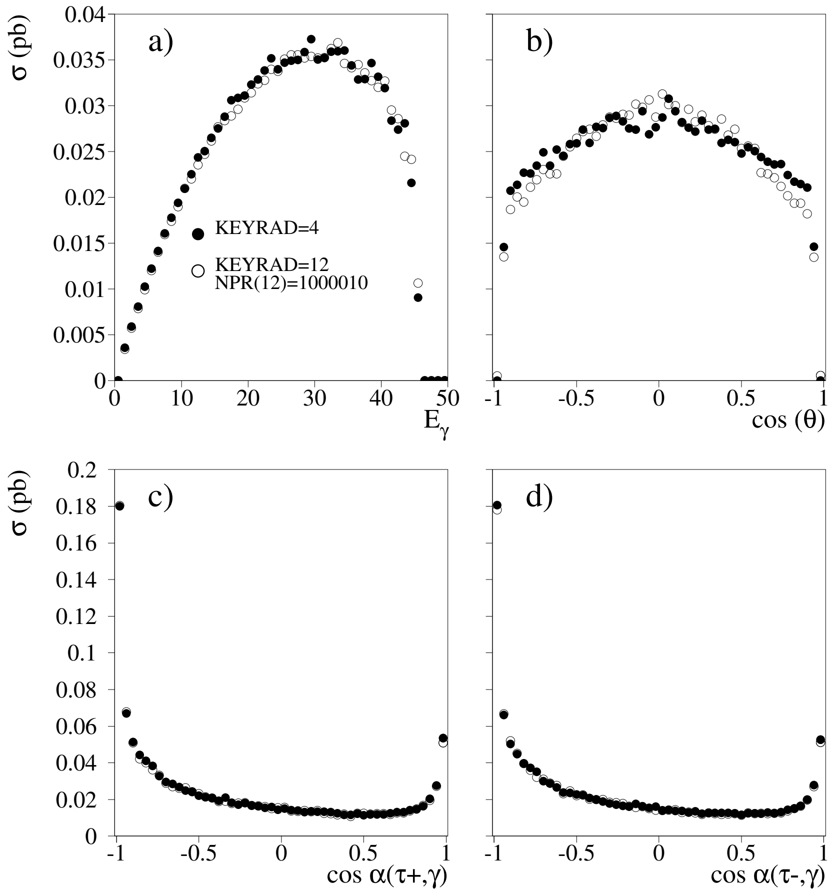

Figure 7: Comparison of the anomalous contribution to the differential cross section for single and multiple bremsstrahlung including only FSR (`KEYRAD=4` compared to KEYRAD=12,NPR(12)=1000010). Differential cross sections are shown as a function of a) photon energy, b) angle between photon and beam electron c) angle between photon and τ^+, d) angle between photon and τ^-. The cuts specified in section 5 have been applied, and the value $F_2(0) = +0.04$ has been used.

where $\mathcal{M}_{\rm ano}$ is the matrix element computed by `TTG` for $F_2(0) \neq 0$ and/or $F_3(0) \neq 0$ using the 4-vectors for the taus and the photon, and $\mathcal{M}_{\rm SM}$ is the matrix element, also computed by `TTG`, for the case of $F_2(0) = F_3(0) = 0$.

The calculation of these weights is activated by setting the card `IFKALIN=2`. This is transmitted from the main program via the `KORALZ` input parameter `NPAR(15)`. If this card is set, then `KORALZ` initializes `TTG` by calling the routine `ANOMINI_L3`. Constants of nature are passed from `KORALZ` to `ANOMINI_L3` with the help of the routine `KZ_STOREPARMS`. The reduction procedure described in section 4 is performed for each event in the routine `WTANOM`. After reduction, the actual weights for anomalous couplings are calculated by calling the routine `FU_L3`. All devices necessary for the importance sampling algorithm which minimizes the statistical divergence on difference in distributions with and without anomalous couplings are in place.

Additional options for `TTG` are anticipated in the common block `TTG_USER`. Such options are currently set in the routine `kzphynew(XPAR,NPAR)` , but there are no connections (yet) to the `KORALZ` matrix input parameters `XPAR, NPAR`, though it is straightforward to implement this. For the moment, one may set

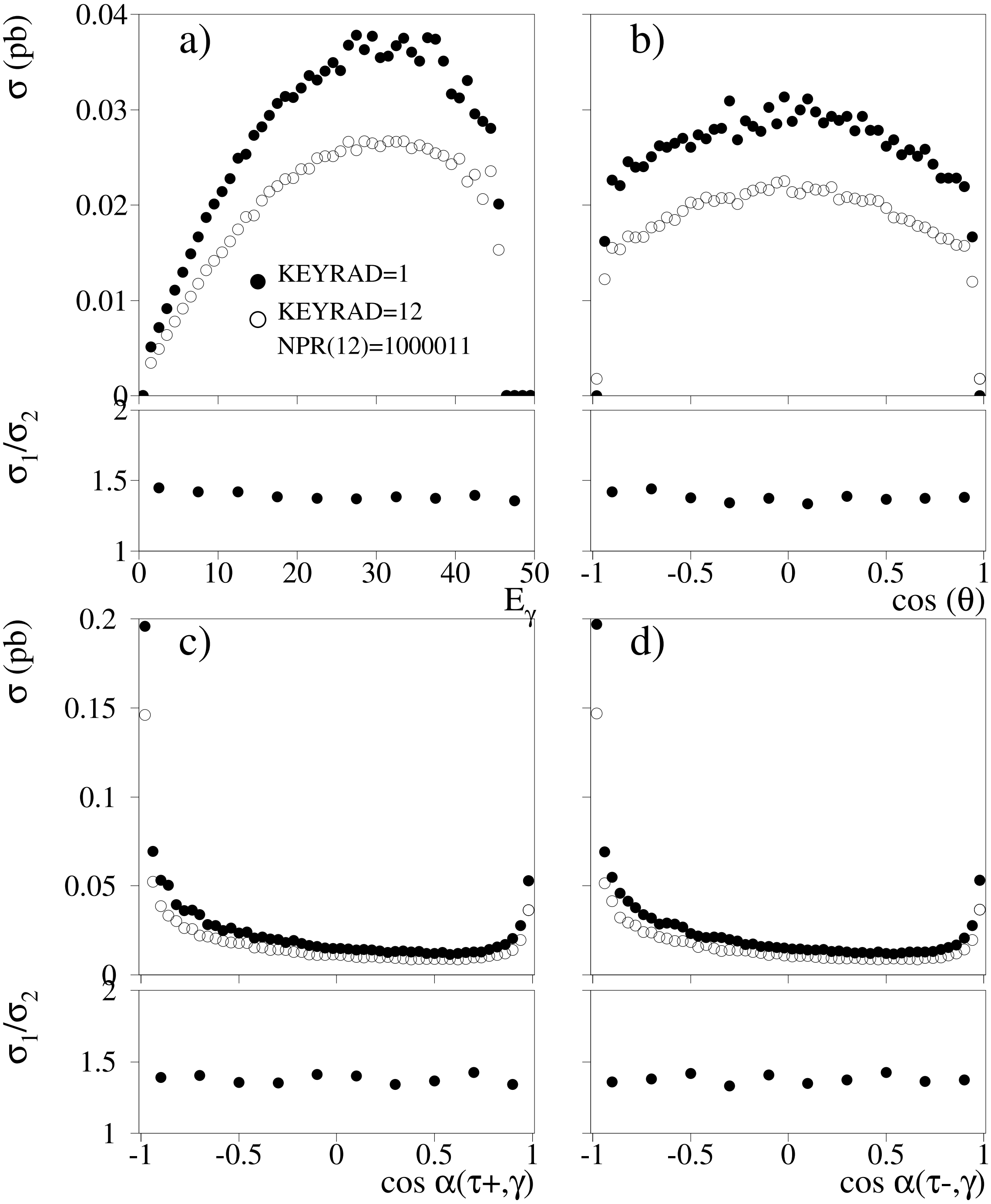

Figure 8: Comparison of the anomalous contribution to the differential cross section for single and multiple bremsstrahlung including both ISR and FSR (`KEYRAD=1` compared to KEYRAD=12,NPR(12)=1000011). The larger plots show these two distributions, while the lower plots show the ratio of the two. Differential cross sections are shown as a function of a) photon energy, b) angle between photon and beam electron c) angle between photon and τ^+, d) angle between photon and τ^-. The cuts specified in section 5 have been applied, and the value $F_2(0) = +0.04$ has been used.

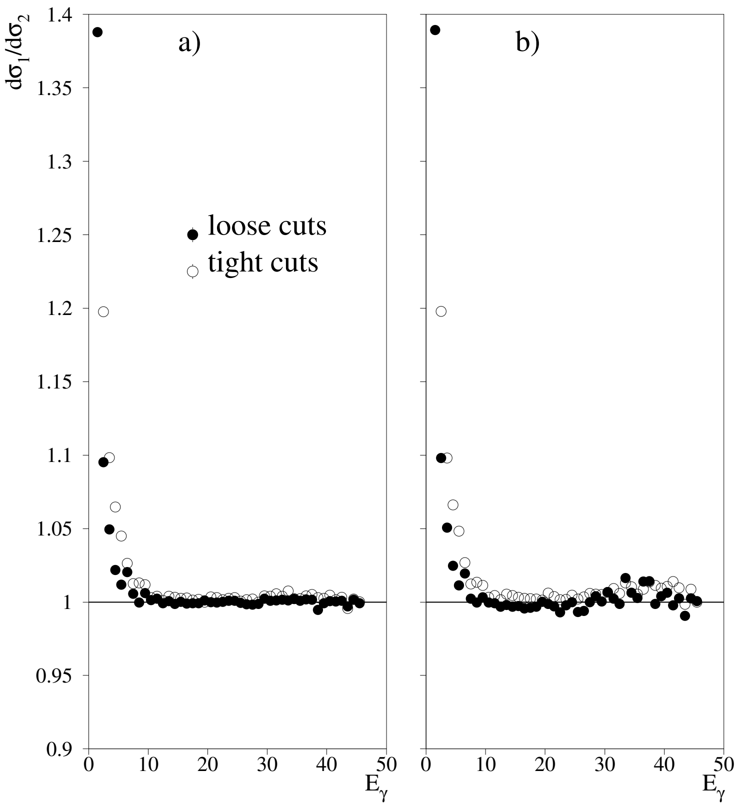

Figure 9: a) Ratio of the anomalous contribution to the differential cross section as a function of photon energy as computed using the reduction scheme to that computed not using it. Exactly one photon is required to satisfy $E_\gamma > 1$GeV, $|cos(\theta)| < 0.9$, $\cos(\alpha_{\tau^+,\gamma}) < 0.995$, and $\cos(\alpha_{\tau^-,\gamma}) < 0.995$ (loose cuts), or $E_\gamma > 2.5$GeV, $|cos(\theta)| < 0.7$, $\cos(\alpha_{\tau^+,\gamma}) < 0.95$, and $\cos(\alpha_{\tau^-,\gamma}) < 0.95$ (tight cuts). b) The same ratio for the case of one or more photons satisfying the loose or tight cuts. Only the highest energy photon is used to compute the weight in the case of no reduction. For loose cuts, 3.9% of *selected* events have more than one photon, and for tight cuts, 1.5% have more than one. The value $F_2(0) = +0.04$ has been used.

the following flags in the `TTG_USER` common:

`IF1`	$= 1$ to compute weights for $F_2(0)$
`IF2`	$= 1$ to compute weights for $F_3(0)$
`ISFL`	`TTG` "simple" flag

where one or of both `IF1` and `IF2` may be set, and where `ISFL` may have the following settings:

`ISFL` $= -1$	`TTG` computes *only* terms with anomalous contributions
`ISFL` $= 0$	`TTG` includes all terms
`ISFL` $= 1$	`TTG` uses the approximation of reference [2]

In order to provide the user with enough information to retrieve w for a given event for any $F_2(0)$ or $F_3(0)$, we take advantage of the fact that, for each event, we may write w as a quadratic function of the anomalous couplings:

$$w = \alpha F_2^2(0) + \beta F_2(0) + \gamma F_3^2(0) + \delta F_3(0) + \epsilon. \qquad (6)$$

When `FU_L3` is called, these 5 constants are stored in the common block[7] `common /kalinout/ wtkal(6)`, with the following assignments:

`wtkal(1)`	not used here (see [13])
`wtkal(2)`	ϵ
`wtakl(3)`	α
`wtkal(4)`	β
`wtkal(5)`	γ
`wtkal(6)`	δ

The user is then free to calculate w for whatever combination of $F_2(0)$ and $F_3(0)$ is desired. Note that in practice we set $\epsilon = 1$, since anomalous terms must vanish for $F_2(0) = F_3(0) = 0$, and $\delta = 0$, as the interference between standard model and anomalous amplitudes vanishes in the case of radiation from an electric dipole moment. These shortcuts save substantial CPU time.

Acknowledgements

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[7] This is similar to the way anomalous $\nu\bar{\nu}\gamma$ information is stored [13].

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