## Supplementary material for LHCb-PAPER-2021-001



Figure 1: Distribution of $e^{+} e^{-}$vertices in $D^{+} \rightarrow \pi^{+} \eta$ candidates in the $r z$ plane integrated over the azimuthal angle, where a positive (negative) value of $r$ denotes a vertex with $x>0(x<0)$. The coordinate system is defined such that $z$ is along the beam axis, with positive $z$ denoting the direction from the $p p$-interaction point into the LHCb detector; $y$ is vertical upwards; and $x$ is horizontal and defined such that the coordinate system is right handed. The contributions from three-body $\eta \rightarrow e^{+} e^{-} \gamma$ decays form the two regions in the centre, whilst $\eta \rightarrow \gamma \gamma$ decays followed by a photon conversion form regions at larger $\left|r_{e^{+} e^{-}}\right|$where the material surrounding the $p p$ interaction point is located.


Figure 2: Compilation of different measurements of $\mathcal{A}_{C P}\left(D^{+} \rightarrow \pi^{+} \pi^{0}\right)$, along with a weighted average of the listed results.

