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# Measurement of the mass of the Z boson and the energy calibration of LEP

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and the LEP collaborations ALEPH, DELPHI, L3 and OPAL[2)]

**Abstract**

The mass of the Z boson has been determined by combining the data from the four LEP experiments ALEPH, DELPHI, L3 and OPAL. The dominant error arises from uncertainties in the calibration of the energy of the beams in LEP. A programme of investigations including energy calibration by resonant depolarization of transversely polarized beams has led to a significant reduction of the uncertainty on the Z mass compared with the precision achieved with the 1990 data. The mass of the Z is measured to be $M_{\mathrm{Z}} = (91.187 \pm 0.007)$ GeV.

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1) Author list see [1]
2) Lists of authors can be found in the references listed under [2]

## 1 Introduction

The mass, $M_Z$, of the neutral weak boson is a fundamental parameter of nature and the large electron-positron collider, LEP, at CERN is the ideal place to measure it precisely. Although the precision of present tests of the standard model requires a knowledge of $M_Z$ to only a few tens of MeV, a direct measurement of the mass of the top quark and the expected improvements on the precision of other LEP measurements will require a better knowledge of $M_Z$. From data taken up to the end of 1990 by the four LEP collaborations ALEPH, DELPHI, L3 and OPAL, a combined value of $M_Z$ had been obtained [3], with the uncertainties in the LEP energy scale contributing the dominant error of 20 MeV in comparison with a statistical error of 5 MeV on the combined result. Since then, the understanding of the energy calibration has progressed [1]. New results from data taken during the 1991 energy scan of seven different energies within $\pm 3$ GeV of the Z mass are now available.

Parameters of the Z are extracted from the energy dependence of the cross section for $e^+e^- \to$ hadrons and $e^+e^- \to$ leptons around the Z resonance. The error on the mass is dominated by the uncertainty in the absolute energy scale and also affected by the uncertainty on the difference between the scan energies. The next sections concentrate on the techniques used to determine the energy of the electron and positron beams, followed by a discussion of the measurement of $M_Z$.

## 2 LEP energy calibration

The momentum of the electron and positron beams circulating in LEP is proportional to the magnetic bending field integrated over the path of the particles. For particles on the central orbit, *i.e.* passing through the centre of the quadrupoles and sextupoles, the momentum is determined by the field of the 3280 main bending magnets and by small contributions from constant fields such as the Earth's magnetic field or remanent fields in the beam pipe. Contributions from the quadrupoles and sextupoles also have to be considered for non-central orbits.

Four techniques provided information on the energy:

*i) The Field Display* [4] uses a rotating coil to measure the magnetic field in a reference dipole powered in series with the main ring magnets. Measurements were performed regularly and are used as a reference value for the energy of each individual fill of the machine, to which corrections are applied based on other calibration techniques. The reproducibility of the field display measurements is about $\pm 2.5\times 10^{-5}$.

*ii) The Flux Loop* [5, 6] consists of closed electrical loops, each threading all the dipoles in one octant of the machine; the integrated induced voltage when altering the dipole currents is a direct measure of the magnetic field generated by the main ring dipoles. Absolute calibrations of the flux loop with a relative precision of $\sim \pm 10^{-4}$ were performed prior to installation of the magnets. However, the flux loop method is insensitive to constant fields and does not take into account additional bending in the quadrupoles and sextupoles on non-central orbits. It therefore needs various corrections.

*iii) Proton Calibrations* [6] were performed to check the flux loop measurements by storing 20 GeV protons in LEP. Protons at that energy are not ultra-relativistic and therefore their momentum can be measured by determination of the frequency of the RF acceleration voltage. This determines the momentum of positrons on the same orbit. The relative precision of this method is high at 20 GeV ($\sim \pm 10^{-4}$), but the determination of the beam energy at 45 GeV depends on extrapolations of the magnetic field with flux-loop measurements and leads to a degradation of precision ($\sim \pm 2\times 10^{-4}$).

*iv) Resonant Depolarization* determines the beam energy by measuring the frequency with which the spins of transversely polarized electrons precess about the vertical bending field. This technique measures the beam energy under conditions very close to those of data-taking runs and is by far the most precise method available. Such measurements were successfully performed during four fills at the end of the 1991 running period.

The depolarization measurements form the cornerstone of the 1991 energy calibration. The energy of each individual fill of the machine was obtained from the field display corrected for the average deviation from the field display value observed during the depolarization calibrations. The local changes of the beam energies in the accelerating cavities also had to be taken into account to obtain the centre-of-mass energies at the interaction regions. Flux loop measurements were used to monitor the stability of the magnets with time.

### 2.1 Energy calibration by resonant depolarization

Transverse polarization in LEP was first observed in 1990 by means of a Compton-scattering laser polarimeter [7]. Towards the end of the 1991 data taking period, transversely polarized electrons at a nominal energy of 46.5 GeV were depolarized in a controlled way by applying an oscillating horizontal magnetic field [8]. Under the influence of such a weak field the spins are slightly rotated away from the vertical axis on each turn, and a depolarizing resonance occurs if the depolarizing field is in phase with the spin precession [9]. The number of spin precessions per revolution, the spin tune $\nu_s$, is related to the beam energy via

$$E_{\rm beam} = \frac{\nu_s \times m_{\rm e} c^2}{(g_{\rm e} - 2)/2} \equiv 0.4406486(1)\ {\rm GeV} \times \nu_s \ ,$$

where $(g_{\rm e} - 2)/2$ is the magnetic moment anomaly of the electron, $m_{\rm e}$ is the electron mass, and $c$ is the speed of light. The depolarizing field is applied once per turn, and therefore the resonance occurs at a frequency which is independent of the integer part of the spin tune: $f_{\rm dep} = (\nu_s - {\rm int}(\nu_s)) \times f_{\rm rev}$ , where $f_{\rm rev}$ =11245.50(4) Hz is the revolution frequency of the beam particles. The integer part of $\nu_s$, 105 at 46.5 GeV, is well known from the other calibration techniques, since a unit tune change corresponds to a $\sim$440 MeV change in beam energy.

The resonance was located by varying the frequency of the depolarizing magnetic field over successively smaller ranges. Six measurements of the beam energy during four fills were obtained, each with a relative resolution of about $\pm 3\times10^{-5}$. The observed variation is larger, about $\pm 8\times10^{-5}$, and is attributed mainly to changes of the beam energy between the measurements.

### 2.2 Stability of the beam energy over time

Periodic flux loop measurements were performed in order to monitor the stability of the magnetic field generated by the dipoles. Unlike the reference magnet, which has a steel inner core, the ring dipoles are made of concrete-steel cores. The properties of the ring dipoles change with time as the cores dehydrate [5] and therefore the field in the ring dipoles is different from the field measured in the reference magnet. In addition, the magnetic field has a temperature coefficient [10], resulting in an energy variation of $\Delta E/E$=$(1.00\pm 0.25)\times10^{-4}/\ ^{o}$C. The error given covers the values of the temperature coefficient determined from laboratory measurements and from flux loop calibrations done at different temperatures. Temperature measurements were obtained from a reference set of eight magnets, estimated to represent the average dipole temperature to within $\pm$0.25 $^{o}$C. The flux loop measurements, after temperature correction, are shown in figure 1 together with the proton and depolarization calibrations. The flux loop calibrations were constant during 1991 up to the start of the energy scan, then they showed a step corresponding to $-15$ MeV in beam energy, and also gave indications for a time dependence with a relative change of $-2.2\times10^{-6}$/day or about $-9$ MeV in beam energy over the entire duration of the energy scan. This slope is indicated by the dashed line on the figure. Proton calibrations performed before and after the step could not confirm the change in beam energy; therefore only one-half the size of the step was taken as correction, with a systematic error of $\pm$10 MeV on the beam energy assigned. Depolarization calibrations performed during the scan could not exclude the existence of the slope, resulting in an additional uncertainty of $\pm$2 MeV on the average centre-of-mass energy.

The scatter of the resonant depolarization measurements gives another estimate of the time dependence of the beam energy. Reasons for the changes of the beam energy between the measurements include temperature effects and contributions to the bending field from correction dipoles used to tune the horizontal orbit of the beams. Tidal forces from the Moon and the Sun also alter the beam energies, as discussed in reference [11]. These forces lead to time-dependent deformations in the shape of the Earth and are expected to cause relative changes of the circumference, $C$, of LEP by a few $10^{-8}$. However, the length of the closed path taken by the particles is determined by the frequency of the voltage driving the acceleration cavities and does not change; therefore the particles are no longer at the central orbit and feel contributions from the quadrupoles and sextupoles to the bending field. The resulting change in beam energy is given by

$$\Delta p/p = -(1/\alpha_c)\Delta C/C,$$

where $\alpha_c = 3.87 \times 10^{-4}$ is the "momentum compaction factor" for the 1991 beam optics. A change of the 27 km circumference of LEP by 1 mm alters the beam energy by 4 MeV.

The six depolarization measurements of the beam energy are shown in figure 2 as a function of the tidal force normalized between -1 and 1. A strong correlation is observed and is confirmed by a controlled experiment performed during the 1992 running period of LEP [12]. The line on figure 2 represents the best fit to the data when fixing the slope to the one measured in 1992; the 1991 polarization calibrations agree well with this prediction.

For the analysis of the 1991 data the full variation observed in the polarization data is used as an estimate of the energy variation from all sources.

### 2.3 Centre-of-mass energy at the experiments

The energy of the beams is not constant as they go around the machine; particles at an average energy of 45.6 GeV lose 124 MeV per turn due to synchrotron radiation and gain the same amount of energy in the radio-frequency (RF) cavities on either side of the L3 and OPAL experiments. This is shown in figure 3 for typical 1991 running conditions. There is a difference between the design and the actual radio-frequency used for physics running such that the cavities appear too far away from the interaction point. Therefore, particles arrive too early in the first set of cavities with respect to the phase of the accelerating voltage and gain an energy which is about 13 MeV greater than the gain in the second set, where they are late in phase. This results in shifts of the centre-of-mass energy of about 13 MeV at the L3 and OPAL interaction points. If the power is not equally distributed between the left- and right-hand sides of an experiment, *e.g.* due to cavities being switched off, additional changes of the centre-of-mass energy by a few MeV occur in all four experiments. The precise value of the correction also depends on other parameters such as the synchrotron tune or the beam optics. Based on the average values of these parameters and their observed fluctuations, the uncertainty introduced on the centre-of-mass energy, averaged over all fills, was estimated to be $\pm 1$ MeV, and fill-to-fill fluctuations were estimated to be $\pm 2$ MeV. Since the energy of the electron and the positron beams averaged in all the bending magnets must be the same, these errors are anti-correlated between experiments on opposite sides of the ring, *i.e.* between ALEPH and DELPHI and between L3 and OPAL.

### 2.4 Calibration results

The centre-of-mass energy in physics runs during the 1991 energy scan was obtained from the depolarization measurements performed at a nominal energy of 93 GeV. The correction to the field display value of the centre-of-mass energy is $(-61.0 \pm 5.3)$ MeV, where the error, $\Delta E^{\mathrm{abs}}$, is composed as follows:

- the spread of the depolarization measurements divided by the square root of the number of measurements ($\pm 3.7$ MeV);
- the effect of a difference in average temperature of $0.71 \pm 0.25$ $^\circ$C between polarization runs and physics runs, including the uncertainty in the temperature coefficient ($\pm 3$ MeV);
- the possible slope seen in the flux loop measurements ($\pm 2$ MeV);
- uncertainties in the average operational parameters which affect the correction due to the RF cavities ($\pm 1$ MeV).

The energy scale of the runs before the energy scan, all at the Z peak, was determined from the average of both the proton calibration and the extrapolation of the polarization results by means of the flux loop, and has an error of $\pm 19$ MeV. This relatively large error has no influence on the final result for the Z mass.

Fills at energies other than 93 GeV have a contribution from an observed non-linearity in the excitation curve of the dipoles leading to a correction of $(2.0 \pm 1.5)$ MeV$\times(93 - E_{cm}/\mathrm{GeV})$. The error on the coefficient, $\Delta E^{\mathrm{non-lin}}$, was estimated by comparing proton calibrations at 20 GeV with depolarization calibrations at 46.5 GeV. In addition, there is an estimated random energy-point-to-energy-point error of $\Delta E^{\mathrm{set}}/E = 3 \times 10^{-5}$ arising from systematically different settings of machine parameters at different energies.

Since only five fills were taken at each off-peak energy point in 1991, it was important to consider fluctuations arising from the non-reproducibility of the beam energy from fill to fill. The spread of the polarization data ($\sim \pm 8\times10^{-5}$), energy changes due to dipole temperature variations ($\pm 3\times10^{-5}$) and RF instabilities ($\pm 2\times10^{-5}$) led to an estimate for the fill-to-fill reproducibility of the energy of $\Delta E^{\mathrm{rep}}/E = 10^{-4}$; this error is reduced according to the number of fills per energy point.

To summarize, the energy error of each scan point at mean centre-of-mass energy $E_i$ with $n_i$ fills contributing to it is described by[1]:

$$\frac{\Delta E_i}{E_i} = \left(\frac{\Delta E}{E}\right)^{\mathrm{abs}} \oplus \frac{|93 - E_i/\mathrm{GeV}|}{E_i} \Delta E^{\mathrm{non-lin}} \oplus \left(\frac{\Delta E}{E}\right)_i^{\mathrm{set}} \oplus \frac{1}{\sqrt{n_i}} \left(\frac{\Delta E}{E}\right)_i^{\mathrm{rep}}$$

where

$$\begin{aligned} (\Delta E/E)^{\mathrm{abs}} &= \pm 5.7\times10^{-5} \\ \Delta E^{\mathrm{non-lin}} &= \pm 1.5 \text{ MeV} \\ (\Delta E/E)_i^{\mathrm{set}} &= \pm 3\times10^{-5} \\ (\Delta E/E)_i^{\mathrm{rep}} &= \pm 10\times10^{-5}. \end{aligned}$$

The last two errors are uncorrelated between different energy points, whereas the first two are fully correlated for all energy points. Except for small effects due to the uncertainties in the correction for the RF cavity position, these errors are the same for all four experiments.

## 3 Determination of $M_\mathrm{Z}$

Each of the four experiments determined the Z mass, together with other electroweak parameters, from combined fits to the hadronic and leptonic cross sections measured during energy scans in 1989, 1990 and 1991, where the 1989 statistics are negligible. The total luminosity was $\sim$20 pb$^{-1}$ per experiment with about two thirds of the luminosity taken at the peak of the Z resonance and one third taken at six off-peak energies[2] within $\pm 3$ GeV of the peak. Information on $M_\mathrm{Z}$ comes mainly from the off-peak data. Since hadronic Z decays are about seven times more frequent than decays into charged leptons, the experimental precision on $M_\mathrm{Z}$ is determined by the precision achieved on the point-to-point multi-hadronic cross section. After unfolding of radiative corrections this cross section at each energy point, $E_i$, is parametrized by a modified Breit-Wigner shape of the form:

$$\sigma(E_i) = \sigma^{\mathrm{pole}} \frac{E_i^2 \Gamma_\mathrm{Z}^2}{(E_i^2 - M_\mathrm{Z}^2)^2 + E_i^4 \Gamma_\mathrm{Z}^2 / M_\mathrm{Z}^2} ,$$

where $\sigma^{\mathrm{pole}}$ represents the cross section at $E_i = M_\mathrm{Z}$ and $\Gamma_\mathrm{Z}$ is the Z width. Further details about the parametrization have been described elsewhere [13]. Small contributions to the cross section from $s$-channel photon exchange and the photon-Z interference were fixed to their Standard Model values. Neglecting these, only the precision on the relative point-to-point cross sections is important for the determination of $M_\mathrm{Z}$.

Uncertainties in the energy calibration were included in the fitting procedures by constructing the error correlation matrix between all scan points in 1990 and 1991. Since the 1990 energy calibration was much less precise than the present one, 1990 data do not contribute significantly to the present value of $M_\mathrm{Z}$. Errors due to the energy calibration are common to all experiments and were determined by taking the difference of the errors on the parameters obtained from fits to the individual data sets of the experiments with and without taking into

[1] '$\oplus$' stands for summing in quadrature

[2] these energies were slightly different in 1990 and 1991

account energy uncertainties. These amount to an error of $\pm 6$ MeV on $M_Z$, to be compared with an uncorrelated error of $\pm 7$ MeV per experiment. Uncertainties on $M_Z$ arising from radiative corrections or from the precision on the point-to-point luminosity are negligible.

The results [2] are shown in table 1. The individual measurements are compatible, as can be seen from the $\chi^2$ value of 2.1 for three degrees of freedom. The combination was performed by averaging over the experiments after subtracting in quadrature the common error due to the relative and absolute energy scale uncertainties. The result, $M_Z$=$(91.187 \pm 0.004_{\rm exp.} \pm 0.006_{\rm LEP})$ GeV, is a considerable improvement over the value of $(91.175 \pm 0.005_{\rm exp.} \pm 0.02_{\rm LEP})$ GeV obtained from the 1989 and 1990 data alone [3].

## 4 Summary

The average absolute energy scale for data taken during the 1991 energy scan around the Z mass was determined with a relative precision of $\pm 5.7 \times 10^{-5}$, corresponding to $\pm 5.2$ MeV at a centre-of-mass energy of $M_Z$. This represents an improvement of a factor four over the precision achieved previously and was made possible by repeated energy measurements using resonant depolarization of transversely polarized electron beams. In addition to the overall scale error, uncertainties in the local energy scale about the normalization point and uncertainties in the fill-to-fill reproducibility of the beam energy led to a total error of $\pm 6$ MeV on the mass of the Z. This is to be compared with a statistical precision of $\pm 4$ MeV on $M_Z$ obtained after combination of the measurements of the four LEP experiments, resulting in a value of $M_Z = (91.187 \pm 0.007)$ GeV.

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| | $M_Z$[GeV] | |
|---|---|---|
| ALEPH | 91.187 | ±0.009 |
| DELPHI | 91.186 | ±0.009 |
| L3 | 91.195 | ±0.009 |
| OPAL | 91.181 | ±0.009 |
| common error due to energy scale uncertainty | | ±0.006 |
| combined result | 91.187 | ±0.004±0.006 |
| $\chi^2$/D.O.F. from independent errors is 2.1/3 | | |

Table 1: Results on the Z mass [2].
Individual experimental results are given including the common error arising from uncertainties in the LEP energy scale.

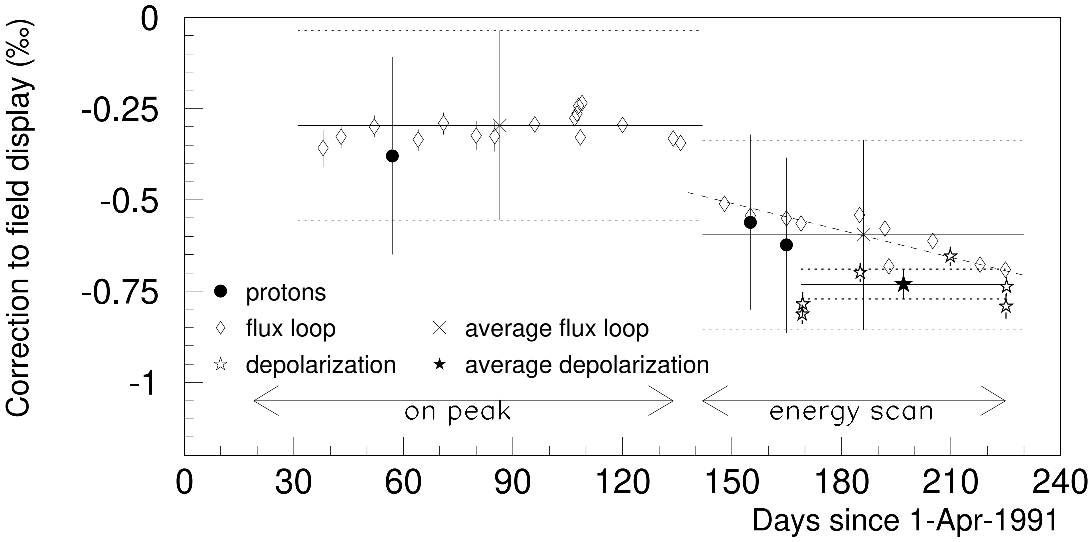


Figure 1: Beam energy calibrations in 1991.
The measurements were corrected to the same temperature as mentioned in the text. Shown as a function of time is the correction to be applied to the energy obtained from measurements of the magnetic field in the reference magnet. The individual flux loop measurements are shown with the error from the temperature correction only; the full error is shown by the horizontal dotted lines representing the average of all measurements before and during the energy scan, respectively. The error bar drawn on the average of the depolarization points represents the r.m.s. spread of the individual measurements.

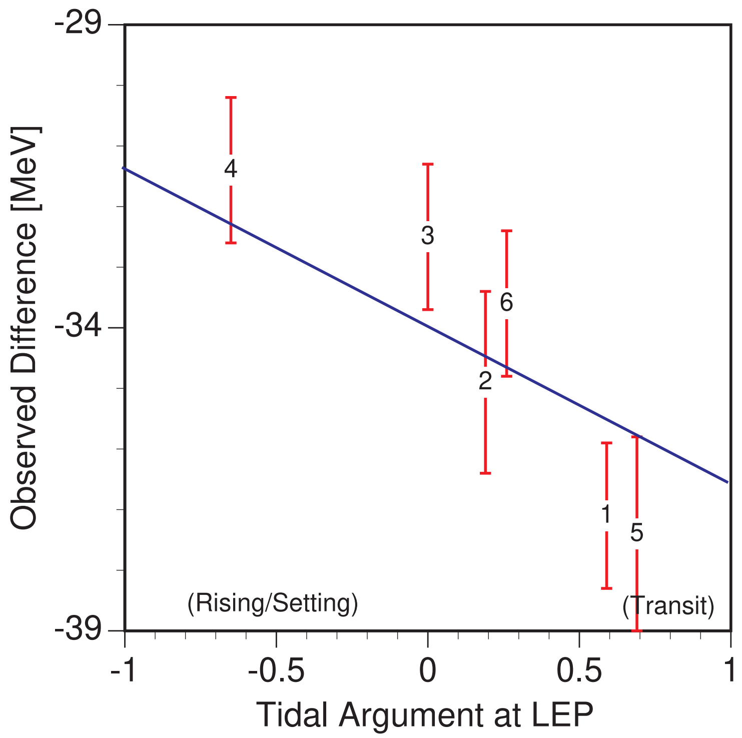


Figure 2: Deviations of the beam energy from the field display value as observed from beam energy measurements with the technique of resonant depolarization. The x-axis represents the tidal force exerted by the Moon and the Sun, normalized between -1 and 1; -1 corresponds to rising or setting of the Moon. The numbering on the measurements represents their sequence in time, and the error bars give the range in beam energy within which resonant depolarization was observed.

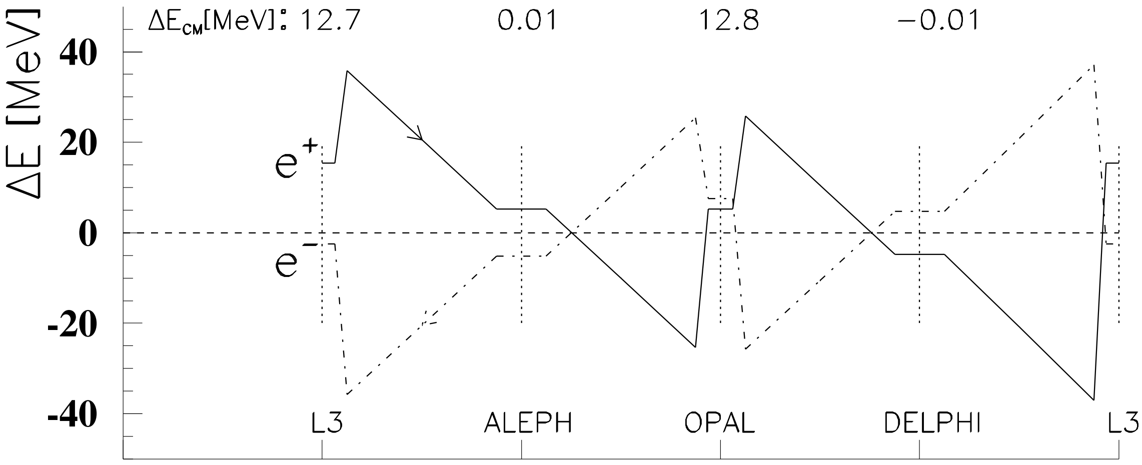


Figure 3: Deviation of particle energies from the average beam energy as a function of the position in LEP, for typical 1991 running conditions. The gain in energy on the left-hand side of the L3 experiment is larger due to additional super-conducting cavities. The numbers '$\Delta E_{CM}$[MeV]' indicate the deviation of the centre-of-mass energy at the interaction points.