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Combined LEP and SLD Electroweak Heavy Flavour Results for Summer 1995 Conferences Internal Note

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Abstract

In this note we combine the LEP and SLD electroweak results from heavy flavour analyses. Average values for the Standard Model parameters R_b, R_c, $A_\mathrm{FB}^{\mathrm{b\overline{b}}}$, $A_\mathrm{FB}^{\mathrm{c\overline{c}}}$, $\mathcal{A}_\mathrm{b}$ and $\mathcal{A}_\mathrm{c}$ are derived. The document is based on the analyses available on 20 July 1995. Each experiment has been responsible for providing their results in a common format.

Using the LEP data only, two fits are investigated: an 11 parameter fit, where the asymmetries are averaged at each of three centre-of-mass energies, and a 7 parameter fit where all the asymmetry measurements are taken to be a measure of the pole asymmetries. Both fits have been repeated including the SLD R_b and left-right forward-backward asymmetry measurements and leaving $\mathcal{A}_\mathrm{b}$ and $\mathcal{A}_\mathrm{c}$ as additional free parameters. The results of the 7+2 parameter fit give the combined LEP/SLD values:

$$\begin{aligned} R_\mathrm{b} &= 0.2219 \pm 0.0017 \\ R_\mathrm{c} &= 0.1540 \pm 0.0074 \\ A_\mathrm{FB}^{0,\,\mathrm{b}} &= 0.0997 \pm 0.0031 \\ A_\mathrm{FB}^{0,\,\mathrm{c}} &= 0.0729 \pm 0.0058 \\ \mathcal{A}_\mathrm{b} &= 0.841 \pm 0.053 \\ \mathcal{A}_\mathrm{c} &= 0.606 \pm 0.090 \end{aligned}$$

There is a large correlation coefficient of -0.35 between R_b and R_c. Fixing R_c to the value predicted by the Standard Model yields:

$$R_\mathrm{b} = 0.2205 \pm 0.0016 \text{ with } R_\mathrm{c} = 0.172 \text{ (fixed)}.$$

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1 Introduction

The LEP Electroweak Heavy Flavour Working Group has developed a procedure for combining measurements of the ratios of partial widths $R_{\mathrm{b}} = \Gamma_{\mathrm{b\overline{b}}}/\Gamma_{\mathrm{had}}$ and $R_{\mathrm{c}} = \Gamma_{\mathrm{c\overline{c}}}/\Gamma_{\mathrm{had}}$ and the forward-backward asymmetries $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$ [1–3]. New results available for the Summer 1995 conferences are included here. Several of the analyses also determine other quantities, in particular the average mixing parameter, $\overline{\chi}$, the fragmentation parameters $\langle x_E\rangle_{\mathrm{b}}$ and $\langle x_E\rangle_{\mathrm{c}}$, and the semileptonic branching fractions for b- and c-quarks. Where these quantities are not measured, they often contribute a systematic uncertainty to the Standard Model measurements. Motivated by the lepton fit analyses, the combined values of seven or eleven parameters have been determined: the parameters of interest in the electroweak sector, R_{b}, R_{c}, $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$, and in addition $\overline{\chi}$, $\mathrm{Br}(\mathrm{b} \rightarrow \ell)$ and $\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{c} \rightarrow \ell^{+})$, to take into account their correlations with the electroweak parameters. The asymmetries vary rapidly with centre-of-mass energy. In a first fit the asymmetry measurements on-peak, above peak and below peak have therefore been combined separately. The centre-of-mass energy dependence of the measured asymmetries agrees with the one predicted by the standard model. To derive the coupling parameters and the electroweak mixing angle ($\sin^2\theta_{eff}^{\mathrm{lept}}$) from the quark asymmetries, in a second fit all off-peak asymmetries have been transported to the peak energy before the fit.

Recently the SLD collaboration has presented precise measurements on R_{b} and on the left-right forward-backward asymmetry for b and c quarks. Since the precision and the dominant sources of systematic uncertainty are similar at LEP and SLC it is useful to produce combined LEP+SLD average. The left-right forward-backward asymmetries are, contrary to the unpolarized asymmetries, only sensitive to the final state couplings ($\mathcal{A}_{\mathrm{b}}$ and $\mathcal{A}_{\mathrm{c}}$) and will be considered as physically independent quantities. However the methods to measure the polarized and unpolarized asymmetries are very similar and thus the systematic uncertainties are potentially correlated. In order to obtain the exact correlation between the SLD and the LEP asymmetries $\mathcal{A}_{\mathrm{b}}$and $\mathcal{A}_{\mathrm{c}}$have been added as additional free parameters and measurements to the averaging procedure.

The understanding of the effects of the semileptonic decay model uncertainty has progressed since last Summer, when there were inconsistencies in the sign convention. We now consider the average values of $\mathrm{Br}(\mathrm{b} \rightarrow \ell)$, $\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{c} \rightarrow \ell^{+})$ and $\overline{\chi}$ to be much more reliable. In addition, the dependence of any of the measurements on the semileptonic branching ratios has been taken into account more completely than before.

In the rest of this note, we first list the new measurements. We then describe the corrections made to the asymmetries to derive the pole asymmetries, and discuss one or two new details of the averaging procedure. Finally the updated averages are presented. The full set of measurements to be averaged are provided in standardized tables by the four LEP experiments and SLD. Where necessary, the experiments applied small corrections to the results quoted in their papers and notes to use agreed default values and ranges for systematic inputs. These tables are all given in the appendices to the note.

2 New Results

The following have been updated since the ICHEP '94 conference in Glasgow:

ALEPH

- New preliminary measurement of Br($b \to \ell$) and Br($b \to c \to \ell^+$) using lifetime tagging from events collected in 92 and 93 [5]. As the published Aleph multiparameter lepton fit [4] is from 90+91 data the new results are statistically uncorrelated to the published values. The dependence of the new branching ratios on the mixing parameter $\overline{\chi}$ has been taken into account in the averaging procedure.

- New preliminary measurement of the charm forward–backward asymmetry with a D* tag (91-94 data) [6].

DELPHI

- The charm asymmetry with a D^* tag contains all data up to 1994 and more decay modes [7].

- The bottom asymmetry from a lifetime tag and jet charge using 1991 to 1994 data was measured using a method similar to those of ALEPH and OPAL independent of lepton tags [7].

- The bottom and charm asymmetry from a lepton tag using the 91+92 data is now published [8]. Preliminary numbers including 93 and 94 data are available [7].

- The lepton fit analysis is published (91+92 data) [9]

- The R_b double tag measurement is published (91+92 data) [9]. A preliminary analysis was performed on 1993 data [10].

- R_c is measured comparing $D^{*\pm}, D^0, D^{\pm}$ production rates at LEP with those at lower energies and by an inclusive D^* double tag using low p_t pions. Both analyses use 1991-1993 data [11].

L3

- R_b and Br($b \to \ell$) – sign of semileptonic decay model error corrected (90-91 prel)

- $A_{FB}^{b\bar{b}}$, $A_{FB}^{c\bar{c}}$ and $\overline{\chi}$ – sign of semileptonic decay model error corrected (90-93 prel)

OPAL

- The double-tag R_b measurement using either displaced reconstructed secondary vertices of high momentum leptons to tag b decays has been updated preliminarily by adding 1994 data [15].

- The measurements of $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ with a lepton tag have been updated preliminarily [13], and now include 1990 to 1994 data.

- The $A_{FB}^{b\bar{b}}$ measurement with a lifetime tag and jet charge measure has been updated and published, including 1991 to 1994 data [19]. The energy dependence is included.

- R_c measured with a D^* tag is now published [20] (90-92 pub)

- Preliminary results are available of a determination of R_c using a double tagging method based on reconstructed D^* mesons and an inclusive charm tag using low p_t pions [21]. This analysis is based on data collected in 1990-1994.

- The rates for all ground state D mesons and baryons has been measured, and is used to derive a measurement of R_c [22]. The data used encompass the years 1991 till 1993.
- The three measurements of R_c have been combined by OPAL to give one number which is used to make LEP averages.
- The $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ measurement using D meson tags has been updated preliminarily to include all data including 1994 and more D states and decay modes [24].

SLD

Heavy flavour electroweak results from the SLD are all preliminary with the inclusion of the 1994-1995 run data :

- R_b measurement from a lifetime double tag is a new analysis using a 3D impact parameter hemisphere probability tag [25].
- $\mathcal{A}_c$ measurement from exclusive D^*, D^+ reconstruction is an update from the 1993 analysis [26].
- $\mathcal{A}_b$ measurement from lifetime tag + jet charge is a new analysis with self-calibration for the jet charge analysing power [27].
- $\mathcal{A}_b$ measurement from lifetime tag + a new technique using $K^{\pm}$ identified by the SLD Cerenkov Ring Imaging Detector for $B/\overline{B}$ discrimination is a new analysis from 94-95 data only [28].
- $\mathcal{A}_b$,$\mathcal{A}_c$ measurements from leptons are update from the 1993 analysis [29].

3 Treatment of the LEP Asymmetry Measurements

Following the procedure adopted last summer, each asymmetry measurement was propagated to the nearest of the following centre-of-mass energies using the predicted dependence from ZFITTER:

$$\begin{aligned}\sqrt{s} &= 89.55\text{GeV (“peak–2”)},\\ \sqrt{s} &= 91.26\text{GeV (“peak”)},\\ \sqrt{s} &= 92.94\text{GeV (“peak+2”)}\end{aligned}$$

These values were chosen as being close to all the $\sqrt{s}$ values at which the measurements were made, and the corrections are in general small. A fit for 11 parameters was made, including $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ at each of these energy points.

An additional 7 parameter fit was also made, in which the off-peak asymmetries were also corrected to $\sqrt{s}$ = 91.26 GeV, using the predicted dependence from ZFITTER [30]. In this case additional information was then used to evaluate the pole asymmetries.

After calculating the overall averages, the corrections described below were made to the combined on-peak asymmetries to derive the pole asymmetries. The input measured asymmetries are all corrected to full acceptance and use the thrust axis as an estimate of the quark direction before gluon radiation. To relate $\sin^2\theta_{\text{eff}}$ to these numbers a few corrections that are summarized in table 1 have to be applied:

- Energy shift to $\sqrt{s} = M_Z$ and QED corrections: The slope of the asymmetry around M_Z depends only on the axial coupling and the charge of the initial and final state fermions and is thus independent of the pole asymmetry itself. The corrections due to the energy shift and initial state radiation have been calculated using ZFITTER [30].

- QCD corrections: The QCD corrections using the thrust axis have been calculated recently by Djouardi, Lampe and Zerwas [31] in first order QCD including mass corrections. They have been found to be $A_{FB}^{QCD} = A_{FB}^{no\,QCD}(1 + c\frac{\alpha_s}{\pi})$ with $c = -0.87$ for charm and $c = -0.79$ for bottom quarks.
  Assuming $\alpha_s(m_Z^2) = 0.12$ and a scale of $\mu^2 = (m_Z/2)^2$ the correction factor is $1 - 0.034 \pm 0.004$ ($1 - 0.037 \pm 0.004$) for b (c) quarks where the error is deduced from a variation of the renormalization scale between $(m_Z/4)^2$ and m_Z^2.
  This result is valid under the assumption of a full acceptance. However it has not yet been studied carefully how far events with very hard gluon radiation contribute in the same way to the asymmetry measurement as two jet events. For that reason another 25% error is assigned to the size of the QCD correction.

- γ exchange, γZ interference: These very small corrections have again been calculated using ZFITTER.

The lifetime/jet-charge measurements of asymmetries take into account QCD effects as an inherent part of the analysis, but the measured asymmetries for the analyses using a lepton or D^* tag need to be corrected for the effects of QCD on the event thrust axis direction. So that the averages could be made at the same level, the jet-charge measurements were adjusted by hand so as to be consistent with the other asymmetry measurements (i.e. 0.0033 was subtracted from each of the three $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk})$ jet-charge measurements before averaging).

source	δA_{FB}^b	δA_{FB}^c
$\sqrt{s} = M_z$	−0.0013	−0.0034
QED corrections	+0.0041	+0.0104
QCD corrections	$+0.0033 \pm 0.0010$	$+0.0023 \pm 0.0007$
γ, γZ	−0.0003	−0.0008
total	$+0.0058 \pm 0.0010$	$+0.0085 \pm 0.0007$

Table 1: Corrections to be applied to the quark asymmetries. The corrections are to be understood as $A_{FB}^0 = A_{FB}^{meas} + \sum_i (\delta A_{FB})_i$

4 Modifications to the Averaging Procedure

Average values for the following 11 parameters, x^μ, are found from the input results, r_i.

$$\begin{aligned} x^\mu \;=\; & R_{\mathrm{b}}, \\ & R_{\mathrm{c}}, \\ & \mathrm{Br}(\mathrm{b} \to \ell), \\ & \mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+) \equiv \text{“Br(bc}\ell\text{)”}, \\ & \overline{\chi}, \\ & A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(-2), \\ & A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(-2), \end{aligned}$$

$$A_{\rm FB}^{\rm b\overline{b}}({\rm pk}),$$
$$A_{\rm FB}^{\rm c\overline{c}}({\rm pk}),$$
$$A_{\rm FB}^{\rm b\overline{b}}(+2),$$
$$A_{\rm FB}^{\rm c\overline{c}}(+2)$$

For the 7 parameter fit, $A_{\rm FB}^{\rm b\overline{b}}(-2)$, $A_{\rm FB}^{\rm c\overline{c}}(-2)$, $A_{\rm FB}^{\rm b\overline{b}}(+2)$ and $A_{\rm FB}^{\rm c\overline{c}}(+2)$ are not averaged separately but are interpreted as additional measurements of $A_{\rm FB}^{\rm b\overline{b}}({\rm pk})$ and $A_{\rm FB}^{\rm c\overline{c}}({\rm pk})$.

The corrections to the asymmetries for QED, QCD and energy dependence effects were discussed in the previous section. There are additional corrections to the results $r_i(\mu)$, where the μ indicates that result i is a measurement of parameter μ. These arise from the fact the measured value may depend on the value assumed for one of the other heavy flavour parameters $x^\nu, \nu \neq \mu$. This dependence is put into the fit explicitly, and no separate error due to the uncertainty in $x^\nu, \nu \neq \mu$ is assigned to $r_i(\mu)$. For example, the value of $R_{\rm b}$ almost always depends on the value of $R_{\rm c}$. If for a particular measurement k, this dependence was quoted as:

$$r_k(R_{\rm b}) = R_{\rm b}(R_{\rm c} = 0.172) + a_k \frac{dR_{\rm c}}{R_{\rm c}},$$

then when constructing the χ^2 for the overall fit we make the substitution:

$$r_k(R_{\rm b}) \rightarrow R_{\rm b}(R_{\rm c} = 0.172) + a_k \frac{R_{\rm c} - 0.172}{0.172}$$

where $R_{\rm c}$ is the current estimate of $R_{\rm c}$. These dependences are given in the tables in the appendices, either by quoting the value of a_k, or by quoting the change in the result $r_i(\mu)$ due to varying one of the other electroweak parameters in a specified range. For the previous results [3] the explicit dependences on $\mathrm{Br}(\mathrm{b} \rightarrow \ell)$ and $\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{c} \rightarrow \ell^+)$ were not included. This has now been rectified, and the dependences taken into account are listed in the tables.

5 Inclusion of Measurements from the SLD Collaboration

SLD measures the left-right forward-backward asymmetry benefitting from the electron beam polarization which is, contrary to the forward-backward asymmetry measured at LEP, only sensitive to the couplings of the Z to the final state fermions. These SLD polarized asymmetry measurements already express results in terms of the coupling parameters corrected for all radiative effects. The fitting procedure is extended to find the average values of these two additional parameters, $\mathcal{A}_{\rm b}$ and $\mathcal{A}_{\rm c}$.

Tables for the SLD results for $R_{\rm b}$, $\mathcal{A}_{\rm b}$ and $\mathcal{A}_{\rm c}$ have been made, following the same format as the LEP tables already used in the averaging procedure. The systematic error is broken down into its components. Just as for the LEP experiment's results, the averaging algorithm recognizes error sources which are common to several SLD measurements, and error sources which are common to SLD and LEP measurements. These are then taken into account in constructing the covariance matrix. The output of the fitting program provides the correlation between the polarized forward-backward asymmetries and the other heavy flavour electroweak parameters. The SLD inputs are adjusted to use the fitted values of $R_{\rm b}$, $R_{\rm c}$ etc., so that the average values from the full fit procedure are not necessarily the values one would obtain by averaging SLD asymmetry results alone. Correlations due to QCD corrections between SLD and LEP asymmetries have been ignored.

6 Results

6.1 Results of the 11 parameter fit to LEP data

Using the averaging procedure defined in the previous section gives the following combined results for the 11 parameter fit:

$$\begin{aligned}
R_{\mathrm{b}} &= 0.2217 \pm 0.0017 \\
R_{\mathrm{c}} &= 0.1540 \pm 0.0074 \\
\mathrm{Br}(\mathrm{b} \to \ell) &= (11.11 \pm 0.23)\% \\
\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^{+}) &= (7.77 \pm 0.37)\% \\
\overline{\chi} &= 0.1149 \pm 0.0062 \\
A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(-2) &= (5.5 \pm 1.4)\% \\
A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(-2) &= (-7.9 \pm 2.8)\% \\
A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk}) &= (9.45 \pm 0.31)\% \\
A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\mathrm{pk}) &= (6.84 \pm 0.60)\% \\
A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(+2) &= (11.3 \pm 1.2)\% \\
A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(+2) &= (8.9 \pm 2.4)\%
\end{aligned}$$

With a χ^2/dof of 47.9/(68 − 11), and the correlation matrix (where $Br(1)$ represents $\mathrm{Br}(\mathrm{b} \to \ell)$ and $Br(2)$ represents $\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^{+})$):

	R_{b}	R_{c}	$Br(1)$	$Br(2)$	$\overline{\chi}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ (−2)	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ (−2)	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ (pk)	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ (pk)	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ (+2)	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ (+2)
R_{b}	1.000	–.342	–.113	–.046	.013	.006	–.026	.008	.066	–.029	.022
R_{c}	–.342	1.000	.020	.157	–.014	–.002	.025	.083	–.072	.029	–.015
$Br(1)$	–.113	.020	1.000	–.255	.184	.003	.017	–.019	.072	–.026	.012
$Br(2)$	–.046	.157	–.255	1.000	–.309	–.006	–.030	–.067	–.169	–.016	–.034
$\overline{\chi}$	.013	–.014	.184	–.309	1.000	.037	.018	.310	–.005	.105	–.017
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(-2)$	.006	–.002	.003	–.006	.037	1.000	.089	.000	.000	–.005	–.003
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(-2)$	–.026	.025	.017	–.030	.018	.089	1.000	–.009	–.029	–.013	–.026
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk})$	.008	.083	–.019	–.067	.310	.000	–.009	1.000	.096	.015	.027
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\mathrm{pk})$	.066	–.072	.072	–.169	–.005	.000	–.029	.096	1.000	.021	.074
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(+2)$	–.029	.029	–.026	–.016	.105	–.005	–.013	.015	.021	1.000	.092
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(+2)$	.022	–.015	.012	–.034	–.017	–.003	–.026	.027	.074	.092	1.000

6.2 Results of the 7 parameter fit to LEP data

If the off-peak asymmetries are corrected to $\sqrt{s} = 91.26$GeV and treated as measurements of the peak asymmetry, the results of the 7 parameter fit are:

$$\begin{aligned}
R_{\mathrm{b}} &= 0.2216 \pm 0.0017 \\
R_{\mathrm{c}} &= 0.1546 \pm 0.0074 \\
\mathrm{Br}(\mathrm{b} \to \ell) &= (11.10 \pm 0.23)\% \\
\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^{+}) &= (7.78 \pm 0.37)\% \\
\overline{\chi} &= 0.1147 \pm 0.0062
\end{aligned}$$

$$A_{FB}^{b\bar{b}}(\mathrm{pk}) = (9.41 \pm 0.30)\%$$
$$A_{FB}^{c\bar{c}}(\mathrm{pk}) = (6.40 \pm 0.58)\%$$

With the χ^2/dof of 54.1/(68 – 7), and a correlation matrix:

	R_b	R_c	$Br(1)$	$Br(2)$	$\overline{\chi}$	$A_{FB}^{b\bar{b}}(\mathrm{pk})$	$A_{FB}^{c\bar{c}}(\mathrm{pk})$
R_b	1.000	–.341	–.114	–.046	.015	.001	.061
R_c	–.341	1.000	.020	.157	–.015	.085	–.066
$Br(1)$	–.114	.020	1.000	–.255	.185	–.023	.075
$Br(2)$	–.046	.157	–.255	1.000	–.309	–.068	–.174
$\overline{\chi}$	.015	–.015	.185	–.309	1.000	.327	–.004
$A_{FB}^{b\bar{b}}(\mathrm{pk})$	.001	.085	–.023	–.068	.327	1.000	.101
$A_{FB}^{c\bar{c}}(\mathrm{pk})$	.061	–.066	.075	–.174	–.004	.101	1.000

There is a correlation of –0.34 between R_b and R_c. The total corrections for the on peak asymmetries to give the pole asymmetries are $+0.0058 \pm 0.0010$ and $+0.0085 \pm 0.0007$ for b and c respectively. In addition there is a correction of +0.0003 (–0.0003) to R_b (R_c) due to photon exchange. The main results to be used in electroweak fits are then:

$$R_b = 0.2219 \pm 0.0017$$
$$R_c = 0.1543 \pm 0.0074$$
$$A_{FB}^{0,b} = 0.0999 \pm 0.0031$$
$$A_{FB}^{0,c} = 0.0725 \pm 0.0058$$

with their correlation matrix:

	R_b	R_c	$A_{FB}^{0,b}$	$A_{FB}^{0,c}$
R_b	1.000	–.341	.001	.061
R_c	–.341	1.000	.085	–.066
$A_{FB}^{0,b}$	.001	.085	1.000	.101
$A_{FB}^{0,c}$	.061	–.066	.101	1.000

If R_c is fixed to 0.172, then the value of R_b is:

$$R_b = 0.2205 \pm 0.0016 \ (R_c = 0.172).$$

6.3 Results of the 13 parameter fit to LEP and SLD data

Including the SLD results on R_b, $\mathcal{A}_b$ and $\mathcal{A}_c$ into the 13 parameter fit the following results are obtained:

$$R_b = 0.2218 \pm 0.0017$$
$$R_c = 0.1537 \pm 0.0074$$
$$\mathrm{Br}(b \to \ell) = (11.12 \pm 0.23)\%$$
$$\mathrm{Br}(b \to c \to \ell^+) = (7.75 \pm 0.36)\%$$
$$\overline{\chi} = 0.1144 \pm 0.0061$$
$$A_{FB}^{b\bar{b}}(-2) = (5.5 \pm 1.4)\%$$
$$A_{FB}^{c\bar{c}}(-2) = (-7.9 \pm 2.8)\%$$

$$
\begin{aligned}
A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk}) &= (9.43 \pm 0.31)\% \\
A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\mathrm{pk}) &= (6.88 \pm 0.60)\% \\
A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(+2) &= (11.2 \pm 1.2)\% \\
A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(+2) &= (8.9 \pm 2.4)\% \\
\mathcal{A}_{\mathrm{b}} &= 0.840 \pm 0.053 \\
\mathcal{A}_{\mathrm{c}} &= 0.609 \pm 0.090
\end{aligned}
$$

With a χ^2/dof of 48.8/(74 − 13), and the correlation matrix (where $Br(1)$ represents $\mathrm{Br}(\mathrm{b} \rightarrow \ell)$ and $Br(2)$ represents $\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{c} \rightarrow \ell^+)$):

	R_{b}	R_{c}	$Br(1)$	$Br(2)$	$\overline{\chi}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ (−2)	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ (−2)	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ (pk)	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ (pk)	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ (+2)	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ (+2)	$\mathcal{A}_{\mathrm{b}}$	$\mathcal{A}_{\mathrm{c}}$
R_{b}	1.000	−.346	−.119	−.046	.018	.006	−.026	.012	.060	−.027	.020	−.069	.047
R_{c}	−.346	1.000	.023	.150	−.007	−.001	.026	.082	−.070	.029	−.015	.075	−.062
$Br(1)$	−.119	.023	1.000	−.255	.190	.002	.018	−.013	.065	−.024	.011	−.002	.060
$Br(2)$	−.046	.150	−.255	1.000	−.299	−.006	−.029	−.065	−.167	−.014	−.034	−.069	−.096
$\overline{\chi}$	.018	−.007	.190	−.299	1.000	.037	.016	.305	−.002	.101	−.016	.115	.015
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(-2)$	.006	−.001	.002	−.006	.037	1.000	.089	.000	.000	−.005	−.003	.002	.002
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(-2)$	−.026	.026	.018	−.029	.016	.089	1.000	−.010	−.029	−.013	−.026	.009	−.001
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk})$	.012	.082	−.013	−.065	.305	.000	−.010	1.000	.103	.012	.029	.058	−.023
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\mathrm{pk})$	.060	−.070	.065	−.167	−.002	.000	−.029	.103	1.000	.025	.073	−.019	.074
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(+2)$	−.027	.029	−.024	−.014	.101	−.005	−.013	.012	.025	1.000	.093	.026	−.015
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(+2)$	.020	−.015	.011	−.034	−.016	−.003	−.026	.029	.073	.093	1.000	−.008	.016
$\mathcal{A}_{\mathrm{b}}$	−.069	.075	−.002	−.069	.115	.002	.009	.058	−.019	.026	−.008	1.000	.073
$\mathcal{A}_{\mathrm{c}}$	.047	−.062	.060	−.096	.015	.002	−.001	−.023	.074	-.015	.016	.073	1.000

6.4 Results of the 9 parameter fit to LEP and SLD data

Including again the offpeak asymmetries into the measurement of the peak asymmetry yields:

$$
\begin{aligned}
R_{\mathrm{b}} &= 0.2216 \pm 0.0017 \\
R_{\mathrm{c}} &= 0.1543 \pm 0.0074 \\
\mathrm{Br}(\mathrm{b} \rightarrow \ell) &= (11.12 \pm 0.23)\% \\
\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{c} \rightarrow \ell^+) &= (7.76 \pm 0.36)\% \\
\overline{\chi} &= 0.1145 \pm 0.0061 \\
A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk}) &= (9.39 \pm 0.30)\% \\
A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\mathrm{pk}) &= (6.44 \pm 0.58)\% \\
\mathcal{A}_{\mathrm{b}} &= 0.841 \pm 0.053 \\
\mathcal{A}_{\mathrm{c}} &= 0.606 \pm 0.090
\end{aligned}
$$

With the χ^2/dof of 55.1/(74 − 9), and a correlation matrix:

	R_b	R_c	$Br(1)$	$Br(2)$	$\overline{\chi}$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{c\bar{c}}(pk)$	$\mathcal{A}_b$	$\mathcal{A}_c$
R_b	1.000	−.345	−.120	−.046	.020	.005	.055	−.068	.046
R_c	−.345	1.000	.023	.150	−.009	.084	−.063	.074	−.061
$Br(1)$	−.120	.023	1.000	−.255	.191	−.017	.068	−.002	.060
$Br(2)$	−.046	.150	−.255	1.000	−.299	−.066	−.171	−.069	−.096
$\overline{\chi}$	.020	−.009	.191	−.299	1.000	.322	−.001	.115	.016
$A_{FB}^{b\bar{b}}(pk)$	.005	.084	−.017	−.066	.322	1.000	.109	.062	−.025
$A_{FB}^{c\bar{c}}(pk)$	.055	−.063	.068	−.171	−.001	.109	1.000	−.018	.073
$\mathcal{A}_b$	−.068	.074	−.002	−.069	.115	.062	−.018	1.000	.074
$\mathcal{A}_c$	.046	−.061	.060	−.096	.016	−.025	.073	.074	1.000

The $\mathcal{A}_b$ and $\mathcal{A}_c$ values are already corrected for all QED and QCD effects. Correcting the LEP measurements for photon exchange etc. yields:

$$\begin{aligned} R_b &= 0.2219 \pm 0.0017 \\ R_c &= 0.1540 \pm 0.0074 \\ A_{FB}^{0,b} &= 0.0997 \pm 0.0031 \\ A_{FB}^{0,c} &= 0.0729 \pm 0.0058 \\ \mathcal{A}_b &= 0.841 \pm 0.053 \\ \mathcal{A}_c &= 0.606 \pm 0.090 \end{aligned}$$

with their correlation matrix:

	R_b	R_c	$A_{FB}^{0,b}$	$A_{FB}^{0,c}$	$\mathcal{A}_b$	$\mathcal{A}_c$
R_b	1.000	−.345	.005	.055	−.068	.046
R_c	−.345	1.000	.084	−.063	.074	−.061
$A_{FB}^{0,b}$	.005	.084	1.000	.109	.062	−.025
$A_{FB}^{0,c}$	.055	−.063	.109	1.000	−.018	.073
$\mathcal{A}_b$	−.068	.074	.062	−.018	1.000	.074
$\mathcal{A}_c$	.046	−.061	−.025	.073	.074	1.000

If R_c is fixed to 0.172, then the value of R_b is:

$$R_b = 0.2205 \pm 0.0016 \; (R_c = 0.172).$$

It is not yet clear how to correlate the uncertainty in the QCD correction between the LEP and SLD results. In the numbers presented above they have been assumed to be independent. If one assumes this error to be fully correlated the correlation between $A_{FB}^{0,b}$ and $\mathcal{A}_b$ would increase by 0.04 and the one $A_{FB}^{0,c}$ and $\mathcal{A}_c$ would increase by 0.008. All other elements of the correlation matrix remain unchanged.

Experimental Input Tables

In the following appendices, the relevant results from the papers and notes by the four LEP experiments and SLD are tabulated. The errors are quoted in the order: uncorrelated, experiment internal (may be correlated with error on other measurements by the same experiments) and finally fully correlated (can lead to a correlation between results by different experiments) Finally, any dependences on other electroweak parameters are given.

The results in these tables have in some cases been corrected **by the experiments** to use agreed default values and ranges for input parameters. The experiments were also responsible for combining where necessary the results of their different analyses and calculating their internal correlation matrices.

A ALEPH tables

	R_b
results	0.2187
statistics	0.00220
Monte Carlo stat	0.00135
track resolution	0.00145
internal	0.00091
$\langle x_E(c) \rangle$	−0.00012
$f(D^+)$	−0.00073
$f(D^0+D^+)$	−0.00041
$f(\Lambda_c)$	−0.00019
D^0 lifetime	−0.00028
D^+ lifetime	−0.00036
D_s lifetime	−0.00022
B decay multip.	−0.00029
D decay multip.	−0.00057
$g \to b\overline{b}$	−0.00027
$g \to c\overline{c}$	−0.00019
Long-lived light hadrons	−0.00024
Total systematic	0.0025
Use $R_c = 0.171$	$-$ 0.01378 dR_c/R_c

Table 2: Error breakdown for ALEPH lifetime double-tag R_b [14]

	R_b
results	0.228
statistics	0.0054
internal	0.0025
Monte Carlo Stat	0.0023
elec effic	−0.0001
elec misid	+0.0010
muon effic	−0.0002
punch	−0.0017
track resoln	0.
Non-hadronic backgr	−0.0003
acceptance corr	−0.0003
Detector effects	−0.0007
c frag function	+0.0008
$Br(c \to \ell)$	+0.0006
Semilept model $b \to \ell$	−0.0013
Semilept model $c \to \ell$	−0.0021
Total systematic	0.0048
Use $R_c = 0.165 \pm 0.020$	−0.0005

Table 3: Error breakdown for ALEPH neural-net R_b [16]; note that this is 20% statistically correlated with the ALEPH R_b from the multiparameter lepton fit.

	$A_{FB}^{b\bar{b}}$(pk)
results	0.0992
roots	91.187
statistics	0.0085
Monte Carlo stat	0.0022
track resoln	0.0017
internal	0.0012
Light quark frag	0.0023
Total systematic	0.0038
Use $R_b = 0.2188$	$-0.0307\ dR_b/R_b$
Use $R_c = 0.171$	$+0.0257\ dR_c/R_c$

Table 4: Error breakdown for ALEPH jet charge $A_{FB}^{b\bar{b}}$(pk) [18]

	R_b	R_c	$\overline{\chi}$	Br(b $\to \ell$)	Br(bcℓ)	$A_{FB}^{b\bar{b}}$(pk)	$A_{FB}^{c\bar{c}}$(pk)
roots						91.26	91.26
results	0.2162	0.1670	0.0993	0.1120	0.0881	8.43	9.1
statistics	0.0062	0.0054	0.0073	0.0033	0.0025	0.68	2.0
Monte Carlo stat	0.	0.	0.0027	0.	0.	0.03	0.
elec effic	0.0002	−0.0047	0.	−0.0018	−0.0015	0.	0.16
muon effic	−0.0002	−0.0040	0.	−0.0015	−0.0018	0.	0.07
elec conversions	−0.0006	−0.0062	0.	0.0013	−0.0006	0.	0.31
elec misid	+0.0002	−0.0024	−0.0002	−0.0008	−0.0001	0.	0.11
pi/K decays	−0.0016	−0.0057	−0.0005	0.0004	−0.0015	0.01	−0.37
punchthrough	−0.0028	−0.0103	−0.0006	0.0016	−0.0028	0.03	+0.42
lepton effic	0.	0.	0.0001	0.	0.	0.	0.
kaon production	0.	0.	−0.0002	0.	0.	+0.01	0.
backgr chrg corr	0.	0.	0.0002	0.	0.	0.	0.
btocbartol model	0.	0.	0.0002	0.	0.	0.	0.
background asy	0.	0.	0.	0.	0.	−0.02	1.38
track resoln	0.	0.	0.	0.	0.	0.03	0.
internal	0.	0.	0.	0.	0.	0.06	0.
$\langle x_E(c)\rangle$	0.	0.	0.	0.	0.	+0.07	0.
b frag function	0.	0.	0.	0.	0.	−0.01	0.
Br(c $\to \ell$)	0.0010	−0.0093	0.	−0.0005	−0.0002	0.05	0.04
Br(b $\to \tau \to \ell$)	−0.0003	−0.0006	0.	−0.0003	−0.0009	0.	0.11
Br(b $\to \bar{c} \to \ell^-$)	−0.0006	−0.0050	0.0002	−0.0003	−0.0020	0.	0.47
Br(b $\to J/\psi \to \ell^+\ell^-$)	+0.0003	+0.0006	0.	−0.0002	−0.0006	0.	−0.02
BR(b $\to$ u)	−0.0022	−0.0006	0.0016	+0.0004	+0.0041	−0.03	−0.15
Semilept model c $\to \ell$	−0.0008	−0.0010	0.0045	−0.0003	+0.0002	−0.05	−0.17
b $\to$ D model.	−0.0005	−0.0012	0.0004	+0.0006	−0.0004	−0.01	−0.08
Semilept model b $\to \ell$	−0.0027	+0.0041	-0.0017	−0.0025	+0.0051	−0.03	−0.90
g $\to$ b$\bar{b}$	0.	0.	−0.0001	0.	0.	+0.01	0.
g $\to$ c$\bar{c}$	0.	0.	−0.0001	0.	0.	+0.01	0.
Total systematic	0.0050	0.0188	0.0058	0.0042	0.0080	0.14	1.9

Table 5: Results of the published ALEPH multiparameter lepton fit and the ALEPH measurement of $A_{FB}^{b\bar{b}}$ and $\overline{\chi}$ using high p_T leptons. These two analyses are combined here by ALEPH for the purpose of averaging with other LEP results [4]

	R_b	R_c	χ	$b \to \ell$	$b \to c \to \ell$	A_b	A_c
R_b	1.00	−0.48	0.00	−0.94	−0.38	0.05	0.05
R_c	−0.48	1.00	0.00	0.47	−0.31	0.00	−0.07
χ	0.00	0.00	1.00	0.27	−0.23	0.25	0.00
$b \to \ell$	−0.94	0.47	0.27	1.00	0.25	−0.04	−0.04
$b \to c \to \ell$	−0.38	−0.31	−0.23	0.25	1.00	−0.01	−0.01
A_b	0.05	0.00	0.25	−0.04	−0.01	1.00	0.01
A_c	0.05	−0.07	0.00	−0.04	−0.01	0.01	1.00

Table 6: ALEPH statistical correlation matrix for lepton measurements.

	$\mathrm{Br}(\mathrm{b}\to\ell)$	$\mathrm{Br}(\mathrm{bc}\ell)$
roots		
results	0.1101	0.0768
statistics	0.0010	0.0018
elec effic	–0.0012	–0.0013
muon effic	–0.0011	–0.0007
elec conversions	0.0002	–0.0008
elec misid	0.	–0.0002
pi/K decays	0.0001	–0.0007
punchthrough	0.0008	–0.0016
backgr chrg corr	–0.0002	0.0005
$\mathrm{Br}(\mathrm{c}\to\ell)$	0.0001	–0.0003
$\mathrm{Br}(\mathrm{b}\to\tau\to\ell)$	–0.0004	–0.0006
$\mathrm{Br}(\mathrm{b}\to\bar{\mathrm{c}}\to\ell^-)$	–0.0001	–0.0014
$\mathrm{Br}(\mathrm{b}\to\mathrm{J}/\psi\to\ell^+\ell^-)$	–0.0002	0.0001
$BR(\mathrm{b}\to\mathrm{u})$	–0.0003	0.0009
Semilept model $\mathrm{c}\to\ell$	–0.0010	-0.0012
$\mathrm{b}\to\mathrm{D}$ model.	+0.0004	–0.0006
Semilept model $\mathrm{b}\to\ell$	–0.0017	+0.0036
udsc impurity	–0.0008	–0.0003
Total systematic	0.0029	0.0049

Table 7: ALEPH preliminary measurement of semileptonic branching ratios using lifetime tagging [5].

	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(\mathrm{pk})$	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(-2)$	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}(+2)$
results	6.4	–4.9	10.9
$\sqrt{s}$	91.2	89.4	93.0
statistics	1.3	7.6	6.1
D* back rate	0.07	–0.10	0.11
D* back asy	–0.16	–0.82	–0.68
Rate c-D* / b-D*	–0.03	0.15	–0.12
W to D*	0.08	–0.02	0.11
effective chi	0.17	–0.04	0.25
Total systematic	0.26	0.84	0.75
Use $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(\mathrm{pk}) = 8.7 \pm 1.4\%$	–0.24		
Use $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(-2) = -1.7 \pm 7.6\%$		–1.3	
Use $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}(+2) = 12.1 \pm 7.0\%$			–1.2

Table 8: Error table for ALEPH D* preliminary measurement of $A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ [6]

	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$
results	–0.031	0.033	0.093	0.051	0.105	0.118
$\sqrt{s}$	88.363	89.424	90.214	92.065	93.023	93.930
statistics	0.110	0.030	0.058	0.049	0.024	0.075
internal	0.001	0.001	0.003	0.002	0.003	0.003

Table 9: ALEPH measurements of $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ off-peak [4]

B DELPHI tables

	R_b
results	0.2210
statistics	0.0016
Hemisph correl	0.0008
detector effects	0.0008
internal	0.0007
$\langle x_E(\mathrm{c})\rangle$	−0.0004
$\mathrm{Br}(\mathrm{c} \to \ell)$	0.0002
Semilept model $\mathrm{b} \to \ell$	0.0002
Semilept model $\mathrm{c} \to \ell$	−0.0002
$f(\mathrm{D}^0)$	−0.0001
$f(\mathrm{D}^+)$	−0.0008
$f(\mathrm{D}^0 + \mathrm{D}^+)$	−0.0005
$f(\mathrm{D_s})$	−0.0004
D^0 lifetime	−0.0002
D^+ lifetime	−0.0002
$\mathrm{D_s}$ lifetime	−0.0002
Λ_c lifetime	0.
D decay multip.	−0.0004
$\mathrm{g} \to \mathrm{b\bar{b}}$	−0.0002
$\mathrm{g} \to \mathrm{c\bar{c}}$	−0.0001
Long-lived light h	−0.0004
$\mathrm{Br}(D \to K^0)$	0.0006
Total systematic	0.0020
Use $R_c = 0.171 \pm 0.014$	−0.0012

Table 10: Error breakdown for DELPHI R_b [10]

	R_c
results	0.152
statistics	0.007
internal	0.0081
b hadron lifetime	0.0050
D meson lifetime	0.0006
$\langle x_E(\mathrm{b})\rangle$	−0.0017
D^* background rate	0.0008
Error from Υ_{4S} data	−0.0058
Total systematic	0.011

Table 11: Breakdown of errors for DELPHI R_c measured using D mesons and inclusive D^* double tag [11]

	R_b	Br(b → ℓ)	Br(bcℓ)	R_c	$\overline{\chi}$
results	0.2145	11.21	7.70	0.1625	0.150
statistics	0.0089	0.45	0.49	0.0085	0.020
Monte Carlo stat	0.	0.	0.	0.	0.
muon backgr	−0.0054	+0.30	−0.91	−0.0160	+0.010
muon effic ec	−0.0004	−0.03	−0.04	−0.0014	0.
muon effic bar	+0.0014	−0.32	−0.25	−0.0038	0.
elec backgr	−0.0029	+0.24	+0.05	−0.0031	−0.003
$BR(\mathrm{b} \to \mathrm{u})$	0	0	−0.03	+0.0002	−0.004
$\langle x_E(\mathrm{c}) \rangle$	+0.0015	−0.17	+0.28	−0.0056	−0.003
b frag function	−0.0008	+0.11	−0.03	+0.0008	−0.002
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	+0.0003	+0.01	−0.33	+0.0013	+0.003
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	−0.0002	−0.03	−0.13	−0.0002	+0.001
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	+0.0002	−0.06	+0.02	0	−0.002
$\mathrm{Br}(\mathrm{c} \to \ell)$	+0.0014	−0.06	−0.04	−0.0098	−0.002
Semilept model b → ℓ	−0.0001	−0.40	+0.69	+0.0004	−0.008
Semilept model c → ℓ	+0.0007	−0.16	−0.03	+0.0049	−0.006
Total systematic	0.0067	0.70	1.26	0.021	0.016

Table 12: DELPHI lepton fit results [9]

	R_b	Br(b → ℓ)	Br(bcℓ)	R_c	$\overline{\chi}$
R_b	1.00	−0.95	−0.38	−0.42	−0.09
Br(b → ℓ)	−0.95	1.00	0.29	0.37	0.11
Br(bcℓ)	−0.38	0.29	1.00	−0.40	−0.17
R_c	−0.42	0.37	−0.40	1.00	0.17
$\overline{\chi}$	−0.09	0.11	−0.17	0.17	1.00

Table 13: DELPHI lepton fit statistical correlation matrix

	$A_{FB}^{b\bar{b}}$(pk)	$A_{FB}^{c\bar{c}}$(pk)	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{b\bar{b}}(+2)$
$\sqrt{s}$	91.23	91.23	89.43	93.017
results	0.1049	0.0837	0.0629	0.1489
statistics	0.0076	0.0139	0.038	0.036
muon id	0.0013	0.0029	0.0013	0.0040
elec id	0.0007	0.0010	0.0	0.0
track resoln	0.0009	0.0006	0.0006	0.0020
backgr asy	−0.0008	0.0079	−0.0001	0.0001
internal	0.0009	0.0033	0.0006	0.0020
$\langle x_E(\mathrm{b})\rangle$	−0.0001	−0.0005	0.0002	−0.0001
$\langle x_E(\mathrm{c})\rangle$	0.0012	−0.0004	−0.0001	0.0014
$\mathrm{Br}(\mathrm{b}\to\bar{\mathrm{c}}\to\ell^-)$	0.0005	0.0038	0.0001	0.0017
$\mathrm{Br}(\mathrm{c}\to\ell)$	0.0004	−0.0035	0.0003	−0.0007
Semilept model $\mathrm{b}\to\ell$	0.0002	0.0051	0.0010	0.0026
Semilept model $\mathrm{c}\to\ell$	0.0018	−0.0014	0.0004	0.0022
Total systematic	0.0035	0.0132	0.0020	0.0065
Use $R_\mathrm{b} = 0.217 \pm 0.003$	−0.0004	+0.0005	−0.0001	−0.0004
Use $R_\mathrm{c} = 0.171 \pm 0.014$	+0.0010	−0.0052	+0.0001	−0.0008
Use $\overline{\chi} = 0.121 \pm 0.0165$	0.0047	0.	+0.0028	+0.0065
Use $\mathrm{Br}(\mathrm{b}\to\ell) = 11.0 \pm 0.34$	−0.0011	0.0015	−0.0003	−0.0010
Use $\mathrm{Br}(\mathrm{b}\to\mathrm{c}\to\ell^+) = 7.9 \pm 0.5$	0.0003	−0.0024	0.0001	0.0003

Table 14: DELPHI measurements of asymmetries using a lepton tag [7]

	$A_{FB}^{b\bar{b}}$(pk)	$A_{FB}^{c\bar{c}}$(pk)	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{b\bar{b}}(+2)$
$A_{FB}^{b\bar{b}}$(pk)	1.00	0.28	0.	0.
$A_{FB}^{c\bar{c}}$(pk)	0.28	1.00	0.	0.
$A_{FB}^{b\bar{b}}(-2)$	0.	0.	1.000	0.
$A_{FB}^{b\bar{b}}(+2)$	0.	0.	0.	1.000

Table 15: DELPHI lepton tag asymmetries statistical correlation matrix

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}$(pk)	$A_{FB}^{c\bar{c}}$(pk)	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$\sqrt{s}$	89.54	89.54	91.23	91.23	92.94	92.94
results	1.94	0.20	7.13	7.52	5.63	7.97
statistics	10.48	5.19	2.50	1.21	9.55	4.55
internal	-.84	.34	-.84	.34	-.84	.34
rate of π_s	.25	-.22	.25	-.22	.25	-.22
rate $c-D^*/b-D^*$	-.14	.16	-.14	.16	-.14	.16
D^* back rate	-.30	-.24	-.30	-.24	-.30	-.24
Afb(bck)uds	-.24	.14	-.24	.14	-.24	.14
Afb(bck)b,c	-.55	-.18	-.55	-.18	-.55	-.18
b hadron lifetime	-.08	.08	-.08	.08	-.08	.08
effective χ	.75	-.08	.75	-.08	.75	-.08
thrust resoln	.10	.10	.10	.10	.10	.10
$\langle x_E(\mathrm{b})\rangle$	.07	.04	.07	.04	.07	.04
$\langle x_E(\mathrm{c})\rangle$	.12	-.05	.12	-.05	.12	-.05
Total systematic	1.36	.57	1.36	.57	1.36	.57

Table 16: Asymmetry measurements with a D* tag from DELPHI [7]

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}(\mathrm{pk})$	$A_{FB}^{c\bar{c}}(\mathrm{pk})$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$A_{FB}^{b\bar{b}}(-2)$	1.0	−.39	0.	0.	0.	0.
$A_{FB}^{c\bar{c}}(-2)$	−.39	1.0	0.	0.	0.	0.
$A_{FB}^{b\bar{b}}(\mathrm{pk})$	0.	0.	1.0	−.44	0.	0.
$A_{FB}^{c\bar{c}}(\mathrm{pk})$	0.	0.	−.44	1.0	0.	0.
$A_{FB}^{b\bar{b}}(+2)$	0.	0.	0.	0.	1.0	−.40
$A_{FB}^{c\bar{c}}(+2)$	0.	0.	0.	0.	−.40	1.0

Table 17: Statistical correlation matrix for asymmetry measurements with a D^* tag from DELPHI

	$A_{FB}^{b\bar{b}}(\mathrm{pk})$
results	0.0990
$\sqrt{s}$	91.23
statistics	0.0072
track resoln	0.0022
vertex resoln	0.0002
internal	0.0028
$f(D^+)$	-0.0001
$f(D^0+D^+)$	-0.0001
D^+ lifetime	-0.0001
D_s lifetime	-0.0001
$g \to b\bar{b}$	-0.0001
$g \to c\bar{c}$	-0.0001
Long-lived light h	-0.0002
Light quark frag	0.0012
Total systematic	0.0038
Use $R_b = 0.221$	$0.006\ dR_b/R_b$
Use $R_c = 0.171$	$-0.0024\ dR_c/R_c$

Table 18: DELPHI $A_{FB}^{b\bar{b}}(\mathrm{pk})$ from jet charge [7]

C L3 tables

	R_b
results	0.222
statistics	0.003
internal	0.0021
$\langle x_E(b) \rangle$	−0.0031
$\langle x_E(c) \rangle$	+0.0018
Light quark frag.	+0.0049
Total systematic	0.007
Use $R_c = 0.171 \pm 0.009$	−0.0011
Use $Br(b \to \ell) = 10.5 \pm 0.5$	−0.0010

Table 19: Error breakdown for L3 neural net R_b [17]

	R_b	$Br(b \to \ell)$
results	0.2187	0.1144
statistics	0.0081	0.0048
internal	0.0047	0.0028
$\langle x_E(b) \rangle$	0.0029	−0.0024
$\langle x_E(c) \rangle$	0.0004	−0.0002
$Br(c \to \ell)$	−0.0012	0.0003
Semilept model $b \to \ell$	+0.0009	−0.0021
Semilept model $c \to \ell$	0.0007	0.0003
Total systematic	0.0058	0.0043
Use $R_c = 0.171 \pm 0.014$	−0.0019	0.0005
Use $Br(b \to c \to \ell^+) = 10.5 \pm 0.5$	0.0052	0.0007

Table 20: Error breakdown for L3 R_b and $Br(b \to \ell)$ measurement using leptons [12]

	R_b	$Br(b \to \ell)$
R_b	1.	−0.98
$Br(b \to \ell)$	−0.98	1.

Table 21: Statistical correlation matrix for L3 R_b and $Br(b \to \ell)$

	R_b	$Br(b \to \ell)$
R_b	1.	−0.32
$Br(b \to \ell)$	−0.32	1.

Table 22: Systematic correlation matrix for L3 R_b and $Br(b \to \ell)$

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(pk)$	$\overline{\chi}$
$A_{FB}^{b\bar{b}}(-2)$	1.00	0.00	0.00	0.0	0.06
$A_{FB}^{b\bar{b}}(pk)$	0.00	1.00	0.00	0.0	0.30
$A_{FB}^{b\bar{b}}(+2)$	0.00	0.00	1.00	0.0	0.10
$A_{FB}^{c\bar{c}}(pk)$	0.0	0.0	0.0	1.00	0.00
$\overline{\chi}$	0.06	0.30	0.10	0.00	1.00

Table 23: Statistical correlation matrix for lepton fit for asymmetries from L3

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(pk)$	$\overline{\chi}$
$A_{FB}^{b\bar{b}}(-2)$	1.00	1.00	1.00	0.00	0.14
$A_{FB}^{b\bar{b}}(pk)$	1.00	1.00	1.00	0.00	0.27
$A_{FB}^{b\bar{b}}(+2)$	1.00	1.00	1.00	0.00	0.27
$A_{FB}^{c\bar{c}}(pk)$	0.00	0.00	0.00	1.00	0.00
$\overline{\chi}$	0.14	0.27	0.27	0.00	1.00

Table 24: Systematic correlation matrix for lepton fit for asymmetries from L3

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(pk)$	$\overline{\chi}$
$\sqrt{s}$	89.56	91.27	92.93	91.24	0.
results	7.02	10.28	11.02	7.84	0.1253
statistics	3.50	1.00	2.90	3.70	0.0110
internal	+0.3500	+0.3600	+0.3610	+2.4000	+0.005160
$\langle x_E(b)\rangle$	+0.0293	+0.0227	+0.0227	−0.4444	−0.001000
$\langle x_E(c)\rangle$	−0.0120	−0.0120	−0.0120	−0.2000	+0.000030
$Br(b \to \bar{c} \to \ell^-)$	−0.0110	−0.0085	−0.0080	0.	+0.000430
$Br(c \to \ell)$	+0.0467	+0.0467	+0.0471	−0.3333	+0.000033
$Br(c \to D^* \to K\pi\pi_s)$	0.	0.	0.	0.	0.
Semilept model $b \to \ell$	−0.0850	−0.0870	−0.0870	0.	−0.000300
Semilept model $c \to \ell$	−0.0550	−0.0430	−0.0420	0.	+0.001860
$b \to D$ model.	+0.0410	+0.0470	+0.0470	0.	+0.001530
Total systematic	0.38	0.38	0.39	2.5	0.0081
Use $R_b = 0.216 \pm 0.005$	−0.0380	−0.0380	−0.0380	+0.1000	+0.000020
Use $R_c = 0.169 \pm 0.005$	+0.0270	+0.0270	+0.0270	−0.2000	+0.000020
Use $A_{FB}^{c\bar{c}}(-2) = -2.90 \pm 1.50$	+0.1380	0.	0.	0.	0.
Use $A_{FB}^{c\bar{c}}(pk) = 6.30 \pm 1.50$	0.	+0.1380	0.	0.	0.
Use $A_{FB}^{c\bar{c}}(+2) = 12.10 \pm 1.50$	0.	0.	+0.1380	0.	0.
Use $A_{FB}^{b\bar{b}}(pk) = 8.8 \pm 1.1$	0.	0.	0.	0.8	0.
Use $Br(b \to \ell) = 10.5 \pm 0.5$	−0.0442	−0.0308	−0.0300	+0.1667	+0.002200
Use $Br(b \to c \to \ell^+) = 7.9 \pm 1.2$	−0.0100	−0.0430	−0.0460	−0.5000	−0.005190

Table 25: Results of the lepton fits from L3 [12]

D OPAL tables

	R_b
Result	0.2197
Statistics	0.00140
Monte Carlo stat	0.00053
Elec id efficiency	0.00012
Elec misid	0.00024
Elec conversions	0.00026
Muon id efficiency	0.00007
Muon id background	0.00040
Track resolution	0.00083
Efficiency correlation	0.00072
Event selection	0.00025
$\langle x_E(\mathrm{c})\rangle$	−0.00075
$\mathrm{Br}(\mathrm{c} \to \ell)$	−0.00028
Semilept model $\mathrm{c} \to \ell$	−0.00025
$f(\mathrm{D}^0)$	+0.00002
$f(\mathrm{D}^+)$	−0.00069
$f(\mathrm{D}^0 + \mathrm{D}^+)$	−0.00038
$f(\mathrm{D_s})$	−0.00052
D^0 lifetime	−0.00023
D^+ lifetime	−0.00029
$\mathrm{D_s}$ lifetime	−0.00017
Λ_c lifetime	−0.00005
D decay multip.	−0.00076
$\mathrm{Br}(D \to \overline{K^0})$	+0.00059
$\mathrm{g} \to \mathrm{b\overline{b}}$	0.
$\mathrm{g} \to \mathrm{c\overline{c}}$	−0.00046
Long-lived light h	−0.00033
Light quark frag	0.0
Total systematic	0.00220
R_c	$-0.01845\ dR_\mathrm{c}/R_\mathrm{c}$

Table 26: Error breakdown for OPAL double-tag R_b. Entries marked “0.0” are negligible. [15]

	R_b	$\mathrm{Br}(\mathrm{b} \to \ell)$	$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$	$\overline{\chi}$
$\sqrt{s}$	–	–	–	–
Results	.225	10.5	8.3	.144
Statistics	.011	.60	.40	.022
Monte Carlo statistics	.003	.22	.14	.003
Λ_b polarisation	0.0	–.01	.08	0.0
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$ syst.	–.0014	.072	–.49	.0042
Muon ID efficiency	.001	.24	.13	.001
Muon polar angle	.001	–.02	–.01	0.0
Elec ID efficiency	–.001	.18	.14	.001
e bremsstrahlung	0.0	0.0	0.0	0.0
Muon ID background	.002	–.09	.06	–.001
Elec misid	.001	–.02	.03	0.0
Elec conversions	.001	–.02	–.09	0.0
e decay background	0.0	.01	–.08	.001
$\langle x_E \rangle_c$	–.002	.04	.05	.001
b frag function	–.001	–.16	–.07	.001
c frag function	0.0	0.0	0.0	0.0
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	.001	–.06	–.07	0.0
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	.001	–.07	–.04	0.0
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	0.0	0.0	0.0	0.0
$\mathrm{Br}(\mathrm{c} \to \ell)$	–.00045	–.0045	.018	.0009
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^* \to \mathrm{K}\pi\pi_s)$	0.0	0.0	0.0	0.0
Semilept model $\mathrm{b} \to \ell$	–.004	–.50	.20	–.002
Semilept model $\mathrm{c} \to \ell$	–.001	–.03	–.31	–.001
Total systematic	.0066	0.66	0.69	0.0062
Use $R_c = 0.171 \pm 0.013$	–0.001	0.017	0.001	–0.05

Table 27: Results of the OPAL multiparameter lepton fit – part I [13]

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}(\text{pk})$	$A_{FB}^{c\bar{c}}(\text{pk})$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$\sqrt{s}$	89.54	89.54	91.24	91.24	92.94	92.94
Results	3.3	-21.1	10.3	5.2	12.1	9.0
Statistics	3.0	4.8	0.9	1.0	2.7	4.0
Λ_b polarisation	−0.02	− 0.05	0.01	− 0.05	0.02	−0.05
$\text{Br}(b \to c \to \ell^+)$ syst.	0.08	0.45	0.05	0.22	0.02	0.18
Muon id background	−0.09	−0.56	0.13	0.31	0.18	0.37
Elec misid	−0.08	−0.84	0.07	0.18	0.23	0.14
Elec conversions	−0.72	−2.52	0.18	0.68	0.52	1.70
e decay background	0.0	0.0	0.0	0.0	0.0	0.0
Elec source dep	0.01	−0.26	0.03	−0.06	0.04	−0.06
Track resolution	−0.01	−0.12	0.02	0.07	0.01	0.12
Track charge error	0.01	0.07	0.03	0.01	0.04	0.03
$\langle x_E \rangle_b$	−0.01	−0.30	−0.04	−0.11	−0.07	−0.14
$\langle x_E \rangle_c$	−0.21	0.29	0.19	−0.17	0.27	−0.18
b frag function	0.0	0.0	0.0	0.0	0.0	0.0
c frag function	0.0	0.0	0.0	0.0	0.0	0.0
$\text{Br}(b \to \bar{c} \to \ell^-)$	−0.05	−0.17	0.07	0.40	0.11	0.52
$\text{Br}(b \to \tau \to \ell)$	0.0	−0.07	0.02	0.18	0.01	0.21
$\text{Br}(b \to J/\psi \to \ell^+\ell^-)$	0.04	0.22	0.01	0.07	−0.03	0.03
$\text{Br}(c \to \ell)$	−0.03	0.59	0.09	−0.20	0.14	−0.23
$\text{Br}(c \to D^* \to K\pi\pi_s)$	0.0	0.0	0.0	0.0	0.0	0.0
Semilept model $b \to \ell$	−0.05	0.20	−0.05	−0.63	0.08	−0.64
Semilept model $c \to \ell$	−0.09	0.27	0.13	−0.06	0.22	0.02
Total systematic	0.8	3.0	0.4	1.2	0.7	2.0
Use $R_c = 0.171 \pm 0.013$	−0.05	+0.91	+0.13	−0.29	+0.20	−0.36

Table 28: Results of the OPAL multiparameter lepton fit – part II [13]

	$A_{FB}^{b\bar{b}}(\text{pk})$	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{b\bar{b}}(+2)$
$\sqrt{s}$	91.25	89.52	92.94
Result	0.0973	0.063	0.173
Statistics	0.0067	0.034	0.029
Monte Carlo stat	0.0013	0.0003	0.0024
Track resolution	0.0021	0.0014	0.0036
Internal	0.0018	0.0011	0.0034
B decay multip.	0.0005	0.0003	0.0006
Light quark frag	0.0024	0.0011	0.0044
Total systematic	0.0039	0.0022	0.0070
Use $R_b = 0.216$	-0.01009 dR_b/R_b	-0.018 dR_b/R_b	-0.183dR_b/R_b
Use $R_c = 0.173$	+0.0011 dR_b/R_c	+0.0001 dR_b/R_c	+0.0016 dR_b/R_c
Use $A_{FB}^{c\bar{c}}(\text{pk})$=0.061	+0.0071 $dA_{FB}^{c\bar{c}}/A_{FB}^{c\bar{c}}$	0.0	0.0
Use $A_{FB}^{c\bar{c}}(-2) = -0.0316$	0.0	-0.0021$dA_{FB}^{c\bar{c}}/A_{FB}^{c\bar{c}}$	0.0
Use $A_{FB}^{c\bar{c}}(+2) = 0.12$	0.0	0.0	+0.0068$dA_{FB}^{c\bar{c}}/A_{FB}^{c\bar{c}}$

Table 29: OPAL Asymmetry measurement using jet charge and a vertex tag [19]

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{c\bar{c}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$\sqrt{s}$	89.54	89.54	91.26	91.26	92.94	92.94
Results	−.0270	−.006	.112	.0680	−.2670	.1710
Statistics	.1690	.0680	.035	.014	.1400	.0580
internal	0.0289	0.0103	0.0139	.0058	.0303	.0114
D* back acceptance	.0028	.0009	.0028	.0009	.0028	.0009
D* backgr rate	.0101	.0038	.0062	.0014	.0113	.0038
D* backgr asymmetry	.0023	.0010	.0023	.0010	.0023	−.0010
W to D*	.0014	0.0	.0061	0.0	-.0130	0.0
Effective mixing χ	.0024	0.0	.0096	0.0	−.023	0.0
Thrust resoln	.0010	.0010	.0010	.0010	.0010	.0010
b hadron lifetime	.0065	−.0017	.0005	−.0003	.0026	−.0021
B decay multip.	.02180	−.0036	.00220	−.0023	.01830	.0046
$g \to c\bar{c}$	0.0008	0.0002	0.0034	0.0020	-0.0078	0.0050
Total systematic	.0385	.012	.0205	.0071	.0495	.0151

Table 30: OPAL Asymmetry measurements with a D* tag [24]

	R_c
Result	0.1525
Statistics	0.0061
Internal	0.0074
c frag function	0.0025
b frag function	−.0009
Effective mixing χ	0.0004
$g \to c\bar{c}$	0.0028
b hadron lifetime	0.0015
$Br(c \to D^*)$	−0.0028
D decay multiplicity	0.0012
D branching ratios	0.0014
Total systematic	0.0092

Table 31: Breakdown of errors for OPAL combined R_c using charm hadron tags [20–22]

	R_b	$Br(1)$	$Br(2)$	$\overline{\chi}$	$A_{FB}^{b\bar{b}}$ (−2)	$A_{FB}^{c\bar{c}}$ (−2)	$A_{FB}^{b\bar{b}}$ (pk)	$A_{FB}^{c\bar{c}}$ (pk)	$A_{FB}^{b\bar{b}}$ (+2)	$A_{FB}^{c\bar{c}}$ (+2)
R_b	1.0	−.93	.07	−.12	0.0	−.12	−.15	.15	−.07	.05
$Br(1)$	−.93	1.0	−.15	.14	0.0	.11	.16	−.14	.07	−.05
$Br(2)$	.07	−.15	1.0	−.14	−.01	−.01	−.10	.01	−.04	0.0
$\overline{\chi}$	−.12	.14	−.14	1.0	.07	.01	.71	−.02	.28	−.01
$A_{FB}^{b\bar{b}}(-2)$	0.0	0.0	−.01	.07	1.0	.31	0.0	0.0	0.0	0.0
$A_{FB}^{c\bar{c}}(-2)$	−.12	.11	−.01	.01	.31	1.0	0.0	0.0	0.0	0.0
$A_{FB}^{b\bar{b}}(pk)$	−.15	.16	−.10	.71	0.0	0.0	1.0	.20	0.0	0.0
$A_{FB}^{c\bar{c}}(pk)$	.15	−.14	.01	−.02	0.0	0.0	.20	1.0	0.0	0.0
$A_{FB}^{b\bar{b}}(+2)$	−.07	.07	−.04	.28	0.0	0.0	0.0	0.0	1.0	.30
$A_{FB}^{c\bar{c}}(+2)$	.05	−.05	0.0	−.01	0.0	0.0	0.0	0.0	.30	1.0

Table 32: Statistical correlation matrix for the OPAL multiparameter fit to lepton data. $Br(1)$ denotes $\mathrm{Br}(b \to \ell)$, $Br(2)$ denotes $\mathrm{Br}(b \to c \to \ell^+)$.

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{c\bar{c}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$A_{FB}^{b\bar{b}}(-2)$	1.00	–.414	0.0	0.0	0.0	0.0
$A_{FB}^{c\bar{c}}(-2)$	–.414	1.00	0.0	0.0	0.0	0.0
$A_{FB}^{b\bar{b}}(pk)$	0.0	0.0	1.00	–.411	0.0	0.0
$A_{FB}^{c\bar{c}}(pk)$	0.0	0.0	–.411	1.00	0.0	0.0
$A_{FB}^{b\bar{b}}(+2)$	0.0	0.0	0.0	0.0	1.00	–.402
$A_{FB}^{c\bar{c}}(+2)$	0.0	0.0	0.0	0.0	–.402	1.00

Table 33: OPAL D^* asymmetry statistical correlation matrix

E SLD Tables

	R_b
results	0.2171
statistics	0.00400
Monte Carlo stat	0.00108
Hemisph correl	0.00099
internal	0.00227
$\langle x_E(c)\rangle$	−0.00122
$f(D^+)$	−0.00109
$f(D^0 + D^+)$	−0.00061
$f(\Lambda_c)$	−0.00007
D^0 lifetime	−0.00024
D^+ lifetime	−0.00015
D_s lifetime	−0.00017
B decay multip.	−0.00000
D decay multip.	−0.00180
$g \to b\overline{b}$	−0.00037
$g \to c\overline{c}$	−0.00017
Long-lived light h	−0.00009
Total systematic	0.0037
Use R_c= 0.171 ± 0.017	−0.00228

Table 34: Error breakdown for SLD R_b [25]

	$\mathcal{A}_c$
results	0.640
statistics	0.110
D* back rate	0.039
D* back asy	0.028
Rate c-D / b-D	0.011
b-D Asy	0.018
$\langle x_E(b)\rangle$	0.011
$\langle x_E(c)\rangle$	0.012
c frag function	0.011
beam pol	0.007
c-asym QCD corr	0.007
Use $\mathcal{A}_b$ = 0.935 ± 0.090	-0.012
Total systematic	0.058

Table 35: Error breakdown for SLD $\mathcal{A}_c$ from exclusive D^{*+}, D^+ asymmetry [26]

	$\mathcal{A}_b$
results	0.843
statistics	0.046
b tag syst	0.017
internal	0.033
jetc hemi correl	0.030
beam pol	0.008
b-asym QCD corr	0.008
Use R_b = 0.2180 ± 0.0020	−0.0012
Use R_c = 0.171 ± 0.017	0.0132
Use $\mathcal{A}_c$ = 0.666 ± 0.070	0.0085
Total systematic	0.051

Table 36: Breakdown of errors for SLD $\mathcal{A}_b$ from jet charge [27]

	$\mathcal{A}_b$
results	0.907
statistics	0.094
b tag syst	0.002
Monte Carlo stat	0.030
internal	0.056
B prod mix	0.038
B-D prod	0.011
B vtx K prod	0.041
charm K prod	0.030
uds K prod	0.011
beam pol	0.007
b-asym QCD corr	0.009
$\langle x_E(\mathrm{b})\rangle$	−0.006
$\langle x_E(\mathrm{c})\rangle$	0.003
$f(\mathrm{D}^+)$	0.
$f(\mathrm{D}^0+\mathrm{D}^+)$	0.004
$f(\Lambda_c)$	0.002
Use R_b = 0.2180 ± 0.0020	−0.0002
Use R_c = 0.171 ± 0.017	0.0003
Use $\mathcal{A}_c$ = 0.666 ± 0.070	−0.014
Use $\overline{\chi}$ = 0.130 ± 0.007	0.012
Total systematic	0.094

Table 37: Breakdown of errors for SLD $\mathcal{A}_b$ from $K^\pm$ tag [28]

	$\mathcal{A}_b$	$\mathcal{A}_c$
results	0.865	0.435
statistics	0.072	0.107
lepton misid	0.020	0.026
lept bckgr asy	0.010	0.026
beam pol	0.011	0.006
b-asym QCD corr	0.008	0.000
c-asym QCD corr	0.000	0.040
internal	0.055	0.046
$\langle x_E(\mathrm{b})\rangle$	−0.004	−0.016
$\langle x_E(\mathrm{c})\rangle$	0.010	−0.026
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	0.003	0.030
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0.002	0.015
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	−0.003	0.001
$\mathrm{Br}(\mathrm{c} \to \ell)$	0.003	−0.023
Semilept model $\mathrm{b} \to \ell$	−0.008	−0.008
Semilept model $\mathrm{c} \to \ell$	0.037	−0.042
Use R_b = 0.2180 ± 0.0020	−0.006	0.006
Use R_c = 0.171 ± 0.014	0.006	−0.037
Use $\overline{\chi}$= 0.120 ± 0.010	0.017	0.
Use $\mathrm{b} \to \ell$ = 10.80 ± 0.78	−0.016	0.030
Use $\mathrm{b} \to \mathrm{c} \to \ell$ = 9.30 ± 1.6	0.011	−0.075
Total systematic	0.078	0.132

Table 38: Breakdown of errors for SLD $\mathcal{A}_b$,$\mathcal{A}_c$ from leptons [29]

	$\mathcal{A}_b$	$\mathcal{A}_c$
$\mathcal{A}_b$	1.00	0.18
$\mathcal{A}_c$	0.18	1.00

Table 39: Statistical correlation matrix for SLD $\mathcal{A}_b$,$\mathcal{A}_c$ from leptons

References

[1] *A Consistent Treatment of Systematic Errors for LEP Electroweak Heavy Flavour Analyses*, LEP Electroweak Working Group, LEPHF/94-01, ALEPH Note 94-30, DELPHI 94-23 Phys 357, L3 Note 1577, OPAL TN213.

[2] *Presentation of LEP Electroweak Heavy Flavour Analyses for Summer 1994 Conferences*, LEP Electroweak Working Group, LEPHF/94-02, ALEPH Note 94-90, L3 Note 1613, OPAL TN237.

[3] *LEP Electroweak Heavy Flavour Results for Summer 1994 Conferences*, LEP Electroweak Working Group, LEPHF/94-03, ALEPH Note 94-119 PHYSIC 94-103, DELPHI 94-108 Phys 425, L3 Note 1630, OPAL TN242.

[4] *Heavy Flavour Lepton Contribution for Summer 1994 Conferences* ALEPH 94-123 PHYSIC 94-107
$\mathrm{B^0\overline{B^0}}$ *mixing and* $\mathrm{b\overline{b}}$ *asymmetry from high* p_t *leptons,* ALEPH 94-036 Contributed Paper to the La Thuile and Moriond Winter conferences, 1994;
ALEPH Collab., D. Buskulic *et al.*, Z. Phys. **C62** (1994) 179.

[5] ALEPH Collab., D. Buskulic et al., *Measurement of the semileptonic b branching ratios from inclusive leptons in Z decays*, **EPS0404**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995 .

[6] ALEPH Collab., D. Buskulic et al., *The Forward–Backward Asymmetry for Charm Quarks at the Z pole : an Update*, **EPS0634**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995 .

[7] DELPHI Collab., *Measurement of the Forward-Backward Asymmetries of* $e^+e^- \rightarrow Z \rightarrow b\bar{b}$ *and* $e^+e^- \rightarrow Z \rightarrow c\bar{c}$ **EPS0571**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995, DELPHI 95-87 PHYS 522

[8] DELPHI Collab., P.Abreu et al., Z.Phys **C65** (1995) 569.

[9] DELPHI Collab., P.Abreu et al., Z. Phys. **C66** (1995) 323.

[10] DELPHI Collab., *Measurement of the partial decay width* $R_b = \Gamma_{b\bar{b}}/\Gamma_{had}$ *with the DELPHI detector at LEP* **EPS0570**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995, DELPHI 95-89 PHYS 524

[11] DELPHI Collab., *Study of Charm Mesons Production in* Z^0 *Decays and Measurement of* Γ_c/Γ_h **EPS0557**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995, DELPHI 95-101 PHYS 536

[12] *L3 Results on* $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$, $A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ *and* χ *for the Glasgow Conference* L3 Note 1624,
Measurement of the $\mathrm{B^0\overline{B^0}}$ *mixing parameter and the* $\mathrm{Z^0 \rightarrow b\overline{b}}$ *Forward-Backward asymmetry,* L3 Collab., CERN-PPE/94-89, Submitted to Phys. Lett. B;
L3 Collab., O. Adriani *et al.*, Phys. Lett. **B292** (1992) 454.
L3 Results on R_{b} *and* $\mathrm{Br(b} \rightarrow \ell)$ *for the Glasgow Conference* L3 Note 1625,
Measurement of R_{b} *and Br(*$\mathrm{b} \rightarrow \ell X$*) from b-quark semi-leptonic decays,* L3 Collab., L3 internal note 1449, July 16 1993;

[13] *Measurement of the forward backward asymmetries of* $\mathrm{e^+e^- \rightarrow Z^0 \rightarrow b\overline{b}}$ *and* $\mathrm{e^+e^- \rightarrow Z^0 \rightarrow c\overline{c}}$ *from events tagged by a lepton, including 1994 data.* **EPS0279**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995, OPAL Physics Note PN165, Feb 27 1995;
OPAL Collab, R. Akers *et al.*, Z. Phys. **C60** (1993) 199.

[14] ALEPH Collab., D. Buskulic *et al.*, Phys. Lett. **B313** (1993) 535.

[15] *An Update of the Measurement of* $\Gamma_{b\overline{b}}/\Gamma_{\rm had}$ *using a Double Tagging Method*, **EPS0278**, Contributed Paper to the EPS conference on High Energy Physics Brussels, Belgium, 27 Jul.–2 Aug. 1995, OPAL Physics Note PN181;
OPAL Collab., Z. Phys. **C65** (1995) 17-30.

[16] ALEPH Collab., D. Buskulic *et al.*, Phys. Lett. **B313** (1993) 549.

[17] L3 Collab., O. Adriani *et al.*, Phys. Lett. **B307** (1993) 237.

[18] *A Measurement of* $A_{\rm FB}^{b\overline{b}}$ *in lifetime tagged heavy flavour Z Decays,* ALEPH Collab., CERN-PPE/94-84, Submitted to Phys. Lett. B

[19] *A Measurement of the Forward-Backward asymmetry of* $e^+e^- \rightarrow b\overline{b}$ *by Applying a Jet Charge Algorithm to Lifetime Tagged Events*, CERN-PPE/95-50, Apr. 1995, submitted to Z. Phys. C.

[20] *A Measurement of the Production of* $D^{*\pm}$ *Mesons on the* Z^0 *Resonance,* OPAL Collab., R. Akers *et al.*., Z. Phys. C. **67** (1995) 27.

[21] *A measurement of* $B(c \rightarrow D^*)$ *and* $\Gamma_{c\overline{c}}/\Gamma_{had}$ *using a double tagging technique.*, OPAL Collaboration, OPAL physics note PN 175, 13 April 1995, contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-289.

[22] *A summary of OPAL measurements of charm production in* Z^0 *decays.* OPAL Collab., OPAL Physics note PN190

[23] ALEPH Collab., D. Buskulic *et al.*, Z. Phys. **C62** (1994) 1-14.

[24] *A Measurement of the Charm and Bottom Forward Backward Asymmetry using D Mesons with the OPAL Detector at LEP* OPAL Physics Note, PN183, 17 July 1995, contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-290.

[25] SLD Collab., *A Measurement of* R_b *using Lifetime Double Tag* contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-222.

[26] SLD Collab., *A Measurement of the Left-Right Forward-Backward Asymmetry for Charm Quarks with* D^{*+} *and* D^+ *Mesons*, contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-248.

[27] SLD Collab., *Measurement of* A_b *at the* Z^0 *Resonance Using a Lifetime Tag and Momentum-Weighted Track Charge*, contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-250.

[28] SLD Collab., *Measurement of* A_b *at the* Z^0 *Resonance Using Identified Charged Kaons*, contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-251.

[29] SLD Collab., *Measurement of* A_b *and* A_c *from the Left–Right Forward–Backward Asymmetry of Leptons in Hadronic Events at the* Z^0 *Resonance*, contributed Paper to the EPS Brussels, Belgium, 27 Jul.–2 Aug. 1995, eps-249.

[30] D. Bardin *et al.*, Z. Phys. **C44** (1989) 493; Comp. Phys. Comm. **59** (1990) 303; Nucl. Phys. **B351**(1991) 1; Phys. Lett. **B255** (1991) 290 and CERN-TH 6443/92 (May 1992).

[31] Djouardi, Lampe and Zerwas *A note on QCD corrections to Forward-Backward Asymmetries of Heavy-Quark Jets in Z Decays*, MPI-Ph/94-81.