

Figure 1: Top: distribution of $m(\pi^+\pi^-)$ (K_s^0) for $D^{\pm} \to K_s^0\pi^{\pm}$ candidates in (a) logarithmic and (b) linear scale. Bottom: distribution of $m(K^+K^-)$ (ϕ) for $D_s^{\pm} \to \phi\pi^{\pm}$ candidates in (c) logarithmic and (d) linear scale.



Figure 2: Distribution of (a) $m(\phi\pi^{\pm})$ and (b) $m(K_{\rm S}^0\pi^{\pm})$ for $D_{(s)}^{\pm} \to \phi\pi^{\pm}$ and $D_{(s)}^{\pm} \to K_{\rm S}^0\pi^{\pm}$ candidates, respectively.



Figure 3: Distribution of $\eta - p_{\rm T}$ of the bachelor pion for $D_{(s)}^{\pm} \to \eta' \pi^{\pm}$ candidates in the (a) *T1*, (b) *T2*, and (c) *T3* samples. The $\eta - p_{\rm T}$ bin edges are marked by dashed lines.



Figure 4: Distribution of $p_{\rm T} - \eta$ of the bachelor pion for $D_s^{\pm} \to \phi \pi^{\pm}$ candidates in the (a) *T1*, (b) *T2*, and (c) *T3* samples. The $p_{\rm T} - \eta$ bin edges are marked by dashed lines.



Figure 5: Distribution of $p_{\rm T} - \eta$ of the bachelor pion for $D^{\pm} \to K_{\rm S}^0 \pi^{\pm}$ candidates in the (a) *T1*, (b) *T2*, and (c) *T3* samples. The $p_{\rm T} - \eta$ bin edges are marked by dashed lines.



Figure 6: Top: normalized distributions of (a) $\log_{10}(p_{\rm T} [\text{MeV}/c])$ and (b) η of the bachelor pion in $D_{(s)}^{\pm} \to \eta' \pi^{\pm}$ (points with errors), $D_s^{\pm} \to \phi \pi^{\pm}$ (red, solid), and $D^{\pm} \to K_{\rm S}^0 \pi^{\pm}$ (blue, dashed) decays. Bottom: normalized distributions of (c) $\log_{10}(p_{\rm T} [\text{MeV}/c])$ and (d) η of the $D_{(s)}^{\pm}$ candidates in $D_{(s)}^{\pm} \to \eta' \pi^{\pm}$ (points with errors), $D_s^{\pm} \to \phi \pi^{\pm}$ (red, solid), and $D^{\pm} \to K_{\rm S}^0 \pi^{\pm}$ (blue, dashed) decays.



Figure 7: Mass distribution of $\eta' \pi^{\pm}$ candidates, combined over all kinematic bins, pp centre-ofmass energies, and hardware trigger selections, for (a) positively and (b) negatively charged $D_{(s)}^{\pm}$ candidates. Points with errors represent data, while the curves represent the fitted model (solid), the $D_s^{\pm} \rightarrow \phi_{3\pi} \pi^{\pm}$ (dashed) and $D^{\pm} \rightarrow \phi_{3\pi} \pi^{\pm}$ (long-dashed) components, and the sum of all background contributions (dotted), including combinatorial background. Residuals divided by the corresponding uncertainty are shown under each plot.